Optimal control law for the concentration of carbon dioxide in a tomato greenhouse and optimal-tuning PI control as LQR for the automatic device.

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Abstract—The tomato model-space has been developed and it is called a big leaf-big fruit model, this model is formed by the mass balances: the non-structural biomass (nutrients) and the structural biomass of fruits and leaves. Also, a model that describes the behaviour on carbon dioxide concentration inside greenhouse is obtained. From the two models we get a new complete crop-greenhouse model. This model allows to get an optimal control for the carbon dioxide enrichment in a tomato greenhouse which gives benefits, because it is possible to achieve a saving for energy consumption and more tomato production. The optimal control theory is applied to the crop-greenhouse integrated system, which is based on four state variables: the consumption of nutrients, the fruits and leaves growth and the carbon dioxide concentration. This work contributes with the optimal control law that gives the desired CO₂ concentration behaviour during the growth time for the crop. This behaviour will be a reference signal for the controller implementation in the electronic device that will be made in a future work. The simulations for the crop-greenhouse system are presented for a two weeks period. This paper also contributes at optimal-tuning PI control as Linear Quadratic Regulator of the electronic device. The simulation of the optimal control PI are showed here.

Index Terms—Optimal control, Optimal-tuning, Linear Quadratic Regulator, Carbon dioxide, structural biomass of leaves, structural biomass of fruit, state space.

I. INTRODUCTION

In past years, researches have proposed different optimal climate control methods for greenhouse systems. These efforts have not been applied in practice because it is difficult for real application [1,2,3,9,12]. The difficulty lies in that the crop growth is based in many different variables and the mathematical analysis with all variables is complicated. In this work, the variable that has our interest is the carbon dioxide.

The carbon dioxide enrichment is practised in the greenhouse crops in order to increase the yield and the benefit. There are studies that demonstrate the CO₂ enrichment improves the net photosynthesis in the plants achieving the increase of the total weight, height, and the number of leaves and branches [9]. Other research has demonstrated that the CO₂ enrichment makes physic-chemical changes in the crop, like color and firmness [8].

Optimization problems with two or more objectives are very common in engineering and many other disciplines. The process of optimizing a collection of objective function is called multi-objective optimization and it is difficult because of the large number of conditions and variables involved in the system [4]. In this work the optimization problem has two objectives, first, decrease the energy consumption for carbon dioxide enrichment and second, increase the tomato production. The search process can be accomplished in two ways: deterministic and stochastic search algorithms [6].

Optimal strategies for CO₂ enrichment can be deduced experimentally or analytically. Experimentation is not able to produce a valid result for all condition set. The mathematical analysis gives us a better option to obtain an optimal strategy because it considers all the variables involved in the system. This method is based on ventilation, photosynthesis, dry matter and production rate models.

One of the main objectives is to contribute with the optimal control problem and its implementation in real time. The tomato crop has been chosen because it is one of the most important crop in our country and is the second farm product consumed in the world. To achieve the objective, we start from the tomato and greenhouse mathematical set model considering
the following variable: plant and fruit dry weight, the nutrients amount and the CO$_2$ concentration.

In this paper, we obtained the behaviour of CO$_2$ concentration inside the greenhouse that is necessary for growth during two weeks. We start from the fact that this behaviour is a reference signal that a classic controller will have to follow for each time instant. In future work, all results in this paper help us to design an electronic device that could be used in real tomato greenhouses.

With the approach of optimal PI tuning for first order processes, also the provision of simultaneously and optimally finding the two parameters of a PI controller (i.e. $K_i$, $K_p$), development has been addressed in this paper.

II. GENERAL FORMULATION OF THE OPTIMAL CONTROL PROBLEM

Optimal control problems appeared as essential tools in modern control theory. Several authors have proposed different basic mathematical formulations of fixed time problems [5]. The optimal control of any system has to be based on three concepts: the dynamic model of the system, a functional and the system restrictions. In matrix notation the state equation is represented as follows:

$$\dot{x} = f(x(t), u(t), t).$$

(1)

Where $x(t)$ is the state vector, $u(t)$ is the control vector and $t$ is the time. A criterion is required to evaluate the performance of the system, normally, the functional is defined by:

$$J = \phi(x(t_f), t_f) + \int_{t_0}^{t_f} L(x(t), u(t), t) \, dt,$$

(2)

Where $t_0$ and $t_f$ are the initial and final time, $\phi$ and $L$ are scalar functions, $t_f$ can be fixed or free. Starting at the initial state $x(t_0) = x_0$ and applying the control signal $u(t)$ for $t \in [t_0, t_f]$, it makes that system follows some trajectory of states, then the functional assigns a unique real number for each trajectory of the system.

The fundamental problem of optimal control is to determine an admissible control $u^*$ which makes that equation (1) follows one admissible trajectory $x^*$ that minimize the value of the functional in equation (2). Then, $u^*$ is named optimal control and $x^*$ is an optimal trajectory.

**Necessary conditions for a solution**

Restrictions (1) are added to the functional (2) with a Lagrange multipliers vector time variant $\Psi(t)$ and the functional is rewritten as follows:

$$J = \phi(x(t_f)) + \int_{t_0}^{t_f} [L(x(t), u(t), t) - \Psi^T f(x(t), u(t), t) - \dot{x}] \, dt,$$

(3)

Then, the Hamiltonian scalar function is defined, which depends on the variable state vector, the control signal and the new vector $\Psi(t)$

$$H(x(t), u(t), \Psi(t), t) = L(x(t), u(t), t) + \Phi(t) f(x(t), u(t), t)$$

(4)

Then, it is possible to write an auxiliary system starting from a new auxiliary vector that depends on time $\Psi(t)$. The new system is formed from Hamiltonian function, as follows:

$$\dot{\Psi}^T = - \frac{\partial H}{\partial x} = - \frac{\partial L}{\partial x} - \Psi^T \frac{\partial f}{\partial x}$$

(5)

The auxiliary system allows to know the final conditions of the general system, which can be written as follows:

$$\Psi^T(t_f) = \frac{\partial \phi}{\partial x}(t_f)$$

(6)

An infinitesimal variation in $u(t)$ denominated $\delta u(t)$ produces a variation in the functional $J$ like $\delta J$. For a stationary solution it is required that arbitrary variation is equal to zero, $\delta J = 0$. This is true when

$$\frac{\partial H}{\partial u} = \frac{\partial L}{\partial u} + \Psi^T \frac{\partial f}{\partial u} = 0$$

(7)

Note that from the Hamiltonian function (4) it is possible to get the control form. Then, to find the vector function of control $u(t)$ that produces a stationary value of the functional we must solve the following differential equation system:

$$\begin{cases}
\dot{x}(t) = f(x(t), u(t), t), \\
\dot{\Psi}(t) = - \frac{\partial H}{\partial x},
\end{cases}$$

(8)

The boundary conditions for this differential equations are separated, it means that some of them are defined in $t = t_0$ and the others in $t = t_f$. This is a problem with boundary values of two points. Note the equations that describe the states $x(t)$ and the auxiliary states $\Psi(t)$ in the equation (8) are coupled, for this reason $u(t)$ depends on $\Psi(t)$ through the stationary condition and the auxiliary states depend on $x(t)$ and $u(t)$. And the first system in (8) has the initial conditions of he system while the last system in (8) has the final condition of the system.
III. Dynamic Models of the Crop and of the Greenhouse

A. Dynamic model of the Crop

The model in space states of the tomato crop has three principles states (Van Straten et al., 2011)[13]:
- Non-structural Biomass (Nutrients).
- Leaves Structural Biomass.
- Fruits Structural Biomass.

1) Biomass balance of nutrients: Nutrients are being produced by photosynthesis. The gross canopy photosynthesis rate in dry matter per unit area is \( P \). Nutrients are converted to leaf and fruits, this is known as growth. Leaf and fruits have a demand for nutrients, which will be honored if there are sufficient nutrients available. We denote \( W_B \) as the total nutrients in the plant and it is expressed as dry weight per area unit. The biomass balance equation of nutrients is the following [13]:

\[
\frac{dW_B}{dt} = P - h() \left[ \frac{(1+h)G_{L}^{dem}}{z} + (1 + \theta_F)G_{F}^{dem} \right] - \left[ \frac{R_F}{z} + R_F \right].
\]

The biomass balance equation of nutrients (9) can take two values depending on the nutrients abundance \( h \), where the first expression is taken when \( h = 1 \) (abundance of nutrients) and the second one is taken when \( h = 0 \) (lack of nutrients).

\[
\frac{dW_B}{dt} = \begin{cases} 
P - \frac{(1+h)G_{L}^{dem}}{z} \left(1 + \theta_F\right)G_{F}^{dem} - \frac{R_F}{z} - R_F, & \text{if } h = 1, \\
\left(1 + \theta_F\right)G_{F}^{dem} - \frac{R_F}{z} - R_F, & \text{if } h = 0. 
\end{cases}
\]

B. Dynamic Model of the Greenhouse

1) Balance of CO₂ energy in the greenhouse: The balance of carbon dioxide energy within greenhouse is given by the equation [13]:

\[
\frac{dC_{CO_2}}{dt} = \left[ \eta_{CO_2/dw}P \right] - \left[ \eta_{CO_2/dw}R \right] - \left[ \varphi_{CO_2, a, o} u_{CO_2} \right].
\]

Where each term is described:
- \( \eta_{CO_2/dw}P \): Carbon dioxide taken from the greenhouse air for plant photosynthesis.
- \( \eta_{CO_2/dw}R \): Carbon dioxide returned to the greenhouse air for plant respiration.
- \( \varphi_{CO_2, a, o} u_{CO_2} \): Carbon dioxide supply on the outside greenhouse.

Depending on the abundance of nutrients \( h() \), the biomass leaf balance equation (11) can take two values:

\[
\frac{dW_L}{dt} = \begin{cases} 
G_{L}^{dem} - H_L, & \text{if } h = 1, \\
- R_L - H_L, & \text{if } h = 0.
\end{cases}
\]

Finally, the equation (13) of biomass balance of fruits can take two different values depending on nutrient abundance \( h() \), where \( H_F \) is the fruit harvest rate:

\[
\frac{dW_F}{dt} = \begin{cases} 
G_{F}^{dem} - H_F, & \text{if } h = 1, \\
- R_F - H_F, & \text{if } h = 0.
\end{cases}
\]
flow rate of carbon dioxide.

In this greenhouse model, the position of the carbon dioxide supply valve is the control input. For this reason, the valve relates directly to the actuator that is present on a physical way in the greenhouse.

C. Integrated Model Crop-Greenhouse

From previous description of greenhouse and crop models is possible to get a complete system formed by three crop equations and greenhouse equation. This new equation system describes the complete system behaviour and it is important to note that all of the equations are related principally by the P element and the state variables of the crop. It is important to say that the three equations related to the crop are taken with the assumption that there is an abundance of nutrients ($h\cdot\cdot\cdot = 1$). Therefore, general system is as follows:

\[
\begin{align*}
\dot{W}_L(t) &= G^{dem}_L - H_L, \\
\dot{W}_F(t) &= G^{dem}_F - H_F, \\
\dot{W}_B(t) &= P(t) - \frac{W_{CO_2}^{dem}}{\theta_F} + (1 + \theta_F)G_{F}^{dem} - \frac{R_B}{\theta_F} + R_F, \\
3C_{CO_2}(t) &= -\eta CO_2/\Delta vP + +\eta CO_2/\Delta vR - \varphi CO_2 - B, \\
&\quad + u_{CO_2}.
\end{align*}
\]  

(16)

IV. SYNTHESIS OF OPTIMAL CONTROL

We consider the system (16). The terms for the equation system are substituted using the equation table of the mathematical model (table 1) and the values are substituted using the table of physical parameters (table 2). The resulting model is:

\[
\begin{align*}
\dot{W}_L(t) &= 2.2996 \times 10^{-6} W_L(t), \\
\dot{W}_F(t) &= 4.3925 \times 10^{-6} W_F(t), \\
\dot{W}_B(t) &= P(t) - 5.39 \times 10^{-6} W_L(t) - 5.92 \times 10^{-6} W_F(t), \\
3C_{CO_2}(t) &= 1.0266(R(t) - P(t)) + 0.155 \times 10^{-10} u_{CO_2}.
\end{align*}
\]

(17)

$P$ and $R$ are the following:

\[
P(t) = \frac{3.7192 \times 10^{-11} W_{P, 511}(t)}{1.6353 \times 10^{-3} - 4.0499 \times 10^{-8} W_{P, 511}(t)},
\]

\[
R(t) = 1.5942 \times 10^{-6} W_F(t) + 0.4856 \times 10^{-6} W_L(t) + 1.668 \times 10^{-7}.
\]

It is important to note that the terms $P(t)$ and $R(t)$ have involved two of the three state variables of the crop and they are time dependent functions, so the entire system is connected and it can be solved simultaneously.

We consider the following functional, which has the same form shown in (3):

\[
J = \frac{1}{2}[W_L^2(t) + W_F^2(t) + W_B^2(t) + C_{CO_2}^2(t)] + \\
\quad + \int_{t_0}^{t_f} [W_L^2(t) + W_F^2(t) + \\
\quad + W_B^2(t) + C_{CO_2}^2(t) + (u_{CO_2}^p)^2(t)]dt
\]

(18)

The first term involves the three first variables at the end time, they are related to the final production and the nutrients, and the integral contains the control input in order to avoid the risk for big control inputs. The idea is to minimize the functional (18), related with the equations system (17).

A. Solution Method Description

The Hamiltonian scalar function is obtained considering the relation (4) with the Lagrange multipliers and the functional (17):

\[
H(x, u, \Psi, t) = \\
\quad = \frac{1}{2}[W_L^2(t) + W_F^2(t) + W_B^2(t) + C_{CO_2}^2(t) + (u_{CO_2}^p)^2(t)] + \\
\quad + 2.2996 \times 10^{-6} W_L(t)\dot{\Psi}_1(t) + 4.3925 \times 10^{-6} W_F(t)\dot{\Psi}_2(t) + \\
\quad + [P - 5.39 \times 10^{-6} W_L(t) - 5.92 \times 10^{-6} W_F(t)]\dot{\Psi}_3(t) + \\
\quad + \frac{1}{3}[1.0266(R(t) - P(t)) + 0.1554 \times 10^{-10} u_{CO_2}^p]\dot{\Psi}_4(t).
\]

(19)

The system of auxiliary variables is formed using the expression (5) and has the following form:

\[
\begin{align*}
\dot{\Psi}_1 &= W_L + 2.2996 \times 10^{-6} \Psi_1 + \frac{R_P}{\theta_F} \Psi_3, \\
\dot{\Psi}_2 &= W_F + 4.3925 \times 10^{-6} \Psi_2, \\
\dot{\Psi}_3 &= W_B, \\
\dot{\Psi}_4 &= C_{CO_2}.
\end{align*}
\]

(20)

The stationary condition give us the following control form, which was obtained from equation (7) and depends on fourth appended state:

\[
u_{CO_2}^p = -\frac{1}{3} 0.1554 \times 10^{-10} \Psi_4(t).
\]

(21)

It is necessary to solve the equation systems (17) and (20), in this way we can know the $\Psi_4$ value and finally we will get the control form. The system (17) has initial condition and the system (20) has final conditions. The systems are coupled because the control form (21) has been substituted. To solve the complete system like a system with initial conditions, the auxiliary equations are considered in reverse time, then the behaviour of the auxiliary variables is returned to the direct time. When we solve the appended equation system in reverse time the system becomes a system with initial conditions. It is important to note that the equation (21) depends on the fourth state but this state depends on
TABLE I
GREENHOUSE AND CROP MATHEMATICAL MODEL EQUATIONS

<table>
<thead>
<tr>
<th>Term</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P = \frac{P_{\text{max}}}{P_{\text{max}} + K_I} \left( \frac{C_{\text{CO}<em>2}}{K</em>{\text{CO}<em>2}} \right) f</em>{m} { } )</td>
<td>Production of assimilates by photosynthesis.</td>
</tr>
<tr>
<td>( R = h { } \left( \frac{P}{2} G_{\text{dem}}^\text{em} + \theta F G_{\text{dem}}^\text{em} \right) + \frac{R_{\ell}}{2} + R_F )</td>
<td>Total amount breathed plant per unit of time.</td>
</tr>
<tr>
<td>( f_{\text{PAR}} = f_{\text{PAR}}/\tau_I )</td>
<td>The PAR light intensity at the crop level.</td>
</tr>
<tr>
<td>( f_{m} { } = \frac{(W_{L}/r_m)^m}{1+(W_{L}/r_m)^m} )</td>
<td>Maturity factor.</td>
</tr>
<tr>
<td>( G_{\text{dem}}^\text{em} = f_{L/F}(T) k_{\text{ref}} G f_{T}(T) f_{D} { } W_L )</td>
<td>Growth leaves demand.</td>
</tr>
<tr>
<td>( G_{\text{dem}}^\text{em} = k_{\text{ref}} G f_{T}(T) f_{D} { } W_F )</td>
<td>Growth fruits demand.</td>
</tr>
<tr>
<td>( f_{L/F}(T) = f_{\text{ref}} L/F e^{2(T-T_{\text{ref}})} )</td>
<td>Temperature-dependent ratio.</td>
</tr>
<tr>
<td>( f_{T}(T) = \frac{Q_T - T_{\text{ref}}}{10} )</td>
<td>Temperature dependent with a ( Q_{10} ) relation.</td>
</tr>
<tr>
<td>( f_{D} { } = c_f^1 - c_f^2 D )</td>
<td>Correction factor for the fruit growth rate.</td>
</tr>
<tr>
<td>( R_{\ell} = k_{\text{HL}} W_L )</td>
<td>Respiration demand of the leaves.</td>
</tr>
<tr>
<td>( R_F = k_{\text{HF}} W_F )</td>
<td>Respiration demand of the fruits.</td>
</tr>
<tr>
<td>( H_{L} = k_{\text{HL}} W_L )</td>
<td>Leaf picking rate.</td>
</tr>
<tr>
<td>( H_{F} = k_{\text{HF}} W_F )</td>
<td>Harvest rate.</td>
</tr>
<tr>
<td>( K_{\text{HL}} = C_{\text{yL}} K_{\text{H}} )</td>
<td>Coefficient of harvest.</td>
</tr>
<tr>
<td>( K_{\text{HF}} = C_{\text{yF}} K_{\text{H}} )</td>
<td>Coefficient of harvest.</td>
</tr>
<tr>
<td>( K_{\text{H}} = C d_1 + C d_2 n(T/C d_3) - C d_3 - C d_4 e_D )</td>
<td>Harvest rate.</td>
</tr>
<tr>
<td>( u_I = \left( \frac{p v_{I}}{1 + p v_{I} + p v_{2} + p v_{3} + p v_{4} + p v_{5}} \right) v + p v_{5} )</td>
<td>Ventilation flow rate.</td>
</tr>
</tbody>
</table>

Fig. 1. Time varying temperature for two weeks.

the other three states. Using MatLab tools we solve the equation systems (17) and (20).

V. SIMULATION OF CONTROL LAW

The MatLab tools were used to elaborate the program that solve the differential equations system formed by equations (17) and (20). The simulation period is for two weeks. The results obtained are described below. It is important to know that in the following results, the temperature and solar radiation are time varying, in order to make the simulation more real. Figures 1 and 2 show the temperature and solar radiation varying in a time of two weeks.

A. Analysis with a Step Input.

In a first simulation we use a step function as control input (\( v_{\text{CO}_2}^0 = 1 \)). We solved the equation system and later we got the graphics that represent the behaviour of most important variables of the system. Figure 3 shows the behaviour of three crop state variables. The red line
because it is observed that the crop is consuming the available nutrients during the growth time. Figure 4 shows the fruit behaviour which has acceptable growth just like the leaves (green color line). The black line represents the nutrients behaviour which is acceptable because it is observed that the crop is consuming the carbon dioxide behaviour. Note the CO₂ concentration increases up to 9000 ppm. This is very

## Table II

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( z )</td>
<td>0.0081</td>
<td>Fraction leaf of total vegetative mass</td>
</tr>
<tr>
<td>( \theta_v )</td>
<td>0.23</td>
<td>Surplus assimilate requirement factor per unit fruit increment.</td>
</tr>
<tr>
<td>( \theta_F )</td>
<td>0.2</td>
<td>Surplus assimilate requirement factor per unit vegetative increment.</td>
</tr>
<tr>
<td>( p_h )</td>
<td>( 2.7 \times 10^{-3} )</td>
<td>Parameter of switching function, ([m^2 kg^{-1}]).</td>
</tr>
<tr>
<td>( p_m )</td>
<td>( 1.8 \times 10^{-2} )</td>
<td>Parameter in maturity factor, ([kg m^{-2}]).</td>
</tr>
<tr>
<td>( m )</td>
<td>2.511</td>
<td>Parameter in maturity factor</td>
</tr>
<tr>
<td>( \rho^{\text{max}} )</td>
<td>( 2.2 \times 10^{-6} )</td>
<td>Maximum gross canopy photosynthesis rate, ([kg m^{-2} s^{-1}]).</td>
</tr>
<tr>
<td>( K_1 )</td>
<td>577</td>
<td>Monod constant for PAR, ([W m^{-2}]).</td>
</tr>
<tr>
<td>( K_c )</td>
<td>0.211</td>
<td>Monod constant for CO₂, ([kg m^{-3}]).</td>
</tr>
<tr>
<td>( f_{\text{PAR}}/1 )</td>
<td>0.475</td>
<td>PAR fraction of global radiation.</td>
</tr>
<tr>
<td>( \tau_r )</td>
<td>0.7</td>
<td>Transmittance of the roof.</td>
</tr>
<tr>
<td>( k_{\text{GF}}^{\text{ref}} )</td>
<td>( 3.8 \times 10^{-6} )</td>
<td>Reference fruit growth rate coefficient, ([s^{-1}]).</td>
</tr>
<tr>
<td>( T_{\text{GF}}^{\text{ref}} )</td>
<td>20</td>
<td>Reference temperature, ([^\circ C]).</td>
</tr>
<tr>
<td>( Q_{100} )</td>
<td>1.6</td>
<td>Temperature function parameter growth.</td>
</tr>
<tr>
<td>( f_{L/F}^{\text{ref}} )</td>
<td>1.38</td>
<td>Reference leaf-fruit partitioning factor.</td>
</tr>
<tr>
<td>( v_2 )</td>
<td>-0.168</td>
<td>( \rho^{\text{ref}} ) Parameter in harvest function (leaf).</td>
</tr>
<tr>
<td>( T_{L/F}^{\text{ref}} )</td>
<td>19</td>
<td>Reference temperature for respiration, ([^\circ C])</td>
</tr>
<tr>
<td>( k_{\text{FL}}^{\text{ref}} )</td>
<td>( 2.9 \times 10^{-7} )</td>
<td>Maintenance respiration coefficient leaf, ([s^{-1}]) Absorbed in relation to the total energy of the net radiation heat received.</td>
</tr>
<tr>
<td>( Q_{10B} )</td>
<td>2</td>
<td>Temperature function parameter respiration.</td>
</tr>
<tr>
<td>( f_{L/F}^{\text{ref}} )</td>
<td>25</td>
<td>Reference temperature for respiration, ([^\circ C]).</td>
</tr>
<tr>
<td>( v_1 )</td>
<td>0.7</td>
<td>Maintenance respiration coefficient leaf, ([s^{-1}]).</td>
</tr>
<tr>
<td>( C_{\text{aL}} )</td>
<td>1.636</td>
<td>Absorbed in relation to the total energy of the net radiation heat received.</td>
</tr>
<tr>
<td>( C_{\text{bF}} )</td>
<td>0.4805</td>
<td>Parameter in development rate function, ( s^{-1}).</td>
</tr>
<tr>
<td>( C_{\text{CO}_2}^{\text{L}} )</td>
<td>1.6037</td>
<td>Parameter in development rate function, ( s^{-1}).</td>
</tr>
<tr>
<td>( C_{\text{CO}_2}^{\text{L}}/\text{d}w )</td>
<td>1.4667</td>
<td>Parameter in development rate function, ( s^{-1}).</td>
</tr>
<tr>
<td>( C_{\text{CO}_2}^{\text{L}}/\text{m}^2 )</td>
<td>2.10 ( \times 10^{-6} )</td>
<td>Parameter in development rate function, ( s^{-1}).</td>
</tr>
<tr>
<td>( V_{\text{aL}} )</td>
<td>3</td>
<td>Parameter.</td>
</tr>
<tr>
<td>( \rho_{1} )</td>
<td>7.17 ( \times 10^{-5} )</td>
<td>Parameter.</td>
</tr>
<tr>
<td>( \rho_{2} )</td>
<td>0.0156</td>
<td>Parameter.</td>
</tr>
<tr>
<td>( \rho_{3} )</td>
<td>2.71 ( \times 10^{-5} )</td>
<td>Parameter.</td>
</tr>
<tr>
<td>( \rho_{4} )</td>
<td>6.32 ( \times 10^{-5} )</td>
<td>Parameter.</td>
</tr>
<tr>
<td>( \rho_{5} )</td>
<td>7.40 ( \times 10^{-5} )</td>
<td>Parameter.</td>
</tr>
</tbody>
</table>
high and it means so much energy consumption.

Fig. 4. Behaviour of CO$_2$ concentration a step as input control.

B. Analysis with the Synthesized Control

For the next simulation, we got results using the variable parameters, but now we simulated the control law (21) deduced in this paper. Figure 5 shows the results. Note that the behaviour of the three crop state variables are similar to the previous cases, but in this case the carbon dioxide behaviour is different. In Figure 6 we can note that CO$_2$ concentration decreases and it reaches 450 ppm, which is very acceptable because it means low energy consumption and an acceptable quantity of carbon dioxide for clean air.

Fig. 5. Behaviour of fruits, leaves and nutrients dry matter with the control deduced.

Fig. 6. Behaviour of CO$_2$ concentration the control input deduced.

VI. AUTOMATIC CONTROL SYSTEM FOR THE CONCENTRATION OF CARBON DIOXIDE INSIDE THE GREENHOUSE

Figure 7 shows the complete automatic system which will control the carbon dioxide concentration. The valve and supply tank are the pneumatic system. And the CO$_2$ sensor will make the reading of current carbon dioxide inside the greenhouse.

Fig. 7. Automatic control system for concentration of carbon dioxide inside the greenhouse.

A. Mathematical Model of Pneumatic Pressure System

The pressure system that will do the carbon dioxide enrichment is formed by a valve and a storage tank, Figure 8. In it, the flux through the restriction is a function of the difference of pressure. This kind of system is characterized in terms of a resistance and a capacitance. The resistance is defined like the change in the differential pressure to make a change in the mass flux [15]:

$$R = \frac{d(\Delta P)}{dq} \quad (22)$$

where $\Delta P$ is a small change in the pressure of the gas and $dq$ is a small change in the flux of the gas.

By the other hand, the capacitance is defined like:

$$C = \frac{dm}{dp} = V\frac{dP}{dp} \quad (23)$$
where

\[ C = \text{Capacitance}, \ \text{lb/ft}^2/\text{lb} \]
\[ m = \text{Gas mass in the tank, lb} \]
\[ p = \text{Pressure of gas, lb/ft}^2 \]
\[ V = \text{Volume of tank, ft}^3 \]
\[ \rho = \text{Density, lb/ft}^3 \]

From the general law of gases, the capacitance is expressed as [15]:

\[ p(V/m)^n = p/\rho - n = \text{constant} \]  

(24)

where \( n \) is the politropic exponent. The expression for ideal gases is:

\[ pv = p/\rho = RMT \]  

(25)

where:

\[ p = \text{Absolute pressure, lb/ft}^2 \]
\[ v = \text{Volume occupied by one mole of gas, ft}^3/\text{lbmol} \]
\[ R = \text{Universal constant gas, ft}^3/\text{lbmolR} \]
\[ M = \text{Molecular weight of the gas, lb/} \text{lbmol} \]
\[ R_{\text{gas}} = \text{Constant of the gas.} \]

Therefore, the capacitance is obtained as:

\[ C = \frac{V}{nR_{\text{gas}}T} \]  

(26)

For the system in this paper, the capacitance was got from the formula (26) using the parameters in Table VI-A. The calculated value was \( 1.3447e^{-3} \text{Kgm}^2/\text{N} \).

### Table III

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>V- Tank volume</td>
<td>0.17657</td>
<td>pie³</td>
</tr>
<tr>
<td>n- Polytropic exponent</td>
<td>0.543544</td>
<td>pie³/ft³</td>
</tr>
<tr>
<td>R- Universal constant of gases</td>
<td>10.73158</td>
<td>pie³/ft³</td>
</tr>
<tr>
<td>M- Molecular weight CO₂</td>
<td>44.01</td>
<td>g/mol</td>
</tr>
<tr>
<td>I- Absolut temperature</td>
<td>538.47</td>
<td>K</td>
</tr>
</tbody>
</table>

VALUES TO CALCULATE THE CAPACITANCE OF THE SYSTEM.

The formula to calculate the resistance is:

\[ R = \frac{8\eta L}{\pi r^4} \]  

(27)

VII. OPTIMAL-TUNING PI CONTROL AS LINEAR QUADRATIC REGULATOR

From analysis controllability we know the system is completely controllable, it means every control will achieve stabilize the system. But the question is what...
control is the best to stabilize the system in less time. In this section a comparison between classical tuning for PI controller and optimal control theory for PI controller is made. PID controllers are most common in process industries due its simplicity. In this research we will work with a PI controller because of the characteristics of the pneumatic system.

Using the Lyapunov method, the optimal control problem is reduced to the Algebraic Riccati Equation (CARE), which is solved to calculate the feedback gains for a set of matrices. These matrices regulate the penalties on the deviation in the trajectories of the state variables and the control signal. Combining the tuning philosophy of PID controllers with the concept of Linear Quadratic Regulators (LQR) allows to get an optimal control for the actuator in our automatic system [16].

![Fig. 9. Block diagram of PI controller.](image)

In figure 9 a PI controller in parallel form has been considered to control the system represented in (28). From the general equation for PI controller and supposing the feedback control system is exited with an external input \( r(t) \) to get a control signal \( u(t) \) and output \( y(t) \), we define the state variables as:

\[
\begin{align*}
  x_1 &= \int e(t) dt, \quad x_2 = e(t) \\
  y &= \frac{k}{RCs+1} u(t)
\end{align*}
\]

From the block diagram presented in figure 9 and the transfer function (28), it is clear that

\[
Y(s) = \frac{k}{RCs+1} U(s)
\]

And then:

\[
(RCs+1)E(s) = -kU(s)
\]

Then, we can transform the equation (32) changing it to time domain:

\[
RC\dot{e}(t) + e(t) = -ku(t)
\]

And finally, we can re-written the equation (33) using the selection of state variables in (30).

\[
RC\dot{x}_2 + x_2 = -ku(t)
\]

State space representation for (34) is:

\[
\begin{align*}
  \dot{x}_1 &= x_2 \\
  \dot{x}_2 &= -\frac{x_2}{RC} - \frac{k}{RC}u(t)
\end{align*}
\]

Using (35), the state space formulation becomes to:

\[
\dot{x}(t) = \begin{pmatrix} 0 & 1 \\ 0 & -1/RC \end{pmatrix} x(t) + \begin{pmatrix} 0 \\ -k/RC \end{pmatrix} u(t)
\]

From the development of the transfer function which describes the behaviour of the pneumatic system we know the \( R \) and \( C \) values. And, with the representation in (36) we can analyse the controllability of the system using the controllability matrix. The determinant of matrix controllability is different to zero, the system is completely controllable.

\[
|B AB| = \begin{vmatrix} 0 & -1/18.022 \\ -1/18.022 & 1/(18.022)^2 \end{vmatrix}
\]

Now, in order to have an optimal formulation with the system (36), the following quadratic function \( J \) is selected [16],

\[
J = \int_0^\infty (xT(t)Qx(t) + uT(t)Ru(t))dt
\]

Minimization of cost function gives the state feedback control law as:

\[
u(t) = -R^{-1}B^TPx(t) = -Fx(t)
\]

Where \( P \) is the solution of the Algebraic Riccati Equation given by:

\[
A^TP + PA - PBR^{-1}B^TP + Q = 0
\]

We know the matrix A and B, Q is a symmetric positive semi-definite matrix and R is a positive number. Both Q and R are selected by the designer. The matrix Q is chosen to be as positive semi-definite diagonal matrix. Matrix P and matrix Q have the following form:

\[
P = \begin{pmatrix} P_{11} & P_{12} \\ P_{12} & P_{22} \end{pmatrix} \quad Q = \begin{pmatrix} Q_1 & 0 \\ 0 & Q_2 \end{pmatrix}
\]

The solution for Riccati equation (39) was obtained from \textit{care} function in MatLab, this function returns the unique solution \( P \) of the algebraic Riccati equation, also it returns the gain matrix.

The solution \( P \) using the arbitrary values for matrix \( Q \) and parameter \( R \) is:

\[
P = \begin{pmatrix} 6.2485 & 18.022 \\ 18.022 & 94.588 \end{pmatrix}
\]

Finally, we can substitute all known parameters in control law equation (38).
Solving the substitution, we got a new matrix which has the proportional and integrate gain for the PI controller.

\[ u(t) = (ki \ kp) \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} \]  

(42)

Control form can be written in this way:

\[ u(t) = ki \int e(t) dt + kp e(t) \]  

(43)

As we know, this form is the representation of general PI controller.

A. PI controller simulation

A program was made using MatLab to solve the Ricatti equation and to make variations in matrix Q. The Figure 10 represents the results for different variations of Q with a step input. From graphic we can chose the best behaviour for the system response.

![Step response for Q matrix variation](image)

Fig. 10. Step responses with variations of matrix Q.

With the variation in elements of matrix Q, the overshoot slightly increases with gradual fall in rise time. In Figure 10 it is possible to see the fifth variation of Q is the best response because it has an overshoot less than two percent and the system stabilized about five seconds.

From the program in MatLab we can know the gains for PI controller that corresponds to the values of Q.

B. Method of Ziegler-Nichols for tuning a PI controller

It is important to make a comparison with other method tuning for PI controller, to make sure the optimal control tuning is the best way to have a control over the actuator. The proportional component of controller PI will make the system reaches the reference signal in less time and the integral component will minimize the stable state error. The Ziegler-Nichols method is based in the analysis response of the system with a step input in open-loop. The response is a reaction curve, called ‘S’ curve.

From the analysis we get the parameters of the model with the equations:

\[ k_0 = \frac{y_\infty - y_0}{u_\infty - u_0}, \quad \tau_0 = t_1 - t_0, \quad \gamma_0 = t_2 - t_1 \]

Using MatLab a program was made to know the parameters and finally we got the gains for the PI controller using the value table which Ziegler and Nichols proposed.

<table>
<thead>
<tr>
<th>Controller</th>
<th>( K_p )</th>
<th>( T_r )</th>
<th>( T_d )</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>( K_0 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PI</td>
<td>( K_0 )</td>
<td>( 3\tau_0 )</td>
<td></td>
</tr>
<tr>
<td>PID</td>
<td>( K_0 )</td>
<td>( 2\tau_0 )</td>
<td>( 0.5\tau_0 )</td>
</tr>
</tbody>
</table>

In figure 11 we use the classical control tuning using the Ziegler-Nichols method. The result is the system is stabilized in about 10 seconds, which means the PI controller is working because it reduces the stabilizing time. But studying the results before where the optimal theory was applied for getting the PI controller we can note the best response was when the system is stabilized in five seconds.

![Step response of classical PI control tuning](image)

Fig. 11. Response of the system, with a classical PI control tuning.

From all results presented before, we can conclude that classical way for PI tuning is working very well but the best result is getting when we applied the optimal tuning to the system to get a good control over the valve in the pneumatic system because the system is stabilized in less time.

VIII. Conclusion

From the integrated tomato-greenhouse model it is possible to do the optimization of all variables. In this case, the most important variable is the carbon dioxide concentration. In a first step, the optimal control theory...
was used to determine the desired behaviour of the variables over a prescribed time period. In a second step, a PI control was designed from optimal control theory to make change in the behaviour of the actuator. All the results obtained in this paper are the base to create an electronic device able to make the automatic control of carbon dioxide concentration inside a real tomato greenhouse.

REFERENCES


