Global interval sensitivity analysis of Hermite probability density function percentiles

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Abstract— The derivation of a Hermite probability density function whose parameters are mean value, standard deviation, skewness and kurtosis is presented in the article. Global interval sensitivity analysis capable of evaluating the effects of all combinations of the intervals of input parameters on the model output is presented. Global interval sensitivity analysis was used to identify the effects of uncertainties of the parameters of Hermite probability density function on its 5 percentile. A strong influence of the interaction between skewness and kurtosis, which would not have been identified using local sensitivity analysis, was identified. The global interval sensitivity analysis and Hermite probability density function presented here provide in combination with Monte Carlo simulation methods a very powerful tool for the probabilistic evaluation of structural reliability.

Keywords— Interval, interaction, kurtosis, skewnes, quantile, probability, reliability, sensitivity analysis.

I. INTRODUCTION

SENSITIVITY analysis is generally considered an important part of high-quality analyses, which provide high degree of transparency to studies [1]. Sensitivity analysis is especially useful in determining which considerations are appropriate candidates for further data collection in order to decrease the degree of uncertainty occurring in the results. Sensitivity analysis is generally considered a minimum, necessary component of a quality risk assessment report [2] and is a desired component of multiple-criteria decision analysis (MCDA) or multiple-criteria decision-making (MCDM), which are sub-disciplines of operations research [3, 4].

Sensitivity analysis has over the past twenty years undergone an evolution from primitive tools based on the "one-factor-at-a-time" method to highly sophisticated variance-based methods. It has been demonstrated that the use of the "one-factor-at-a-time" method is irresponsible and unjustified if the analysed model has not been proven to be linear [5]. Recently, sensitivity analysis has become an increasingly important application in scientific disciplines in which it was previously not used or was used only sporadically [6-12]. Reviews of sensitivity analysis approaches across various scientific disciplines are discussed in [13]. Bibliometrics of the trends of various practices of sensitivity analysis over the past decade is published in [14]. Sensitivity analysis is most commonly applied in medicine and chemistry [14]. The application of sensitivity analysis in economics and decision making sciences is not as widespread as in biology, mathematics or physics [14]. It is interesting that sensitivity analysis is used relatively less in technical fields, particularly in analyses aimed at probabilistic evaluations of reliability. Reliability sensitivity analysis plays an important role in reliability design, reliability-based optimization design and reliability-based robust design [15].

A highly preferred method is the variance-based sensitivity analysis, which is frequently used in modelling within the Monte Carlo framework [16-18]. Variance-based measures of sensitivity are attractive due to their ability to handle non-linear responses and measure interaction effects in non-additive systems [1]. The method is based on the decomposition of the variance of the output, which is the basis for measuring sensitivity across the whole input space (global method). Variance-based sensitivity analysis examines the effects of the variances of input variables on the variance of the model output.

The usual focus of sensitivity analysis is to examine the influence of the variability of input variables on the variability of the model output, see, e.g., [19-21]. However, in technical practice we often need to measure the effect of the variability of input variables on the characteristic or design values of the model output. Characteristic and design values are usually quantiles of probability density functions that describe the relative likelihood for the output random variable. Classic variance-based measures do not provide complex and satisfactory solutions of sensitivity, because the value of the quantile is not merely a function of the variance of the model output, but is also influenced by additional statistical characteristics, such as mean value, skewness, kurtosis, etc.

The concept of interval sensitivity analysis capable of studying the effects of the statistical characteristics of the input variables on the quantile of the model output is presented in this paper. The output of the sensitivity analysis is the ordering of sensitivity coefficients. Interval sensitivity analysis is ranked among the methods of global sensitivity analysis, because the order of dominant influences includes pairs and triples, and not just the individual factors. This can be used to improve our knowledge of models based on non-linear variants of FEM, see, e.g. [22-26], to identify the most influential input factors and to propose dominant checks of model behaviour.

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II. Hermite Probability Density Function

During the statistical evaluation of experimental data we often encounter situations where the statistical set exhibits small skewness and kurtosis, see, e.g. [27]. Conventional pdfs like Gauss or lognormal do not have skewness and kurtosis as optional parameters. In the event that approximation using customary pdf is not sufficiently concise, it is necessary to seek a better solution. A suitable pdf is the four-parameter Hermite pdf $\varphi_H(x)$, which respects not only the arithmetic mean and standard deviation, but also the skewness and kurtosis of valid observations.

A. Hermite pdf - First Variant

The Hermite pdf consists of the standard Gauss pdf (1), which is multiplied by the Hermite polynomial (2). Parameters c_1, c_2, c_3, c_4, c_5 are the solutions of equations (3) to (7), which express conditions that the Hermite pdf must fulfil.

$$\varphi_G(x) = \exp(-0.5x^2)/\sqrt{2\pi}, \tag{1}$$

$$\varphi_H(x) = \varphi_G(x)(c_1x^4 + c_2x^3 + c_3x^2 + c_4x + c_5), \tag{2}$$

where $x \in \mathbf{R}$. Equation (3) expresses the condition that the area beneath $\varphi_H(x)$ is equal to 1. Equations (4), (5), (6), (7) represent the mean, variance, skewness and kurtosis.

$$\int_{-\infty}^{\infty} \varphi_H(x)dx = 1, \tag{3}$$

$$\int_{-\infty}^{\infty} x\varphi_H(x)dx = 0, \tag{4}$$

$$\int_{-\infty}^{\infty} x^2\varphi_H(x)dx = 1, \tag{5}$$

$$\int_{-\infty}^{\infty} x^3\varphi_H(x)dx = a, \tag{6}$$

$$\int_{-\infty}^{\infty} x^4\varphi_H(x)dx = k. \tag{7}$$

Integration of (3) to (7) leads to five linear equations whose solutions are parameters c_1, c_2, c_3, c_4, c_5, see (8) to (12). The system of linear equations is clearly written in (13).

$$3c_1 + c_3 + c_5 = 1, \tag{8}$$

$$3c_2 + c_4 = 0, \tag{9}$$

$$15c_1 + 3c_3 + c_5 = 1, \tag{10}$$

$$15c_2 + 3c_4 = a, \tag{11}$$

$$105c_1 + 15c_3 + 3c_5 = k, \tag{12}$$

$$\begin{bmatrix} 3 & 0 & 1 & 0 & 1 \\ 0 & 3 & 0 & 1 & 0 \\ 15 & 0 & 3 & 0 & 1 \\ 0 & 15 & 0 & 3 & 0 \\ 105 & 0 & 15 & 0 & 3 \end{bmatrix} \cdot \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \\ a \\ k \end{bmatrix}, \tag{13}$$

$$c_1 = (k-3)/24, \tag{14}$$

$$c_2 = a/6, \tag{15}$$

$$c_3 = (3-k)/4, \tag{16}$$

$$c_4 = -a/2, \tag{17}$$

$$c_5 = (k+5)/8. \tag{18}$$

Upon substitution of c_1, c_2, c_3, c_4, c_5 into (2), the equation of the Hermite pdf can be rewritten in the form (19).

$$\varphi_H(x) = \varphi_G(x)\left[1 + \frac{a}{6}(x^3 - 3x) + \frac{(k-3)}{24}(x^4 - 6x^2 + 3)\right]. \tag{19}$$

Function (19) can not be used for arbitrary parameters a, k, because the condition $\varphi_H(x) \geq 0$ is not automatically fulfilled for $\forall x \in (-\infty, \infty)$.

If a=0 and $\varphi_H(x)>0$ then (20) must hold.

$$1 + \frac{(k-3)}{24}(x^4 - 6x^2 + 3) > 0. \tag{20}$$

Inequality (20) is fulfilled if $k \in [3, 7) \cup x \in (-\infty, \infty)$ or $k \in (0, 3) \cup x \in [x_L, x_R]$ where

$$x_{L,R} = \mp\sqrt[4]{\frac{3}{3-k}\left(\sqrt{24}\sqrt{k^2 - 10k + 21} - 5k + 23\right)}. \tag{21}$$

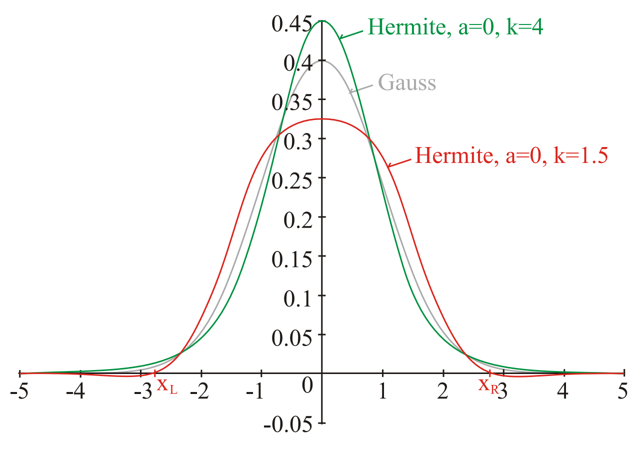

Fig. 1 Examples of $\varphi_H(x)$ with a=0

Examples of pdfs (19) with a=0 are shown in Fig. 1. Interval [x_L, x_R] defines the region around the mean value where $\varphi_H(x) \geq 0$. If the interval [x_L, x_R] is sufficiently wide, (19) can be replaced with a truncated Hermite pdf (22) defined on the interval $x \in$ [x_L, x_R].

$$\overline{\varphi}_H(x) = c_6 \varphi_H(x) \quad \begin{cases} \overline{\varphi}_H(x) \geq 0 \ \ \forall\ x \in \langle x_L, x_R \rangle \\ \overline{\varphi}_H(x) = 0 \ \ \text{otherwise} \end{cases} \tag{22}$$

Statistical moments from (22) are only approximately equal to the required statistical moments, which is due to the truncation of the pdf, see (24) to (27). For larger deviations of (22) from the Gauss pdf (1) the parameters of (22) can be numerically set so that the statistical moments are as close as possible to the required statistical moments and (28) is simultaneously fulfilled. Parameter c_6 is approximately equal to 0.99 for common values of a, k. The closer c_6 is to 1, the more (22) approaches the Gauss pdf (19) and parameters a, k are closer to the required values of skewness and kurtosis.

$$\int_{x_L}^{x_R} \overline{\varphi}_H(x)dx = 1, \tag{23}$$

$$\int_{x_L}^{x_R} x\overline{\varphi}_H(x)dx \approx 0, \tag{24}$$

$$\int_{x_L}^{x_R} x^2\overline{\varphi}_H(x)dx \approx 1, \tag{25}$$

$$\int_{x_L}^{x_R} x^3\overline{\varphi}_H(x)dx \approx a, \tag{26}$$

$$\int_{x_L}^{x_R} x^4\overline{\varphi}_H(x)dx \approx k, \tag{27}$$

$$\overline{\varphi}_H(x) \geq 0 \ \ \forall\ x \in [x_L, x_R]. \tag{28}$$

Examples of parameters a, k of the pdf (22) and their obtained statistical moments are shown in Fig. 2 and Fig. 3. Fig. 2 depicts functions with relatively high values of parameters a, k. The mean value, standard deviation, skewness and kurtosis are numerically evaluated on the interval [x_L, x_R] where $\varphi_H(x) \geq 0$. It is apparent that parameters a, k are only approximately equal to the values of skewness and kurtosis, which is especially observed in Fig. 2. The agreement between the parameters used in (22) and the required statistical moments decreases with increasing truncation.

Fig. 3 shows (22) with relatively small values of parameters a, k. If parameter a is close to zero and parameter k is close to three, then the skewness approaches a and the kurtosis approaches k relatively accurately.

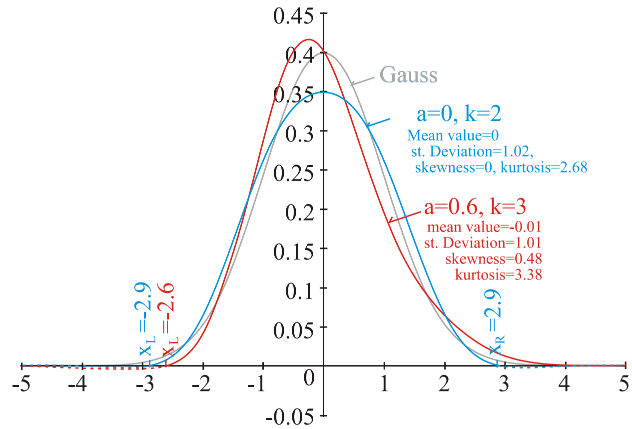

Fig. 2 Examples of $\overline{\varphi}_H(x)$ with high values of a, k

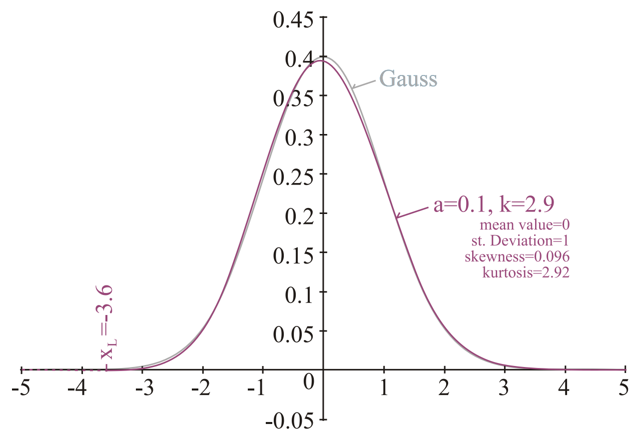

Fig. 3 Examples of $\overline{\varphi}_H(x)$ with small values of a, k

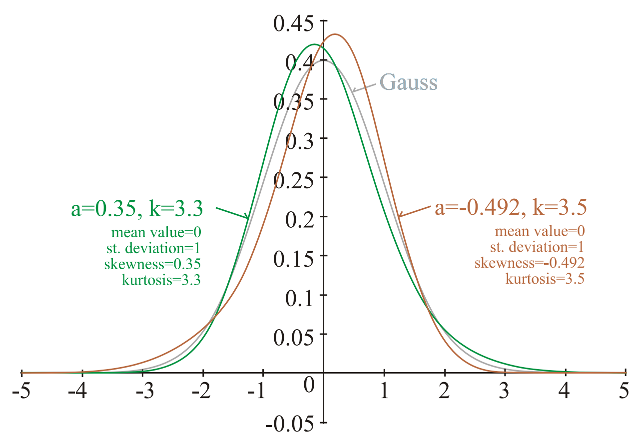

Fig. 4 Examples of $\varphi_H(x)$ with k>3

Examples of (19) with k>3 are shown in Fig. 4. It is apparent that for k>3 certain values of a exist, for which the truncation need not be applied and (19) can be used without further modification.

The boundary between input spaces $\varphi_H(x)$ and $\overline{\varphi}_H(x)$ is defined by pairs of parameters a, k, which were numerically evaluated for 15 points on the boundary. The boundary is shown in Fig. 5. It is clear that $\overline{\varphi}_H(x)$ must always be applied for $k<3$. Conversely, for $k \geq 3$ certain values of parameter a exist, for which $\varphi_H(x)$ can be applied, see Fig. 5.

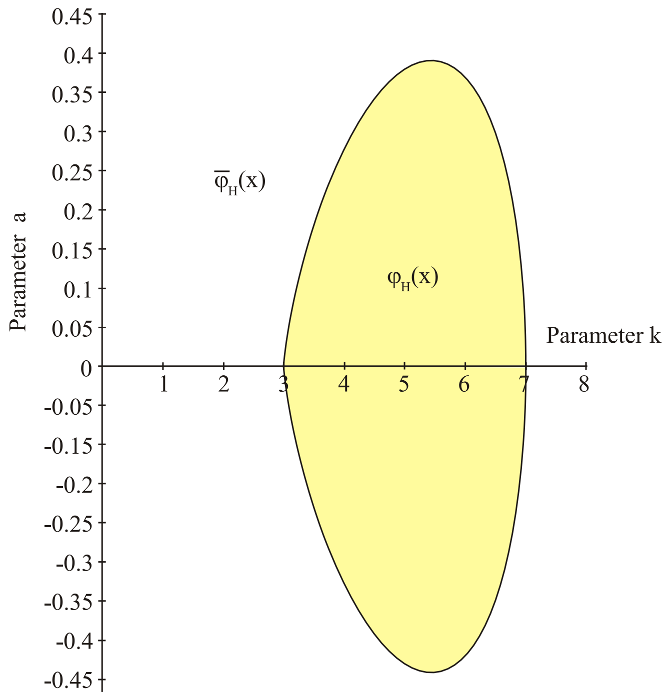

Fig. 5 Input space for the application of $\varphi_H(x)$ and $\overline{\varphi}_H(x)$

The effects of parameters a, k on the skewness and kurtosis and values of x_L and x_R that determine the domain of (22) are shown in Fig. 6 to Fig. 9. The results in Fig. 6 and Fig. 7 were obtained using numerical integration. Thresholds x_L and x_R were sought on the interval [-10, 10] using the bisection method.

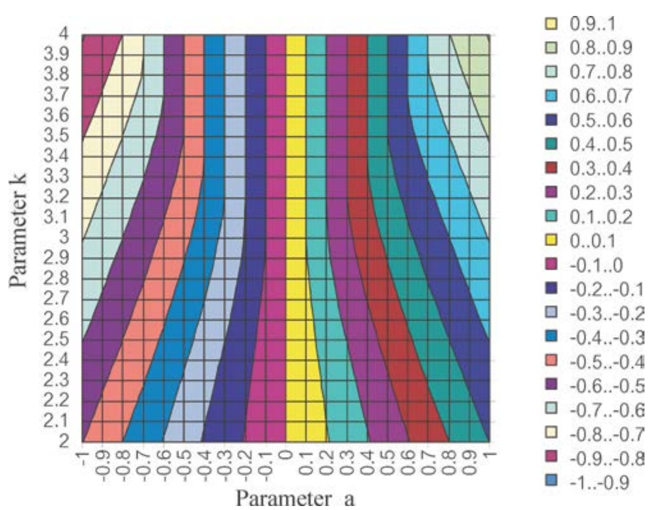

Fig. 6 Skewness of $\overline{\varphi}_H(x)$ for parameters a, k

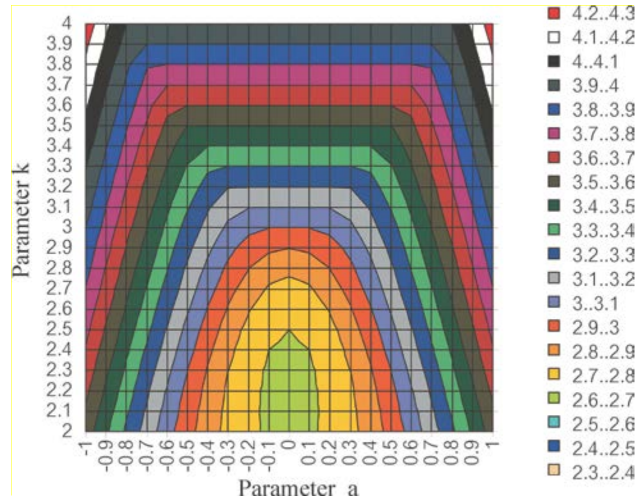

Fig. 7 Kurtosis of $\overline{\varphi}_H(x)$ for parameters a, k

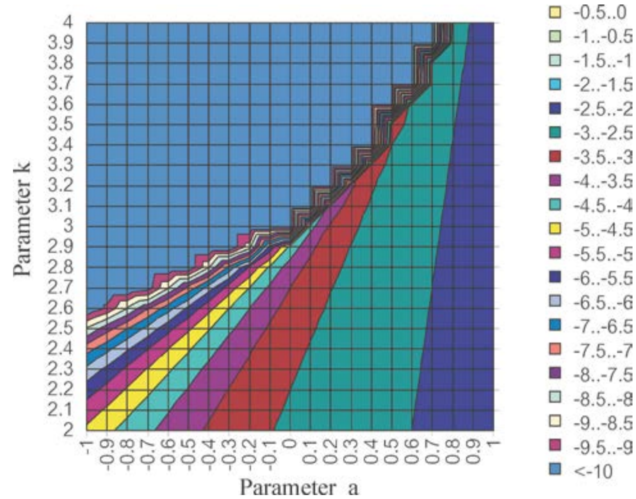

Fig. 8 Left threshold x_L of $\overline{\varphi}_H(x)$ for parameters a, k

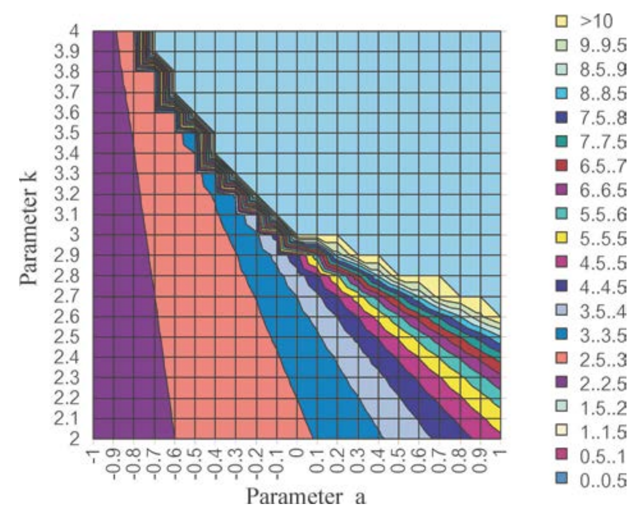

Fig. 9 Right threshold x_R of $\overline{\varphi}_H(x)$ for parameters a, k

B. Hermite pdf - Second Variant

Assuming the Hermite pdf in the form (2) is not the only way of introducing skewness and kurtosis as parameters of the pdf. With regard to the requirement that the exponent of the polynomial is at most three, we can write:

$$\varphi_{H2}(x) = \varphi_G(x)(c_1x^3 + c_2x^2 + c_3x + c_4). \tag{29}$$

Parameters c_1 to c_4 can be calculated from the four conditions.

$$\int_{-\infty}^{\infty} \varphi_{H2}(x)dx = 1, \tag{30}$$

$$\int_{-\infty}^{\infty} x\varphi_{H2}(x)dx = 0, \tag{31}$$

$$\int_{-\infty}^{\infty} x^3\varphi_{H2}(x)dx = a, \tag{32}$$

$$\int_{-\infty}^{\infty} x^4\varphi_{H2}(x)dx = k. \tag{33}$$

Equations (31) to (33) express the conditions for the mean value, skewness and kurtosis, but do not prescribe the condition for the standard deviation. Integration of (30) to (33) results in four linear equations (34) to (37), whose solutions are parameters c_1, c_2, c_3, c_4, see (39) to (42). The system of linear equations is written in (38).

$$c_2 + c_4 = 1, \tag{34}$$

$$3c_1 + c_3 = 0, \tag{35}$$

$$3(5c_1 + c_3) = a, \tag{36}$$

$$3(5c_2 + c_4) = k, \tag{37}$$

$$\begin{bmatrix} 0 & 1 & 0 & 1 \\ 3 & 0 & 1 & 0 \\ 15 & 0 & 3 & 0 \\ 0 & 15 & 0 & 3 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ a \\ k \end{bmatrix}, \tag{38}$$

$$c_1 = a/6, \tag{39}$$

$$c_2 = (k-3)/12, \tag{40}$$

$$c_3 = -a/2, \tag{41}$$

$$c_4 = (15-k)/12. \tag{42}$$

Substitution of c_1, c_2, c_3, c_4 into (29) yields (43), which is an alternative to (19).

$$\varphi_{H2}(x) = \varphi_G(x)\left[1 + \frac{a}{6}(x^3 - 3x) + \frac{(k-3)}{12}(x^2 - 1)\right]. \tag{43}$$

The variance of (43) is calculated in (44).

$$\int_{-\infty}^{\infty} x^2\varphi_{H2}(x)dx = \frac{k+3}{6}, \tag{44}$$

The Hermite pdf is also used in the software Statrel 3.10, which is a tool for reliability-oriented statistical analysis of data including simulation and analysis of time series. Failure to set the standard deviation in the parametric pdf does not present a limitation for application within the framework of Monte Carlo type simulation methods. For random realizations the mean value and standard deviation can be additional set through the transformation of the random realizations.

$$\bar{\varphi}_{H2}(x) = c_6\varphi_H(x) \quad \begin{cases} \bar{\varphi}_{H2}(x) \geq 0 \;\; \forall\, x \in [x_L, x_R] \\ \bar{\varphi}_{H2}(x) = 0 \;\; \text{otherwise.} \end{cases} \tag{45}$$

If $\varphi_{H2}(x)$<0 truncation of (43) yields (45). It may be noted that, neither (45) nor (43) provide too many advantages in practical applications in comparison to (22) or (19). In practical applications it is necessary to perform distrubution tests such as the Kolmogorov-Smirnov test, which compares the empirical cumulative distribution function of the data to the fitted cumulative distribution function, see, e.g., [28]. Due to the great adaptability of (22) and (19), the distribution tests reject distribution fitting less frequently than when applied to two or three-parameter pdfs.

III. Interval Sensitivity Analysis

Sensitivity analysis is the study of how uncertainty in the output of a model (numerical or otherwise) can be apportioned to different sources of uncertainty in the model input factors [1]. The notion of interval sensitivity analysis is used differently in various scientific disciplines, see e.g. [29-31]. The global interval sensitivity analysis presented in this article is based on the work [32].

The uncertainty of input factors x_i, for i=1, 2, ...n, can be expressed by the intervals [${}^{left}x_i$, ${}^{right}x_i$]. Interval sensitivity analysis can be used to estimate the effect of the size of the endpoints of interval [${}^{left}x_i$, ${}^{right}x_i$] on the size of the interval of the model output [${}^{left}y$, ${}^{right}y$]. The effect of each interval [${}^{left}x_i$, ${}^{right}x_i$] on interval [${}^{left}y$, ${}^{right}y$] can be evaluated using so-called factorial designs [32-34].

Each input interval can be assumed to have two possible values, called 'levels' (lower and upper endpoints of the interval), which are denoted as '-' (${}^{left}x_i$) or '+' (${}^{right}x_i$). Sensitivity analysis is used to simulate all possible combinations of lower '-' and upper '+' endpoints of the intervals on the model output. The computational cost is 2^n runs.

Let us consider three variables x_1, x_2, x_3, which define three intervals $[{}^{left}x_1,\ {}^{right}x_2]$, $[{}^{left}x_2,\ {}^{right}x_2]$, $[{}^{left}x_3,\ {}^{right}x_3]$. There exist $k=2^3=8$ combinations of all endpoints of input intervals. We denote the outcome of the j-th combination as ${}^{j}y$, with $j=1, \ldots, k$. The design matrix determines the $k=8$ endpoint combinations, see Table 1.

TABLE I

2^3 OUTPUT COMBINATIONS

Run j	1	2	3	4	5	6	7	8
x_1	-	-	-	-	+	+	+	+
x_2	-	-	+	+	-	-	+	+
x_3	-	+	-	+	-	+	-	+
x_1-x_2	+	+	-	-	-	-	+	+
x_1-x_3	+	-	+	-	-	+	-	+
x_2-x_3	+	-	-	+	+	-	-	+
x_1-x_2-x_3	-	+	+	-	+	-	-	+
${}^{j}y$	${}^{1}y$	${}^{2}y$	${}^{3}y$	${}^{4}y$	${}^{5}y$	${}^{6}y$	${}^{7}y$	${}^{8}y$

The design matrix listed in Table 1 is used to calculate the main and interactions effects. The difference in output values, e.g. ${}^{5}y$ and ${}^{1}y$, arises solely from variations on the level of variable x_1, whereas the other two variables x_2, x_3 are kept the same. Four individual measures exist for x_1. The main effect of a variable is defined as the average effect of that variable over all conditions of other factors. The main effect, e.g. C_1 of x_1, is calculated as the average of four individual measures. Main effects C_2 and C_3 are determined analogously.

$$C_1 = \frac{\left({}^{5}y-{}^{1}y\right)+\left({}^{6}y-{}^{2}y\right)+\left({}^{7}y-{}^{3}y\right)+\left({}^{8}y-{}^{4}y\right)}{4}, \qquad (46)$$

$$C_2 = \frac{\left({}^{3}y-{}^{1}y\right)+\left({}^{4}y-{}^{2}y\right)+\left({}^{7}y-{}^{5}y\right)+\left({}^{8}y-{}^{6}y\right)}{4}, \qquad (47)$$

$$C_3 = \frac{\left({}^{2}y-{}^{1}y\right)+\left({}^{4}y-{}^{3}y\right)+\left({}^{6}y-{}^{5}y\right)+\left({}^{8}y-{}^{7}y\right)}{4}, \qquad (48)$$

The interaction between x_1-x_2 is evaluated by coefficient $C_{1\text{-}2}$, the interaction between x_1-x_3 is evaluated by coefficient $C_{1\text{-}3}$, the interaction between x_2-x_3 is evaluated by coefficient $C_{2\text{-}3}$. Coefficient $C_{1\text{-}2\text{-}3}$ is determined using the sign combination, which has not been previously used.

$$C_{1-2} = \frac{{}^{1}y+{}^{2}y-{}^{3}y-{}^{4}y-{}^{5}y-{}^{6}y+{}^{7}y+{}^{8}y}{4}, \qquad (49)$$

$$C_{1-3} = \frac{{}^{1}y-{}^{2}y+{}^{3}y-{}^{4}y-{}^{5}y+{}^{6}y-{}^{7}y+{}^{8}y}{4}, \qquad (50)$$

$$C_{2-3} = \frac{{}^{1}y-{}^{2}y-{}^{3}y+{}^{4}y+{}^{5}y-{}^{6}y-{}^{7}y+{}^{8}y}{4}, \qquad (51)$$

$$C_{1-2-3} = \frac{-{}^{1}y+{}^{2}y+{}^{3}y-{}^{4}y+{}^{5}y-{}^{6}y-{}^{7}y+{}^{8}y}{4}. \qquad (52)$$

IV. SENSITIVITY ANALYSIS OF HERMITE PDF PERCENTILE

The influence of parameters a, k and the standard deviation σ on the 5 percentile of the Hermite pdf was investigated next. The following intervals are considered: $\sigma\in[0.97,\ 1.03]$, $a\in[-0.2,\ 0.2]$, $k\in[2.6,\ 3.4]$. Eight values of the 5 percentile were calculated for the evaluation of the sensitivity analysis, thus taking into account all combinations of the endpoints of the interval on the size of the 5 percentile. Results of the global interval sensitivity analysis are shown n Fig. 10.

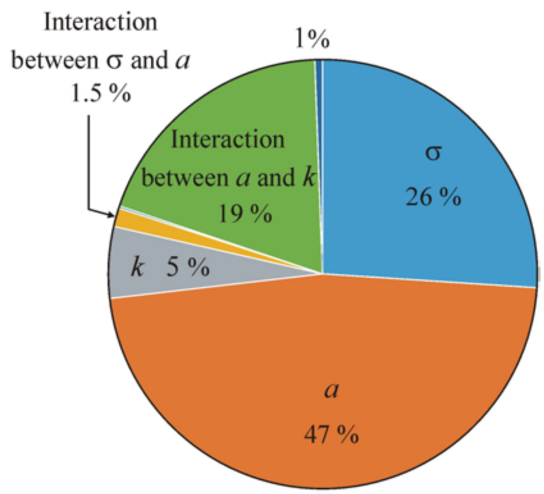

Fig. 10 Result of interval sensitivity analysis of 5 percentile

Fig. 10 shows that the skewness has a dominant influence on the 5 percentile. Conversely, the influence of kurtosis is relatively small. A strong interaction effect between skewness and kurtosis is apparent. This is very interesting if we consider that the actual influence of kurtosis is relatively small. Kurtosis acts as an accelerator of the effect of skewness and is an important parameter in approximation problems of statistical observations using parametric pdfs, and the calculation of quantiles.

V. CONCLUSION

A four-parameter Hermite pdf (19), which takes into account the effects of the mean value, standard deviation, skewness and kurtosis was derived in the article. Hermite pdf is very valuable, particularly when calculating quantiles, which are sensitive to skewness and kurtosis.

The concept of interval sensitivity analysis based on [32], which can be used to study the effects of all combinations of endpoints of intervals of deterministic parameters or statistical parameters of pdfs on the interval of the model output, was published. Interval sensitivity analysis is very useful for examining the effect of changes in pdf parameters on their quantiles. Quantiles are often considered as characteristic or design values [35, 36], reliability is often analyzed using probability [37]. Interval sensitivity analysis can be applied in structural reliability analysis to study the influence of partial safety factors on design reliability [38].

An example, which examines the sensitivity of the 5 percentile to changes in the standard deviation, skewness and kurtosis of a Hermite pdf, was illustrated. Interval sensitivity analysis is global, therefore, it can also quantify the effects of higher order interactions between input variables on the studied output. Results of the interval sensitivity analysis of the 5 percentile revealed an interesting interaction between the skewness and kurtosis. These interactions illustrate the importance of kurtosis as a significant statistical characteristic, which alone has little influence on the size of the quantile, but in combination with skewness has a significant influence.

Interval sensitivity analysis has great potential in many scientific and engineering applications that study the ranking performance of MCDM methods [39-42]. Numerous methods and techniques for solving MCDM problems exist, however, the use of global sensitivity analysis is sporadic, see, e.g. [43]. The application of the presented global interval sensitivity analysis in a fuzzy environment could be used to verify the stability and validity of results of MCDM methods and provide the results a high degree of transparency.

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