Temporary Change Detection on ARMA(1,1) Data

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***Abstract*—**The aim of this research is to carry out appropriate method to detect Temporary Change on Autoregressive Moving Average (ARMA) (1,1) data. Estimation of model parameters and outlier effects are used to iteratively for joint estimation procedure. Simulation data were generated from ARMA (1,1) model. The ARMA consists of 4 models which were produced by parameters combination of Autoregressive (AR) and Moving Average (MA). Residuals were estimated by Conditional Least Square (CLS) and Median Absolute Deviation (MAD). Removing outliers' effect used two ways: replacing data which containing outlier and omitting. The observation contains outlier replaces by other value, namely replacement procedure and omitting the observation contains the outlier, namely omit one procedure. The result shows omit one procedure detect outliers better than replacement procedure for all cases. Moreover, MAD and Omit one combination is slightly better than CLS and Omit one combination. This method was implemented to Surabaya's Air Pollutant (Sulfur Trioxide) data and produced similar result. Joint Estimation method using combination MAD and omit one procedure obtain more accurate to detect Temporary Change than three others procedures.

***Keywords*—**Joint Estimation, Median Absolute Deviation, Omit-one, Single Outlier Detection, Temporary Change
Mathematics Subject Classification: 62M10, 37M05, 37M10

I. Introduction

OUTLIER is observation stand apart from the bulk of data [1],[2]. Outlier(s) data often influence for model or common pattern, therefore the effect have to be reduced [3]. Conversely, outliers can provide useful information. The outliers are valuable to be ignored as their presence signifies important events. The examples are an intervention or an unexpected incident such as strike, new regulation, natural disasters, wars, change of political leaders and currency crises [4],[5],[6], hence change point gives better explanation using other natural phenomenon [7].

There are four types of outlier i.e. Aberrant Outlier or Additional Outlier (AO), Aberrant Innovation or Innovation Outlier (IO), Level Shift or Level Change (LC), and Transient of Level Change or Temporary Change (TC). AO and IO were specifically called outlier, but TC and LC as change point or structure change [2],[8],[9].

[10] was the earliest to carry out a study on outliers in time series, especially for non-seasonal ARIMA(p,0,0), p=1,2,3,…. He proposed detection and removing outlier effect. Later, many studies of outliers in ARIMA (p,0,q) or ARMA(p,q) p,q=1,2,3,… were conducted as continuation for his work [11],[12],[13],[14], and [15]. Studies of single outlier detection were carried out [9],[16],[17],[18],[19]. [20] studied structural break, as continuation [8].

Handling outliers using omit one is a one of popular procedure in time series [2]. [21] suggested this method to remove outliers' effect. The researchers earlier used omit one method that combined with Least Square as method to estimate variance of residuals. Median Absolute Deviation (MAD) is an alternative method to estimate variance. This method was introduced by [22]. [23] proposed some variation of MAD. [21] applied the method in case of outlier detection. MAD can be written as follows:

$$\mathrm{MAD} = 1.483\,\mathrm{median}\left(\left|e_t - \mathrm{median}(e)\right|\right),\ \text{that e is residuals.}$$

This paper discuss about choosing appropriate TC Detection on ARMA (1,1) using simulation study. Afterward, this method was implemented to Surabaya's Air Pollutant data. Aim of this study is to find appropriate method especially method of variance estimation in Single Outlier Detection using Joint Estimation Method. This study extends to effect of critical value, series length and magnitude of outlier factors to accuracy of outlier detection.

II. Data Generating

Simulation data was generated from ARIMA (1,1) model. The pure data were contaminated by Temporary change. Four models were determined. The models were representing all

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possible Autoregressive (AR) and Moving Average (MA) parameters combination: positive and negative. Series length (n) 100, 200, 500 and 1000 were chosen. Critical values were determined 2, 2.5, 3, 3.5 and 4 therefore magnitude of outlier is determined as 2, 4 and 6. Data simulating use R which is developed by R Development Core Team [24].

Stationer and invertible time series ARIMA (1,1) model can be written as follows [2] and [8]:

$$\phi(B)Z_t = \theta(B)a_t \tag{1}$$

which $\phi(B) = \left(1 - \phi B - \phi^2 B^2 - .. - \phi^p B^p\right)$,

$\theta(B) = \left(1 - \theta B - \theta^2 B^2 - ... - \theta^q B^q\right)$, B is the backshift operator and a_t is white noise residuals.

Eq. 1 can be written as

$$Z_t = \frac{\theta(B)}{\phi(B)} a_t \tag{2}$$

ARIMA (1,0,1) can be formulated to

$$Z_t = \frac{\theta(B)}{\phi(B)} a_t = \frac{(1-\theta B)}{(1-\phi B)} a_t = (1-\theta B)(1+\phi B + ...)a_t$$

$$= \left(1 + (\phi - \theta)B + (\phi - \theta)\phi B^2 + ... + (\phi - \theta)\phi^{i-1}B^i + ...\right)a_t$$

ARIMA Model presence outlier is

$$Y_t = Z_t + \omega \frac{1}{1-\delta B} I_t^{(T)} \qquad ,0 < \delta < 1. \tag{3}$$

From Eq. 2 and Eq. 3 obtain

$$Y_t = \frac{\theta(B)}{\phi(B)} a_t + \omega \frac{1}{(1-\delta B)} I_t^{(T)} \tag{4}$$

where Y_t is the observed series, Z_t is the pure ARIMA without outlier series in Eq. 1, ω represents the magnitude of the outlier, δ is the pace of the dampening effect and $I_t^{(T)}$ is a time indicator variable signifying the occurrence of an outlier. It is clear from Eq. 4 that the effect of temporary change is temporary, exponential decay depending on δ value.

III. Test Statistics for TC Detection

From Eq.3 and Eq.4 can be concluded that TC affect to the observation at t=T and r subsequent observations. In this section the topic is discussed.

A. Residual Estimation

To make easy understanding of how residual estimates are used in the TC detection procedure, look as a simple case when T and all parameters in Eq. 1 are known [2]. Let

$$\pi(B) = 1 - \pi_1 B - \pi_2 B^2 - ... \tag{5}$$

Where π_j represent the weights for *j* and the value similar to 0 for *j* moderately large when the roots of $\theta(B)$ lie outside of the unit circle. The estimates residuals ($\hat{e}_t$) which may be outliers exist can be written as,

$$\hat{e}_t = \pi(B)Y_t$$

$$= \pi(B)\left(\frac{\theta(B)}{\phi(B)} a_t + \omega \frac{1}{(1-\delta B)} I_t^{(T)}\right)$$

$$= \frac{\phi(B)}{\theta(B)}\left(\frac{\theta(B)}{\phi(B)} a_t + \omega \frac{1}{(1-\delta B)} I_t^{(T)}\right)$$

$$= a_t + \omega \frac{\phi(B)}{\theta(B)} \frac{1}{(1-\delta B)} I_t^{(T)}$$

$$= \omega \frac{\pi(B)}{(1-\delta B)} I_t^{(T)} + a_t \tag{6}$$

B. Estimation of Temporary Change Effect

[21] rewrite Eq. 6 as a simple linear regression equation as follows:

$$\hat{e}_t = \omega x_t + a_t. \tag{7}$$

The series x_t assumes the value 0 for *t*< T; the value 1 for *t* = T and for *t*= T + *j* (*j*=1, 2,…, n-*j*) the value for x_t is computed as follows:

From Eq. 6, the estimated residuals of a Temporary Change are described as,

$$\hat{e}_t = \omega \frac{\pi(B)}{(1-\delta B)} I_t^{(T)} + a_t$$

$$= \omega\left(1 - \pi_1 B^1 - \pi_2 B^2 - ...\right)(1-\delta B)^{-1} I_t^{(T)} + a_t$$

$$= \omega\left(I_t^{(T)} - \pi_1 B^1 I_t^{(T)} - ...\right)\left(1 + (\delta B) + (\delta B)^2 + ...\right) + a_t$$

but, $B^j I_t^{(T)} = I_{t-j}^{(T)}$ therefore, at time *t* = T + *j*,

$$\hat{e}_t = \omega\left(1 - B^j\left(\delta^j - \sum_{k=1}^{j-1} \delta^{j-1}\pi_k - \pi_j\right) I_t^{(T)}\right) + a_t \tag{8}$$

and thus, $x_{(T+j)} = \delta^j - \sum_{k=1}^{j-1} \delta^{j-1}\pi_k - \pi_j$

IV. Illustrations Case

Maximum Likelihood Estimation (MLE) as parameter estimation was used in this research as [9],[14] and [16] conducted. Moreover Estimation of TC effect applied Least Square (CLS) and Median Absolute Deviation (MAD) methods. Detection outlier used single outlier detection procedure. Since an outlier was detected, removing the outlier has to be done. Detection next outlier will be conducted. The procedure is repeated until no outlier can be detected. Removing outlier after detection use two procedure: the first procedure is removing data at time the occurrence of outlier and other procedure is replacing data with predicted data which was obtain on modelling before.

There are four combinations as a result of two approaches above, the first combination is CLS as variance estimation as part of TC estimation and replacement the data occurrence of outlier with the fit namely CLS-Fit. The second combination is MAD as method as part of the TC estimation and replacement the data occurrence of outlier with the fit as procedure of removing outlier effect, namely MAD-Fit. The third is Least Square as part of TC estimation combined with removing the data occurrence of outlier as method to remove the outlier effect, namely CLS-Omit and the fourth is combining the MAD and Omit abbreviate as MAD-Omit.

A. Sampling Behavior

To represent the all models of ARIMA (1,1), four models from parameters combination of ARIMA (1,1) were used. The parameters combination of AR (1) and MA (1) is positive-positive, positive-negative, negative-positive and negative-negative such as models in Table 1. In this simulation, there are four magnitudes of outlier: 2, 4 and 6. The pace of the dampening effect is given δ =0.7. Critical values are given for 5 various numbers such as: 2, 2.5, 3, 3.5 and 4 based on [21]. Number of observation or series length (n) use four different numbers such as: 100, 200, 500 and 1000.

Table 1 Four Models for simulation study

No	Models
1	$Z_t = 0.8Z_{t-1} + a_t + 0.3a_{t-1}$
2	$Z_t = 0.8Z_{t-1} + a_t - 0.3a_{t-1}$
3	$Z_t = -0.8Z_{t-1} + a_t + 0.3a_{t-1}$
4	$Z_t = -0.8Z_{t-1} + a_t - 0.3a_{t-1}$

Analysis was conducted on two steps. First step to find out effect of different critical values and different TC magnitude to percentage of outlier which was detected. Using optimal critical value was obtained in first step, second step find out the best method. The main objective of first step is to find optimal critical value of each model in term of series length 100, 200, 500 and 1000. [21] suggested an interval of critical value for certain series length. However in practice several critical values for optimal result are needed. Secondly, investigate series length and different method effects for accurateness of outlier detection.

B. Obtaining Optimal Critical Value

First step of this study was finding out effect of the different critical value to measure performance of TC detection. Figs 1-4 draw relationship between outlier magnitude and critical values for percentage of Temporary Change detection on each method.

Figure 1 shows there are significant different percentage of Temporary Change detection for different critical values or different outlier magnitude for all methods in term of length series 100. The optimal critical values (critical value which yield the highest percentage of TC detection) are 2 and 2.5. Other models for each method have similar result, but critical value=2 result slightly higher percentage than 2.5 for few cases. Critical value 2 is the best choice in term of series length 100. The simulation has similar result with [21].

From the other point of view, increasing outlier magnitude yields increasing performance of TC detection. The performance of TC detection is represented by the number of percentages of outlier detection.

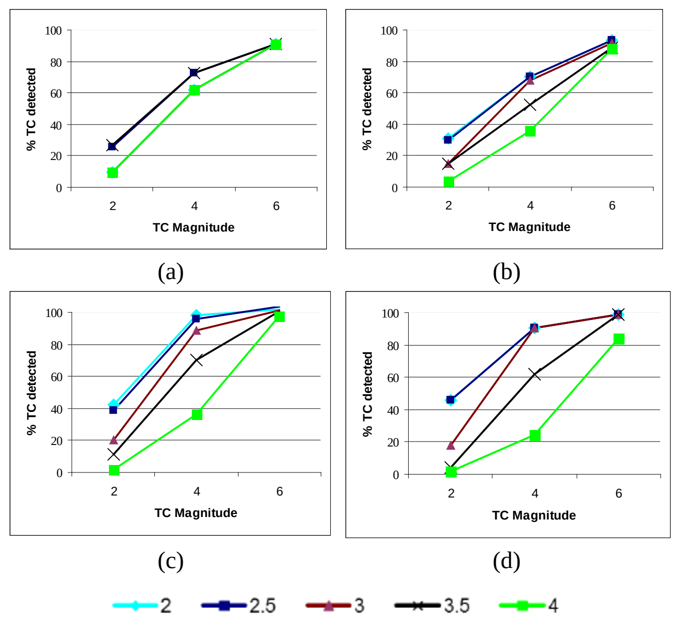

Fig. 1 Percentage of Outlier was detected by TC detection on various outlier magnitude and critical value in term of series length=100 for four models (a) model 1 (b) model 2 (c) model 3 (d) model 4

Figure 2 exhibits different percentage of Temporary Change detection is affected by different critical values and different outlier magnitude. The optimal critical values for series length=200 are 2 and 2.5 for small outlier magnitude, but critical value=2.5 gives the best result for outlier magnitude=4 and 6. In general, critical value=2.5 is the optimal value to obtain the highest performance. Furthermore, increasing outlier magnitude gives increasing percentage of outlier can be detected.

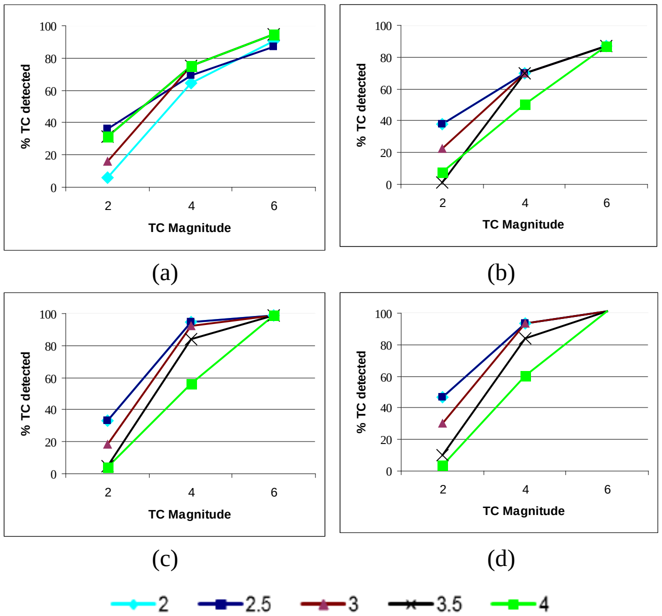

Fig. 2 Percentage of Outlier was detected by TC detection on various outlier magnitude and critical value in term of series length=200 for four models (a) model 1 (b) model 2 (c) model 3 (d) model 4

Fig. 3 displays different critical values or different outlier magnitude yields different percentage of Temporary Change detection. Percentage of Temporary detection is function of outlier magnitude, higher outlier magnitude result higher performance of TC detection.

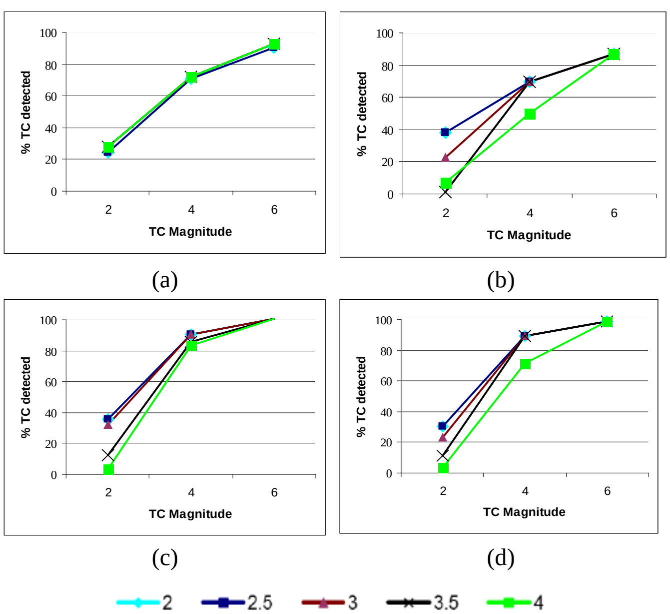

Fig. 3 Percentage of Outlier was detected by TC detection on various outlier magnitude and critical value in term of series length=500 for four models (a) model 1 (b) model 2 (c) model 3 (d) model 4

The optimal critical values are 2 and 2.5 for small outlier magnitude, less than 4, but all critical values give similar result for big outlier magnitude. Commonly, critical value=2.5 is the optimal value to obtain the highest percentage of Temporary Change detection in term of series length=500.

Figure 4 presents different critical values or different outlier magnitudes affect to different percentage of Temporary Change can be detected for series length=1000. The optimal critical values are 3 for small outlier magnitude, but all critical values give similar result for outlier magnitude=6. Critical value=3 is always obtain the highest percentage of Temporary Change detection for all magnitude of outlier. Furthermore increasing outlier magnitude gives increasing performance.

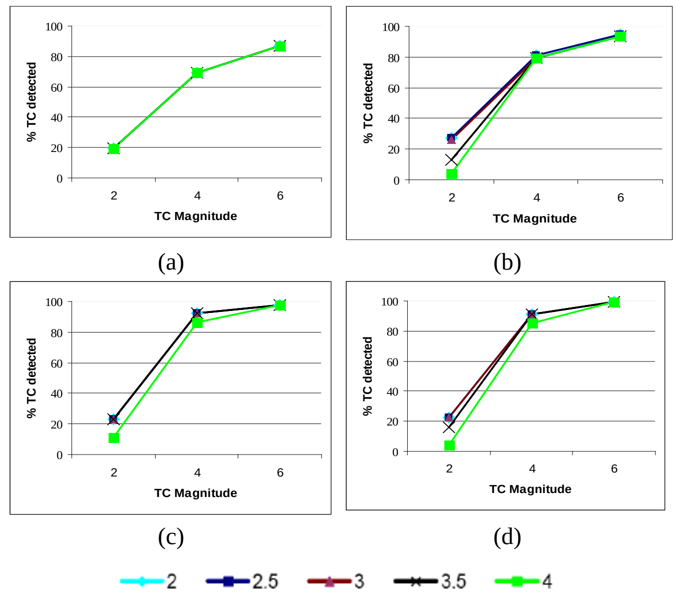

Fig. 4 Percentage of Outlier was detected by TC detection on various outlier magnitude and critical value in term of series length=1000 for four models (a) model 1 (b) model 2 (c) model 3 (d) model 4

Figure 1-4 shows, increasing of outlier magnitude affect to better performance of outlier detection. Appropriate magnitudes of critical value depend on series length, longer series length need higher critical value to obtain the best performance of outlier detection. Fig. 1-4 show, length series 100, 200, 500 and 1000 are suggested to choose critical value 2, 2.5, 2.5 and 3 respectively.

C. *Obtaining the Best Method*

Using optimal critical value for each series length, the best method can be determined. All methods for each series length compare to each other using percentage of Temporary Change detection as selection criterion. Figure 5 shows percentage of detection is function of outlier magnitude. It means increasing magnitude of outlier affects to increase performance (percentage of outlier can be detected).

Outlier was detected sequentially using single outlier detection method. Detection first outlier is followed by modifying data before detection another outlier. Omit-one method was omitting one observation which result a biggest outlier, whereas Replace-method was replacing the observation which result biggest outlier with certain values, such as similar value of the data which presence outlier or average of data which underlying before and after presence of outlier. In this research, the value was produced by average two observations, before and after outlier point. Since Z_t is observation t-th, $Z_t^* = (Z_{t-1} + Z_{t+1})/2$, Z_t^* is a new value of Z_t.

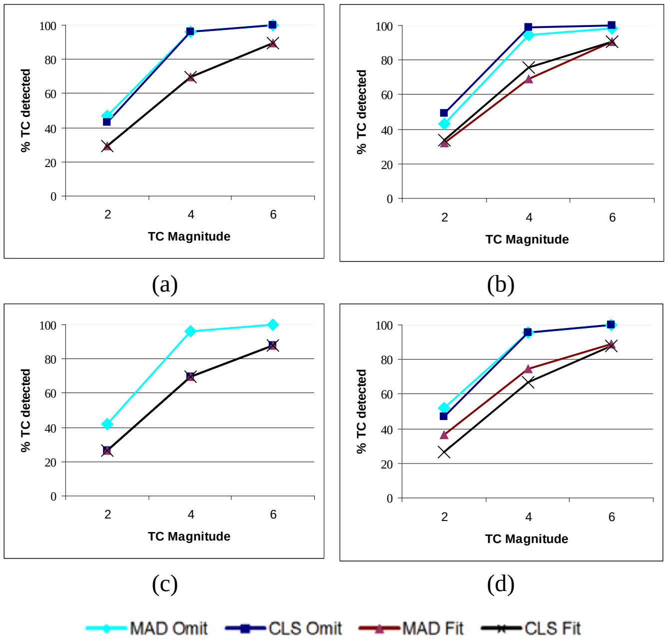

Fig. 5 Percentage of Outlier was detected by TC detection on various outlier magnitude and Model in term of series length=100 for four models (a) model 1 (b) model 2 (c) model 3 (d) model 4

Figure 5 shows that comparison between Omit-one method and Fit method yields performance of Omit-one method higher than Fit method for all of outlier magnitudes. Both Omit-one methods (CLS-Omit and MAD-Omit) are better than both Fit methods (CLS-Fit and MAD-Fit). It means omitting outlier is better choice than replacing one using data fit, in term of series length=100. For size of outlier =4, all outliers could be detected using Omit method except Model 3, therefore Fit method achieve less than 80%. Performance of Fit method is similar each other, except model 4, CLS Fit is worse than MAD Fit.

Figure 6 displays outlier magnitude affect to percentage of detection. Escalating magnitude of outlier affects to bigger percentage of temporary change detection. Both CLS-Omit and MAD-Omit methods can detect temporary change better than both Fit methods, CLS-Fit and MAD-Fit. Result of MAD-Omit is similar to CLS-Omit; moreover result of MAD-Fit similar to CLS-Fit. It means omitting outlier is better choice than replacing one using data fit, especially for series length=200.

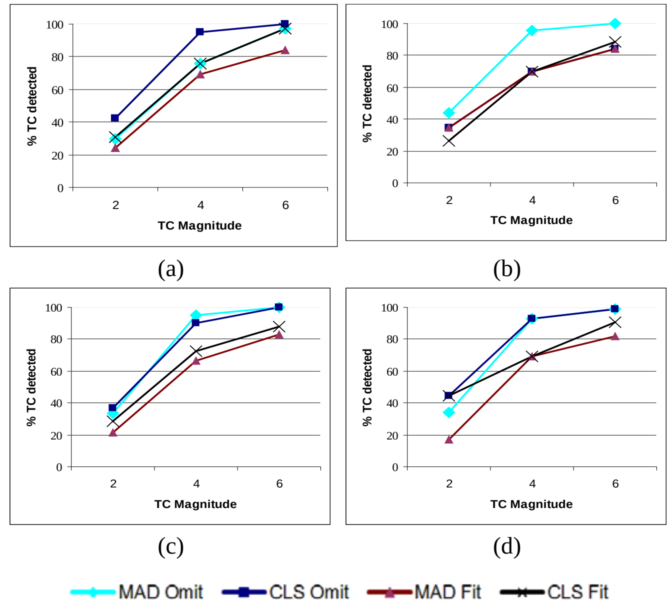

Fig. 6 Percentage of Outlier was detected by TC detection on various outlier magnitude and Model in term of series length=200 for four models (a) model 1 (b) model 2 (c) model 3 (d) model 4

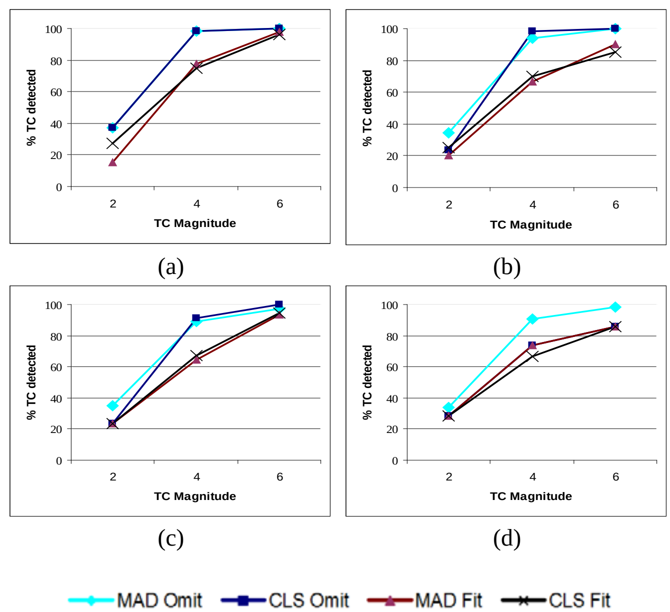

Fig. 7 Percentage of Outlier was detected by TC detection on various outlier magnitude and Model in term of series length=500 for four models (a) model 1 (b) model 2 (c) model 3 (d) model 4

Figure 7 exhibits increasing outlier magnitude influence to increase percentage of outlier detection. It is simple to be seen that the greater outlier easier to be detected. Accuracy of detection is as function of outlier magnitude, as shown Figure 7. Increasing magnitude of outlier affect to increase performance (accuracy of outlier detection), moreover difference methods affect different result. Estimation outlier using MAD is better than CLS. Moreover MAD-Omit method is slightly better than MAD-fit method. In contrast, comparison between Omit-one methods and Fit methods yields Omit-one is better performance. Percentage of outlier could be detected by Omit-one method is higher than fit method for outlier magnitude equal or more than 2. It means omitting outlier is appropriate method.

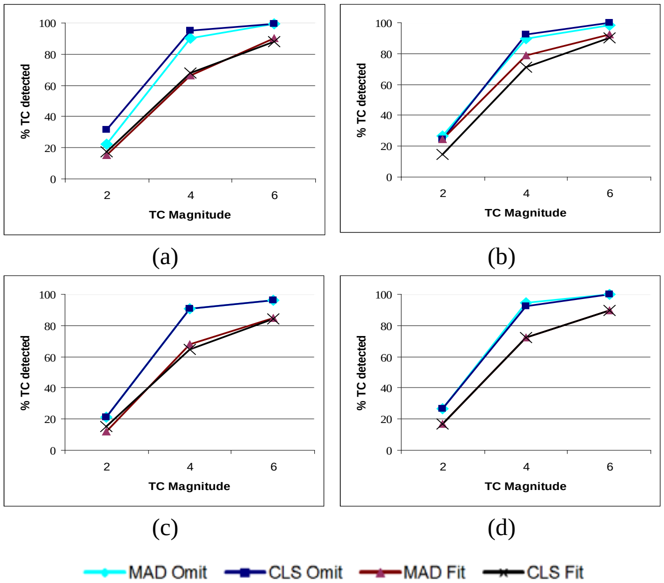

Fig. 8 Percentage of Outlier was detected by TC detection on various outlier magnitude and Model in term of series length=1000 for four models (a) model 1 (b) model 2 (c) model 3 (d) model 4

Figure 8 exhibits MAD-Omit method is slightly better than MAD-fit method for three models: model 2, model 3 and model 4, conversely MAD-fit is slightly better than MAD-omit method for model 1. Outlier detection using MAD-Omit for length series 500 result better performance than three others methods for all models, except model 4 CLS-Omit is slightly better than MAD-Omit.

Generally Figure5-8 can be interpreted that performance is affected by outlier magnitude and method but not by various models. In general increasing magnitude of outlier affects to better performance, furthermore MAD-Omit method is better than three other methods. In the point of view series length, performance of outlier detection not depends on the series length. Commonly, omitting data which presence of outlier is better than replacing data using a value was produced by average two observations, before and after outlier point.

V. Case Study: Surabaya's Air Pollutant (Sulfur Trioxide)

A. Outlier Detection

Pollutant data in this case study is Sulfur Trioxide (SO_3) at Surabaya city. Surabaya is a big city which has three billion citizens and pollution is one of the critical problems. One of the significant pollutant component is Sulfur Trioxide (SO_3) that be produced by transportation and industry. Data was taken every half hour on February 1st and 2nd 2005. From this data, outliers are needed to detect for knowing the extreme data or extreme change of the data.

Firstly, assuming that there is no outlier on pollutant data and obtain appropriate model. Using autocorrelation function (ACF) and partial autocorrelation function (PACF), tentative model can be determined. Fig. 9 presents the ACF and PACF. There are three available models: AR(2), MA(1) and ARMA(1,1).

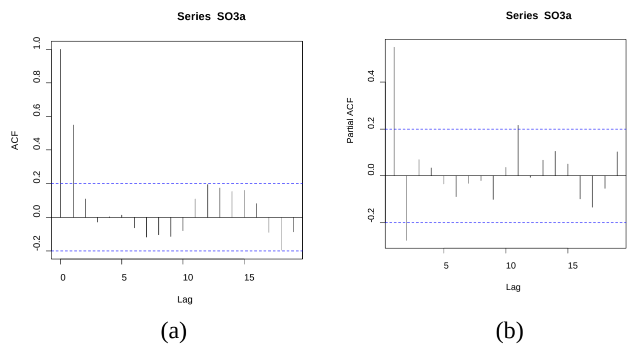

Fig. 9 (a) ACF and (b) PACF to determine tentative model

Secondly, estimate model parameter for each tentative model. Table 2 shows intercept, AR and MA parameter each model. All tentative models converge; therefore Ljung-Box test shows the models can be used to data representation.

Table 2 Parameter model estimation, assumption test and obtaining the best model

Model	Intercept	AR(1)	AR(2)	MA(2)	Ljung-Box Test	AIC
AR(2)	-0.2989	0.5703	-0.292		Not significant	273.59
MA(1)	-0.3019			0.6582	Not significant	267.60
ARMA(1,1)	-0.299	0.286		0.4855	Not significant	266.19

Table 3 Outlier detection of air pollutant data

i-th Outlier Detection	CLS-FIT		MAD-FIT		CLS-OMIT		MAD-OMIT	
	Outlier Magnitude	T	Outlier Magnitude	T	Outlier Magnitude	T	Outlier Magnitude	T
1	**2.59705**	**7**	**2.75177**	**7**	**2.59705**	**7**	**2.75177**	**7**
2	2.90302	8	3.10922	8	3.3174	8	3.58531	8
3					3.0323	9	3.41053	9
4					3.64021	10	4.15626	10
5					**3.86029**	**11**	**4.47418**	**11**
6					2.90461	12	3.35804	12
7							2.58453	1

Thirdly, obtaining the best model; using Akaike's Information Criterion (AIC) the best model is the smallest AIC's value. The best model is ARMA(1,1).

Fourthly, outlier detection use Temporary Change Detection method. Four steps are conducted iteratively. The result of the statistical output can be shown as Table 3.

Table 3 shows that the four methods yield similar result for the first step, they show same observation point as outlier. It means that the all methods are good approach to detect outlier. MAD-Omit find out seven outliers, and follow by CLS-Omit, CLS-Fit and MAD-Fit. Whereas for further detection CLS-Omit and MAD-Omit succeed to discover masking effect at 11th data and the CLS-Fit and MAD-Fit fail. Masking effect is the biggest outlier after other outliers remove before.

Furthermore the value of masking effect which be detected by MAD-Omit is better than CLS-Fit because the outlier magnitude was produced by MAD-Omit is greater than CLS-Fit.

Figure 10 shows outlier data from plot fits with residuals, number of outlier of the MAD-Omit and CLS-Omit are larger than CLS-Fit and MAD-Fit. Therefore, the biggest value of outlier by MAD-Omit methods is larger than the biggest of each other methods, as shown at Table 3.

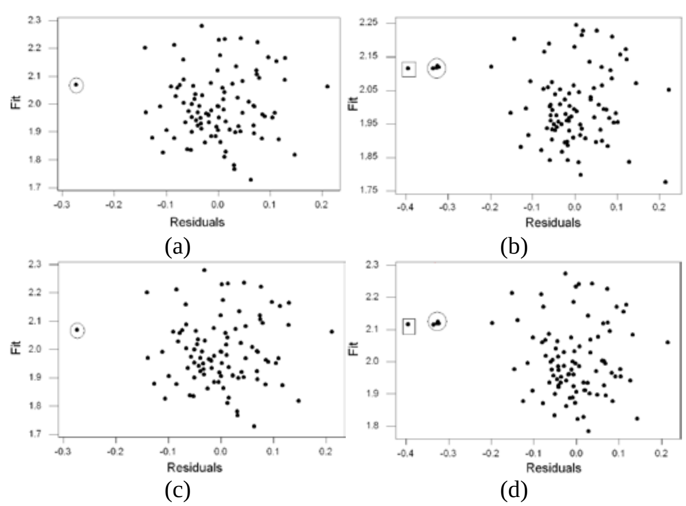

Fig. 10. Fitting data vs. Residuals for all methods

B. *Obtaining the best method*

To confirmation the result above, the fittest model should be checked. 15 series are taken on February 3rd as out of sample. The best model is the model which obtains the best forecast. Akaike's Information Criterion (AIC) is used as criteria to model selection. The smallest AIC value shows the fittest model.

Table 4 AIC of four methods for choosing the fittest model.

	AIC
CLS-Fit	50.4761
MAD-Fit	50.4761
CLS-Omit	43.1535
MAD-Omit	39.2006

Table 4 shows, using AIC as model selection, MAD-Omit detect Temporary Change better than three others method. Generally, TC detection using MAD-Omit is better method compare with MAD-Fit, CLS-Fit and CLS-Omit.

VI. Conclusions

Detecting Temporary Change in time series ARMA (1,1) model for simulation data shows Omit-one Method result better performance compare with Replacement, especially for large number data. Estimation variance use MAD method is slightly better than CLS method. Moreover, MAD-Omit combination method is the appropriate method to detect Temporary Change compare with MAD-Fit, CLS-Omit and CLS-Fit. Result of Surabaya's Air Pollutant data support the conclusion is taken from simulation data that MAD-Omit method is the best method to detect Temporary Change cases.

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