Optimal control law for the concentration of carbon dioxide in a tomato greenhouse and optimal-tuning PI control as LQR for the automatic device.

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***Abstract*—The tomato model-space has been developed and it is called a big leaf-big fruit model, this model is formed by the mass balances: the non-structural biomass (nutrients) and the structural biomass of fruits and leaves. Also, a model that describes the behaviour on carbon dioxide concentration inside greenhouse is obtained. From the two models we get a new complete crop-greenhouse model. This model allows to get an optimal control for the carbon dioxide enrichment in a tomato greenhouse which gives benefits, because it is possible to achieve a saving for energy consumption and more tomato production. The optimal control theory is applied to the crop-greenhouse integrated system, which is based on four state variables: the consumption of nutrients, the fruits and leafs growth and the carbon dioxide concentration. This work contributes with the optimal control law that gives the desired CO_2 concentration behaviour during the growth time for the crop. This behaviour will be a reference signal for the controller implementation in the electronic device that will be made in a future work. The simulations for the crop-greenhouse system are presented for a two weeks period.**
This paper also contributes at optimal-tuning PI control as Linear Quadratic Regulator of the electronic device. The simulation of the optimal control PI are showed here.

***Index Terms*—Optimal control, Optimal-tuning, Linear Quadratic Regulator, Carbon dioxide, structural biomass of leaves, structural biomass of fruit, state space.**

I. INTRODUCTION

In past years, researches have proposed different optimal climate control methods for greenhouse systems. These efforts have not been applied in practice because it is difficult for real application [1,2,3,9,12]. The difficulty lies in that the crop growth is based in many different variables and the mathematical analysis with all variables is complicated. In this work, the variable that has our interest is the carbon dioxide.

The carbon dioxide enrichment is practised in the greenhouse crops in order to increase the yield and the benefit. There are studies that demonstrate the CO_2 enrichment improves the net photosynthesis in the plants achieving the increase of the total weight, height, and the number of leaves and branches [9]. Other research has demonstrated that the CO_2 enrichment makes physic-chemical changes in the crop, like color and firmness [8].

Optimization problems with two or more objectives are very common in engineering and many other disciplines. The process of optimizing a collection of objective function is called multi-objective optimization and it is difficult because of the large number of conditions and variables involved in the system [4]. In this work the optimization problem has two objectives, first, decrease the energy consumption for carbon dioxide enrichment and second, increase the tomato production. The search process can be accomplished in two ways: deterministic and stochastic search algorithms [6].
Optimal strategies for CO_2 enrichment can be deduced experimentally or analytically. Experimentation is not able to produce a valid result for all condition set. The mathematical analysis gives us a better option to obtain an optimal strategy because it considers all the variables involved in the system. This method is based on ventilation, photosynthesis, dry matter and production rate models.

One of the main objectives is to contribute with the optimal control problem and its implementation in real time. The tomato crop has been chosen because it is one of the most important crop in our country and is the second farm product consumed in the world. To achieve the objective, we start from the tomato and greenhouse mathematical set model considering the following variable: plant and fruit dry weight, the nutrients amount and the CO_2 concentration.

In this paper, we obtained the behaviour of CO_2 concentration inside the greenhouse that is necessary for growth during two weeks. We start from the fact that this behaviour is a reference signal that a classic controller will have to follow for each time instant. In future work, all results in this paper help us to design an electronic device that could be used in real tomato greenhouses.

With the approach of optimal PI tuning for first order processes, also the provision of simultaneously and optimally finding the two parameters of a PI controller $(i.e. Ki, Kp)$, development has been addressed in this paper.

II. General Formulation of the Optimal Control Problem

Optimal control problems appeared as essential tools in modern control theory. Several authors have proposed different basic mathematical formulations of fixed time problems [5].
The optimal control of any system has to be based on three concepts: the dynamic model of the system, a functional and the system restrictions. In matrix notation the state equation is represented as follows:

$$\dot{x} = f\left(x(t), u(t), t\right). \tag{1}$$

Where $x(t)$ is the state vector, $u(t)$ is the control vector and t is the time. A criterion is required to evaluate the performance of the system, normally, the functional is defined by:

$$J = \phi\left(x(t_f), t_f\right) + \int_{t_0}^{t_f} L\left(x(t), u(t), t\right)\, dt, \tag{2}$$

Where t_0 and t_f are the initial and final time, ϕ and L are scalar functions, t_f can be fixed or free. Starting at the initial state $x(t_0) = x_0$ and applying the control signal $u(t)$ for $t \in [t_0, t_f]$, it makes that system follows some trajectory of states, then the functional assigns a unique real number for each trajectory of the system.

The fundamental problem of optimal control is to determine an admissible control u^* which makes that equation (1) follows one admissible trajectory x^* that minimize the value of the functional in equation (2). Then, u^* is named optimal control and x^* is an optimal trajectory.

Necessary conditions for a solution

Restrictions (1) are added to the functional (2) with a Lagrange multipliers vector time variant$\Psi(t)$ and the functional is rewritten as follows:

$$\begin{aligned} J = \phi(x(t_f)) + \int_{t_0}^{t_f} [L(x(t), u(t), t) \\ -\Psi^T f(x(t), u(t), t) - \dot{x}]dt, \end{aligned} \tag{3}$$

Then, the Hamiltonian scalar function is defined, which depends on the variable state vector, the control signal and the new vector $\Psi(t)$

$$H(x(t), u(t), \Psi(t), t) =$$

$$= L(x(t), u(t), t) + \Psi(t) f(x(t), u(t), t) \tag{4}$$

Then, it is possible to write an auxiliary system starting from a new auxiliary vector that depends on time $\Psi(t)$. The new system is formed from Hamiltonian function, as follows:

$$\dot{\Psi}^T = -\frac{\partial H}{\partial x} = -\frac{\partial L}{\partial x} - \Psi^T \frac{\partial f}{\partial x} \tag{5}$$

The auxiliary system allows to know the final conditions of the general system, which can be written as follows:

$$\Psi^T(t_f) = \frac{\partial \phi}{\partial x}(t_f) \tag{6}$$

An infinitesimal variation in $u(t)$ denominated $\delta u(t)$ produces a variation in the functional J like δJ. For a stationary solution it is required that arbitrary variation is equal to zero, $\delta J = 0$. This is true when

$$\frac{\partial H}{\partial u} = \frac{\partial L}{\partial u} + \Psi^T \frac{\partial f}{\partial u} = 0 \tag{7}$$

Note that from the Hamiltonian function (4) it is possible to get the control form. Then, to find the vector function of control $u(t)$ that produces a stationary value of the functional we must solve the following differential equation system:

$$\begin{cases} \dot{x}(t) = f\left(x(t), u(t), t\right), \\ \dot{\Psi}(t) = -\frac{\partial H^T}{\partial x}, \end{cases} \tag{8}$$

The boundary conditions for this differential equations are separated, it means that some of them are defined in $t = t_0$ and the others in $t = t_f$. This is a *problem with boundary values of two points.* Note the equations that describe the states $x(t)$ and the auxiliary states $\Psi(t)$ in the equation (8) are coupled, for this reason $u(t)$ depends on $\Psi(t)$ through the stationary condition and the auxiliary states depend on $x(t)$ and $u(t)$. And the first system in (8) has the initial conditions of he system while the last system in (8) has the final condition of the system.

III. Dynamic Models of the Crop and of the Greenhouse

A. Dynamic model of the Crop

The model in space states of the tomato crop has three principles states (Van Straten et al., 2011)[13]:

- Non-structural Biomass (Nutrients).
- Leaves Structural Biomass.
- Fruits Structural Biomass.

1) Biomass balance of nutrients: Nutrients are being produced by photosynthesis. The gross canopy photosynthesis rate in dry matter per unit area is P. Nutrients are converted to leaf and fruits, this is known as growth. Leaf and fruits have a demand for nutrients, which will be honored if there are sufficient nutrients available. We denote W_B as the total nutrients in the plant and it is expressed as dry weight per area unit. The biomass balance equation of nutrients is the following [13]:

$$\frac{dW_B}{dt} = P - h\{\cdot\}\left(\frac{(1+\theta_V)}{z}G_L^{dem} + (1+\theta_F)\,G_F^{dem}\right) - \\ -h\{\cdot\}\left(\frac{R_L}{z} + R_F\right). \tag{9}$$

The biomass balance equation of nutrients (9) can take two values depending on the nutrients abundance $h\{\cdot\}$, where the first expression is taken when $h\{\cdot\}=1$ (abundance of nutrients) and the second one is taken when $h\{\cdot\}=0$ (lack of nutrients).

$$\frac{dW_B}{dt} = \begin{cases} P - \frac{(1+\theta_V)}{z}G_L^{dem} - (1+\theta_F)\,G_F^{dem} - \\ -\frac{R_L}{z} - R_F, \\ P, \end{cases} \tag{10}$$

R_F.- Respiration needs of fruits
θ_V.- Additional amount of nutrients needs for one unit of structural vegetative parts.
G_L^{dem}.- Unit area growth demand of leaves.
θ_F.- Additional amount of nutrients needs for one unit of structural fruit parts.
G_F^{dem}.- Unit area growth demand of fruit.
z.- Total vegetative parts.
$h\{\cdot\}$.- Nutrients abundance.

2) Biomass balance of leaves: The leaf growth is equal to the amount of nutrients converted to structural leaf biomass in the plant and it is given by $h\{\cdot\}G_L^{dem}$. The model does not incorporate an extra state for stem and roots, but the factor z assumes that each increment in leaf will be accompanied by an increment in stem and roots. If there are no sufficient assimilates (nutrients), growth stops, normally the assimilates are used for the maintenance, but in lack of nutrients, maintenance in the model goes at the expense of structural parts (leaves and fruit). The biomass balance of leaves is expressed in the form [13]:

$$\frac{dW_L}{dt} = h\{\cdot\}G_L^{dem} - (1-h\{\cdot\})\,R_L - H_L, \tag{11}$$

Depending on the abundance of nutrients $h\{\cdot\}$, the biomass leaf balance equation (11) can take two values:

$$\frac{dW_L}{dt} = \begin{cases} G_L^{dem} - H_L, & \text{if } h\{\cdot\}=1, \\ -R_L - H_L, & \text{if } h\{\cdot\}=0. \end{cases} \tag{12}$$

where
H_L is the leaf picking rate.

3) Biomass balance of fruit: Similarly to the biomass of leaf case, the growth of fruits in the plant from the nutrients is given by $h\{\cdot\}G_F^{dem}$. The term G_F^{dem} depends principally on the pivotal temperature, cultivation temperature level and the reference temperature [13].

$$\frac{dW_F}{dt} = h\{\cdot\}G_F^{dem} - (1-h\{\cdot\})\,R_F - H_F, \tag{13}$$

Finally, the equation (13) of biomass balance of fruits can take two different values depending on nutrient abundance $h\{\cdot\}$, where H_F is the fruit harvest rate:

$$\frac{dW_F}{dt} = \begin{cases} G_F^{dem} - H_F, & \text{if } h\{\cdot\}=1, \\ -R_F - H_F, & \text{if } h\{\cdot\}=0. \end{cases} \tag{14}$$

B. Dynamic Model of the Greenhouse

1) Balance of CO_2 energy in the greenhouse: The balance of carbon dioxide energy within greenhouse is given by the equation [13]:

$$\frac{V_g}{A_g}\frac{dC_{CO_2}}{dt} = -\eta_{CO_2/dw}P + \eta_{CO_2/dw}R - \\ -\varphi_{CO_2,g\_o}^{vent} + u_{CO_2}, \tag{15}$$

Then each term is described.

- ∗ Carbon dioxide taken from the greenhouse air for plant photosynthesis:
$$\eta_{CO_2/dw}P,$$
- ∗ Carbon dioxide returned to the greenhouse air for plant respiration:
$$\eta_{CO_2/dw}R,$$

The term $\frac{V_g}{A_g}$ is the reason of the volume of greenhouse per unit of area.
R is the total respiration plant per unit of time.

- ∗ Lost of carbon dioxide mass by ventilation:
$$\varphi_{CO_2,g\_o}^{vent} = u_V(C_{CO_2} - C_{CO_2\_o}),$$

where: u_V is the ventilation flow rate per unit of area.
C_{CO_2} (kgm^{-3}) is the carbon dioxide concentration within greenhouse.
$C_{CO_2,o}$ (kgm^{-3}) is the carbon dioxide concentration on the outside greenhouse.

- ∗ Carbon dioxide supply:
$$u_{CO_2} = u_{CO_2}^{Vp}\varphi_{CO_2,in\_g}^{max},$$

where $u_{CO_2}^{Vp}$ is the opening supply valve.
$\varphi_{CO_2,in\_g}^{max}$ $\left(kg[CO_2]m^{-2}[gh]s^{-1}\right)$ is the maximum

flow rate of carbon dioxide.

In this greenhouse model, the position of the carbon dioxide supply valve is the control input. For this reason, the valve relates directly to the actuator that is present on a physical way in the greenhouse.

C. Integrated Model Crop-Greenhouse

From previous description of greenhouse and crop models is possible to get a complete system formed by three crop equations and greenhouse equation. This new equation system describes the complete system behaviour and it is important to note that all of the equations are related principally by the P element and the state variables of the crop. It is important to say that the three equations related to the crop are taken with the assumption that there is an abundance of nutrients ($h\{\cdot\} = 1$). Therefore, general system is as follows:

$$\begin{cases} \dot{W}_L(t) = G_L^{dem} - H_L, \\ \dot{W}_F(t) = G_F^{dem} - H_F, \\ \dot{W}_B(t) = P - \frac{1+\theta_v}{z} G_L^{dem} + (1+\theta_F) G_F^{dem} - \frac{R_L}{z} + R_F, \\ 3\dot{C}_{CO_2}(t) = -\eta CO_{2/dw} P + +\eta CO_{2/dw} R - \varphi_{CO_{g-0}}^{vent} \\ \qquad +uCO_2, \end{cases} \tag{16}$$

IV. Synthesis of Optimal Control

We consider the system (16). The terms for the equation system are substituted using the equation table of the mathematical model (table 1) and the values are substituted using the table of physical parameters (table 2). The resulting model is:

$$\begin{cases} \dot{W}_L(t) = 2.2996 \times 10^{-6}\ W_L(t), \\ \dot{W}_F(t) = 4.3925 \times 10^{-6}\ W_F(t), \\ \dot{W}_B(t) = P(t) - 5.39 \times 10^{-6} W_L(t) - 5.92 \times 10^{-6} W_F(t), \\ 3\dot{C}_{CO_2}(t) = 1.0266(R(t) - P(t)) + 0.155 \times 10^{-10} u_{CO_2}^{v_p}, \end{cases} \tag{17}$$

P and R are the following:

$$P(t) = \frac{3.7192 \times 10^{-11}\ W_L^{2.511}(t)}{1.6353 \times 10^{-9} + 4.0439 \times 10^{-5}\ W_L^{2.511}(t)},$$

$$R(t) = 1.5942 \times 10^{-6} W_F(t) + 0.4856 \times 10^{-6} W_L(t) + 1.668 \times 10^{-7}.$$

It is important to note that the terms P(t) and R(t) have involved two of the three state variables of the crop and they are time dependent functions, so the entire system is connected and it can be solved simultaneously.

We consider the following functional, which has the same form shown in (3):

$$J = \tfrac{1}{2}[W_L^2(tf) + W_F^2(tf) + W_B^2(tf) + C_{CO_2}^2(tf) + \\ + \int_{t_0}^{t_f} [W_L^2(t) + W_F^2(t) + \\ + W_B^2(t) + C_{CO_2}^2(t) + (u_{CO_2}^{v_p})^2(t)]dt] \tag{18}$$

The first term involves the three first variables at the end time, they are related to the final production and the nutrients, and the integral contains the control input in order to avoid the risk for big control inputs. The idea is to minimize the functional (18), related with the equations system (17).

A. Solution Method Description

The Hamiltonian scalar function is obtained considering the relation (4) with the Lagrange multipliers and the functional (17):

$$H(\mathbf{x}, \mathbf{u}, \mathbf{\Psi}, t)) = \\ = \frac{1}{2}[W_L^2(t) + W_F^2(t) + W_B^2(t) + C_{CO_2}^2(t) + (u_{CO_2}^{v_p})^2(t)] + \\ + 2.2996 \times 10^{-6} W_L(t)\Psi_1(t) + 4.3925 \times 10^{-6} W_F(t)\Psi_2(t) + \\ + \left[P - 5.39 \times 10^{-6} W_L(t) - 5.92 \times 10^{-6} W_F(t)\right]\Psi_3(t) + \\ + \frac{1}{3}\left[1.0266(R - P) + 0.1554 \times 10^{-10} u_{CO_2}^{v_p}\right]\Psi_4(t). \tag{19}$$

The system of auxiliary variables is formed using the expression (5) and has the following form:

$$\begin{cases} \dot{\Psi}_1 = W_L + 2.2996 \times 10^{-6}\ \Psi_1 + \frac{\partial P}{\partial W_L}\Psi_3 - \\ \quad -5.39 \times 10^{-6}\Psi_3 + \frac{1}{3}\frac{\partial (R-P)}{\partial W_L}\Psi_4(1.0266) \\ \dot{\Psi}_2 = W_F + 4.3925 \times 10^{-6}\ \Psi_2 - \\ \quad -5.92 \times 10^{-6}\ \Psi_3 + \frac{1}{3}\frac{\partial R}{\partial W_F}\Psi_4(1.0266), \\ \dot{\Psi}_3 = W_B, \\ \dot{\Psi}_4 = C_{CO_2}, \end{cases} \tag{20}$$

The stationary condition give us the following control form, which was obtained from equation (7) and depends on fourth appended state:

$$u_{CO_2}^{v_p} = -\frac{1}{3} 0.1554 \times 10^{-10} \Psi_4(t). \tag{21}$$

It is necessary to solve the equation systems (17) and (20), in this way we can know the Ψ_4 value and finally we will get the control form. The system (17) has initial condition and the system (20) has final conditions. The systems are coupled because the control form (21) has been substituted. To solve the complete system like a system with initial conditions, the auxiliary equations are considered in reverse time, then the behaviour of the auxiliary variables is returned to the direct time. When we solve the appended equation system in reverse time the system becomes a system with initial conditions. It is important to note that the equation (21) depends on the fourth state but this state depends on

TABLE I
GREENHOUSE AND CROP MATHEMATICAL MODEL EQUATIONS

Term	Description
$P = P^{max}\left(\frac{I^{PAR}}{I^{PAR}+K_I}\right)\left(\frac{C_{CO_2}}{C_{CO_2}+K_C}\right)f_m\{\cdot\}$	Production of assimilates by photosynthesis.
$R = h\{\cdot\}\left(\frac{\theta_V}{z}G_L^{dem} + \theta_F G_F^{dem}\right) + \frac{R_L}{z} + R_F$	Total amount breathed plant per unit of time.
$I^{PAR} = f_{PAR/I}\tau_r I_o$	The PAR light intensity at the crop level.
$f_m\{\cdot\} = \frac{(W_L/p_m)^m}{1+(W_L/p_m)^m}$	Maturity factor.
$G_L^{dem} = f_{L/F}(T)k_{GF}^{ref}f_{TG}(T)f_D\{\cdot\}W_L$	Growth leaves demand.
$G_F^{dem} = k_{GF}^{ref}f_{TG}(T)f_D\{\cdot\}W_F$	Growth fruits demand.
$f_{L/F}(T) = f_{L/F}^{ref}e^{v_2(T-T_{L/F}^{ref})}$	Temperature-dependent ratio.
$f_{TG}(T) = Q_{10R}^{T-T_G^{ref}/10}$	Temperature dependent with a Q_{10G} relation.
$f_{TR}(T) = Q_{10R}^{T-T_R^{ref}}$	Function of temperature with a Q_{10G} relation.
$f_D\{\cdot\} = \frac{c_{f1}-c_{f2}D}{c_{f1}-c_{f2}}$	Correction factor for the fruit growth rate.
$R_L = k_{RL}^{ref}f_{TR}(T)W_L$	Respiration demand of the leaves.
$R_F = k_{RF}^{ref}f_{TR}(T)W_F$	Respiration demand of the fruits.
$H_L = k_{HL}W_L$	Leaf picking rate.
$H_F = k_{HF}W_F$	Harvest rate.
$K_{HL} = C_{yL}K_H$	Coefficient of harvest.
$K_{HF} = C_{yF}K_H$	Coefficient of harvest.
$K_H = Cd1 + Cd2ln(T/Cd3) - Cd3 - Cd4e_D$	Harvest rate.
$u_V = \left(\frac{p_{V1}u_V^{A_{plsd}}}{1+p_{V2}u_V^{A_{plsd}}} + p_{V3} + p_{V4}u_V^{A_{pwsd}}\right)v + p_{V5}$	Ventilation flow rate.

the other three states. Using MatLab tools we solve the equation systems (17) and (20).

V. SIMULATION OF CONTROL LAW

The MatLab tools were used to elaborate the program that solve the differential equations system formed by equations (17) and (20). The simulation period is for two weeks. The results obtained are described below. It is important to know that in the following results, the temperature and solar radiation are time varying, in order to make the simulation more real. Figures 1 and 2 show the temperature and solar radiation varying in a time of two weeks.

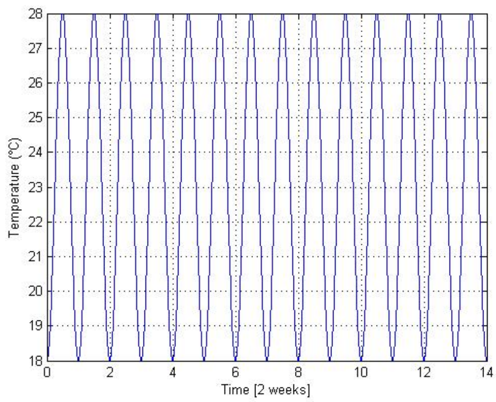

Fig. 1. Time varying temperature for two weeks.

A. Analysis with a Step Input.

In a first simulation we use a step function as control input ($u_{CO_2}^{v_p} = 1$). We solved the equation system and later we got the graphics that represent the behaviour of most important variables of the system. Figure 3 shows the behaviour of three crop state variables. The red line

TABLE II
PHYSIC PARAMETERS.

Variable	Value	Description
z	0.6081	Fraction leaf of total vegetative mass
θ_v	0.23	Surplus assimilate requirement factor per unit fruit increment.
θ_F	0.2	Surplus assimilate requirement factor per unit vegetative increment.
p_h	2.7×10^{-3}	Parameter of switching function, $[m^2\ kg^{-1}]$
p_m	1.8×10^{-2}	Parameter in maturity factor, $[kg\ m^{-2}]$
m	2.511	Parameter in maturity factor
p^{max}	2.2×10^{-6}	Maximum gross canopy photosynthesis rate, $[kg\ m^{-2}\ s^{-1}]$
K_1	577	Monod constant for PAR, $[W\ m^{-2}]$
K_c	0.211	Monod constant for CO_2, $[kg\ m^{-3}]$
$f_{PAR/I}$	0.475	PAR fraction of global radiation
τ_r	0.7	Transmittance of the roof
k_{GF}^{ref}	3.8×10^{-6}	Reference fruit growth rate coefficient, $[s^{-1}]$
T_{GF}^{ref}	20	Reference temperature, $[^0C]$
Q_{10G}	1.6	Temperature function parameter growth
$f_{L/F}^{ref}$	1.38	Reference leaf-fruit partitioning factor
v_2	-0.168	Parámetro de partición de fruta-hoja, $[K^{-1}]$
$T_{L/F}^{ref}$	19	Fruit-leaf partitioning reference temperature, $[^0C]$
k_{RL}^{ref}	2.9×10^{-7}	Maintenance respiration coefficient leaf, $[s^{-1}]$
Q_{10R}	2	Temperature function parameter respiration
T_R^{ref}	25	Reference temperature for respiration, $[^0C]$
k_{RF}^{ref}	1.2×10^{-7}	Maintenance respiration coefficient leaf, $[s^{-1}]$
η	0.7	Absorbed in relation to the total energy of the net radiation heat received.
$Cd1$	$2.13x10^{-7}$	Parameter in development rate function, s^{-1}
$Cd2$	$2.47x10^{-7}$	Parameter in development rate function, s^{-1}
$Cd4$	$7.50x10^{-11}$	Parameter in development rate function, s^{-1}
C_{yL}	1.636	Parameter in harvest function (fruit)
C_{yF}	0.4805	Parameter in harvest function (leaf)
$C_{CO_{2,0}}$	1.6637	
$C_{CO_{2/dw}}$	1.4667	Ratio CO_2 per unit dry weight, $Kg[CO_2]Kg^{-1}[dw]$
$C_{CO_{2,ing}}$	$2.10x^{-6}$	Ratio CO_2 per unit dry weight, $Kg[CO_2]m^{-2}[gh]s^{-1}$
$\frac{V_g}{A_g}$	3	Volume per unit greenhouse area
p_{v1}	$7.17x10^{-5}$	Parameter.
p_{v2}	0.0156	Parameter.
p_{v3}	$2.71x10^{-5}$	Parameter.
p_{v4}	$6.32x10^{-5}$	Parameter.
p_{v5}	$7.40x10^{-5}$	Parameter.

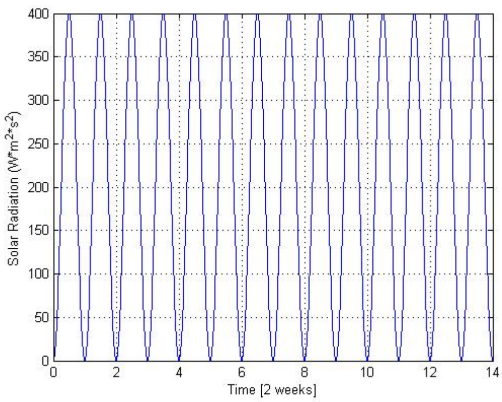

Fig. 2. Time varying solar radiation for two weeks.

shows the fruit behaviour which has acceptable growth just like the leafs (green color line). The black line represents the nutrients behaviour which is acceptable because it is observed that the crop is consuming the available nutrients during the growth time. Figure 4

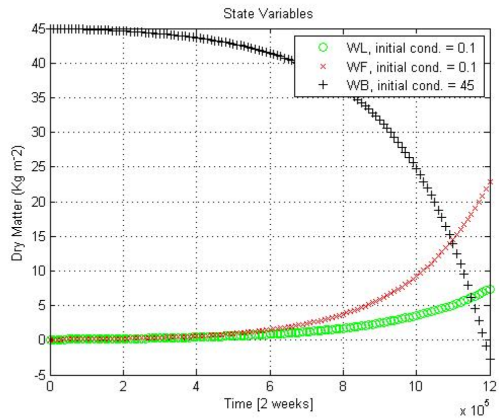

Fig. 3. Behaviour of fruits, leaves and nutrients dry matter with an input step.

shows the carbon dioxide behaviour. Note the CO_2 concentration increases up to 9000 ppm. This is very high and it means so much energy consumption.

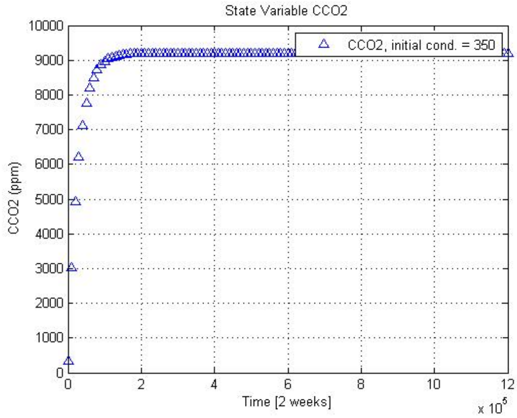

Fig. 4. Behaviour of CO_2 concentration a step as input control.

B. Analysis with the Synthesized Control

For the next simulation, we got results using the variable parameters, but now we simulated the control law (21) deduced in this paper. Figure 5 shows the results. Note thet the behaviour of the three crop state variables are similar to the previous cases, but in this case the carbon dioxide behaviour is different. In Figure 6 we can note that CO_2 concentration decreases and it reaches 450 ppm, which is very acceptable because it means low energy consuption and an acceptable quantity of carbon dioxide for clean air.

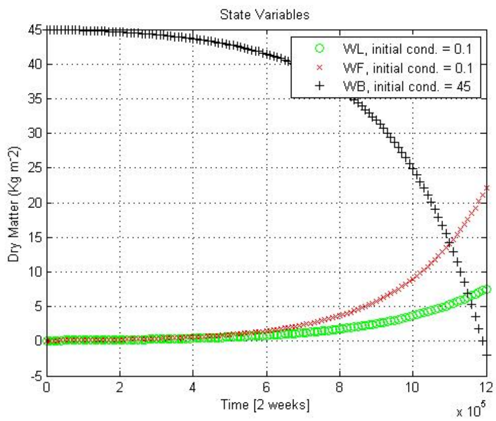

Fig. 5. Behaviour of fruits, leaves and nutrients dry matter with the control deduced.

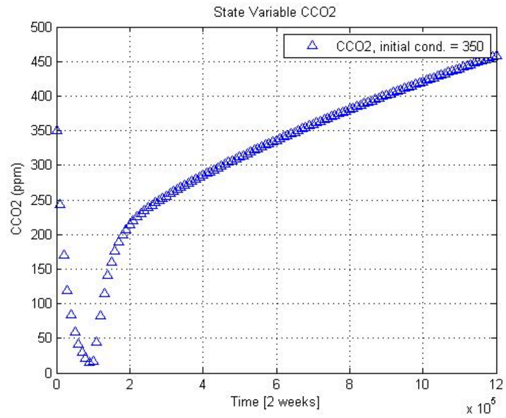

Fig. 6. Behaviour of CO_2 concentration the control input deduced.

VI. Automatic Control System for the Concentration of Carbon Dioxide Inside the Greenhouse

Figure 7 shows the complete automatic system which will control the carbon dioxide concentration. The valve and supply tank are the pneumatic system. And the CO_2 sensor will make the reading of current carbon dioxide inside de greenhouse.

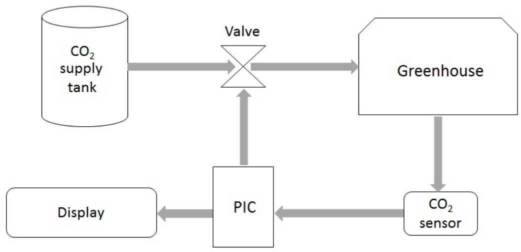

Fig. 7. Automatic control system for concentration of carbon dioxide inside the greenhouse.

A. Mathematical Model of Pneumatic Pressure System

The pressure system that will do the carbon dioxide enrichment is formed by a valve and a storage tank, Figure 8. In it, the flux through the restriction is a function of the difference of pressure. This kind of system is characterized in terms of a resistance and a capacitance. The resistance is defined like the change in the differential pressure to make a change in the mass flux [15]:.

$$R = \frac{d(\Delta P)}{dq} \tag{22}$$

where ΔP is a small change in the pressure of the gas and dq is a small change in the flux of the gas.

By the other hand, the capacitance is defined like:

$$C = \frac{dm}{dp} = V\frac{d\rho}{dp} \tag{23}$$

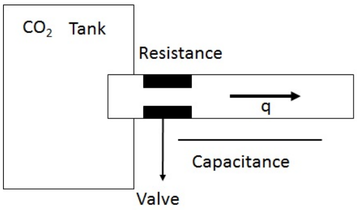

Fig. 8. Pressure system with tank and restriction.

where
$C =$ Capacitance, $lbfth^2/lb$
$m =$ Gas mass in the tank, lb
$p =$ Pressure of gas, lb/fth^2
$V =$ Volume of tank, fth^3
$\rho =$ Density, lb/fth^3

From the general law of gases, the capacitance is expressed as [15]:

$$p(\frac{V}{m})^m = \frac{p}{\rho - n} = constant \tag{24}$$

where n is the politropic exponent. The expression for ideal gases is:

$$pv = \frac{p}{\rho} = \frac{\overline{R}}{M}T = R_{gas}T \tag{25}$$

where:
$p =$ Absolute pressure, lb/fth^2
$\overline{v} =$ Volume occupied by one mole of gas, $fth^3/lbmol$
$\overline{R} =$ Universal constant gas, $fthlb/lbmolR$
$v =$ Specific volume of the gas, fth^3/lb
$M =$ Molecular weight of the gas, $lb/lbmol$
$R_{gas} =$ Constant of the gas.

Therefore, the capacitance is obtained as:

$$C = \frac{V}{nR_{gas}T} \tag{26}$$

For the system in this paper, the capacitance was got from the formula (26) using the parameters in Table VI-A. The calculated value was $1.3447e^{-3}Kgm^2/N$

Description	Value	Units
V- Tank volume	0.17657	pie^3
n- Politropic exponent	1	
R_{gas}- Gas constant	0.243844	$pielb/lbR$
$\overline{R}$- Universal constant of gases	10.73158	$pielb/lbmolR$
M- Molecular weight CO_2	44.01	g/mol
T- Absolut temperature	538.47	R

TABLE III
VALUES TO CALCULATE THE CAPACITANCE OF THE SYSTEM.

The formula to calculate the resistance is

$$R = \frac{8\eta L}{\pi r^4} \tag{27}$$

where:
ΔP is the pressure difference.
η is the viscosity of the gas, N/m^2.
r inner radius of the pipe in meters.
L length of the pipe expressed in meters.

For our particular system we use the values in table VI-A and the result is 13402.7370 pneumatic ohmnios.

Description	**Value**	**Units**
Viscosity of CO_2	$1.3711e^{-5}$	N/m^2
Inner of pipe	0.0127	m
Lenght of pipe	1	m

TABLE IV
VALUES TO CALCULATE THE RESISTANCE FOR THE SYSTEM

To obtain the mathematical model of the system small variations are considered and the system is considered linear. The following terms are defined:

$\overline{P} =$ Pressure of the gas in stable state, lb/pie^2
$p_i =$ small change in the gas that enter ld/pie^2
$p_o =$ small change in the pressure gas in the thank lb/pie^2
$V =$ volume of the thank pie^3
$m =$ mass of the gas in the thank, lb
$q =$ flux gas, ld/seg
$\rho =$ density gas, lb/pie^3

The resistance is $R = (p_i - p_o)/q$ and the capacitance is obtained from (24), in this way:

$$Cdp_o = qdt \quad \text{or}$$
$$Cdp_o = \frac{p_i - p_o}{R}dt$$

which is written like $RC\frac{dp_o}{dt} + p_o = pi$

If p_i and p_o are the input and output, respectively, the transfer function is:

$$\frac{P_o(s)}{P_i(s)} = \frac{1}{RCs + 1} \tag{28}$$

And, with the values of resistance and capacitance calculated previously, the transfer function can be written as follows:

$$\frac{P_o(s)}{P_i(s)} = \frac{1}{18.022s + 1} \tag{29}$$

From, now we will working with the transfer function to get a PI controller which will have an action over the actuator.

VII. OPTIMAL-TUNING PI CONTROL AS LINEAR QUADRATIC REGULATOR

From analysis controllability we know the system is completely controllable, it means every control will achieve stabilize the system. But the question is what

control is the best to stabilize the system in less time. In this section a comparison between classical tuning for PI controller and optimal control theory for PI controller is made.
PID controllers are most common in process industries due its simplicity. In this research we will work with a PI controller because of the characteristics of the pneumatic system.

Using the Lyapunov method, the optimal control problem is reduced to the Algebraic Riccati Equation (CARE), which is solved to calculate the feedback gains for a set of matrices. These matrices regulate the penalties on the deviation in the trajectories of the state variables and the control signal. Combining the tuning philosophy of PID controllers with the concept of Linerar Quadratic Regulators (LQR) allows to get an optimal control for the actuator in our automatic system [16].

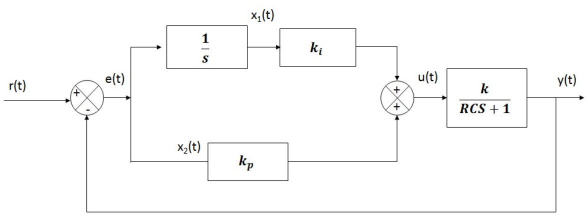

Fig. 9. Block diagram of PI controller.

In figure 9 a PI controller in parallel form has been considered to control the system represented in (28). From the general equation for PI controller and supposing the feedback control system is exited with an external input $r(t)$ to get a control signal $u(t)$ and output $y(t)$, we define the state variables as:

$$x_1 = \int e(t)dt, \quad x_2 = e(t) \tag{30}$$

From the block diagram presented in figure 9 and the transfer function (28), it is clear that

$$\frac{Y(s)}{U(s)} = \frac{k}{RCs+1} = \frac{-E(s)}{U(s)} \tag{31}$$

And then:

$$(RCs+1)E(s) = -kU(s) \tag{32}$$

Then, we can transform the equation (32) changing it to time domain:

$$RC\dot{e}(t) + e(t) = -ku(t) \tag{33}$$

And finally, we can re-written the equation (33) using the selection of state variables in (30).

$$RC\dot{x}_2 + x_2 = -ku(t) \tag{34}$$

State space representation for (34) is:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\frac{x_2}{RC} - \frac{k}{RC}u(t) \end{cases} \tag{35}$$

Using (35), the state space formulation becomes to:

$$\dot{x}(t) = \begin{pmatrix} 0 & 1 \\ 0 & -1/RC \end{pmatrix} x(t) + \begin{pmatrix} 0 \\ -k/RC \end{pmatrix} u(t) \tag{36}$$

From the development of the transfer function which describes the behaviour of the pneumatic system we know the R and C values. And, with the representation in (36) we can analyse the controllability of the system using the controllability matrix. The determinant of matrix controllability is different to zero, the system is completely controllable.

$$|B \quad AB| = \begin{vmatrix} 0 & -1/18.022 \\ -1/18.022 & 1/(18.022)^2 \end{vmatrix} =$$

Now, in order to have an optimal formulation with the system (36), the following quadratic function (J) is selected [16],

$$J = \int_0^\infty (x^T(t)Qx(t) + u^T(t)Ru(t))dt \tag{37}$$

Minimization of cost function gives the state feedback control law as:

$$u(t) = -R^{-1}B^TPx(t) = -Fx(t) \tag{38}$$

Where P is the solution of the Algebraic Riccati Equation given by:

$$A^TP + PA - PBR^{-1}B^TP + Q = 0 \tag{39}$$

We know the matrix A and B, Q is a symmetric positive semi-definite matrix and R is a positive number. Both Q and R are selected by the designer. The matrix Q is chosen to be as positive semi-definite diagonal matrix. Matrix P and matrix Q have the following form:

$$P = \begin{pmatrix} P_{11} & P_{12} \\ P_{12} & P_{22} \end{pmatrix} \qquad Q = \begin{pmatrix} Q_1 & 0 \\ 0 & Q_2 \end{pmatrix} \tag{40}$$

The solution for Riccati equation (39) was obtained from $care$ function in MatLab, this function returns the unique solution P of the algebraic Riccati equation, also it returns the gain matrix.
The solution P using the arbitrary values for matrix Q and parameter R is:

$$P = \begin{pmatrix} 6.2485 & 18.022 \\ 18.022 & 94.588 \end{pmatrix} \tag{41}$$

Finally, we can substitute all known parameters in control law equation (38).

Solving the substitution, we got a new matrix which has the proportional and integrate gain for the PI controller.

$$u(t) = (ki \quad kp) \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} \tag{42}$$

Control form can be written in this way:

$$u(t) = ki \int e(t)dt + kpe(t) \tag{43}$$

As we know, this form is the representation of general PI controller.

A. PI controller simulation

A program was made using MatLab to solve the Ricatti equation and to make variations in matrix Q. The Figure 10 represents the results for different variations of Q with a step input. From graphic we can chose de best behaviour for the system response.

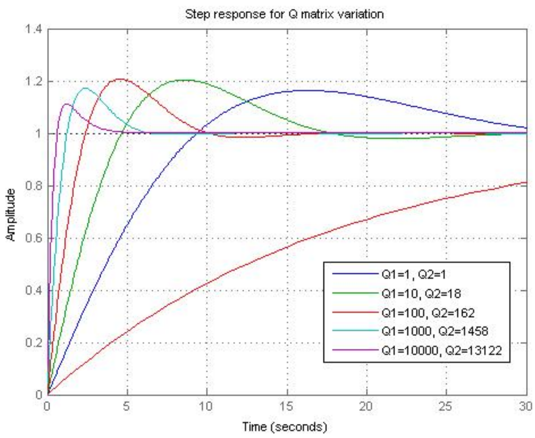

Fig. 10. Step responses with variations of matrix Q.

With the variation in elements of matrix Q, the overshoot slight increases with gradual fall in rise time. In Figure 10 it is possible to see the fifth variation of Q is the best response because it has an overshot less than two percent and the system stabilized about five seconds.

From the program in MatLab we can know the gains for PI controller that corresponds to the values of Q.

B. Method of Ziegler-Nichols for tuning a PI controller

It is important to make a comparison with other method tuning for PI controller, to make sure the optimal control tuning is the best way to have a control over the actuator. The proportional component of controller PI will make the system reaches the reference signal in less time and the integral component will minimize the stable state error. The Ziegler-Nichols method is based in the analysis response of the system with a step input in open-loop, The response is a reaction curve, called 'S' curve.

From the analysis we get the parameters of the model with the equations:

$$k_0 = \frac{y_\infty - y_0}{u_\infty - u_0}, \quad \tau_0 = t_1 - t_0, \quad \gamma_0 = t_2 - t_1$$

Using MatLab a program was made to know the parameters and finally we got the gains for the PI controller using the value table which Ziegler and Nichlos proposed.

	K_p	T_r	T_d
P	$\frac{\gamma_0}{K_0\tau_0}$		
PI	$\frac{0.9\gamma_0}{K_0\tau_0}$	$3\tau_0$	
PID	$\frac{1.2\gamma_0}{K_0\tau_0}$	$2\tau_0$	$0.5\tau_0$

In figure 11 we use the classical control tuning using the Ziegler-Nichols method. The result is the system is stabilized in about 10 seconds, which means the PI controller is working because it reduces the stabilizing time, But studying the results before where the optimal theory was applied for getting the PI controller we can note the best response was when the system is stabilized in five seconds.

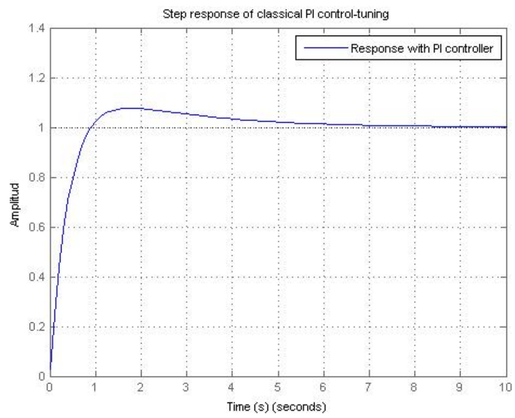

Fig. 11. Response of the system, with a classical PI control tuning.

From all results presented before, we can conclude that classical way for PI tuning is working very well but the best result is getting when we applied the optimal tuning to the system to get a good control over the valve in the pneumatic system because the system is stabilized in less time.

VIII. Conclusion

From the integrated tomato-greenhouse model it is possible to do the optimization of all variables. In this case, the most important variable is the carbon dioxide concentration. In a first step, the optimal control theory was used to determine the desired behaviour of the variables over a prescribed time period. In a second step, a PI control was designed form optimal control theory to make change in the behaviour of the actuator. All the results obtained in this paper are the base to create an electronic device able to make the automatic control of carbon dioxide concentration inside a real tomato greenhouse.

REFERENCES

[1] Aikman D. P., 1996. A procedure for optimizing carbon dioxide enrichment of a glasshouse tomato crop. Journal of agricultural engineering research 63, No. 2, 171–183.

[2] Ioslovich I., Gutman P. and Linker R., 2009. Hamilton-Jacobi-Bellman formalism for optimal climate control of greenhouse crop. Automatica, 1227–1231.

[3] Ioslovich I. and Seginer I., 1995. Normalized co-state variable for seasonal optimization of greenhouse tomato production. Acta horticulturae 417, 87–94.

[4] Alexander Zemiliak, Tatiana Markina (2013) . Preliminary Structure of Quasi Optimal Algorithm of Optimization of Analog Circuits. WSEAS Transactions on CIRCUITS and SYSTEMS ISSUE 7, volume 12.

[5] Hull D.G., 2013 Optimal control theory for applications, Springer-Verlag, New York, USA.

[6] B. Fahimnia, R. Molae, M. Ebrahimi(2008) . Genetic Algorithm Optimization of Fuel Consumption in Compressor Stations. WSEAS Transactions on CIRCUITS and SYSTEMS ISSUE 1, Volume 3.

[7]Ioslovich I. and Seginer I., 1998. Approximate seasonal optimization of the greenhouse environment for a multi-state-variable tomato model. Transactions of the ASAE 41.

[8] Md. Shahidul I., Toshiyuki M., Yuichi Y., 1996. Effect of carbon dioxide enrichment on physico-chemical and enzymatic changes in tomato fruits at various stages of maturity. Scientia horticulturae 65, 137–149.

[9] Mortensen M. L., 1987. CO2 enrichment in greenhouse. Sci. Horticulturae 33, 1–25.

[10]Seginer I. and Ioslovich I., 1998. Seasonal optimization of the greenhouse environment for a simple two-stage crop growth model. Journal of agricultural engineering research 70, 145–155.

[11]Tchamitchian M. and Ioslovich I., 1998. Equivalence of the temperature integral and the carbon dynamics concepts in plants: Utility for controll. Acta horticulturae 519, 171–180.

[12] Van Straten G., Challa H. and Buwalda F., 2000. Towards user accepted optimal control of greenhouse climate. Computers and electronics in agriculture 26, No. 3, 221–238.

[13] Van Straten G., van Willigenburg L. G., van Henten E. and van Ooteghem R., 2011. Optimal control of greenhouse cultivation. CRC. Press, United States of America.

[14] Van Henten E. J., 1994. Greenhouse climate management: An optimal control approach. Doctorate dissertation. Agricultural University, Wageningen.

[15] Robert L. Mott, Mecánica de Fluidos, 6ta edición.

[16] Saptarshi das, Indranil Pan, Kaushik Halder, Shantanu Das, Amitava Gupta. LQR based improved discrete PID controller design via optimum selection of weighting matrices using fractional order integral performance index. Journal: Applied mathematical Modelling 37 (2013) 4253-4268.