Stabilization of convective instability in micropolar fluid model by feedback control strategy subjected to internal heat source

N.F.M. Mokhtar, and I.K. Khalid

***Abstract*—**This investigation reports on a stability analysis of Rayleigh-Benard convection in a horizontal of micropolar fluid layer heated from below. The effect of a feedback control strategy on the onset of steady convection in the presence of internal heat source is investigated theoretically using Galerkin technique. The eigenvalues are obtained for free-free, rigid-rigid, free-rigid boundary combination with isothermal temperature boundary condition. The influence of various micropolar parameters on the onset of convection has also been analyzed. The onset of motion is found to depend on the feedback control parameter, K and internal heat source, Q and the micropolar parameter N_i.

***Keywords*—**Convection, feedback control, internal heat source, micropolar fluid model.

I. INTRODUCTION

RESEARCH in heating of fluids that consist of dumbbell molecules or short rigid cylindrical elements like polymeric fluids, colloidal fluids and liquid crystal is vital for the processing industries where the flow behavior in shear cannot be characterized by Newton relationship. These fluids can be theoretically characterized as micropolar fluid and the thermal conductivity of some of these fluids plays an important role in the development of energy coefficient heat transfer equipment. Eringen [1-3] was first developed the theory of micropolar fluids and extended the theory to the thermomicropolar fluids. The theory is analytical tractability and thus it has been the subject of numerous investigations. Hudimoto and Tokuoka [4], Ariman [5] and Datta [6] had proved that micropolar fluid can successfully described the non-Newtonian behavior of certain real fluids.

Thermal instability of a micropolar fluid between two horizontal planes heated from below (or above) with various effects have been studied by Datta and Sastry [7], Ahmadi [8], Investigation on the onset of instability in a heat conducting micropolar fluid layer between rigid boundaries has been done by Rama Rao [9], Lebon and Parez-Garcia [10] and by Payne and Straughan [11]. Depending on the energy equation, differences in the results were obtained. The magnetoconvection in micropolar fluid has been investigated by Siddheswar and Pranesh [12]. The micropolar fluid layer was found to be more stable than the classical pure fluid layer. The effects of through flow and magnetic field on the onset of benard convection in micropolar fluid have been studied by Narasimha Murty [13]. Using the Keller-box method, Roslinda et al. [14] solved the problem of unsteady boundary layer flow of a micropolar fluid over a stretching sheet. Dufour and Soret effects on heat and mass transfer in a micropolar fluid in a horizontal channel is investigated by Awad and Sibanda [15]. The study uses the homotopy analysis method to find approximate analytical series solutions for the governing system of nonlinear differential equations.

The internal heat source and heat controller play a critical role in the materials and chemical processing industries. Chemical, petrochemical and refining engineers as well as equipment designers recognize that they need to understand and manage source of heat to maximize the performance of their processes. The onset of thermal instability in a horizontal fluid layer, subject to an internal heat generation has been analysed by Sparrow et al. [16] and Roberts [17]. The effect of quadratic basic state temperature gradient caused by uniform internal heat generation was studied by Char and Chiang [18] for Benard-Marangoni convection. Wilson [19] used analytical and numerical techniques to analyse the effect of internal heat source. Gasser and Kazimi [20], Kaviany [21] and Mokhtar et al. [22] studied the effect of internal heat generation on the onset of convection in a porous medium. Latest, Khalid et al. [23] reported that the effect of magnetic field has a stability effect on the convection in micropolar fluid system in the presence of internal heating. The effects of linear and nonlinear feedback control strategies on the steady and oscillatory stability thresholds have been studied both experimentally and theoretically [24-26]. Bau [27] applied a linear control feedback on Marangoni-Benard convection and found that the critical Marangoni number can be increased using the feedback control strategy. The uniform solution on feedback control with variety of effects in horizontal fluid layer has been reported to stabilize the system [28,29].

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In this work we investigated the stabilization of Rayleigh-Benard convection in micropolar fluid by feedback control strategy subjected to internal heat generation. We employed the stability analysis based on the linear stability theory and the resulting eigenvalue problem is solved using the Galerkin method.

II. Mathematical Formulation

We select a coordinate frame in which the z-axis is aligned vertically upwards. We consider a horizontal layer of fluid confined between the planes z = 0 and z = 1. The temperatures at the lower and upper boundary are taken to be T0 + ΔT and T0 respectively. The Oberbeck-Boussinesq approximation is employed and following Siddheshwar and Pranesh [12] after neglecting the magnetic effect and consider the internal heat source, the conservation equations take the form

$$\nabla \cdot \mathbf{v} = 0\,, \tag{1}$$

$$\rho_0\left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{v}\right) = -\nabla p - \rho g \boldsymbol{k} + \zeta\nabla\times\boldsymbol{\omega} + (2\zeta+\eta)\nabla^2\mathbf{v}\,, \tag{2}$$

$$\rho_0 I\left(\frac{\partial \boldsymbol{\omega}}{\partial t} + (\mathbf{v}\cdot\nabla)\boldsymbol{\omega}\right) = (\lambda+\eta)\nabla(\nabla\cdot\boldsymbol{\omega}) + (\eta\nabla^2\boldsymbol{\omega}) + \zeta(\nabla\times\mathbf{v} - 2\boldsymbol{\omega})\,, \tag{3}$$

$$\frac{\partial T}{\partial t} + (\mathbf{v}\cdot\nabla)T = \frac{\beta}{\rho_0 C_v}(\nabla\times\boldsymbol{\omega})\cdot\nabla T + \kappa\nabla^2 T + h_g \tag{4}$$

where $\mathbf{v} = (u, v, w)$, ρ_0 is the density, t is time, p is the pressure, g is the acceleration due to the gravity, $\boldsymbol{k}$ is the unit vector in the z-direction, ζ is the coupling viscosity coefficient for vortex viscosity, λ and η are the bulk and shear spin viscosity coefficients respectively, $\boldsymbol{\omega}$ is the micro rotation, I is the moment of inertia, T is the temperature, β is the micropolar heat conduction coefficient, C_v is the specific heat, κ is the thermal conductivity and h_g is the overall uniformly distributed volumetric internal heat source within the micropolar fluid layer. The basic state of the fluid is quiescent and is described by

$$\boldsymbol{q}_b = (0,0,0),\ \boldsymbol{\omega}_b = (0,0,0),\ p = p_b(z),\ \rho = \rho_b(z),\ T = T_b(z) \tag{5}$$

where the subscript b denotes the basic state. Substituting Eq. (5) into Eqs. (2) and (4), we get the basic state governing equations as

$$\frac{dp_b}{dz} = -\rho_b g\,, \tag{6}$$

$$\frac{d^2 T_b}{dz^2} = -\frac{h_g}{\kappa}\,, \tag{7}$$

with $\rho_b = \rho_0\left[1 - \alpha_t(T_b - T_0)\right]$.

Subject to the boundary conditions $T_b = T_0$ at $z = 0$ and $T_b = T_0 - \Delta T$ at $z = d$, Eq. (7) is solved and we obtained

$$T_b(z) = -\frac{h_g}{2\kappa}z^2 + \left(\frac{h_g d}{2\kappa} - \frac{\Delta T}{d}\right)z + T_0\,. \tag{8}$$

Take note that Eq. (8) is a parabolic distribution with the liquid layer height due to the existence of the internal heat generation. Without the internal heat generation; $Q = 0$, the basic state temperature distribution in the fluid layer is linear. Let the basic state be disturbed by an infinitesimal thermal perturbation and we now have

$$q = q_b + q',\quad \omega = \omega_b + \omega',\quad p = p_b + p',\quad \rho = \rho_b + \rho',\quad T = T_b + T'\,, \tag{9}$$

where the primes indicate that the quantities are infinitesimal perturbations. Substituting Eq. (9) into Eqs. (1) – (4) and linearized in the usual manner, we obtained the linearised equations in the form

$$\nabla\cdot\boldsymbol{q}' = 0, \tag{10}$$

$$0 = -\nabla p' - \rho' g\hat{k} + (2\zeta+\eta)\nabla^2\boldsymbol{q}' + \zeta(\nabla\times\boldsymbol{\omega}'), \tag{11}$$

$$-W\frac{\Delta T}{d} = \frac{\beta}{\rho_0 C_v}\left(\nabla\times\boldsymbol{\omega}'\cdot\left[-\frac{\Delta T}{d}\right]\hat{k}\right)\cdot\nabla T + \kappa\nabla^2 T' + \left(\frac{z h_g}{\kappa} - \frac{h_g d}{2\kappa} + \frac{\Delta T}{d}\right)W\,, \tag{12}$$

$$(\lambda'+\eta')\nabla(\nabla\cdot\boldsymbol{\omega}') + \eta'\nabla^2\boldsymbol{\omega}' + \zeta(\nabla\times q' - 2\boldsymbol{\omega}') = 0\,. \tag{13}$$

In the present problem, we assume the principle of exchange of stability is valid and deal only with stationary convection. Hence the time derivatives have been dropped in Eqs. (10) – (13). The perturbation equations are non-dimensionalized using the following definitions

$$(x^*, y^*, z^*) = \left(\frac{x}{d}, \frac{y}{d}, \frac{z}{d}\right), W^* = \frac{W'}{x/d}, T^* = \frac{T'}{\Delta T},\quad \Omega^* = \frac{(\nabla\times\omega')z}{x/d^3}\,. \tag{14}$$

Substituting Eq. (14) into Eqs. (11) - (13), eliminating the pressure term by operating curl twice on the resulting equation of (11), operating curl once on Eq. (13) and non-dimensionalising we get

$$(1+N_1)\nabla^4 W + N_1\nabla^2\Omega + Ra\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) = 0\,, \tag{15}$$

$$\nabla^2 T + (1 - Q(1-2z))W - N_5\Omega = 0 \tag{16}$$

$$N_3\nabla^2\Omega - 2N_1\Omega - N_1\nabla^2 W = 0\,, \tag{17}$$

where the asterisks have been dropped for simplicity. Here,

$N_1 = \dfrac{\zeta}{\eta+\zeta}$, is the coupling parameter,

$N_3 = \dfrac{\eta'}{(\eta+\zeta)d^2}$, is the couple stress parameter,

$N_5 = \dfrac{\beta}{\rho_0 C_v d^2}$ is the heat conduction parameter,

$Ra = \dfrac{\alpha g \Delta T \rho_0 d^3}{(\eta+\zeta)\chi}$ is the Rayleigh number,

and $Q = \dfrac{h_g d^2}{2\kappa \Delta T}$ is the heat source strength.

The perturbation quantities in a normal mode form are

$$\left(W,\ T,\ \Omega\right) = \left[W(z),\ \Theta(z),\ G(z)\right]\exp\left[i(a_x x + a_y y)\right] \quad (18)$$

where $W(z)$, $\Theta(z)$, $G(z)$ are amplitudes of the perturbations of vertical velocity, temperature and spin, and $a = \sqrt{a_x^2 + a_y^2}$ is the wavenumber of the disturbances at the liquid layer. Substituting Eq. (18) into Eqs. (15) – (17) we get

$$\left(1+N_1\right)\left(D^2 - a^2\right)^2 W + N_1\left(D^2 - a^2\right)G = Raa^2\Theta, \quad (19)$$

$$\left(D^2 - a^2\right)\Theta = (Q(1-2z)-1)W + N_5 G, \quad (20)$$

$$N_1\left[\left(D^2 - a^2\right)W + 2G\right] - N_3\left(D^2 - a^2\right)G = 0, \quad (21)$$

where $D = d/dz$.

We assume that the temperature is constant on the boundaries. We set the boundary condition for the uniform temperature at the bottom boundary to include the controller rule and following Bau [27], we use thermal feedback control mechanism to modify the heated surface temperature in proportion to the deviation of the fluid's temperature from its conductive value. The determination of a control; $q(t)$ can be accomplished using the proportional integral differential (PID) controller of the form

$$q(t) = r + K\,[e(t)],\ e(t) = \hat{h}(t) + h(t) \quad (22)$$

where r is the calibration of the control, $e(t)$ an error or deviation from the measurement, $\hat{h}$ from some desired or reference value; $h(t)$, $K = K_p + K_d\, d/dt + K_i \int dt$ with K_p is the proportional gain, K_d is the differential gain and K_i is the integral gain. Based on equation (22), for one sensor plane and proportional feedback control, the actuator modifies the heated surface temperature using a proportional relationship between the upper, $z = 1$ and the lower, $z = 0$ thermal boundaries for perturbation field

$$T'(x, y, 0, t) = -KT'(x, y, 1, t), \quad (23)$$

where T' denotes the deviation of the fluid's temperature from its conductive state and K is the scalar controller gain in which it will be used to control our system.

Equations (20) – (22) are solved subject to appropriate boundary conditions that are

$$W(0) = DW(0) = G(0) = \Theta(0) + K\Theta(1) = 0, \quad (24)$$

$$W(1) = D^2W(1) = G(1) = D\Theta(1) = 0, \quad (25)$$

where K is the controller parameter. Equation (24) indicates the use of rigid and isothermal for lower boundary, and for equation (25) the indicates stress free and insulating. The condition on G is the no-spin boundary condition for both boundaries.

III. Method of Solution

Equations (19) – (21) together with the boundary conditions (24) and (25) constitute a Sturm-Liouville problem with the Rayleigh number Ra as an eigenvalue while keeping other physical parameters fixed. The Galerkin method is used to solve the resulting eigenvalue problem. Accordingly, the variables are written in a series of basis function as

$$W = \sum_{i=1}^{n} A_i W_i(z)\ \ \Theta(z) = \sum_{i=1}^{n} C_i \Theta_i(z),\ G(z) = \sum_{i=1}^{n} D_i \Gamma_i(z) \quad (26)$$

where the trial functions $W_i(z)$, $\Theta_i(z)$ and $\Gamma_i(z)$ will be chosen in such a way that they satisfy the respective boundary conditions and A_i, C_i and D_i are constants. Substituting Eq. (26) into Eqs. (19) – (21), multiplying the resulting Eq. (19) by $W_j(z)$, Eq. (20) by $\Theta_j(z)$ and Eq. (21) by $\Gamma_j(z)$; performing the integration by parts with respect to z between $z = 0$ and 1 and using the boundary conditions (24) and (25), we obtain the following system of linear homogeneous algebraic equations

$$C_{ji}A_i + D_{ji}C_i + E_{ji}D_i = 0, \quad (27)$$

$$F_{ji}A_i + G_{ji}C_i + H_{ji}D_i = 0, \quad (28)$$

$$I_{ji}A_i + J_{ji}D_i = 0. \quad (29)$$

The coefficients $C_{ji} - J_{ji}$ are given by

$C_{ji} = -(1+N_1)[< (D^2W)^2 > + 2a^2 < DW^2 > + a^4 < W^2 >]$,
$D_{ji} = -a^2 Ra < W\Theta >$,
$E_{ji} = N_1[< DG\, DW > + a^2 < WG >]$,
$F_{ji} = < [1 - Q(1-2z)]W\Theta >$,
$G_{ji} = < (D\Theta)^2 > + a^2 < \Theta^2 >$,
$Hji = -N_5 < \Theta G >$,
$Iji = -N_1 [< DW\, DG > + a^2 < WG >]$,
$Jji = N_3 < DG^2 > + (2N_1 + a^2 N_3) < G^2 >$,

where the angle bracket $\langle \cdots \rangle$ denotes the integration with respect to z from 0 to 1. The above set of homogeneous algebraic equations can have a non-trivial solution if and only if

$$\begin{vmatrix} C_{ji} & D_{ji} & E_{ji} \\ F_{ji} & G_{ji} & H_{ji} \\ I_{ji} & 0 & J_{ji} \end{vmatrix} = 0. \quad (30)$$

The eigenvalue has to be extracted from the characteristic Eq. (30). Now, we choose the trial functions as

$$W_i = z^2\left(1 - z^2\right)T_{i-1}^*,\ G_i = z\left(1 - z\right)T_{i-1}^*,\ \Theta_i = z\left(z - 2\right)T_{i-1}^*$$

where T_i^* are the Chebyshev polynomials of the second kind, such that W_i, Θ_i and G_i satisfy the corresponding boundary conditions.

IV. RESULTS AND DISCUSSION

The onset of Rayleigh-Benard convection in micropolar fluid in the presence of feedback control and internal heat generation is investigated theoretically using Galerkin method. The sensitiveness of critical Rayleigh number; Ra_c to the changes in the micropolar fluid parameters; N_1, N_3 and N_5 are also studied. Three lower-upper cases are considered in this investigation which are rigid-free, rigid-rigid and free-free surfaces.

To verify our results, we compare our eigenvalue solution with Siddheswar and Pranesh [12] as can be seen in Table 1, by considering the upper surface to be free and lower boundary to be rigid. The results are in a good agreement and thus validate our solution. From this table we found that the critical Rayleigh number; Ra_c decreases as the value of internal heating; Q increase. This shows that the effect of internal heat source in the micropolar fluid system trigger the onset of convection rapidly for all coupling stress parameter; N_1 considered. As the effect of N_1, increasing the N_1 values help to slow down the destabilizing process.

TABLE I. COMPARISON OF CRITICAL RAYLEIGH NUMBER FOR DIFFERENT VALUES OF Q.

N_1	Ra_c				
	[12]	*Present study*			
	$Q = 0$	$Q = 0$	$Q = 1$	$Q = 3$	$Q = 5$
0.5	2700.1	2700.1	1483.1	1022.3	780
1	4743.5	4743.5	2276.7	1497.7	1115.9
1.5	8466.9	8466.9	3319.6	2064.2	1497.7
2	16976	16976	4727.7	2743.2	1932

Figure 1 indicates the variation values of Rayleigh number; Ra for K= 0, 2, 4 for three different cases. The parameters chosen are N_1 = 0.5, N_3 = 2, N_5 = 1 and Q = 0. It is found that as Ra is increases, the value of gain controller; also increases and thus stabilize the system for all different surfaces considered. This revealed that the use of controller stabilize the system. If we compare the three different surface curves, we can clearly see that the rigid-rigid curve has the highest plotted Ra values in the graph. It is interesting to take note here that the use of the rigid-rigid surface, is the most stable compare with the other surfaces considered in this investigation.

The variation values of Rayleigh number with wave number; a when Q = 0, 2 and 4 can be seen in Figure 2. The parameters chosen are N_1 = 0.5, N_3 = 2 and N_5 = 1. It is found that the internal heat generation has a rapid impact on the stability of the system where increasing Q, decrease the Ra values for all cases considered. It is proven that the internal heat generation is a destabilizing factor.

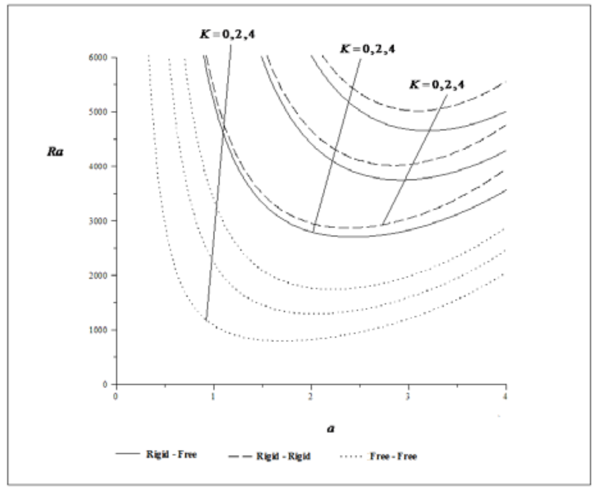

Fig. 1. Variation of *Ra* and *a* with different values of *K*

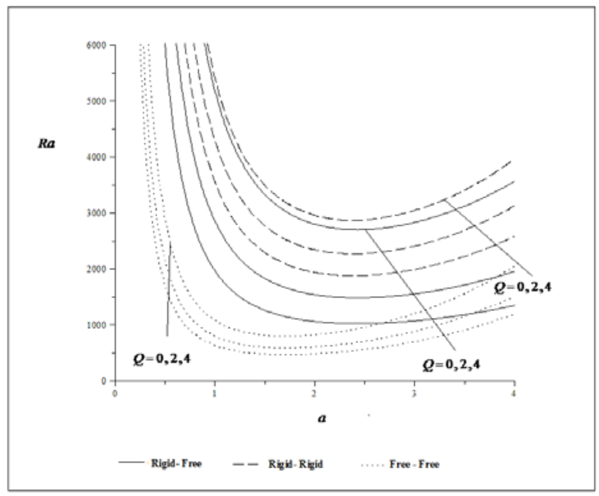

Fig. 2. Variation of *Ra* and *a* with different values of *Q*

Figure 3 show the plot of the critical Rayleigh number; Ra_c versus the coupling parameter; N_1 for various values of controller; K when Q = 2, N_3 = 2 and N_5 = 1. In each of these plots, the critical number increases with increasing of N_1 for all values of K. N_1 indicates the concentration of microelements, and increasing of N_1 is to elevate the concentration of microelements number. When this happened, a greater part of the energy of the system is consumed by these elements in developing gyrational velocities of the fluid and thus delayed the onset of convection. Increasing of controller; K increase the critical Rayleigh number and thus making the system stable.

The graph of the critical Rayleigh number; Ra_c versus the coupling parameter; N_1 for various values of internal heat generation; Q when K = 1, N_3 = 2 and N_5 = 1 is shows in figure 4. In each of these plots, the critical number decreases with increasing of Q and this physically describes that the micropolar system become more prone to instability. However, it is found that by elevating the values of N_1 parameter can help to slow down the process of convection.

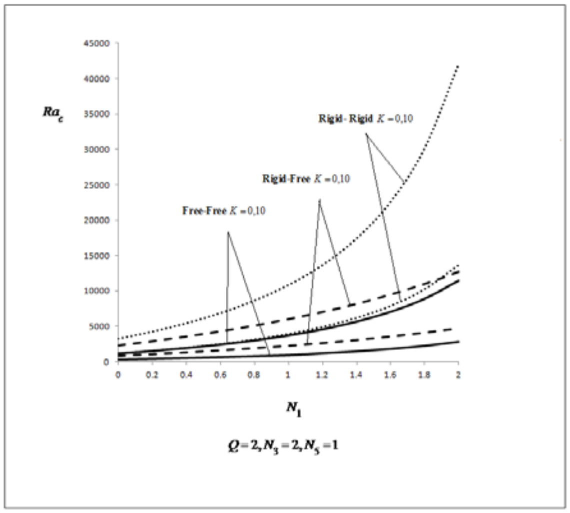

Fig. 3. Variation of Ra_c and N_1 with different values of K

The illustration of the couple stress parameter; N_3 can be seen in figure 5 by substituting N_1 = 0.1, N_5 = 10 and Q = 2. From the graph, it can be clearly seen that in the presence of controller, an increase of N_3 decrease the values of Ra_c for all K in the three cases considered. This situation revealed that the system become more unstable much faster when the couple stress parameter is increasing. The inspection on the boundary surfaces disclosed that the use of rigid-rigid surface can promote stability in the micropolar fluid system whereas the lowest values of the critical Rayleigh number can be found in the free-free surface.

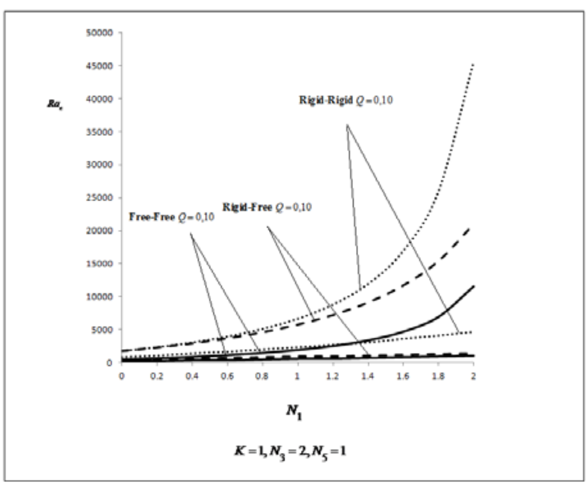

Fig. 4. Variation of Ra_c and N_1 with different values of Q

Figure 6 illustrates the curve of couple stress parameter; N_3 and Ra_c when K = 1, N_1 = 0.1 and N_5 = 10 for various values of internal heat generation Q. This observation shows that for the micropolar fluid system that holds internal heat source, the micropolar fluid system become more unstable when increase the value of internal heat generation. Align with the effect of Q, elevating the couple stress parameter values decrease the critical Rayleigh number.

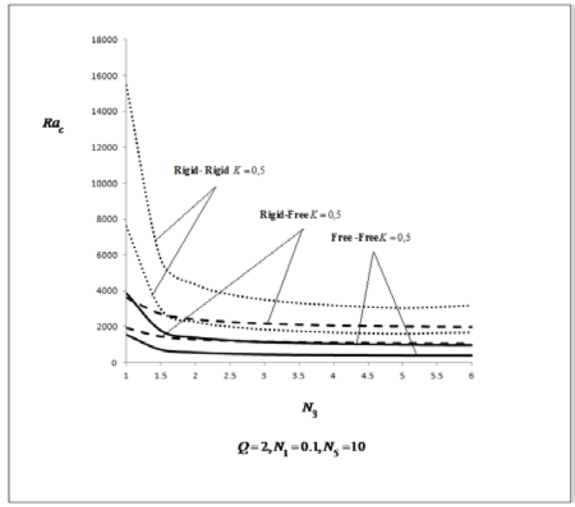

Fig. 5. Variation of Ra_c and N_3 with different values of K

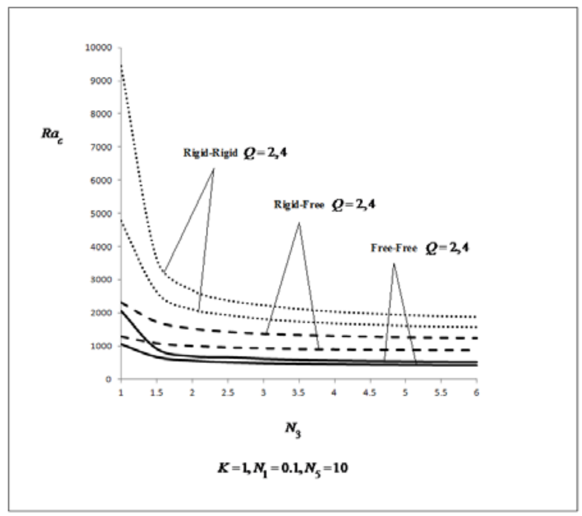

Fig. 6. Variation of Ra_c and N_3 with different values of Q.

Figure 7 and Figure 8 show the plot of Ra_c versus micropolar heat conduction parameter; N_5 when N_1 = 0.1 and N_3 = 2 with different values of K and Q respectively. Scrutinizing on the values of N_5 disclosed that the Ra_c increases when increasing the values of N_5 in both graphs for all cases considered. The reason behind this is, when N_5 increases, the heat induced into the fluid due to the microelements is also increased an thus reducing the heat transfer from the bottom to the top of the system. When this happened, the micropolar fluid system sustain its stability and delay the onset of convection. As the effects of K and Q, it is still identical with the previous results that increasing the K, elevate the Ra_c and increasing the Q makes the system become more unstable.

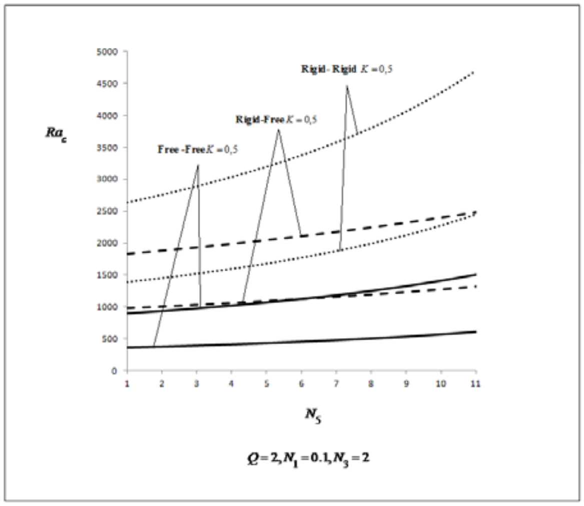

Fig. 7. Variation of Ra_c and N_5 with different values of K.

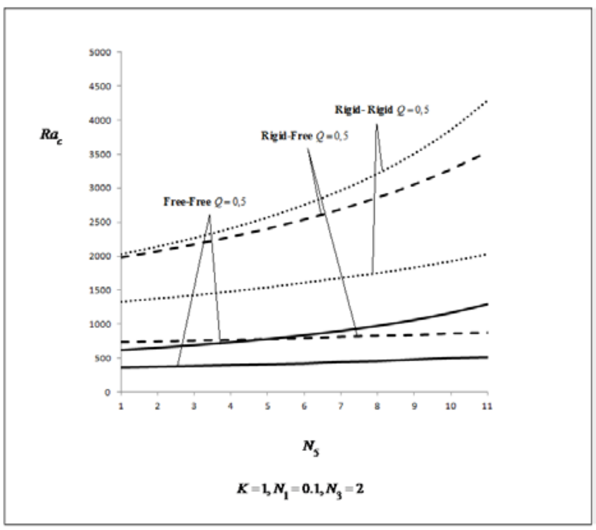

Fig. 8. Variation of Ra_c and N_5 with different values of Q

V. Conclusion

This investigation is particularly notable for scientists and engineers who are dealing with the heat and stability industries. Three types of boundary surfaces are considered in this investigations that are rigid-rigid, rigid-free and free-free. It is found that the effect of controller; K in the micropolar fluid is clearly has a stabilizing effect while the effect of internal heat generation; Q in the micropolar fluid system has a significant influence on the Rayleigh-Benard convection and is clearly a destabilizing factor to make the system more unstable. These both inspection results reported in this study are indistinguishable with the previous investigation reported in the literature. For the three types of surface cases considered that are rigid-free, rigid-rigid and free-free surfaces, it is found that the critical values of the Rayleigh number in rigid-rigid surfaces are the highest followed by rigid-free and free-free. This shows that the use of rigid-rigid surface can delay the onset of convection. As the effect of micropolar parameter; the increase of the coupling parameter; N_1 and heat conduction; N_5 helps to slow down the process of destabilizing. Contrast with the effects of N_1 and N_5, increasing the values of couple stress parameter; N_3 promotes instability in the micropolar fluid system.

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