Performance of macrodiversity system in the presence of Gamma long term fading and different short term fading

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Abstract—In this paper, the performance of macrodiversity system with macrodiversity selection reception and two microdiversity maximum ratio combining (MRC) receivers are derived. In the first microdiversity receiver, received signal suffers correlated Gamma long term fading and Rayleigh short term fading, and in the second microdiversity receiver, received signal experiences correlated Gamma long term fading and κ - μ short term fading. Macrodiversity (MAD) receiver is of selection combining (SC) type and it mitigates Gamma long term fading effects; the first microdiversity (MID) MRC receiver reduces Rayleigh short term fading effects and the second one eliminates κ - μ short term fading effects on system performance. Cumulative distribution functions (CDF) and level crossing rates of signals at outputs of MID receivers are determined. Based on them,

the level crossing rate (LCR) of MAD Selection Combining (SC) receiver output signal is calculated. Some figures are presented to show the influence of severity parameter of Nakagami-*m* short term fading, Rician factor of Rician fading, Rician factor of κ - μ fading, severity parameter and correlation coefficient of Gamma long term fading on LCR.

Keywords—Cumulative distribution function (CDF), macrodiversity reception, microdiversity reception, Gamma fading, κ - μ short term fading, Rayleigh fading, level crossing rate.

I. INTRODUCTION

MACRODIVERSITY system can be used to reduce large scale fading effects and smale scale fading effects on the system performance [1]. Macrodiversity system has MAD SC receiver and two MID MRC receivers. MID MRC receivers study signal envelopes with multiple antennas at base station resulting in reduction of short term fading effects on system performance. MAD SC receiver considers and combines signal envelopes with antennas distributed at base stations resulting in long term fading effects on system performance reduction

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[2] [3].

In the first MID MRC receiver, received signal envelope is under the influence of correlated Gamma large scale fading and κ - μ small scale fading. In the second MID MRC receiver signal is subjected to correlated Gamma large scale fading and Rayleigh small scale fading effects.

The κ - μ fading is general fading with two parameters [4] [5]. One parameter is Rician factor which can be evaluated as ratio of dominant component power and scattering components power, and severity parameter μ which is in relation to the number of clusters in propagation channel. The κ - μ fading channel reduces to Rician fading channel for μ =1 and κ - μ fading channel becomes Nakagami-*m* fading channel for κ =0 and κ - μ fading channel becomes Rayleigh fading channel for κ =0 and μ =1.

Long term fading symbol envelope average power variation can be described by using log-normal distribution and Gamma distribution. Log-normal distribution describes better variations of signal envelope average power then Gamma distribution, but the expressions for system performance of wireless system can be calculated in the closed form when Gamma distribution describes signal envelope average power variations [2].

There are more works in open technical literature considering level crossing rate and average fade duration of radio mobile communication wireless systems in the presence of long term fading and short term fading.

In [6] and [7], MAD system with MAD SC receiver and two MID MRC receivers is analyzed. Level crossing rate and average fade duration of proposed radio system are evaluated for case when received signal experiences Gamma large scale fading and Nakagami-m small scale fading. In [8], MAD reception with MAD SC reception and two MID MRCs are analyzed and the second order statistics are evaluated when received signal is subjected to Gamma long term fading and Rician short term fading. This MAD system with three branches MID reception is considered in [9] and LCR of system is found.

Macrodiversity reception in the presence of Gamma long term fading and κ - μ short term fading is analyzed and AFD and LCR of proposed system are evaluated in [10]. In work [11], LCR is evaluated for macrodiversity reception with MAD SC reception and two MID MRC reception operating over Gamma shadowed Weibull short term fading channel under presence of Weibull cochannel interference.

MAD system including MAD SC receiver and two MID SC receivers is considered in [12]. Received signal experiences both, long term fading and short term fading, whereby MID SC receivers reduce Rayleigh fading effects on system performance and MAD SC receivers mitigate Gamma shadow effects on system performance. Closed form expressions for LCR of MID SC receivers output signals envelopes are calculated and used for evaluation of LCR of MAD SC receiver output signal envelope.

In this paper, MAD system with MAD SC receiver and two MID MRC receivers when received signal experiences correlated Gamma long term fading and κ - μ short term fading at inputs of the first MID MRC receiver and correlated Gamma long term fading and Rayleigh short term fading at inputs of the second MID MRC receiver is examined. MAD SC receiver reduces Gamma long term fading effects on system performance, the first MID MRC receiver reduces κ - μ short term fading and the second MID MRC receiver reduces Rayleigh short term fading effects on system performance.

II. MODEL OF MACRODIVERSITY SYSTEM WITH TWO MRC MICRODIVERSITY RECEIVERS

Macrodiversity system with MAD SC receiver and two MID MRC receivers is described in this paper. Received signal at inputs in the first MID MRC receiver experiences correlated Gamma long term fading and Rayleigh short term fading, and received signal at inputs in the second MID MRC receiver experiences correlated Gamma long term fading and κ - μ short term fading. Model of MAD receiver is shown in Fig. 1.

Signal envelopes at inputs in the first MID MRC receiver are denoted with x_{11} and x_{12} , and at inputs in the second MID MRC receiver with x_{21} and x_{22} . Signal envelopes at outputs of MID MRC receivers are denoted with x_1 and x_2 , and signal envelope at output of MAD SC receiver is denoted with x.

Signal envelopes x_{11} and x_{12} have Rayleigh distribution:

$$p_{x_{li}}(x_{li}) = \frac{2x_{li}}{\Omega_l} \cdot e^{-\frac{x_{li}}{\Omega_l}}, \qquad x_{li} \ge 0, \ i = 1, 2; \quad (1)$$

 Ω_1 is average power of x_{1i} .



Fig.1. System model

Signal envelopes x_{21} and x_{22} follow κ - μ distribution:

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$$p_{x_{2i}}(x_{1i}) = \frac{2\mu(k+1)^{\frac{\mu+1}{2}}x_{2i}^{\mu}}{k^{\frac{\mu-1}{2}}e^{k\mu}\Omega_{2}^{\frac{\mu+1}{2}}} \cdot e^{-\frac{\mu(k+1)}{\Omega_{2}}x_{2i}^{2}}$$

$$I_{\mu-1}\left(2\mu\sqrt{\frac{k(k+1)}{\Omega_{2}}}x_{2i}\right) = \frac{2\mu(k+1)^{\frac{\mu+1}{2}}x_{2i}^{\mu}}{k^{\frac{\mu-1}{2}}e^{k\mu}\Omega_{2}^{\frac{\mu+1}{2}}} \cdot \frac{1}{k^{\frac{\mu-1}{2}}e^{k\mu}\Omega_{2}^{\frac{\mu+1}{2}}} \cdot \frac{1}{k^{\frac{\mu+1}{2}}e^{k\mu}\Omega_{2}^{\frac{\mu+1}{2}}} \cdot \frac{1}{k^{\frac{\mu+1}{2}}e^{k\mu}\Omega_{2}^{\frac{\mu+1}{2}}} \cdot \frac{1}{k^{\frac{\mu+1}{2}}e^{k\mu}\Omega_{2}^{\frac{\mu+1}{2}}} \cdot \frac{1}{k^{\frac{\mu+1}{2}}e^{k\mu}\Omega_{2}^{\frac{\mu+1}{2}}} \cdot \frac{1}{k^{\frac{\mu+1}{2}}e^{k\mu}\Omega_{2}^{\frac{\mu+1}{2}}} \cdot \frac{1}{k^{\frac{\mu+1}{2}}e^{k\mu}\Omega_{2}^{\frac{\mu+1}{2}}} \cdot \frac{$$

μ is severity parameter of κ-μ multipath fading, and $Ω_2$ is average power of x_{2i} .

Squared random variable x_i is equal to the sum of squared variables x_{i1} and x_{i2} :

$$x_2^2 = x_{i1}^2 + x_{i2}^2, \ i = 1, 2.$$
 (3)

Probability density function (PDF) of the first MID MRC output signal envelope x_1 is:

$$p_{x_{1}}(x_{1}) = \frac{2}{\Gamma(2)} \left(\frac{2}{\Omega_{1}}\right)^{2} x_{1}^{3} \cdot e^{-\frac{2}{\Omega_{1}}x_{1}^{2}} =$$
$$= \frac{8}{\Omega_{1}^{2}} x_{1}^{3} \cdot e^{-\frac{2}{\Omega_{1}}x_{1}^{2}}, x_{1} \ge 0.$$
(4)

PDF of the second MID MRC output signal envelope x_2 is:

$$p_{x_{2}}(x_{2}) = \frac{4\mu(k+1)^{\frac{2\mu+1}{2}}}{k^{\frac{2\mu-1}{2}}e^{2k\mu}\Omega_{2}^{\frac{2\mu+1}{2}}2^{\frac{2\mu+1}{2}}} \cdot \frac{1}{\sum_{i_{2}=0}^{\infty}} \left(2\mu\sqrt{\frac{k(k+1)}{2\Omega_{2}}}\right)^{2i_{2}+2\mu-1}} \cdot \frac{1}{i_{2}!\Gamma(i_{2}+2\mu)} \cdot \frac{1}{x_{2}^{2i_{1}+4\mu-1}}e^{-\frac{\mu(k+1)}{\Omega_{2}}x_{2}^{2}}, x_{2} \ge 0.$$
(5)

Cumulative distribution function (CDF) of x_1 is:

$$F_{x_{1}}(x_{1}) = \frac{8}{\Omega_{1}^{2}} \frac{1}{2} \left(\frac{\Omega_{1}}{2}\right)^{2} \gamma \left(2, \frac{2}{\Omega_{1}} x_{1}^{2}\right) = \gamma \left(2, \frac{2}{\Omega_{1}} x_{1}^{2}\right)$$
(6)

CDF of x_2 is:

$$F_{x_2}(x_2) = \frac{4\mu(k+1)^{\frac{2\mu+1}{2}}}{k^{\mu-1/2}e^{2k\mu}\Omega_2^{\mu+1/2}2^{\mu+1/2}} \cdot \frac{1}{\sum_{i_2=0}^{\infty} \left(2\mu\sqrt{\frac{k(k+1)}{2\Omega_2}}\right)^{2i_2+2\mu-1}} \cdot \frac{1}{i_2!\Gamma(i_2+2\mu)} \cdot \frac{1}{\sum_{i_2=0}^{\infty} \left(2\mu\sqrt{\frac{k(k+1)}{2\Omega_2}}\right)^{2i_2+2\mu-1}} \cdot \frac{1}{\sum_{i_2=0}^{\infty} \left(2\mu\sqrt{\frac{k(k+1)}{2\Omega_2}\right)^{2i_2+2\mu-1}} \cdot \frac{1}{\sum_{i_2=0}^{\infty} \left(2\mu\sqrt{\frac{k(k+1)}{2\Omega_2}}\right)^{2i_2+2\mu-1}} \cdot \frac{1}{\sum_{i_2=0}^{\infty} \left(2\mu\sqrt{\frac{k(k+1)}{2\Omega_2}}\right)^{2i_2+2\mu-1}} \cdot \frac{1}{\sum_{i_2=0}^{\infty} \left(2\mu\sqrt{\frac{k(k+1)}{2\Omega_2}\right)^{2i_2+2\mu-1}} \cdot \frac{1}{\sum_{i_2=0}^{\infty} \left(2\mu\sqrt{\frac{k(k+1)}{2\Omega_2}}\right)^{2i_2+2\mu-1}} \cdot \frac{1}{\sum_{i_2=0}^{\infty} \left(2\mu\sqrt{\frac{k(k+1)}{2\Omega_2}\right)^{2i_2+2\mu-1}} \cdot \frac{1}{\sum_{i_2=0}^{\infty} \left(2\mu\sqrt{\frac{k(k+1)}{2\Omega_2}\right)^{2i_2+2\mu-1}} \cdot \frac{1}{\sum_{i_2=0}^{\infty} \left(2\mu\sqrt{\frac{k(k+1)}{2\Omega_2}\right)^{2i_2+2\mu-1}} \cdot \frac$$

$$\cdot \frac{1}{2} \left(\frac{\Omega_2}{\mu(k+1)} \right)^{i_2+2\mu} \gamma \left(i_2 + 2\mu, \frac{\mu(k+1)}{\Omega_2} x_1^2 \right)$$
(7)

Signal envelope average powers Ω_1 and Ω_2 has JPDF:

$$p_{\Omega_{1}\Omega_{2}}(\Omega_{1}\Omega_{2}) = \frac{1}{\Gamma(c)(1-\rho^{2})\rho^{c-1}\Omega_{0}^{c+1}} \cdot \frac{1}{\Gamma(c)(1-\rho^{2})\rho^{c-1}\Omega_{0}^{c+1}} \cdot \frac{1}{i_{3}!\Gamma(i_{3}+c)} \cdot \frac{1}{i_{3}!\Gamma(i_{3}+c)} \cdot \frac{1}{\Omega_{1}^{i_{3}+c-1}\Omega_{2}^{i_{3}+c-1}} \cdot e^{-\frac{\Omega_{1}+\Omega_{2}}{\Omega_{0}(1-\rho^{2})}}, \ \Omega_{1} \ge 0, \ \Omega_{2} \ge 0$$
(8)

where *c* is Gamma long term fading severity parameter, ρ is Gamma long term fading correlation coefficient and Ω_0 is average power of Ω_1 and Ω_2 .

Probability density function of *x* is:

$$p_{x}(x) = \int_{0}^{\infty} d\Omega_{1} \int_{0}^{\Omega_{1}} d\Omega_{2} p_{x_{1}}(x/\Omega_{1}) p_{\Omega_{1}\Omega_{2}}(\Omega_{1}\Omega_{2}) +$$
$$+ \int_{0}^{\infty} d\Omega_{2} \int_{0}^{\Omega_{2}} d\Omega_{1} p_{x_{2}}(x/\Omega_{2}) p_{\Omega_{1}\Omega_{2}}(\Omega_{1}\Omega_{2}) = J_{I} + J_{2} \qquad (9)$$

Now, we need to solve the integrals J_1 and J_2 . The integral J_1 is [13]:

$$\begin{split} J_{1} &= \int_{0}^{\infty} d\Omega_{1} \int_{0}^{\Omega_{1}} d\Omega_{2} p_{x_{1}/\Omega_{1}} \left(x_{1} \right) p_{\Omega_{1}\Omega_{2}} \left(\Omega_{1}\Omega_{2} \right) = \\ &= \int_{0}^{\infty} d\Omega_{1} \int_{0}^{\Omega_{1}} d\Omega_{2} \frac{8}{\Omega_{1}^{2}} x^{3} \cdot e^{-\frac{2}{\Omega_{1}}x^{2}} \cdot \frac{1}{\Gamma(c)(1-\rho^{2})\rho^{c-1}\Omega_{0}^{c+1}} \cdot \\ &\quad \cdot \sum_{i_{3}=0}^{\infty} \left(\frac{\rho}{\Omega_{0}(1-\rho^{2})} \right)^{2^{i_{3}+c-1}} \cdot \frac{1}{i_{3}!\Gamma(i_{3}+c)} \cdot \\ &\quad \Omega_{1}^{i_{3}+c-1}\Omega_{2}^{i_{3}+c-1} \cdot e^{-\frac{\Omega_{1}+\Omega_{2}}{\Omega_{0}(1-\rho^{2})}} = \\ &= 8x^{3} \frac{1}{\Gamma(c)(1-\rho^{2})\rho^{c-1}\Omega_{0}^{c+1}} \cdot \\ &\quad \cdot \sum_{i_{3}=0}^{\infty} \left(\frac{\rho}{\Omega_{0}(1-\rho^{2})} \right)^{2^{i_{3}+c-1}} \cdot \frac{1}{i_{3}!\Gamma(i_{3}+c)} \cdot \\ &\quad \int_{0}^{\infty} d\Omega_{1}\Omega_{1}^{i_{3}+c-1-2} \cdot e^{-\frac{2}{\Omega_{1}}x^{2} - \frac{\Omega_{1}}{\Omega_{0}(1-\rho^{2})}} \cdot \int_{0}^{\Omega_{1}} d\Omega_{2}\Omega_{2}^{i_{3}+c-1} e^{-\frac{\Omega_{2}}{\Omega_{0}(1-\rho^{2})}} = \\ &= 8x^{3} \frac{1}{\Gamma(c)(1-\rho^{2})\rho^{c-1}\Omega_{0}^{c+1}} \cdot \end{split}$$

$$\begin{split} &\cdot \sum_{i_{j}=0}^{\infty} \left(\frac{\rho}{\Omega_{0}\left(1-\rho^{2}\right)} \right)^{2i_{j}+c-1} \cdot \frac{1}{i_{3}!\Gamma(i_{3}+c)} \cdot \\ &\quad \int_{0}^{\infty} d\Omega_{1}\Omega_{1}^{i_{j}+c-l-2} \cdot e^{-\frac{2}{\Omega_{1}}x^{2} - \frac{\Omega_{1}}{\Omega_{0}\left(1-\rho^{2}\right)}} \\ &\quad \left(\Omega_{0}\left(1-\rho^{2}\right)\right)^{i_{j}+c} \gamma \left(i_{3}+c, \frac{\Omega_{1}}{\Omega_{0}\left(1-\rho^{2}\right)}\right)^{0} \right) = \\ &= \frac{8x^{2}}{\Gamma(c)\left(1-\rho^{2}\right)\rho^{c-1}\Omega_{0}^{c+1}} \cdot \sum_{j_{j}=0}^{\infty} \left(\frac{\rho}{\Omega_{0}\left(1-\rho^{2}\right)}\right)^{2i_{j}+c-1} \cdot \frac{1}{i_{3}!\Gamma(i_{3}+c)} \cdot \\ &\quad \int_{0}^{\infty} d\Omega_{1}\Omega_{1}^{i_{j}+c-l-2} \cdot e^{-\frac{2}{\Omega_{1}}x^{2} - \frac{\Omega_{1}}{\Omega_{0}\left(1-\rho^{2}\right)}} \\ &\quad \left(\Omega_{0}\left(1-\rho^{2}\right)\right)^{i_{3}+c} \frac{\Omega_{1}^{i_{j}+c}}{\left(\Omega_{0}\left(1-\rho^{2}\right)\right)^{i_{j}-c}} - \frac{1}{i_{3}+c} \cdot \\ &\quad e^{-\frac{\Omega_{1}}{\Omega_{0}\left(1-\rho^{2}\right)}} \sum_{j_{j}=0}^{\infty} \frac{1}{\left(i_{3}+c+1\right)\left(j_{1}\right)} \frac{\Omega_{1}^{i_{j}}}{\left(\Omega_{0}\left(1-\rho^{2}\right)\right)^{j_{1}}} = \\ &= \frac{8x^{2}}{\Gamma(c)\left(1-\rho^{2}\right)\rho^{c-1}\Omega_{0}^{c+1}} \cdot \sum_{i_{j}=0}^{\infty} \left(\frac{\rho}{\Omega_{0}\left(1-\rho^{2}\right)}\right)^{2i_{j}+c-1} \cdot \frac{1}{i_{3}!\Gamma(i_{3}+c)} \cdot \\ &\quad \frac{1}{i_{3}+c} \sum_{j_{i}=0}^{\infty} \frac{1}{\left(i_{3}+c+1\right)\left(j_{1}\right)} \frac{\Omega_{1}^{j_{1}}}{\left(\Omega_{0}\left(1-\rho^{2}\right)\right)^{j_{1}}} = \\ &= \frac{8x^{2}}{\Gamma(c)\left(1-\rho^{2}\right)\rho^{c-1}\Omega_{0}^{c+1}} \cdot \sum_{i_{j}=0}^{\infty} \left(\frac{\rho}{\Omega_{0}\left(1-\rho^{2}\right)}\right)^{2i_{j}+c-1} \cdot \frac{1}{i_{3}!\Gamma(i_{3}+c)} \cdot \\ &\quad \frac{1}{i_{3}+c} \sum_{j_{i}=0}^{\infty} \frac{1}{\left(i_{3}+c+1\right)\left(j_{1}\right)} \frac{\Omega_{1}^{j_{1}}}{\left(\Omega_{0}\left(1-\rho^{2}\right)\right)^{j_{1}}} \cdot \\ &\quad \frac{1}{i_{3}+c} \sum_{j_{i}=0}^{\infty} \frac{1}{\left(i_{3}+c+1\right)\left(j_{i}\right)} \frac{\Omega_{1}^{j_{1}}}{\left(\Omega_{0}\left(1-\rho^{2}\right)\right)^{j_{1}}} \cdot \\ &\quad \frac{1}{i_{3}+c} \sum_{j_{i}=0}^{\infty} \frac{1}{\left(i_{3}+c+1\right)\left(i_{i}\right)} \frac{\Omega_{1}^{j_{1}}}{\left(\Omega_{0}\left(1-\rho^{2}\right)\right)^{j_{1}}} \cdot \\ &\quad \frac{1}{i_{3}+c} \sum_$$

 $K_n(x)$ is the modified Bessel function of the second kind [14], order *n* and argument *x*.

The integral J_2 is:

$$J_{2} = \int_{0}^{\infty} d\Omega_{2} \int_{0}^{\Omega_{2}} d\Omega_{1} \frac{4\mu(k+1)^{\frac{2\mu+1}{2}}}{k^{\frac{2\mu-1}{2}}e^{2k\mu}\Omega_{2}^{\frac{2\mu+1}{2}}2^{\frac{2\mu+1}{2}}}$$

$$\begin{split} & \sum_{i_{2}=0}^{\infty} \Biggl(2\mu \sqrt{\frac{k(k+1)}{2\Omega_{2}}} \Biggr)^{2i_{2}+4\mu-1} \cdot \frac{1}{i_{2}!\Gamma(i_{2}+2\mu)} \cdot \\ & x^{2i_{2}+4\mu-1} e^{-\frac{\mu(k+1)}{\Omega_{2}}x_{2}^{2}} \frac{1}{\Gamma(c)(1-\rho^{2})\rho^{c-1}\Omega_{0}^{c+1}} \cdot \\ & \sum_{i_{4}=0}^{\infty} \Biggl(\frac{\rho}{\Omega_{0}(1-\rho^{2})} \Biggr)^{2i_{4}+c-1} \cdot \frac{1}{i_{3}!\Gamma(i_{3}+c)} \cdot \\ & \Omega_{1}^{i_{4}+c-1}\Omega_{2}^{i_{4}+c-1} \cdot e^{-\frac{\Omega_{1}+\Omega_{2}}{\Omega_{0}(1-\rho^{2})}} = \frac{4\mu(k+1)^{\frac{2\mu+1}{2}}}{k^{\frac{2\mu-1}{2}}e^{2k\mu}2^{\mu+1/2}} \cdot \\ & \sum_{i_{2}=0}^{\infty} \Biggl(2\mu \sqrt{\frac{k(k+1)}{2}} \Biggr)^{2i_{2}+4\mu-1} \cdot \frac{1}{i_{2}!\Gamma(i_{2}+2\mu)} \cdot \\ & x^{2i_{2}+4\mu-1} \cdot \frac{1}{\Gamma(c)(1-\rho^{2})\rho^{c-1}\Omega_{0}^{c+1}} \cdot \\ & \sum_{i_{3}=0}^{\infty} \Biggl(\frac{\rho}{\Omega_{0}(1-\rho^{2})} \Biggr)^{2i_{3}+c-1} \cdot e^{-\frac{\mu(k+1)x^{2}}{\Omega_{2}} \cdot \frac{\Omega_{2}}{\Omega_{0}(1-\rho^{2})}} \\ & \Biggl(\Omega_{0}\left(1-\rho^{2}\right) \Biggr)^{i_{3}+c} \gamma \Biggl(i_{3}+c, \frac{\Omega_{2}}{\left(\Omega_{0}\left(1-\rho^{2}\right)\right)} \Biggr) \Biggr) = \\ & = \frac{4\mu(k+1)^{\frac{2\mu+1}{2}}}{k^{-\frac{2\mu-1}{2}}e^{2k\mu}2^{\mu+1/2}} \cdot \\ & \sum_{i_{3}=0}^{\infty} \Biggl(2\mu \sqrt{\frac{k(k+1)}{2}} \Biggr)^{2i_{3}+c-1} \cdot \frac{1}{i_{2}!\Gamma(i_{2}+2\mu)} \cdot \\ & x^{2i_{2}+4\mu-1} \cdot \frac{1}{\Gamma(c)(1-\rho^{2})\rho^{c-1}\Omega_{0}^{c+1}} \cdot \\ & \frac{1}{i_{3}+c} \sum_{i_{1}=0}^{\infty} \frac{1}{\left(i_{3}+c+1\right)\left(j_{1}\right)} \Biggr)^{2i_{3}+c-1} \cdot \frac{1}{i_{3}!\Gamma(i_{3}+c)} \cdot \\ & \frac{1}{i_{3}+c} \sum_{i_{1}=0}^{\infty} \frac{1}{\left(i_{3}+c+1\right)\left(j_{1}\right)} \frac{1}{\left(\Omega_{0}\left(1-\rho^{2}\right)\right)^{h}} \cdot \\ & \int_{0}^{\infty} d\Omega_{2}\Omega_{2}^{2i_{3}+2c-1+j_{1}-3\mu-i_{2}} \cdot e^{-\frac{\mu(k+1)x^{2}}{\Omega_{2}} - \frac{2\Omega_{2}}{\Omega_{0}(1-\rho^{2})} = \\ \end{split}$$

$$\begin{split} &= \frac{4\mu(k+1)^{\frac{2\mu+1}{2}}}{k^{\frac{2\mu-1}{2}}e^{2k\mu}2^{\mu+1/2}} \cdot \\ &\cdot \sum_{i_2=0}^{\infty} \left(2\mu\sqrt{\frac{k(k+1)}{2}}\right)^{2i_2+4\mu-1} \cdot \frac{1}{i_2!\Gamma(i_2+2\mu)} \cdot \\ &\quad x^{2i_2+4\mu-1} \cdot \frac{1}{\Gamma(c)(1-\rho^2)\rho^{c-1}\Omega_0^{c+1}} \cdot \\ &\quad \cdot \sum_{i_3=0}^{\infty} \left(\frac{\rho}{\Omega_0(1-\rho^2)}\right)^{2i_3+c-1} \cdot \frac{1}{i_3!\Gamma(i_3+c)} \cdot \\ &\quad \frac{1}{i_3+c} \sum_{j_1=0}^{\infty} \frac{1}{(i_3+c+1)(j_1)} \frac{1}{(\Omega_0(1-\rho^2))^{j_1}} \cdot \\ &\quad \cdot \left(\frac{\mu(k+1)x^2\Omega_0(1-\rho^2)}{2}\right)^{-3\mu/2-i_2/2+i_3+c+j_1/2} \\ &\quad \cdot K_{-3\mu-i_2+2i_3+2c+j_1} \left(2\sqrt{\frac{\mu(k+1)x^22}{\Omega_0(1-\rho^2)}}\right) \end{split}$$
(11)

Probability density function of x is obtained as a sum of integrals J_1 and J_2 from expressions (10) and (11).

Cumulative distribution function of *x* is:

$$F_{x}\left(x\right) = \int_{0}^{\infty} d\Omega_{1} \int_{0}^{\Omega_{1}} d\Omega_{2} F_{x_{1}}\left(x/\Omega_{1}\right) p_{\Omega_{1}\Omega_{2}}\left(\Omega_{1}\Omega_{2}\right) + \int_{0}^{\infty} d\Omega_{2} \int_{0}^{\Omega_{2}} d\Omega_{1} F_{x_{2}}\left(x/\Omega_{2}\right) p_{\Omega_{1}\Omega_{2}}\left(\Omega_{1}\Omega_{2}\right) = J_{3} + J_{4} \quad (12)$$

Then we solve the integral J_3 :

$$\begin{split} J_{3} &= \int_{0}^{\infty} d\Omega_{1} \int_{0}^{\Omega_{1}} d\Omega_{2} \, \gamma \left(2, \frac{2}{\Omega_{1}} \, x^{2} \right) \frac{1}{\Gamma(c) \left(1 - \rho^{2} \right) \rho^{c-1} \Omega_{0}^{c+1}} \cdot \\ & \cdot \sum_{i_{3}=0}^{\infty} \left(\frac{\rho}{\Omega_{0} \left(1 - \rho^{2} \right)} \right)^{2i_{3}+c-1} \cdot \frac{1}{i_{3}! \Gamma(i_{3}+c)} \cdot \\ & \Omega_{1}^{i_{3}+c-1} \Omega_{2}^{i_{3}+c-1} \cdot e^{-\frac{\Omega_{1}+\Omega_{2}}{\Omega_{0} \left(1 - \rho^{2} \right)}} = \\ &= \int_{0}^{\infty} d\Omega_{1} \int_{0}^{\Omega_{1}} d\Omega_{2} \, \frac{1}{2} \left(\frac{2}{\Omega_{1}} \right)^{2} \, x^{4} e^{-\frac{2}{\Omega_{1}} x^{2}} \sum_{j_{1}=0}^{\infty} \frac{1}{3(j_{1})} \left(\frac{2}{\Omega_{1}} \, x^{2} \right)^{j_{1}} \\ & \frac{1}{\Gamma(c) \left(1 - \rho^{2} \right) \rho^{c-1} \Omega_{0}^{c+1}} \cdot \sum_{i_{3}=0}^{\infty} \left(\frac{\rho}{\Omega_{0} \left(1 - \rho^{2} \right)} \right)^{2i_{3}+c-1} \cdot \frac{1}{i_{3}! \Gamma(i_{3}+c)} \cdot \end{split}$$

Volume 11, 2017

$$\begin{split} \Omega_{1}^{i_{1}+c-1}\Omega_{2}^{i_{1}+c-1} \cdot e^{-\frac{\Omega_{1}+\Omega_{2}}{\Omega_{0}\left(1-\rho^{2}\right)}} = \\ &= \frac{1}{2}2^{2}x^{4}\sum_{h=0}^{\infty} \frac{1}{3(j_{1})}2^{j_{1}}x^{2h} \cdot \frac{1}{\Gamma(c)\left(1-\rho^{2}\right)\rho^{c-1}\Omega_{0}^{c+1}} \cdot \\ &\quad \cdot \frac{\sum_{i_{1}=0}^{\infty} \left(\frac{\rho}{\Omega_{0}\left(1-\rho^{2}\right)}\right)^{2i_{1}+c-1} \cdot \frac{1}{i_{1}!\Gamma(i_{5}+c)} \cdot \\ &\quad \int_{0}^{\infty} d\Omega_{1}\Omega_{1}^{i_{3}+c-1-2-j_{1}} \cdot e^{-\frac{\Omega_{1}}{\Omega_{1}}x^{2}} \frac{\Omega_{1}}{\Omega_{0}\left(1-\rho^{2}\right)} \\ &\quad \left(\Omega_{0}\left(1-\rho^{2}\right)\right)^{i_{1}+c}\gamma\left(i_{3}+c,\frac{\Omega_{1}}{\left(\Omega_{0}\left(1-\rho^{2}\right)\right)}\right) = \\ &= 2x^{4}\sum_{j_{1}=0}^{\infty} \frac{1}{3(j_{1})}2^{j_{1}}x^{2j_{1}} \cdot \frac{1}{\Gamma(c)\left(1-\rho^{2}\right)\rho^{c-1}\Omega_{0}^{c+1}} \cdot \\ &\quad \cdot \frac{\sum_{i_{1}=0}^{\infty} \left(\frac{\rho}{\Omega_{0}\left(1-\rho^{2}\right)}\right)^{2j_{1}+c-1} \cdot \frac{1}{i_{3}!\Gamma(i_{5}+c)} \cdot \\ &\quad \frac{1}{i_{3}+c} \cdot \sum_{j_{2}=0}^{\infty} \frac{1}{(i_{3}+c+1)(j_{2})} \cdot \frac{1}{\left(\Omega_{0}\left(1-\rho^{2}\right)\right)^{j_{2}}} \cdot \\ &\quad \int_{0}^{\infty} d\Omega_{1}\Omega_{1}^{2j_{1}+2c+j_{2}-1-2-j_{1}} \cdot e^{-\frac{2}{\Omega_{1}}x^{2}} \cdot \frac{\Omega_{1}}{\Omega_{0}\left(1-\rho^{2}\right)} = \\ &\quad \left(\Omega_{0}\left(1-\rho^{2}\right)\right)^{i_{1}+c}\gamma\left(i_{3}+c,\frac{\Omega_{1}}{\left(\Omega_{0}\left(1-\rho^{2}\right)\right)}\right) = \\ &= 2x^{4}\sum_{j_{n}=0}^{\infty} \frac{1}{3(j_{1})}2^{j_{1}}x^{2j_{1}} \cdot \frac{1}{\Gamma(c)\left(1-\rho^{2}\right)\rho^{c-1}\Omega_{0}^{c+1}} \cdot \\ &\quad \cdot \sum_{j_{n}=0}^{\infty} \left(\frac{\rho}{\Omega_{0}\left(1-\rho^{2}\right)}\right)^{2j_{1}+c-1} \cdot \frac{1}{i_{3}!\Gamma(i_{5}+c)} \cdot \\ &\quad \frac{1}{i_{3}+c} \cdot \sum_{j_{2}=0}^{\infty} \frac{1}{(i_{3}+c+1)(j_{2})} \cdot \frac{1}{\left(\Omega_{0}\left(1-\rho^{2}\right)\right)^{j_{2}}} \cdot \\ \cdot \left(\frac{2x^{2}\Omega_{0}\left(1-\rho^{2}\right)}{2}\right)^{i_{1}+c+j_{2}/2-j_{1}/2} \cdot K_{2j_{1}+2c+j_{2}-j_{1}-2}\left(2\sqrt{\frac{4x^{2}}{\Omega_{0}(1-\rho^{2})}\right) \end{array}$$
(13)

The integral J_4 will be:

$$\begin{split} J_4 &= \int_0^\infty d\Omega_2 \int_0^{\Omega_2} d\Omega_1 \frac{4\mu(k+1)^{\mu+1/2}}{k^{\mu-1/2}e^{2k\mu}\Omega_2^{\mu+1/2}2^{\mu+1/2}} \cdot \\ &: \sum_{i_2=0}^\infty \left(2\mu \sqrt{\frac{k(k+1)}{2\Omega_2}} \right)^{\sum_{j=1}^{2i_2+2\mu-1}} \cdot \frac{1}{i_2!\Gamma(i_2+2\mu)} \cdot \\ &: \frac{1}{2} \left(\frac{\Omega_2}{\mu(k+1)} \right)^{i_2+2\mu} \frac{1}{i_2+2\mu} \left(\frac{\mu(k+1)}{\Omega_2} \right)^{i_2+2\mu} x^{2(i_2+2\mu)} \\ &: e^{-\frac{\mu(k+1)x^2}{\Omega_2}} \sum_{j_1=0}^\infty \frac{1}{(i_2+2\mu+1)(j_1)} \frac{\left(\mu(k+1)x^2\right)^{j_1}}{\Omega_2^{j_1}} \cdot \\ &: \frac{1}{\Gamma(c)(1-\rho^2)\rho^{c-1}\Omega_0^{c+1}} \cdot \frac{s}{i_1=0}^\infty \left(\frac{\rho}{\Omega_0(1-\rho^2)} \right)^{2i_3+c-1} \cdot \frac{1}{i_3!\Gamma(i_3+c)} \cdot \\ &: \Omega_1^{i_3+c-1}\Omega_2^{i_3+c-1} \cdot e^{-\frac{\Omega_1+\Omega_2}{\Omega_0(1-\rho^2)}} = \frac{4\mu(k+1)^{\mu+1/2}}{k^{\mu-1/2}e^{2k\mu}\Omega_2^{\mu+1/2}2^{\mu+1/2}} \cdot \\ &: \frac{S}{\sum_{i_2=0}^\infty} \left(2\mu \sqrt{\frac{k(k+1)}{2}} \right)^{2i_2+2\mu-1} \frac{1}{\sqrt{\frac{1}{2!\Gamma(i_2+2\mu)}}} \cdot \\ &: \frac{1}{2} \frac{1}{i_2+2\mu} x^{2(i_2+2\mu)} \sum_{j_1=0}^\infty \frac{1}{(i_2+2\mu+1)(j_1)} \frac{\left(\mu(k+1)x^2\right)^{j_1}}{\Omega_2^{j_1}} \cdot \\ &: \frac{1}{\Gamma(c)(1-\rho^2)\rho^{c-1}\Omega_0^{c+1}} \cdot \sum_{i_3=0}^\infty \left(\frac{\rho}{\Omega_0(1-\rho^2)} \right)^{2i_3+c-1} \cdot \frac{1}{i_3!\Gamma(i_3+c)} \cdot \\ & \left(\Omega_0(1-\rho^2) \right)^{i_3+c} \gamma \left(i_3+c, \frac{\Omega_2}{\left(\Omega_0(1-\rho^2)\right)} \right)^{2i_3+c-1} \cdot \frac{1}{i_2!\Gamma(i_2+2\mu)} \cdot \\ &: \frac{S}{\sum_{i_2=0}^\infty} \left(2\mu \sqrt{\frac{k(k+1)}{2}} \right)^{2i_2+2\mu-1} \cdot \frac{1}{i_2!\Gamma(i_2+2\mu)} \cdot \\ &: \frac{S}{\sum_{i_2=0}^\infty} \left(2\mu \sqrt{\frac{k(k+1)}{2}} \right)^{2i_2+2\mu-1} \cdot \frac{1}{i_2!\Gamma(i_2+2\mu)} \cdot \\ &: \frac{S}{\sum_{i_2=0}^\infty} \left(2\mu \sqrt{\frac{k(k+1)}{2}} \right)^{2i_2+2\mu-1} \cdot \frac{1}{i_2!\Gamma(i_2+2\mu)} \cdot \\ &: \frac{S}{\sum_{i_2=0}^\infty} \left(2\mu \sqrt{\frac{k(k+1)}{2}} \right)^{2i_2+2\mu-1} \cdot \frac{1}{i_2!\Gamma(i_2+2\mu)} \cdot \\ &: \frac{S}{\sum_{i_2=0}^\infty} \left(2\mu \sqrt{\frac{k(k+1)}{2}} \right)^{2i_2+2\mu-1} \cdot \frac{1}{i_2!\Gamma(i_2+2\mu)} \cdot \\ &: \frac{1}{2} \frac{1}{i_2+2\mu} x^{2(i_2+2\mu)} \sum_{j_1=0}^\infty \frac{1}{(i_2+2\mu+1)(j_1)} \frac{\left(\mu(k+1)x^2\right)^{j_1}}{\Omega_2^{j_1}} \cdot \\ &: \frac{1}{\Gamma(c)(1-\rho^2)} \rho^{c-1}\Omega_0^{c+1} \cdot \sum_{i_3=0}^\infty \left(\frac{\rho}{\Omega_0(1-\rho^2)} \right)^{2i_3+c-1} \cdot \frac{1}{i_3!\Gamma(i_3+c)} \cdot \\ &: \frac{1}{\Gamma(c)(1-\rho^2)} \rho^{c-1}\Omega_0^{c+1} \cdot \sum_{i_3=0}^\infty \left(\frac{\rho}{\Omega_0(1-\rho^2)} \right)^{2i_3+c-1} \cdot \\ &: \frac{1}{\Gamma(c)(1-\rho^2)} \rho^{c-1}\Omega_0^{c+1} \cdot \sum_{i_3=0}^\infty \left(\frac{\rho}{\Omega_0(1-\rho^2)} \right)^{2i_3+c-1} \cdot \frac{1}{i_3!\Gamma(i_3+c)} \cdot \\ &: \frac{1}{\Gamma(c)(1-\rho^2)} \rho^{c-1}\Omega_0^{c+1} \cdot \\ &: \frac{1}{\Gamma(c)} \left(\frac{\rho}{\Omega_0^{c+1}} \right)^{2i_3+c-1} \cdot \\ &: \frac{1}{\Gamma(c)(1-\rho^2)} \rho^{c-1}\Omega_0^{c+1} \cdot \\ &: \frac{1}{\Gamma(c)} \left(\frac{\rho}{\Omega_0^{c+1}} \right)^$$

$$\begin{split} \left(\Omega_{0}\left(1-\rho^{2}\right)\right)^{l_{1}+c} \frac{1}{l_{3}+c} \frac{1}{\left(\Omega_{0}\left(1-\rho^{2}\right)\right)^{l_{3}+c}} \\ \cdot \sum_{j_{2}=0}^{\infty} \frac{1}{(l_{3}+c+1)(j_{2})} \cdot \frac{1}{\left(\Omega_{0}\left(1-\rho^{2}\right)\right)^{l_{2}}} \cdot \\ \cdot \sum_{j_{2}=0}^{\infty} d\Omega_{2} \Omega_{2}^{2l_{3}+2c+j_{2}-1-2\mu-l_{2}-j_{1}} \cdot e^{-\frac{\mu(k+1)x^{2}}{\Omega_{2}} - \frac{2\Omega_{2}}{\Omega_{0}\left(1-\rho^{2}\right)}} = \\ &= \frac{4\mu(k+1)^{\mu+1/2}}{k^{\mu-1/2}e^{2k\mu}\Omega_{2}^{\mu+1/2}2^{\mu+1/2}} \cdot \\ \cdot \sum_{l_{2}=0}^{\infty} \left(2\mu\sqrt{\frac{k(k+1)}{2}}\right)^{2l_{2}+2\mu-1} \cdot \frac{1}{l_{2}!\Gamma(l_{2}+2\mu)} \cdot \\ \cdot \sum_{l_{2}=0}^{\infty} \left(2\mu\sqrt{\frac{k(k+1)}{2}}\right)^{2l_{2}+2\mu-1} \cdot \frac{1}{l_{2}!\Gamma(l_{2}+2\mu)} \cdot \\ \frac{1}{2}\frac{1}{l_{2}+2\mu}x^{2(l_{2}+2\mu)}\sum_{j_{1}=0}^{\infty} \frac{1}{(l_{2}+2\mu+1)(j_{1})} \frac{\left(\mu(k+1)x^{2}\right)^{j_{1}}}{\Omega_{2}^{l_{1}}} \cdot \\ \frac{1}{\Gamma(c)\left(1-\rho^{2}\right)\rho^{c-1}\Omega_{0}^{c+1}} \cdot \sum_{l_{3}=0}^{\infty} \left(\frac{\rho}{\Omega_{0}\left(1-\rho^{2}\right)}\right)^{2l_{3}+c-1} \cdot \frac{1}{l_{3}!\Gamma(l_{3}+c)} \cdot \\ \frac{1}{l_{3}+c} \cdot \sum_{j_{2}=0}^{\infty} \frac{1}{(l_{3}+c+1)(j_{2})} \cdot \frac{1}{\left(\Omega_{0}\left(1-\rho^{2}\right)\right)^{l_{2}}} \cdot \\ \frac{\int_{0}^{\infty} d\Omega_{2}\Omega_{2}^{2l_{3}+2c+j_{2}-1-2\mu-l_{2}-j_{1}} \cdot e^{-\frac{\mu(k+1)x^{2}}{\Omega_{2}} - \frac{2\Omega_{2}}{\Omega_{0}\left(1-\rho^{2}\right)}} = \\ \cdot \left(\frac{\mu(k+1)x^{2}\Omega_{0}\left(1-\rho^{2}\right)}{2}\right)^{l_{3}+c+j_{2}/2-\mu-l_{2}/2-j_{1}/2}} \\ \cdot K_{2l_{3}+2c+j_{2}-2\mu-l_{2}-j_{1}}} \left(2\sqrt{\frac{\mu(k+1)x^{2}2}{\Omega_{0}\left(1-\rho^{2}\right)}}\right) \tag{14}$$

Finally, the cumulative distribution function of x is the sum of integrals J_3 and J_4 from the expressions (13) and (14).

III. LEVEL CROSSING RATE OF SIGNAL ENVELOPES AT OUTPUTS OF MICRODIVERSITY MRC RECEIVERS

The first derivative of Rayleigh random variable has Gaussian distribution. Rayleigh random variable and its first derivative are independent. Therefore, the joint probability density function (JPDF) of Rayleigh random variable and its first time derivative is:

$$p_{x_{li}\dot{x}_{li}}\left(x_{li}\dot{x}_{li}\right) = \frac{2x_{li}}{\Omega_2} \cdot e^{-\frac{x_{li}^2}{\Omega_2}} \cdot \frac{1}{\sqrt{2\pi\beta_2}} e^{-\frac{\dot{x}_{li}^2}{2\beta_2^2}},$$
$$x_{2i} \ge 0, \ i = 1, 2$$
(15)

with
$$\beta_1^2 = \pi^2 f_m^2 \Omega_1$$
.

JPDF of x_1 and \dot{x}_1 is:

$$p_{x_{1}\dot{x}_{1}}\left(x_{1}\dot{x}_{1}\right) = \frac{8}{\Omega_{1}}x_{1}^{3} \cdot e^{-\frac{2}{\Omega_{1}}x_{1}^{2}} \cdot \frac{1}{\sqrt{2\pi}\beta_{1}}e^{-\frac{\dot{x}_{1}^{2}}{2\beta_{1}^{2}}}.$$
 (16)

Level crossing rate of random variable x_1 is:

$$N_{x_{1}} = \int_{0}^{\infty} dx_{1} \dot{x}_{1} p_{x_{1} \dot{x}_{1}} \left(x_{1} \dot{x}_{1} \right) =$$

$$= \frac{8}{\Omega_{1}^{2}} x_{1}^{3} \cdot e^{-\frac{2}{\Omega_{1}} x_{1}^{2}} \cdot \int_{0}^{\infty} d\dot{x}_{1} \dot{x}_{1} \frac{1}{\sqrt{2\pi} \beta_{1}} e^{-\frac{\dot{x}_{1}^{2}}{2\beta_{1}^{2}}} =$$

$$= \frac{8}{\Omega_{1}^{2}} x_{1}^{3} \cdot e^{-\frac{2}{\Omega_{1}} x_{1}^{2}} \frac{1}{\sqrt{2\pi}} \beta_{1} =$$

$$= \frac{8}{\Omega_{1}^{2}} x_{1}^{3} \cdot e^{-\frac{2}{\Omega_{1}} x_{1}^{2}} \frac{1}{\sqrt{2\pi}} \pi f_{m} \frac{\Omega_{1}^{1/2}}{2^{1/2}} =$$

$$= \frac{2\sqrt{2}\sqrt{2\pi} f_{m}}{\Omega_{1}^{3/2}} x_{1}^{3} \cdot e^{-\frac{2}{\Omega_{1}} x_{1}^{2}} . \quad (17)$$

The joint probability density function of x_2 and \dot{x}_2 is:

$$p_{x_{2}\dot{x}_{2}}\left(x_{2}\dot{x}_{2}\right) = \frac{4\mu\left(k+1\right)^{\frac{2\mu+1}{2}}}{k^{\frac{2\mu-1}{2}}e^{2k\mu}\Omega_{1}^{\frac{2\mu+1}{2}}} \cdot \frac{1}{\sum_{i_{1}=0}^{\infty}\left(2\mu\sqrt{\frac{k(k+1)}{2\Omega_{1}}}\right)^{2i_{1}+4\mu-1}} \cdot \frac{1}{i_{1}!\Gamma(i_{1}+2\mu)} \cdot \frac{1}{x_{2}^{2i_{1}+4\mu-1}}e^{-\frac{2\mu(k+1)}{\Omega_{1}}x_{2}^{2}} \cdot \frac{1}{\sqrt{2\pi}\beta_{2}}e^{-\frac{\dot{x}_{2}^{2}}{2\beta_{2}^{2}}}, \quad (18)$$

where $\beta_2^2 = \pi^2 f_m^2 \frac{\Omega_2}{\mu(k+1)}$.

Level crossing rate of random variable x_2 is [3]:

$$N_{x_{2}} = \int_{0}^{\infty} dx_{2} \dot{x}_{2} p_{x_{2} \dot{x}_{2}} \left(x_{2} \dot{x}_{2} \right) = \frac{2\sqrt{2\pi} f_{m} \mu^{1/2} \left(k+1 \right)^{\mu}}{k^{\mu-1/2} e^{2k\mu} \Omega_{2}^{\mu}} \cdot \frac{1}{\sum_{i_{1}=0}^{\infty} \left(2\mu \sqrt{\frac{k(k+1)}{2\Omega_{2}}} \right)^{2i_{1}+2\mu-1}} \cdot \frac{1}{i_{1}!\Gamma(i_{1}+2\mu)} \cdot \frac{1}{x_{2}^{2i_{1}+4\mu-1}} e^{-\frac{2\mu(k+1)}{\Omega_{2}} x_{2}^{2}} \cdot \frac{1}{\sqrt{2\pi}\beta_{2}} e^{-\frac{\dot{x}_{2}^{2}}{2\beta_{2}^{2}}} .$$
(19)

The κ - μ random variable and the first derivative of κ - μ random variable are independent. This first derivative has Gaussian distribution.

Thus, the joint probability density function of κ - μ random variable and its first derivative is:

$$p_{x_{2i}\dot{x}_{2i}}\left(x_{2i}\dot{x}_{2i}\right) = \frac{2\mu\left(k+1\right)^{\frac{\mu+1}{2}}}{k^{\frac{\mu-1}{2}}e^{k\mu}\Omega_{2}^{\frac{\mu+1}{2}}} \cdot \frac{1}{k^{\frac{\mu-1}{2}}e^{k\mu}\Omega_{2}^{\frac{\mu+1}{2}}} \cdot \frac{1}{k^{\frac{\mu}{2}}\left(1+\mu\right)} \cdot \frac{1}{\sqrt{2\pi}\beta_{2}}e^{-\frac{k^{\frac{\mu}{2}}}{2\beta_{2}^{\frac{\mu}{2}}}}, \quad (20)$$

where f_m is maximal Doppler frequency.

IV. LEVEL CROSSING RATE OF SIGNAL ENVELOPE AT OUTPUT OF MAD SC RECEIVER

Macrodiversity SC receiver selects MID MRC receiver with higher signal envelope average power at its inputs. Consequently, level crossing rate of signal envelope at output of MAD SC receiver is:

$$N_{x} = \int_{0}^{\infty} d\Omega_{1} \int_{0}^{\Omega_{1}} d\Omega_{2} N_{x_{1}/\Omega_{1}} p_{\Omega_{1}\Omega_{2}} \left(\Omega_{1}\Omega_{2}\right) + \int_{0}^{\infty} d\Omega_{2} \int_{0}^{\Omega_{2}} d\Omega_{1} N_{x_{2}/\Omega_{2}} p_{\Omega_{1}\Omega_{2}} \left(\Omega_{1}\Omega_{2}\right) = J_{5} + J_{6}. \quad (21)$$

The integral J_6 is [13] [15]:

+

$$\begin{split} J_{6} &= \int_{0}^{\infty} d\Omega_{1} \int_{0}^{\Omega_{1}} d\Omega_{2} N_{x_{1}/\Omega_{1}} p_{\Omega_{1}\Omega_{2}} \left(\Omega_{1}\Omega_{2}\right) = \\ &= \frac{2\sqrt{2\pi} f_{m} \mu^{1/2} \left(k+1\right)^{\mu}}{k^{\mu-1/2} e^{2k\mu}} \cdot \\ &\cdot \sum_{i_{1}=0}^{\infty} \left(2\mu \sqrt{\frac{k(k+1)}{2\Omega_{1}}}\right)^{2i_{1}+2\mu-1} \cdot \frac{1}{i_{1}!\Gamma(i_{1}+2\mu)} \cdot x_{1}^{2i_{1}+4\mu-1} \\ &\cdot \frac{2}{\Gamma(c)\left(1-\rho^{2}\right)\rho^{c-1}\Omega_{0}^{c+1}} \cdot \\ &\cdot \sum_{i_{2}=0}^{\infty} \left(\frac{\rho}{\Omega_{0}\left(1-\rho^{2}\right)}\right)^{2i_{2}+\mu-1} \cdot \frac{1}{i_{2}!\Gamma(i_{2}+c)} \cdot \\ &\int_{0}^{\infty} d\Omega_{1}\Omega_{1}^{i_{1}+c-1-\mu-i_{1}-\mu+1/2} e^{-\frac{\mu(k+1)}{\Omega_{1}} - \frac{\Omega_{1}}{\Omega_{0}\left(1-\rho^{2}\right)}} \\ &\int_{0}^{\infty} d\Omega_{2}\Omega_{2}^{i_{1}+c-1} \cdot e^{-\frac{\Omega_{2}}{\Omega_{0}\left(1-\rho^{2}\right)}} = \frac{2\sqrt{2\pi} f_{m} \mu^{1/2} \left(k+1\right)^{\mu}}{k^{\mu-1/2} e^{2k\mu}} \cdot \\ &\cdot \sum_{i_{1}=0}^{\infty} \left(2\mu \sqrt{\frac{k(k+1)}{2\Omega_{1}}}\right)^{2i_{1}+2\mu-1} \cdot \frac{1}{i_{1}!\Gamma(i_{1}+2\mu)} \cdot x_{1}^{2i_{1}+4\mu-1} \end{split}$$

$$\begin{split} & \cdot \frac{2}{\Gamma(c)(1-\rho^2)\rho^{c-1}\Omega_0^{c+1}} \cdot \\ & \cdot \sum_{i_1=0}^{\infty} \left(\frac{\rho}{\Omega_0(1-\rho^2)} \right)^{2^{i_1+\mu-1}} \cdot \frac{1}{i_2!\Gamma(i_2+c)} \cdot \\ & \int_{0}^{\infty} d\Omega_1 \Omega_1^{c-1-2\mu+1/2} e^{-\frac{\mu(k+1)}{\Omega_1} - \frac{\Omega_1}{\Omega_0(1-\rho^2)}} \\ & \left(\Omega_0 \left(1-\rho^2\right)\right)^{i_1+c} \gamma \left(i_1+c, \frac{\Omega_1}{\Omega_0(1-\rho^2)}\right) = \\ & = \frac{2\sqrt{2\pi} f_m \mu^{1/2} (k+1)^{\mu}}{k^{\mu-1/2} e^{2k\mu}} \cdot \\ & \cdot \sum_{i_1=0}^{\infty} \left(2\mu \sqrt{\frac{k(k+1)}{2\Omega_1}}\right)^{2^{i_1+2\mu-1}} \cdot \frac{1}{i_1!\Gamma(i_1+2\mu)} \cdot x_1^{2i_1+4\mu-1} \\ & \cdot \frac{\sqrt{2}}{\Gamma(c)(1-\rho^2)} \rho^{c-1}\Omega_0^{c+1}} \cdot \\ & \cdot \sum_{i_2=0}^{\infty} \left(\frac{\rho}{\Omega_0(1-\rho^2)}\right)^{2^{i_2+\mu-1}} \cdot \frac{1}{i_2!\Gamma(i_2+c)} \cdot \\ & \int_{0}^{\infty} d\Omega_1 \Omega_1^{c-1-2\mu+1/2} e^{-\frac{\mu(k+1)}{\Omega_1} - \frac{\Omega_1}{\Omega_0(1-\rho^2)}} \\ & \left(\Omega_0 \left(1-\rho^2\right)\right)^{i_1+c} \frac{1}{i_1+c} \frac{\Omega_1^{i_1+c}}{\left(\Omega_0 \left(1-\rho^2\right)\right)^{i_1+c}} e^{-\frac{\Omega_1}{\Omega_0(1-\rho^2)}} \\ & \sum_{j_1=0}^{\infty} \frac{1}{(i_1+c+1)(j_1)} \frac{\Omega_1^{i_1}}{\left(\Omega_0 \left(1-\rho^2\right)\right)^{i_1}} = \\ & = \frac{2\sqrt{2\pi} f_m \mu^{1/2} (k+1)^{\mu}}{k^{\mu-1/2} e^{2k\mu}} \cdot \\ \cdot \sum_{i_2=0}^{\infty} \left(2\mu \sqrt{\frac{k(k+1)}{2\Omega_1}}\right)^{2^{i_1+2\mu-1}} \cdot \frac{1}{i_1!\Gamma(i_1+2\mu)} \cdot x_1^{2i_1+4\mu-1} \\ & \cdot \frac{2}{\Gamma(c)(1-\rho^2)} \rho^{c-1}\Omega_0^{c+1}} \cdot \\ & \frac{1}{i_1+c} \sum_{j_1=0}^{\infty} \frac{1}{(i_1+c+1)(j_1)} \frac{1}{\left(\Omega_0 \left(1-\rho^2\right)\right)^{i_1}} \end{bmatrix}$$

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The integral J_5 can be solved in the similar way. After that, the level crossing rate of MAD SC receiver output signal envelope is calculated by summing solved integrals J_5 and J_6 from (22).

V. GRAPHICAL RESULTS

The level crossing rate of macrodiversity SC receiver output signal envelope depending on the output signal envelope is shown in Figs. 2 to 5. The graphs for LCR of MAD SC receiver output signal envelope are plotted for several values of Gamma long term fading severity parameter, κ - μ short term fading severity parameter, Rician factor of κ - μ multipath fading and envelope average powers Ω_1 and Ω_2 . The system performance is better for lower values of LCR.

Level crossing rate of MAD SC receiver output signal versus SC receiver output signal envelope for variable Rician factor and signal envelope average powers Ω_1 and Ω_2 is shown in Fig. 2. Changeable parameter is also parameter β . In Figs. 4 and 5, $\mu=c=2$, and parameter β is variable.

Level crossing rate decreases for higher values of Gamma long term fading severity parameter and κ - μ short term fading severity parameter. Level crossing rate increases for lower values of output signal envelope and level crossing rate decreases for higher values of output signal envelope.



Fig. 2. Level crossing rate of MAD SC receiver output signal versus SC receiver output signal envelope



Fig. 3. LCR of MAD SC receiver output signal versus the SC receiver output signal envelope



Fig. 4. LCR of MAD SC receiver output signal versus the SC receiver output signal envelope for μ =c=2, variable Rician factor and parameters Ω_1 , Ω_2 , β_1 and β_2



Fig. 5. LCR of MAD SC receiver output signal versus the SC receiver output signal envelope for μ =c=2, and changeable other fading parameters

Maximum of curve goes to lower values of signal envelopes when the κ - μ severity parameter increases. The influence of signal envelopes on LCR is higher for lower values of signal envelope. Also, the influence of Gamma severity parameter on LCR is higher for lower values of output signal envelopes. The system performance is better for lower values of LCR.

Level crossing rate increases when correlation coefficient increases. Correlation coefficient has values of zero to one. The influence of correlation coefficient on LCR is higher for lower values of output signal envelope. When correlation coefficient goes to one, MAD reception becomes MID reception. The lowest signal envelope occurs on both MID MRC receivers simultaneously. When correlation coefficient goes to zero, diversity gain increases. With increasing of Rician factor of κ - μ multipath fading, the LCR decreases and diversity gain increases. Diversity gain increases when dominant component power increases or scattering components power decrease. The influence of Rician factor on level crossing rate is higher for higher values of correlation coefficient.

VI. CONCLUSION

Macrodiversity system with MAD selection combining receiver and two microdiversity MRC receivers is considered in this paper. Received signal in the first MRC receiver suffers Rayleigh short term fading and Gamma long term fading and received signal in the second MRC receiver experiences correlated Gamma long term fading and κ -µ short term fading.

MAD SC receiver is used to reduce Gamma long term fading effects; the first MID MRC receiver mitigates Rayleigh short term fading effects and the second MID MRC receiver reduced κ - μ short term fading effects on system performance. The closed form expressions for LCR of signal envelopes at outputs of MID MRC receivers are calculated and used for evaluation the LCR of MAD SC receiver output signal envelope. The influence of Rician factor of κ - μ short term fading, κ - μ short term fading severity parameter, Gamma long term fading severity parameter and Gamma long term fading correlation coefficient on level crossing rate is presented and discussed. Further, the expression for LCR of MAD SC receiver output signal envelope derived here can be used for evaluation of the average fade duration for wireless communication system operating over shadowed multipath fading channels.

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