Control of Time-Delay Systems by Means of Modified Smith Predictors: A Simple Matlab Toolbox

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Abstract—This paper is focused on description of a simple Matlab toolbox for control of time-delay systems by means of modified Smith predictors. More specifically, the program contains three methods, namely modified Smith predictor for unstable and integrating processes with time delay, modified PI-PD Smith predictor for processes with long dead time, and modified Smith predictor design by Coefficient Diagram Method (CDM). The software offers Graphical User Interface (GUI) for convenient controller design with subsequent direct applicability in prepared Simulink schemes. Its capabilities are demonstrated on several simulation examples. The described product is downloadable on the Internet and can be freely used for research and educational purposes.

Keywords—Time-delay systems, modified Smith predictors, control design, toolbox, Matlab, Simulink.

I. INTRODUCTION

TIME-DELAY systems commonly appear in many practical control applications and thus also in a lot of related research works, e.g. [1] – [7]. They are also called dead-time systems, systems with aftereffect, etc. [8]. Unfortunately, the presence of time delay can severely complicate control design because it impairs the stability and performance of the control loops.

One of the classical tools for overcoming the time delay is known as the Smith predictor. This effective compensation structure has been published as early as in the late 1950s [9], [10]. Since that time, a number of various modifications with improved features have been introduced. Some of them can be found in [11] – [17]. Besides, the adaptive version of Smith predictor is presented e.g. in [18], [19].

The principal aim of this paper is to present a simple Matlab program which is suitable for controller design and simulation under assumption of three modifications of Smith predictor. The package includes the following methods:

- modified Smith predictor for unstable and integrating processes with time delay [14], modified PI-PD Smith predictor for processes with long dead time [15], and modified Smith predictor design by Coefficient Diagram Method (CDM) [16]. The software is freely downloadable from the web site [20]. Actually, it is a translated and a bit improved version of the program created under the scope of the Master’s Theses [21]. The capabilities are demonstrated by means of three control examples.

The previous works describing this program was already presented at conferences [22], [23] and as a part of the monograph chapter [24]. However, this paper deals with the topic in more detail. Furthermore, application of several modified Smith predictors to control of systems with time-varying delay and comparison with another control approach are provided e.g. in [25] – [27].

The paper is organized as follows. In Section 2, basic theoretical background for three modifications of Smith predictor is provided. The Section 3 then describes the program itself. Further, the three simulation examples illustrating the capabilities of the toolbox are presented in Section 4. And finally, Section 5 offers some conclusion remarks.

II. MODIFIED SMITH PREDICTORS

The classical Smith predictor represents well-known tool for compensation of time-delay term [9], [10]. Nevertheless, the Smith predictor fails for unstable or integrating processes under presence of disturbance and it is very sensitive to modelling errors. Some of the original Smith predictor imperfections have been reduced by its modifications which appeared in the scientific literature. Among many others, works [11] – [17] contributed to embellishment of the Smith predictor. Consequently, these modifications have brought the improvement in setpoint and disturbance responses for various scenarios.

Within this paper, three selected modified Smith predictors [14] – [16] which have been implemented into the program are going to be briefly described. The common feature of all these modifications is that they utilize more sophisticated and complicated structure with additional controllers when compared with the original Smith predictor. However, the use of mathematical model of the controlled plant including time-
delay term in the inner loop is preserved in all three modifications. Furthermore, the same model is supposed as a nominal system during controller design.

The provided description of the methods is more or less general. Then, the specific relations for design of controllers have been usually pre-derived for first and second order time-delay plants. They can be found either in the related literature [14]–[16] or directly in the source code of the program [20].

A. Method 1: Modified Smith Predictor for Unstable and Integrating Processes with Time Delay

First of the implemented modified Smith predictors was presented in [14]. It is useful for unstable and integrating processes and its structure is shown in fig. 1.

![Fig. 1 modified Smith predictor structure [14]](image)

The really controlled system is formally separated into two blocks representing time-delay-free transfer function $G_r(s)$ and time-delay term $\Theta_n(s)$. Analogically, its mathematical model in the inner loop consists of $G_m(s)$ and $\Theta_m(s)$. Signals $w$, $n$, $y$ stand for reference value, disturbance in the input of the controlled plant, and output signal, respectively. The same notation will be adopted also for the other two modifications.

The controller $G_{c1}(s)$ is utilized to stabilize the unstable pole. The other controllers $G_{c2}(s)$ and $G_{c3}(s)$ then ensure reference tracking and disturbance rejection, respectively, by considering the inner loop as an open-loop stable system. Moreover, the signal outgoing from the controller $G_{c2}(s)$ can be interpreted as an estimation of the disturbance $n$.

The design approach is based on standard forms of the closed loop system response in the meaning of Integral Squared Time Error (ISTE) criterion and on the Nyquist stability analysis [14].

B. Method 2: Modified PI-PD Smith Predictor for Processes with Long Dead Time

The second modification of the Smith predictor, suggested in [15], uses the structure with trio of controllers as shown in Fig. 2, where $G_{c1}(s)$ is a PI controller, $G_{c2}(s)$ is a PD (or only P where it is appropriate) controller and $G_{c3}(s)$ is the disturbance controller introduced in [13].

![Fig. 2 modified Smith predictor structure [15]](image)

Actually, the structures displayed in Figs. 1 and 2 are practically the same from the block algebra point of view. However, the paper as well as the program itself has adopted the original visualizations of the authors [14], [15].

The structure from Fig. 2, which replaces the conventional controller by a PI-PD structure, should outperform a PID controller in some common SISO systems. Generally, the synthesis is based on usage of standard forms for obtaining the optimal closed-loop transfer function parameters in the meaning of ISTE criterion, i.e. an algebraically simple approach to control system design is applied [15].

C. Method 3: Modified Smith Predictor Design by Coefficient Diagram Method (CDM)

Finally, the third modified Smith predictor utilizes the design by CDM. It was published in [16] and it takes advantage of the control structure with trio of controllers depicted in Fig. 3.

![Fig. 3 modified Smith predictor structure [16]](image)

The CDM design itself is based on the following four studies [16]:

- Coefficient diagram: It is a semilogarithmic diagram which allows investigating the stability and response of systems in a single graph. The vertical axis logarithmically shows the coefficients of characteristic polynomial, stability indices, stability limits and equivalent time constant while the horizontal axis represents the order values corresponding to each of coefficients.
- Modification of Kessler standard form: The form developed by Kessler in 1960 has decreased the oscillations and overshoots compared to the original Graham’s ITAE form. In this approach, a new form called “Standard Manabe Form” is used. This design
should result in quite stable and robust responses with small settling time.

- Lipatov stability analysis: The effect of coefficient variations can not be seen clearly for higher order systems. The conditions for stability or instability of such systems, based on Lipatov’s work, are included in CDM design technique.

- Obtaining characteristic polynomial: A method similar to pole placement is applied. However, the main difference is the Manabe form.

The reader interested in CDM can find all necessary background and details of the procedure in [16], [28], [29] and related literature.

III. DESCRIPTION OF THE PROGRAM

The program can be freely downloaded from the web page [20]. It is a one “zipped” file which contains 36 “m-files”, 10 “mdl-files” and 1 text file with basic instructions and references. The software was lastly modified and verified under Matlab 7.9.0 (R2009b) + Simulink. According to tests, it should work properly in Matlab 7 (R14) and higher (there can arise problems with Simulink schemes in lower versions). After decompression, it can be executed by “go.m” file. The main window of the Graphical User Interface (GUI) is depicted in Fig. 4. Here, a user can select the modification of Smith predictor which should be used for a whole control experiment.

With regard to the first and second method, it should be added that comparison of delay free part of setpoint response with a closed-loop standard form for a minimum ISTE criterion as suggested in [30] (according to reference from [14]) leads to graphical relation(s) between some parameters. These graphs can be found e.g. in [15]. As these relations are necessary to use during controller computation, their graphical versions have been approximated by sixth order polynomials for the purpose of the program (see the m-codes).

A. Method 1

After the selection of the first modification “For Unstable and Integrating Time-Delay System” [14], the window with type of controlled plant and fundamental properties of the experiment (simulation time, reference signal, disturbances) will appear – see Fig. 5.

There are three sorts of controlled systems available for this method. The pop-up menu with them is shown in Fig. 6. As can be seen, the first method allows choosing from unstable first order time-delay plant, unstable second order time-delay plant, and (first order) integrating time-delay plant.

In the next window, coefficients of the controlled system of specific type and some other additional parameters depending on the used method can be set as depicted in Fig. 7. Here, the case of unstable second order time-delay plant is illustrated. Nevertheless, the program allows not only definition of nominal system, which is considered as a model for control design as well as a model in the inner control loop (according to Figs. 1-3), but also adjustment of the perturbed system, which is used as a really controlled plant (with potentially different coefficients). If these systems are the same, the parameters of nominal system can be copied to the perturbed one using a single button.
Fig. 7 definition of parameters for nominal and perturbed system for the first method [14] and unstable second order time-delay plant

After confirming the data from Fig. 7, the parameters of controllers are calculated and visualized (Fig. 8).

Fig. 8 controller parameters – an example for the first method [14] with parameters from Fig. 7.

Potentially, some additional information can appear – see the example in Fig. 9.

Fig. 9 an example of warning for the first method [14]

Finally, a Simulink scheme (see an example in Fig. 10) is opened together with the controller parameters. The control loop utilizes preset and computed parameters so the simulation can be performed simply by “Start simulation” button. Then, the results can be seen directly in the “Scope” block or exported for future processing in a standard manner.

Fig. 10 simulation scheme for the first method [14] and unstable second order time-delay plant

B. Method 2

Fig. 11 possible types of controlled system for the second method [15]
Regarding to the second method “PI-PD Modification for Systems with Long Dead Time” [15], the window with type of controlled plant and fundamental properties of the experiment is basically the same as in the previous case (see Fig. 5). The only alteration consists in the selection of controlled system type. Now, the three choices according to Fig. 11 are available.

Subsequently, setting the parameters of controlled system is similar to Fig. 7 and controller parameters are also presented in the analogical form as in Fig. 8. The example of Simulink control loop scheme for the second method is provided in Fig. 12.

![Fig. 12 simulation scheme for the second method [15] and second order time-delay plant](image)

**C. Method 3**

Finally, the third method “Control Design Using Coefficient Diagram Method” has also the same window for adjustment of basic properties of control experiment as in Fig. 5. However, a user can select only from the first and second order time-delay plant as depicted in Fig. 13.

![Fig. 13 possible types of controlled system for the third method [16]](image)

In the following step, the window with specific parameters of the nominal and perturbed plants is a bit different than for other two implemented methods, because the preferred type of controller, in the meaning of without or with disturbance rejection feature, can be chosen by user. Moreover, the setting time (directly in seconds) is also adjustable parameter here – see Fig. 14.

![Fig. 14 definition of parameters for nominal and perturbed system for the third method [16] and second order time-delay plant](image)

Finally, the simulation scheme for this modification under assumption of second order time-delay plant and controller with disturbance rejection is shown in Fig. 15.

![Fig. 15 simulation scheme for the third method [16] and second order time-delay plant](image)

**IV. SIMULATION EXAMPLES**

The capabilities of the toolbox and the effectiveness of the methods are going to be illustrated on the three simulation examples. In all cases, the reference signal is considered with the step change from 1 to 2 in a third of a simulation time and then some disturbance is injected to the input of the controlled plant during the last third of the simulation time. All time constants are supposed to be in seconds.
A. Method 1

In the first example, the modification [14] is employed. The controlled system is supposed to be described by unstable first order time-delay transfer function:

\[ G(s) = \frac{3}{10s - 1} e^{-5s} \]  \tag{1}

The simulation time is 300 seconds and the load disturbance \( n = -0.1 \). The calculations lead to the trio of controllers:

\[ G_c(s) = \frac{s + 10}{s} \]
\[ G_{c1}(s) = 7.1911 \]
\[ G_{c2}(s) = 0.4714 \]  \tag{2}

with presumption (see the program):

\[ K_p = 1 \]
\[ T_s = 0.1 \]  \tag{3}

The control results obtained from the program are provided in Fig. 16.

Subsequently, the same controller has been applied to the perturbed plants. First to the plant with 10% higher gain:

\[ G_{p1}(s) = \frac{3.3}{10s - 1} e^{-5s} \]  \tag{4}

Then to the plant with 10% higher time constant:

\[ G_{p2}(s) = \frac{3}{11s - 1} e^{-5s} \]  \tag{5}

And finally to the plant with 10% higher time-delay term:

\[ G_{p3}(s) = \frac{3}{10s - 1} e^{-5.5s} \]  \tag{6}

The control results are visualized in Fig. 17 (for perturbed gain), Fig. 18 (for perturbed time constant) and Fig. 19 (for perturbed time-delay term).
B. Method 2

The second example uses the modification [15]. The controlled object is now given in the form of ideal first order time-delay integrator:

\[ G(s) = \frac{1}{s} e^{-2s} \]  

(7)

The simulation time is now 120 seconds and the load disturbance remains the same as in the previous example \( n = -0.1 \). The computed controllers are as follows:

\[ G_{c1}(s) = \frac{s+10}{s} \]
\[ G_{c2}(s) = 3.4053 \]
\[ G_{c3}(s) = 0.25 \]

(8)

with presumption, again:

\[ K_c = 1 \]
\[ T_c = 0.1 \]  

(9)

The Fig. 20 shows the corresponding output controlled signal.

Analogically to the first example, the controllers (8) have been applied also to a perturbed plant. Now, it has been the integrator with 10% higher both coefficient in numerator and time-delay term.

\[ G_{p1}(s) = \frac{1.1}{s} e^{-2.2s} \]  

(10)

The results of control are presented in Fig. 21.

C. Method 3

Finally, the third example is focused on the application of modification [16]. Here, it has been supposed the second order time-delay plant:

\[ G(s) = \frac{5}{(10s + 1)(2s + 1)} e^{-10s} = \frac{5}{20s^2 + 12s + 1} e^{-10s} \]

(11)

The simulation time was preset to 300 seconds and the load disturbance still remained \( n = -0.1 \). As the method allows prescribing the setting time directly, it was chosen \( T_s = 10 [s] \). Furthermore, the controllers with disturbance rejection ability were preferred. The resulting regulators are then:

\[ G_{c1}(s) = 1 \]
\[ G_{c2}(s) = \frac{1}{0.92941s^2 + 1.4441s} \]
\[ G_{c3}(s) = 4.971s^2 + 4.3541s + 1 \]

(12)

The control simulation output is depicted in Fig. 22.
In the final simulation, all the perturbed system parameters $K$, $a_2$, $a_1$, $TD$ (see Fig. 14) are assumed 10% higher. So, the perturbed plant is:

$$G_p(s) = \frac{5.5}{22s^2 + 13.2s + 1} e^{-11s}. \quad (13)$$

The results of control are presented in Fig. 23.

![Fig. 23 method 3 – control results for the perturbed system (13)](image)

**V. CONCLUSION**

The paper has described the simple Matlab toolbox for control of time-delay systems using modified Smith predictors. Three methods (modified Smith predictor for unstable and integrating processes with time delay, modified PI-PD Smith predictor for processes with long dead time, and modified Smith predictor design by CDM) have been implemented into GUI for convenient control design with direct utilization in Simulink environment. The basic capabilities are shown by means of simulation examples. The toolbox is free to use for research and educational purposes.

**REFERENCES**


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