Numerical simulation of the disturbances development in a supersonic boundary layer

S. A. Gaponov, A. N. Semenov

Abstract—For solution of hydrodynamic stability problems the evolutionary method is offered. The essence of this method consists that an arbitrary initial disturbance is described by one wave with the greatest increment on large times, which varies according to the law $\exp(-i\omega \tau)$. In order to verify the new method and to work out the numerical scheme the stability calculations were carried out also on the base of the classical theory. The evolutionary method is used to study the effects of the gas injection direction through a porous surface on stability of a supersonic boundary layer at the Mach number $M=2$. Results of the evolutionary method coincide with data of the classical theory very well.

The boundary layer receptivity process due to the interaction of three-dimensional slow acoustic disturbances is numerically investigated at a free stream Mach number of 2.0. Problem is solved in the linear approximation relatively excited disturbances by an acoustic wave. Numerical simulations were conducted with using the program complex Ansys. In general, matching the results of the approximate method (based on stability equations for low-frequency fluctuations) with direct numerical simulation data is satisfactory. Normalized solutions on the corresponding maxima of the velocity perturbations amplitudes are coincided well enough about a wall. The greatest discrepancy occurs in the area of the boundary layer edge where the approximation theory is inapplicable.

Keywords— supersonic boundary layer, establishing method, acoustic waves, interaction, receptivity, numerical simulations

I. INTRODUCTION

This paper is the extended version of the presentation at the 12th International conference on applied and theoretical mechanics (Prague, March 18-20, 2016) which was published in [1].

The problems on the hydrodynamic stability and the interaction of a supersonic boundary layer with acoustic waves which are considered in the present paper were raised mainly in connection with the problem on the turbulence formation. Transition from laminar to turbulent state in shear flows occurs due to evolution of different disturbances inside the shear layer. Though there are several mechanisms and routes to go from a laminar to a turbulent state, most of them generally follow these fundamental processes: receptivity, linear instability, nonlinear instability and breakdown to turbulence. Here the first two problems will be considered: the linear instability, and the receptivity of a supersonic boundary layer to the external acoustic waves.

As for the linear instability of a supersonic boundary layer it was solved by many authors. A detailed review of their works can be found in [2-4]. It is necessary to notice that, as a rule, all authors use the standard method of elementary waves leading to the solution of the eigenvalue problem of the homogeneous system of ordinary differential equations with homogeneous boundary conditions. The lack of this method is in the difficulty to find the waves with the highest increment. Its search comes to the end successfully under a condition if its approximate value is known. Therefore, growing in time waves given the front direction and wave number, depending on the incoming values of main flow, such as Mach number, Reynolds number and others, are calculated on the base of small changes of determinative parameters. However, the wave with the maximum increment for some basic terms will not be the determinative one (with a maximum growth factor), for the other flow parameters. Therefore, it is desirable to have such a calculation method which would guarantee uniquely obtaining of the wave with the highest increment. For linear problems, this can be achieved by an evolutionary method by the integration over time of partial differential equations. Because any disturbance, satisfying uniform boundary conditions can be decomposed into the sum of the waves with different increments, the wave with the largest increment will dominate at large times. This method can be called by the usual term - the establishing method. In contrast to the generally accepted method of establishing when the solution goes to the constant, in our case the solution goes to the exponential dependence on time. In the hydrodynamic stability theory there are the temporary instability (wave number on uniform spatial coordinates are real) and the spatial instability (when perturbations with real frequencies growth in the space). At low amplification rate in the space and time, which is characteristic for the boundary layers, temporal and spatial increments are associated with the simple approximate relation: the amplification rate in space equals the negative temporary divided by the wave group velocity [2]. If necessary, the more precise value of the spatial amplification rate can be obtained by the classical method. In this paper
theoretically investigates the influence of the direction of the blowing gas through a porous surface on the stability of a supersonic boundary layer using the classical method of elementary waves and evolutionary method.

At present the most complex problem on the prediction of the transition position in the boundary layer flows is related to the receptivity of these flows to the external effects. This problem was discussed in detail for the first time in [5]. A review of the early works of the acoustic field effect on the transition from a laminar supersonic boundary layer to the turbulent one has been given in [2]. The problems on the supersonic flow aeroacoustics were studied mainly within the framework of the investigations on the conditions for the onset of auto-oscillations and sound generation by the supersonic shear flows in the jets and mixing layers. The advanced approach based on the idea of the possibility of the mutual influence of the acoustic and hydrodynamic waves has been demonstrated in [6].

The first attempts to investigate the interaction of the sound waves and supersonic boundary layer on the basis of the stability theory were undertaken in [7,8]. The problem on the excitation of unstable waves by sound was considered in [9]. The interaction of sound with a supersonic boundary layer experimentally was studied in [10–11] where the main results of theory [8] were confirmed.

In [12] the theory of interaction of external high-frequency acoustics of small amplitudes with a supersonic boundary layer on the basis of the linear equations of hydrodynamic stability is stated in detail. Its essence consists in the assumption that an acoustic wave length differs from a boundary layer thickness a little. Then in the first approximation the main flow current in a stationary boundary layer can be accepted as plane-parallel flow. The problem is reduced to integration of the ordinary differential equations which coefficients depend on the longitudinal speed and temperature in a stationary flow. At finite incidence angle from the solution of stability equations it is possible to receive a distribution of the disturbance amplitude in a boundary layer excited by an acoustic wave and a reflection coefficient. At an approach based on the idea of the possibility of the mutual influence of the acoustic and hydrodynamic waves has been demonstrated in [6].

The flow is described by the known Navier - Stokes, continuity, energy and state equations [9]:

\[
\begin{align*}
\rho \frac{\partial v^*}{\partial t} + \rho \frac{\partial (\rho u^*)}{\partial x} &= -\nabla p^* + \frac{1}{2} \nabla (\rho u^*)^2 + 2 \mu \nabla^2 v^* + \rho \mathbf{f}, \\
\rho \frac{\partial T^*}{\partial x} &= \frac{1}{Pr} \frac{\partial (\rho \nabla T^* - \nabla T^*)}{\partial t} + \frac{1}{Pr} \frac{\partial (\rho u^* \nabla T^*)}{\partial x} + 2 \mu \nabla^2 T^* + \rho \mathbf{g},
\end{align*}
\]

Here \( v^* \) – velocity with components \( (u^*, v^*, w^*) \) in \( x, y, z \) directions, \( p^*, \rho^*, T^* \) – pressure, density and temperature, \( c_p \) – specific heat at constant pressure, \( R \) – gas constant, \( \mathbf{f} \) – body forces, \( \mathbf{g} \) – gravity acceleration, \( \mu^* \) – dynamic viscosity.

II. BASIC EQUATIONS

The gas flow is described by the known Navier - Stokes, continuity, energy and state equations [9]:

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\begin{align*}
\rho \frac{\partial v^*}{\partial t} &= -\nabla p^* + \frac{1}{2} \nabla (\rho u^*)^2 + 2 \mu \nabla^2 v^* + \rho \mathbf{f}, \\
\rho \frac{\partial T^*}{\partial t} &= \frac{1}{Pr} \frac{\partial (\rho \nabla T^* - \nabla T^*)}{\partial t} + \frac{1}{Pr} \frac{\partial (\rho u^* \nabla T^*)}{\partial x} + 2 \mu \nabla^2 T^* + \rho \mathbf{g},
\end{align*}
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Here \( v^* \) – velocity with components \( (u^*, v^*, w^*) \) in \( x, y, z \) directions, \( p^*, \rho^*, T^* \) – pressure, density and temperature, \( c_p \) – specific heat at constant pressure, \( R \) – gas constant, \( \mathbf{f} \) – body forces, \( \mathbf{g} \) – gravity acceleration, \( \mu^* \) – dynamic viscosity.

III. THE BOUNDARY LAYER STABILITY WITH VEKTORED INJECTION

A. The stability theory of plane-parallel flows

In this paper we will explore the disturbance in supersonic boundary layers on a flat plate at high Reynolds numbers \( Re_x = u_x / \nu \) where \( u_x, \rho_x, \mu_x \) - velocity, density and dynamic viscosity in a free flow, \( x \) - distance from the front edge of the plate to researched area. In this case, the main flow
is independent on the transverse, z-coordinate, weakly dependent from x-coordinate and velocity in y-direction is low. Therefore, the main (stationary) flow can be considered for a plane-parallel, depending only on y, only the velocity in the x-direction \( u'(y) \) is unequal to zero.

If to introduce the dimensionless coordinates, time, and parameters of flow in the form: \( dX = dx / \delta, \ dY = dy / \delta, \ dZ = dz / \delta, \ dT = u_{*}dt / \delta, \ v = v_{*} / u_{*}, \ p = p_{*} / p_{e}, \ T = T^{*} / T_{e}, \ \rho = \rho^{*} / \rho_{e} \) where \( \delta = \sqrt{\mu u_{*} / \rho_{e} u_{*}^{2}} \) - the thickness of the boundary layer, the velocity, density, pressure and temperature of the compressible gas in the boundary layer can be represented in the form:

\[
\begin{align*}
u & = U(Y) + \varepsilon u', \ v = \varepsilon v', \ w = \varepsilon w', \ p = P(Y) + \varepsilon p' , \\
T & = T_{0}(Y) + \varepsilon \theta', \ \rho = 1 / T_{0}(Y) + \varepsilon \rho', \\
U(Y), P, T_{0}(Y) & - velocity, pressure, and temperature in the unperturbed laminar boundary layer. Flow parameters disturbances which depend on \( X, Y, Z \) and \( \tau \) are marked by the prime.
\end{align*}
\]

Equations for linear disturbances in the approximation of Dana-Lin [3] and Alekseev [23] in the two-dimensional boundary layer have the form:

\[
\begin{align*}
\frac{1}{T_{0}(Y)} \left( \partial u' / \partial \tau + U \partial u' / \partial Y + \partial U / \partial Y \right) & = - \frac{1}{\gamma M_{w}^{2} \partial X} \partial p' / \partial Y + \mu \partial^{2} u' / \partial Y^{2} , \\
\frac{1}{T_{0}(Y)} \left( \partial v' / \partial \tau + U \partial v' / \partial Y + \partial U / \partial Y \right) & = - \frac{1}{\gamma M_{w}^{2} \partial Z} \partial p' / \partial Y + \mu \partial^{2} w' / \partial Y^{2} , \\
\frac{1}{T_{0}(Y)} \left( \partial \theta' / \partial \tau + U \partial \theta' / \partial Y + \partial U / \partial Y \right) & = \gamma - 1 \left( \partial \rho' / \partial \tau + U \partial \rho' / \partial Y \right) + \\
\frac{\mu}{PrRe} \partial^{2} \theta' / \partial Y^{2} , \ & \ p' / P = T_{0} \varepsilon^{*} + \theta' / T_{0} .
\end{align*}
\]

The system (1) is solved with the boundary conditions, [2]:

\[
\begin{align*}
u' = v' = w' = \theta' = 0 , \ at \ Y = 0, \infty .
\end{align*}
\]

B. The classical stability theory

The classical stability theory is founded on the method of elementary waves \( a', p' = (a(Y), \pi) \exp(i(\alpha X + \beta Z - \omega \tau)) \).

In this case, vector components \( a = (f, \varphi, \rho, \theta, \zeta) \) amplitudes of perturbations \( u', v', w', \theta', \zeta' \). Equations (1) are given to system of the linear ordinary differential equations:

\[
\begin{align*}
\frac{d\zeta}{d\tau} & = \frac{i\alpha}{\gamma M_{w}^{2} \partial X} f + \varphi \frac{dU}{dY} + i\beta \frac{d\varphi}{dY} , \\
\frac{d\varphi}{d\tau} & = \frac{i\beta}{\gamma M_{w}^{2} \partial Y} f + \varphi \frac{dU}{dY} + i\alpha \frac{d\varphi}{dY} , \\
\frac{d\theta}{d\tau} & = \frac{i\alpha}{\gamma M_{w}^{2} \partial X} h + \varphi \frac{dU}{dY} + i\beta \frac{d\varphi}{dY} , \\
\frac{d\zeta}{d\tau} & = \frac{i\beta}{\gamma M_{w}^{2} \partial Y} h + \varphi \frac{dU}{dY} + i\alpha \frac{d\varphi}{dY} .
\end{align*}
\]
\[ \frac{\partial \theta}{\partial \tau} = \left[ U \frac{\partial \theta}{\partial X} + \varphi \frac{\partial T}{\partial Y} \right] - \]
\[ - (\gamma - 1) T_0 \left( \frac{\partial f}{\partial X} + i \beta h + \frac{\partial \varphi}{\partial Y} \right) + \frac{\gamma \mu T_0}{PrRe} \frac{\partial^2 \theta}{\partial Y^2}. \]
\[ f(0, \infty) = \varphi(0, \infty) = h(0, \infty) = \theta(0, \infty) = 0. \]

The computational domain and the numerical scheme.

The problem was solved for the periodic perturbation in the coordinate \( x \), i.e. \( a(Y, X, \tau) = a(Y, X + L, \tau) \). The region of an integration in the normal direction was enclosed interval \( 0 < Y < Y^* \). We took into account the conditions of equality to zero disturbances at \( Y = 0 \). Value \( Y^* \) was accepted rather large that its additional increase did not lead to essential change of disturbances increments.

For the integration of the system (5) we used 2-step finite-difference scheme [24]:
\[ f^{n+1} - f^n \over \Delta \lambda = -U \varphi^n + \frac{\mu T_0}{Re} f^{n+1}_j - f^n_{j-1} - \]
\[ \frac{2}{h_x} \frac{\partial h^n}{\partial \tau} \]
\[ \frac{\partial h}{\partial \tau} \]
\[ \frac{\partial \varphi}{\partial \tau} \]
\[ \frac{\partial \varphi}{\partial \tau} \]
\[ \frac{\partial^2 \theta}{\partial \tau^2} + \frac{\partial^2 \theta}{\partial Y^2} \]
\[ + \frac{\gamma \mu T_0}{PrRe} \frac{\partial^2 \theta}{\partial Y^2} \]
\[ \frac{\partial \varphi}{\partial \tau} \]
\[ \frac{\partial \varphi}{\partial \tau} \]
\[ \frac{\partial \varphi}{\partial \tau} \]
\[ \frac{\partial^2 \theta}{\partial \tau^2} + \frac{\partial^2 \theta}{\partial Y^2} \]
\[ + \frac{\gamma \mu T_0}{PrRe} \frac{\partial^2 \theta}{\partial Y^2} \]
\[ \frac{\partial \varphi}{\partial \tau} \]
\[ \frac{\partial \varphi}{\partial \tau} \]
\[ \frac{\partial \varphi}{\partial \tau} \]
\[ \frac{\partial^2 \theta}{\partial \tau^2} + \frac{\partial^2 \theta}{\partial Y^2} \]
\[ + \frac{\gamma \mu T_0}{PrRe} \frac{\partial^2 \theta}{\partial Y^2} \]

Unknown values at the boundary were obtained by interpolating on three neighboring points.

The value \( \omega \) was determined by the formula:
\[ \omega = \frac{1}{iN2\Delta} \ln \left( \frac{\pi^{n+1} \pi^n}{\pi^0} \right). \]
Calculations were performed as long until it was constant with the acceptable accuracy. In this case the real or imaginary part \( q(Y, X, \tau) \) were changed as the \( a_{i-1}(Y, X, \tau) = a_{i-1} \sin(\alpha X + \omega \tau) \). The value of \( \alpha = 2\pi n / L \), where \( n \)- number of periods stacked on the calculating range \( L \).

IV. BOUNDARY LAYER EQUATIONS

In self-similar variables boundary layer equations have the form [25]:
\[ \frac{d}{dY} \left( \mu \frac{dU}{dY} \right) + \frac{dU}{dY} = 0, \quad \frac{dg}{dY} = \frac{U}{2T_0}. \]
\[ \frac{d}{dY} \left( \frac{dT}{Pr \, dY} \right) + \frac{dT}{Pr \, dY} + (\gamma - 1) M^2 \frac{dU}{dY} = 0, \]
Here \( \gamma = C_p / C_v \) - ratio of specific heats, \( M_x = u_x / a_x \) - Mach number and \( a_x \) - sound velocity at the boundary layer edge. At a uniform gas blowing through a wall at an angle \( \lambda \) to the main flow direction the velocity components on a wall are defined as follows: \( V(0) = G \sin \lambda \), \( U(0) = G \cos \lambda \). Because \( g(0) = -V(0) Re / T_w \), \( g(0) = -G Re \sin \lambda / T_w \). Introducing the parameter \( C_q = -G Re / T_w \), characterizing the intensity of the blowing or suction through the surface, the boundary conditions on thermally insulated surface can be written as:
\[ g(0) = C_q \sin \lambda, \quad U(0) = T_w C_q \cos \lambda / Re, \quad \frac{dT}{dY}(0) = 0; \]
\[ T_0(\infty) = U(\infty) = 1; \]

The dependence of \( \mu^* \) on temperature was adopted in accordance with the Sutherland's law, which in dimensionless form can be written as follows:
\[ \mu^* = \mu^* (T_x) \left( \frac{T_0}{T_x} \right)^{3/2} \frac{T_x}{T_0} + T_x, \]
where \( T_x \) - Sutherland's constant. In wind tunnels without heating at a constant stagnation temperature \( T_x \), \( T_x = T^* / (1 + (\gamma - 1) M^2) \).

V. THE BOUNDARY LAYER INTERACTION WITH ACOUSTIC WAVES

Scheme of an interaction of the sliding along the surface of monochromatic sound waves with the boundary layer of the streamline surface (plate) is shown in Fig. 1. Wave front section which is parallel to normal coordinate \( y \) is represented by the letter \( S \). Wave vector of the sound wave \( \alpha \) with projections \( \alpha_1 \) and \( \alpha_2 \) on \( x \) and \( z \) respectively is parallel to the
plane \((x, z)\). Velocity of the running flow is parallel to an axis \(x\) and is perpendicular the planes \((y, z)\). The figure shows a conditional boundary layer, its thickness \(\delta\) and the sliding angle \(hi = \arctg(\beta / \alpha)\). The acoustic wave is periodic on \(z\) with period \(\lambda_z = 2\pi / \beta\) and on \(x\) with period \(\lambda_x = 2\pi / \alpha\).

Thus, the external parameters of the acoustic wave is changed by law: \(q_i = q_i^0 \cos(\alpha x + \beta z - \omega t)\), were \(q_i^0\) - oscillation amplitude, \(\omega\) – angular frequency. Problem is solved in the linear approximation relatively excited disturbances by an acoustic wave.

\[ h = \frac{\gamma}{\gamma^r} \frac{d^2\theta}{dt^2} = 0. \]

Distributions of the perturbations amplitude and constant \(\varrho_x\) are determined from the solution of the differential equations with given boundary conditions.

### B. The direct numerical simulation

The evolution of disturbances in the layer was simulated numerically using the ANSYS Fluent software package.

In Fig. 2 the computational domain is shown. The flow direction is shown in Fig. 1. The height of the parallelogram AE was selected to avoid of the interaction of shock waves, which forms in the leading edge vicinity of the plate due to viscous-inviscid interaction, with the top side (EHGF). The width AD is taken equal to the wavelength in \(z\)-direction \(\lambda_z = 2\pi / \beta\).

On an entrance (side AEHD) and on the top side flow parameters are set which are composed of the stationary part and parameters of a sound wave:

\[ p^* = p^*_x + Ap^*_x \cos(\alpha x + \beta z - \omega t) \]

\[ T^* = T^*_x + AT^*_x \frac{(\gamma - 1)}{\gamma} \cos(\alpha x + \beta z - \omega t) \]

\[ u^* = u^*_x + A\sqrt{\frac{RT_x^*}{\gamma}} \cos(hi) \cos(\alpha x + \beta z - \omega t) \]

\[ v^* = A\sqrt{\frac{RT_x^*}{\gamma}} \sin(hi) \cos(\alpha x + \beta z - \omega t) \],

where \(A\) is a dimensionless pressure amplitude.

The Mach number \(M_x = \sqrt{(u^2 + v^2) / \gamma RT_x^*}\), wave vectors:

\( \alpha = \alpha \cos(hi), \beta = \alpha \sin(hi), \varphi = \omega / [(M_x \cos(hi) - 1)\sqrt{\gamma RT_x^*}] \).

The nonslip boundary conditions \((v^* = 0)\) are imposed on the plate surface (PLCB) and the plate temperature corresponds to the adiabatic condition \((\partial T^*/\partial y = 0)\). Flow parameters on the surface ADLP were taken from a symmetry condition. Flow parameters on sides AEFB and DHGC were taken from a periodicity condition of a solution in \(z\)-direction. On the outflow boundary, the unknown variables are extrapolated from an internal domain.

The computational domain consisted of two subdomains, which are divided from each other by the plane A'B'C'D'. The solution in a wall area (bottom subdomain) strongly changes
along the coordinate $y$ in comparison with a change in the top subdomain. Therefore in the top subdomain, which height is $A'E$, the computational grid step on coordinate $y$ ($h_y$) may exceed the corresponding step in the bottom subdomain at several times. The computational steps in $z$, $x$-directions may be identical in the both domains.

It is supposed that free flow disturbances are rather small ($A<<1$) that the development of disturbances in all domains satisfies to linear laws. Disturbances are remained monochromatic on a time and a lateral coordinate.

VI. RESULTS

A. Stationary flow parameters in the boundary layer

The calculations results of longitudinal velocity profiles at Mach number $M_\infty = 2$, $T_e = 110^\circ K$, $T_0 = 300^\circ K$ for different values of the normal injection parameter $C_q$ are presented in Fig 3. Note, that approaching to value of velocity to unit is slowed with increasing injection rates. Thus one can clearly see that normal blowing leads to increasing of boundary layer thickens. Furthermore, an inflection point is appeared in the velocity profile which can contribute to destabilization of the boundary layer.

![Fig. 3 Distribution of longitudinal velocities for various values of the parameter Cq](image)

Influence of blowing direction in the distribution of longitudinal velocity is shown in Fig. 4. The velocity distribution without blowing is marked by symbols. From these data it follows that the stationary flow parameters are dependent on the tangential injection weakly. The normal velocity component plays a decisive role in this respect.

B. Results on the stability theory

Main results were obtained on the basis of equations (5). Stability calculations were carried out by the classical theory (3)-(4) for processing of the settlement scheme (6). The rectangular mesh with 240 points in the $X$- coordinate and 400 point in $Y$- coordinate with the time step $\Delta = 0.001$ was used.

As already mentioned, at large times the solution of equations (5) are described by an exponential dependence on will not dwell on the initial data which were set arbitrarily. the time, regardless of the initial data. Therefore, we

![Fig. 4 Dependeences of longitudinal velocities on the normal coordinate for the different λ.](image)

![Fig. 5 The dependence of the real part of the pressure perturbation near the walls over time](image)

![Fig. 6 The dependence of the real part of the pressure perturbation on the coordinate X](image)

Fig. 5 shows the time variation of the real part of the pressure amplitude near the wall, $Y=0$, when $Y=40$. In the graph B the result is shown in the time interval $2000<\tau<3000$. Initial values of $\pi_\tau$ of the graph B increased in one thousand
times are shown on the top graph A, from which one can see that in the initial time moments there are several frequencies. However, over time the most growing frequency is allocated which changes under the law \( \cos(\omega \tau + \psi_0) \exp(\omega \tau) \).

Fig. 6 shows the dependence of the real part of the pressure amplitude on space coordinate \( X \) at the two times analogously to Fig. 5. It is seen that at large times the spatial dependence is described by a harmonic dependence with the wave number \( \alpha = 2\pi / L \) rather well.

The necessary computational domain is determined empirically by comparing of the growth rates for different values of \( Y^* \) with the data of the classical theory. From Fig. 7 it is clearly visible that at the thickness \( Y^* = 40 \) results of numerical modeling differ from data of the classical theory a little.

\[
\omega = \omega_0 \exp(\omega_1 \tau) \cos(\omega_2 \tau + \psi_0) \exp(\omega_3 \tau).
\]

C. The numerical simulation interaction of a boundary layer with acoustic waves

The results of the numerical simulation of the boundary layer interaction with acoustic waves were conducted for the Mach number \( M_\infty = 2.0 \), Prandtl number \( Pr = 0.72 \), ratio of specific heats \( \gamma = 1.4 \). The dependence of the dynamic viscosity \( \mu^* \) on temperature was adopted in accordance with the Sutherland’s law.

The dimensions of the computational domain and its subdomains (Fig.2) were the following: \( AB=65mm, AA'=3mm, AD=2\pi/\beta \). The plate PLCB was located at a distance of 5mm from the entrance side (AEHD) to the computational domain. So, the plate length was equal to 60mm.

At a fine-tuning of the calculated scheme the comparison of numerical results (NS) with data of the parallel flow theory (BL) at a low-frequency approach was carried out. Such comparison for the fluctuations amplitude of the longitudinal velocity related to its value at \( Y^* = 10 \) (a) and maxima (b)
velocity related to its value on the boundary layer edge, \( Y = y/\delta = 8 \), at \( hi = 0 \), \( Re = 300 \), \( F = \omega \rho \mu / \mu_\infty = 1.26 \times 10^{-4} \) is given in fig. 9a. Data of the numerical simulation was obtained at free stream parameters: \( M_\infty = 2.0 \), \( v_\infty = 0 \), \( p_\infty^* = 5600 \text{ Pa}, T_\infty^* = 164 \text{ K} \). In general, matching the results of the approximate method (based on stability equations for low-frequency fluctuations) with direct numerical simulation data is satisfactory. If to normalize solutions on the corresponding maxima (Fig. 9b), it is possible to notice very good coincidence of results near a wall. From this comparison it is possible to conclude, that the computational grid used in numerical simulation is good enough. The greatest discrepancy occurs in the area of the boundary layer edge where the approximation theory is inapplicable.

Fig. 10 Instantaneous velocity perturbation contours induced by planar free-stream acoustic wave (\( F = 10^{-4} \) and \( hi = 0^\circ \))

The main results were obtained with the following computational subdomain grid. Both in the bottom subdomain and in the top subdomain (Fig.2) the number of nodes was 1500 on \( x \)-coordinate, 300 on \( y \)-coordinate and 40 on \( z \)-coordinate.

In Fig. 10a the instantaneous contour of the full velocity perturbation induced by planar free-stream acoustic wave (\( F = 10^{-4} \) and \( hi = 0^\circ \)) is shown in the area between the surface plate and the top side of computational domain. This contour is shown more detail near the plate in Fig. 10b. It can be seen that at the some distance from the plate edge a periodic structure under shock wave coincides with the structure above the shock wave practically. It tells about a weak intensity of the jump created by a leading edge area of the thin plate.

Within the boundary layer (Fig.10b) almost periodic structure on longitudinal coordinate with the period of a little bigger in the comparison with the period near a jump is visible. It follows from this that the phase velocity of the perturbation exceeds value of \( u^* _c (1 - 1/M_\infty) \), and it corresponds to eigen fluctuations (“Tollmien-Schlichting” waves) of a boundary layer [2]. Along with the strong change of the velocity perturbation intensity in the boundary layer the weak dependence of the perturbations amplitude in the area between the shock wave and the boundary layer edge is observed. Perturbations amplitude changing within this area is explained by the interference of an external acoustics with the acoustics which was created by the non-stationary boundary layer.

The dependence of the maximum disturbance amplitudes of a full velocity (\( \bar{u}^* = u^* \cos(hi) + w^* \sin(hi) \)) in the boundary layer, \( A_{\max}^u \) on the Reynolds number at the different sliding angles is shown in Fig. 11 (\( F = 0.445 \times 10^{-4} \)). At the small sliding angles perturbations amplitudes monotonously increases, at least up to \( Re = 500 \). With an increase of a sliding angle (\( hi = 30^\circ \)) there is a maximum in this dependence. It is displaced to a leading edge of a plate with the increase of \( hi \). The amplitude increase in the field of Reynolds numbers \( Re > 450 \) is connected with the boundary layer instability.

Fig. 11 Amplitude maximum of the full velocity in depending on a Reynolds number at different sliding angle

\[
A_{\max}^u \quad \text{hi= 0°} \quad 20° \quad 30° \quad 40° \quad 50°
\]

Fig. 12 Amplitudes maximum of the full velocity in depending on the orientation angle.

Finally, the dependence of maximum amplitude on the sliding angle at fixed positions \( x \) is shown in Fig. 12.
VII. CONCLUSION

In the paper disturbances inside the boundary layer are investigated at a free stream Mach number of 2.0.

The new method of stability problem solving of the boundary layer is proposed, which is based on an evolutionary perturbations development in time. This method can be called by the usual term - the establishing method. In contrast to the generally accepted method of establishing when the solution goes to the constant, in our case the solution goes to the exponential dependence on time. This method allows determine parameters of a disturbance with the greatest increment.

Influence of the gas blowing direction through a porous surface on the supersonic boundary layer stability was studied for the first time. In the contrast to the strong influence of normal blowing on the boundary layer stability, tangential blowing has a little effect on it.

The boundary layer receptivity process due to the interaction of three-dimensional acoustic waves with a flat plate and the evolution of disturbances inside the boundary layer are investigated numerically. Both the steady and unsteady solutions are obtained by solving the full Navier-Stokes equations using the ANSYS Fluent software package.

The simulation data for low-frequency disturbances near the plate surface well agree with the approximate results which were obtained on a base of the equations of the classical stability theory. The greatest discrepancy occurs in the area of the boundary layer edge where the approximation theory is inapplicable.

It has been found that interaction of sound with the boundary layer lead to an increase of the amplitude of disturbances inside the boundary. It has been shown that the amplitude of fluctuations inside the boundary layer may exceed the amplitude of the external acoustic field in many times and it depends on the parameters of the wave and the main flow. At a given position (Reynolds's number) there is an optimum sliding angle at which the maximum fluctuations of a boundary layer are excited. With increasing of the Reynolds number a critical value of a sliding angle decreases and the fluctuations intensity increases in a boundary layer.

The simulation of interaction of acoustic waves with the flat plate shows that “Tollmien-Schlichting” waves are generated in the boundary layer.

REFERENCES


