Bayesian Estimations in the Generalized Lindley Model

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Abstract— The generalized Lindley distribution offers an important and a reliable tool for modelling and testing lifetime data. In this paper, the Bayesian analysis of generalized Lindley distribution model is considered under type II censured data. Bayes estimator and corresponding risks were derived using different loss functions such us squared error loss (SELF), Linex loss (LF) and entropy loss function (EF). We define two criteria which are the Pitman closeness criterion and the integrated mean square error (IMSE) to compare the Bayesian and the maximum likelihood estimators (MLE). A real data example is given for illustration.

Keywords— Lindley Model, Maximum likelihood method, The Bayesian approach, loss function, TypeII censored data.

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I. Introduction

THE Lindley distribution was originally proposed by Lindley [13] in the context of Bayesian statistics, as a counter example of fiducial statistics which can be seen that as a mixture of Exp (θ) and gamma(2,θ). More details on the Lindley distribution can be found in Ghitany et al. [7]. A random variable X is said to have Lindley distribution with a parameter if its probability density function is defined as:

$$f(x;\theta) = \frac{\theta^2(1+x)e^{-\theta x}}{1+\theta}\ ; x,\theta > 0 \qquad (*)$$

M.E. Ghitany and co-workers have discussed various properties of this distribution and showed that in many ways (∗) provides a better model for waiting times and survival times data than the exponential distribution. This distribution provides a fits better to the empirical data, which considered all the negative binomial distributions and Hermit. Recently, a good part of attention was paid to this probability density function (pdf) in the statistic literature. For example, Ghitany et al. (2008) and Al-Mutairi and Ghitany (2009) studied the distribution of certain properties of discrete Poisson-Lindley proposed Sankaran (1970). Recently, this work was extended by Mahmoudi and Zakerzadeh (2010). Other articles on this continuous distribution include Hernndez Bastida et al. (2011) and Gmez-Déniz and Caldern-Ojeda [4]. Ghitany et al. [8] developed a two-parameter weighted Lindley distribution and discussed its applications to survival data. Recently, Zakerzadah and Dolati [21] and Elbatal et al. [6] have discussed its applications to survival data. Recently, Zakerzadah and Dolati [21] and Elbatal et al. [6] have obtained the generalized Lindley distribution and the Kumaraswamy Quasi Lindley distribution, respectively. In this article, we study the estimation of the generalized Lindley distribution which depends two parameters with type II censored data. Two approaches are proposed, the first is the classical maximum likelihood estimation (MLE). The second is the Bayesian procedure performed under three loss function (Quadratic, entropy and linex). Using an exhaustive Monte Carlo study, these three Bayesian estimators are compared to the maximum likelihood estimator (MLE) using Pitman closeness criterion and the integrated mean square error (IMSE).

The rest of this paper is organized as follows. In Section 2, we define the model. The section 3, study estimating parameters by the classical maximum likelihood estimation (MLE) with type II censored data. In Section 4, the Bayesian procedure performed under three loss function. In section 5, we are interested in the simulation study and the comparison between the two approaches presiding in the case of date censored type II, using the criterion of Pitman and the Integrated Mean Square Error. An application with real data was provided in section 6.

II. The Model

A model of Lindley a two parameters generalized (LG) is be defined by Shanker and al. (2013):

$$f_{LG}(x,\theta,\beta) = \frac{\theta}{\theta+\beta}\theta e^{-\theta x} + \frac{\beta}{\theta+\beta}\theta^2 x e^{-\theta x} \qquad (1)$$

$$= \frac{\theta^2(1+\beta x)}{\theta+\beta}e^{-\theta x};\ x,\theta,\beta > 0$$

The distribution function is defined as follows:

$$F_{LG}(x,\theta,\beta) = \int_0^{+\infty} f(t)dt = 1 - \left(1 + \frac{\theta\beta x}{\theta+\beta}\right)e^{-\theta x} \qquad (2)$$

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III. MAXIMUM LIKELIHOOD ESTIMATORS

In this section we obtain the maximum likelihood estimates (MLE) parameters of the generalized law Lindley, we use type II censored data.

Consider a n-sample $(X_1, X_2, \ldots, X_n)$ generated from (1). Let $(X_1, X_2, \ldots, X_m)$, m-censured sample from (1) m∈{1,2,3,.....,n}, the likelihood is then given n data censored type II, the $LG\ (\theta, \beta)$ sample m size, probability function can be constructed as follows:

$$L(\theta, \beta|\underline{X}) = \frac{n!}{n-m!} \prod_{i=1}^{m} f(x_i)[1 - F(x_m)]^{n-m} \qquad (3)$$

with $x_1 \leq x_2 \leq \ldots\ldots \leq x_m$ .The likelihood function (1) can be written:

$$L(\theta, \beta|\underline{X}) \propto \prod_{i=1}^{m} \frac{\theta^2(1+\beta x_i)}{\theta+\beta} e^{-\theta x} \left[\left(1 + \frac{\theta \beta x_m}{\theta+\beta}\right) e^{-\theta x_m}\right]^{n-m}$$

$$L(\theta, \beta|\underline{X}) \propto \frac{\theta^{2m}}{(\theta+\beta)^m} e^{-\theta \sum_{i=1}^{m} x_i} \left[\left(1 + \frac{\theta\beta x_m}{\theta+\beta}\right) e^{-\theta x_m}\right]^{n-m} \prod_{i=1}^{m} (1+\beta x_i)$$

$$L(\theta, \beta|\underline{X}) \propto \frac{\theta^{2m}}{(\theta+\beta)^m} e^{-\theta T_m} \left(1 + \frac{\theta\beta x_m}{\theta+\beta}\right)^{n-m} \prod_{i=1}^{m} (1+\beta x_i) \qquad (4)$$

To $T_m = \sum_{i=1}^{i=m} x_i + x_m(n-m)$

We set $LnL(\theta, \beta|\underline{x}) = l(\theta, \beta|\underline{x})$ and the corresponding logarithm is :

$$l(\theta, \beta|\underline{X}) \propto ln\left[\frac{\theta^{2m}}{(\theta+\beta)^m} e^{-\theta T_m} \left(1 + \frac{\theta\beta x_m}{\theta+\beta}\right)^{n-m} \prod_{i=1}^{m}(1+\beta x_i)\right]$$

$$l(\theta, \beta|\underline{X}) \propto 2m\, ln\theta - m ln(\theta+\beta) - \theta T_m + \sum_{i=1}^{m} \ln(1+\beta x_i)$$

$$+(n-m)\ln\left(\frac{\theta+\beta+\theta\beta x_m}{\theta+\beta}\right)$$

The likelihood function is:

$$l(\theta, \beta|\underline{X}) \propto 2m\, ln\theta - n ln(\theta+\beta) - \theta T_m + \sum_{i=1}^{m} \ln(1+\beta x_i)$$

$$+(n-m)\ln(\theta+\beta+\theta\beta x_m) \qquad (5)$$

The MLE estimators of the two parameters θ, β are solutions of the vectorial equation $l(\theta, \beta|\underline{X}) = 0$

Then, the likelihood equations are as follows:

$$\begin{cases} \dfrac{\partial l(\theta, \beta|\underline{X})}{\partial \theta} = \dfrac{2m}{\theta} - \dfrac{n}{\theta+\beta} - T_m + \dfrac{(n-m)(1+\beta x_m)}{\theta+\beta+\theta\beta x_m} = 0 \\ \dfrac{\partial l(\theta, \beta|\underline{X})}{\partial \beta} = -\dfrac{n}{\theta+\beta} + \sum_{i=1}^{m} \dfrac{x_i}{1+\beta x_i} + \dfrac{(n-m)(1+\theta x_m)}{\theta+\beta+\theta\beta x_m} = 0 \end{cases} \qquad (6)$$

The system of equations (6) cannot be solved directly, we can use an iterative method .We then obtained $(\hat{\theta}_{MLE}, \hat{\beta}_{MLE})$.
In this paper, we will use the R package BB which has high capabilities for solving a nonlinear system of equations (Vara han and Gilbert, 2009).

IV. BAYESIAN ESTIMATION UNDER DIFFERENT LOSS FUNCTIONS

a- Prior and Posterior distributions

In the presence of a priori information on the settings, it can be assumed that the parameter θ follows a law $G(a, b)$ density $\pi(\theta)$ and the parameter β admits prior density $\pi(\beta) = \frac{1}{\beta}$.
Moreover θ and β are independent.

$$\pi(\theta, \beta) = \pi(\theta)\pi(\beta) = \frac{b^a}{\Gamma(a)} \theta^{a-1} e^{-b\theta} \frac{1}{\beta}$$

The posterior density is then written:

$$\pi(\theta, \beta|X) = \frac{L(\theta, \beta|\underline{X})\pi(\theta, \beta)}{\iint_0^{+\infty} L(\theta, \beta|\underline{X})\pi(\theta, \beta) d\theta d\beta}$$

$$\pi(\theta, \beta|X) = \frac{e^{-\theta(T_m+b)} \cdot \dfrac{\theta^{2m+a-1}}{\beta(\theta+\beta)^m} \cdot \left(1 + \dfrac{\theta\beta x_m}{\theta+\beta}\right)^{n-m} \prod_{i=1}^{m}(1+\beta x_i)}{\iint_0^{+\infty} e^{-\theta(T_m+b)} \cdot \dfrac{\theta^{2m+a-1}}{\beta(\theta+\beta)^m} \cdot \left(1 + \dfrac{\theta\beta x_m}{\theta+\beta}\right)^{n-m} \prod_{i=1}^{m}(1+\beta x_i)\ d\theta d\beta} \qquad (7)$$

b- Loss function

We consider in this work three loss functions (Quadratic, Linex, entropy).

Under quadratic loss function; Bayesian estimators of θ and β noted by respectively $\hat{\theta}_{BQ}, \hat{\beta}_{BQ}$ match their medium a posteriori, are:

$$\hat{\theta}_{BQ} = \frac{\iint_0^{+\infty} \dfrac{\theta^{2m-a}}{(\theta+\beta)^m} e^{-\theta(T_m+b)} \left(1+\dfrac{\theta\beta x_m}{\theta+\beta}\right)^{n-m} \prod_{i=1}^{m}(1+\beta x_i)\ d\theta d\beta}{\iint_0^{+\infty} \dfrac{\theta^{2m-a-1}}{(\theta+\beta)^m} e^{-\theta(T_m+b)} \left(1+\dfrac{\theta\beta x_m}{\theta+\beta}\right)^{n-m} \prod_{i=1}^{m}(1+\beta x_i)\ d\theta d\beta}$$

$$\hat{\beta}_{BQ} = \frac{\iint_0^{+\infty} \dfrac{\theta^{2m-a-1}}{(\theta+\beta)^m} e^{-\theta(T_m+b)} \left(1+\dfrac{\theta\beta x_m}{\theta+\beta}\right)^{n-m} \prod_{i=1}^{m}(1+\beta x_i)\ d\theta d\beta}{\iint_0^{+\infty} \dfrac{\theta^{2m+a-1}}{(\theta+\beta)^m} e^{-\theta(T_m+b)} \left(1+\dfrac{\theta\beta x_m}{\theta+\beta}\right)^{n-m} \prod_{i=1}^{m}(1+\beta x_i)\ d\theta d\beta} \qquad (8)$$

The corresponding Posterior risks are then:

$$PR(\hat{\theta}_{BQ}) = E_\pi(\theta^2) - 2\hat{\theta}_{BQ}E_\pi(\theta)$$
$$PR(\hat{\beta}_{BQ}) = E_\pi(\beta^2) - 2\hat{\theta}_{BQ}E_\pi(\beta)$$

With loss of function Entropy Bayesian estimators of θ and β noted by respectively $\hat{\theta}_{BE}$ and $\hat{\beta}_{BE}$ defined as:

$$\hat{\theta}_{BE} = \left[E\left(\frac{1}{\theta^p}\right)\right]^{-\frac{1}{p}}$$
$$\hat{\beta}_{BE} = \left[E\left(\frac{1}{\beta^p}\right)\right]^{-\frac{1}{p}} \qquad (9)$$

The posterior risks are:

$$PR(\hat{\theta}_{BE}) = pE_\pi(ln(\theta) - ln(\hat{\theta}_{BE})$$
$$PR(\hat{\theta}_{BE}) = pE_\pi(ln(\beta) - ln(\hat{\beta}_{BE}))$$

Under the Linex loss function, the Bayesian estimators of θ and β noted by respectively $\hat{\theta}_{BL}$ and $\hat{\beta}_{BL}$ defined as:

$$\hat{\theta}_{BL} = \frac{-1}{a} logE(e^{-a\theta})$$
$$\hat{\beta}_{BL} = \frac{-1}{a} logE(e^{-a\beta})$$

To proceed to the calculation by simulation, the recommended method is the particular method MCMC as the Metropolis-Hastings algorithm.

V. SIMULATIONS

In this section, we are interested in the simulation study. This study for the parameters estimates $LG(\theta;\beta)$ function, which developed in the previous sections. The estimate of maximum likelihood and Bayesian by the three loss functions (Quadratic-Entropy-Linex) are obtained for censored type II data. All calculations are performed on R 3.1.3. We took three sample sizes: (i) the small sample size n=10, (ii) moderate sample size n=30 and (iii) large sample size n=100 and $\theta = 1, \beta = 0{,}5$. For each combination (n, θ, β), we generated N=5000 and calculate the maximum likelihood estimators and Bayes by the three loss function (Quadratic-Entropy-Linex) of θ, β, we propose to perform a Monte Carlo study assuming that $a = 1, b = 4$, we obtain the following results.

a- Likelihood estimation

We used the R 3.1.3 package BB which presents very high performances for nonlinear systems. The results are as follows:

n	m	$\hat{\theta}_{MLE}$	$\hat{\beta}_{MLE}$
10	10	1.04484(0.011338)	0.48783(0.02046)
	8	0.987191(0.16938)	0.43010(0.02469)
	5	0.79549(0.33932)	0.36819(0.20081)
30	30	1.00416(0.02048)	0.485222(0.01148)
	24	0.94029(0.04385)	0.44499(0.01856)
	15	0.83036(0.11811)	0.42870(0.01994)
100	100	1.00444(0.00447)	0.50399(0.00555)
	80	0.942841(0.01458)	0.49637(0.01791)
	50	0.78915(0.08105)	0.47399(0.01791)

Table1: The MLE of the parameters with Quadratic Error.

b-Bayesian estimation

The Bayesian estimators are obtained with performing the MCMC methods. Table 2 presents the Bayesian estimations with quadratic loss function and the corresponding posterior risk.

- With the entropy loss function, we obtain the following (table 3).

- Under the linex loss function, the results are given in table 4

- Noting that the best Bayesian estimators, in the sense of the smallest square errors are obtained with a pert Linex function or entropy with p=-0.5 and a=-0.5.

n	m	$\hat{\theta}_{BQ}$	$\hat{\beta}_{BQ}$
10	10	1.03167(0.04820)	0.54559(0.05771)
	8	0.99542(0.00446)	0.49602(0.00438)
	5	0.96524(0.00616)	0.46544(0.00605)
30	30	0.98352(0.00350)	0.46622(0.00320)
	24	1.03851(0.00360)	0.53861(0.00355)
	15	0.96043(0.00332)	0.46203(0.00310)
100	100	1.00891(0.00743)	0.559007(0.00668)
	80	0.92410(0.00342)	0.43010(0.00319)
	50	0.90983(0.00466)	0.41263(0.00374)

Table2: Bayesian estimation of the parameters under Quadratic Loss Function

n	m	P=-2		P=-0.5		P=0.5		P=2	
		$\hat{\theta}_{BE}$	$\hat{\beta}_{BE}$	$\hat{\theta}_{BE}$	$\hat{\beta}_{BE}$	$\hat{\theta}_{BE}$	$\hat{\beta}_{BE}$	$\hat{\theta}_{BE}$	$\hat{\beta}_{BE}$
10	10	1.03064	0.54749	1.02777	0.54231	1.02595	0.53916	1.02338	0.53483
		0.00737	0.02491	0.00044	0.00147	0.00043	0.00143	0.00676	0.02185
	8	0.93935	0.44036	0.93876	0.43913	0.93838	0.43833	0.93781	0.43716
		0.00165	0.00744	0.00010	0.00045	0.00010	0.00045	0.00162	0.00714
	5	0.92634	0.42743	0.92579	0.42625	0.92542	0.42547	0.92488	0.42432
		0.00159	0.00736	0.00009	0.00045	0.00009	0.00045	0.00157	0.00725
	30	1.00449	0.50653	1.00245	0.50255	1.00119	0.50022	0.99942	0.49712
		0.00534	0.02051	0.00031	0.00118	0.00031	0.00113	0.00477	0.01696
	24	0.97090	0.47336	0.96938	0.47037	0.96844	0.46861	0.96713	0.46627
		0.00410	0.01652	0.00024	0.00095	0.00023	0.00091	0.00368	0.01368
	15	0.95153	0.45625	0.95036	0.45397	0.94962	0.45262	0.94858	0.45078
		0.00325	0.01303	0.00019	0.00076	0.00019	0.00073	0.00296	0.01106
	100	0.97433	0.52481	0.96919	0.51969	0.96600	0.51651	0.96153	0.51206
		0.01351	0.01346	0.00080	0.00080	0.00078	0.00078	0.01208	0.01204
	80	0.96158	0.46359	0.96012	0.46219	0.95922	0.46132	0.95797	0.46012
		0.00383	0.00368	0.00022	0.00021	0.00022	0.00021	0.00338	0.00326
	50	0.94370	0.45758	0.94261	0.45609	0.94194	0.45520	0.94101	0.45398
		0.00286	0.00387	0.00016	0.00022	0.00016	0.00021	0.00251	0.00330

Table3: Bayesian estimation and PR under entropy loss function

n	m	a=-2		a=-0.5		a=0.5		a=2	
		$\hat{\theta}_{BL}$	$\hat{\beta}_{BL}$	$\hat{\theta}_{BL}$	$\hat{\beta}_{BL}$	$\hat{\theta}_{BL}$	$\hat{\beta}_{BL}$	$\hat{\theta}_{BL}$	$\hat{\beta}_{BL}$
10	10	1.03294	0.54804	1.02972	0.54494	1.02772	0.54302	1.02492	0.54033
		0.00848	0.00814	0.00202	0.00194	0.00049	0.00047	0.00756	0.00727
	8	0.93970	0.44027	0.93914	0.43972	0.93878	0.43935	0.93823	0.43882
		0.00148	0.00147	0.00092	0.00091	0.00091	0.00090	0.00145	0.00143
	5	0.92666	0.42467	0.92346	0.42415	0.92311	0.42381	0.92259	0.4233
		0.00139	0.00137	0.00008	0.00008	0.00008	0.00008	0.00138	0.00136
	30	1.02025	0.50094	1.01759	0.49830	1.01597	0.49670	1.01376	0.49450
		0.00697	0.00691	0.00041	0.00040	0.00039	0.00039	0.00601	0.00595
	24	1.00784	0.50903	1.00484	0.50607	1.00304	0.50430	1.00060	0.50190
		0.00784	0.00771	0.00045	0.00045	0.00043	0.00043	0.00665	0.00654
	15	0.98061	0.48304	0.97829	0.48082	0.97688	0.47947	0.97492	0.47759
		0.00607	0.00582	0.00035	0.00034	0.00034	0.00033	0.00530	0.00508
	100	0.97433	0.52481	0.96919	0.51969	0.96600	0.51651	0.96153	0.51206
		0.01351	0.01346	0.00080	0.00080	0.00078	0.00078	0.01208	0.01204
	80	0.96158	0.46359	0.96012	0.46219	0.95922	0.46132	0.95797	0.46012
		0.00383	0.00368	0.00022	0.00021	0.00022	0.00021	0.00338	0.00326
	50	0.94370	0.45758	0.94261	0.45609	0.94194	0.45520	0.94101	0.45398
		0.00286	0.00387	0.00016	0.00022	0.00016	0.00021	0.00251	0.00330

Table4 : Bayesian estimation of the parameters and PR under the Linex loss function

c- Comparison with the maximum likelihood Estimator

In this section, we propose to compare the best Bayesian estimators previously with the maximum likelihood estimator. For this, we propose to use the following criterions: the Pitman closeness (Pitman, 1937, Fuller, 1982 and Jozani, 2012) and the integrated mean square error (IMSE) defined as follows:

Definition 1: An estimator $\hat{\theta}_1$ of a parameter θ dominates in the sense of Pitman closeness criterion another estimator $\hat{\theta}_2$, if for all $\theta \in \Theta$ $P_\theta\left[\left|\hat{\theta}_1 - \theta\right| < \left|\hat{\theta}_2 - \theta\right|\right] > 0.5$

Consider the estimates $\hat{\theta}_i (i = 1, \dots, N)$ (i=1,....,N) obtained with N samples of the model.

Definition 2: The integrated mean square error is defined as:

$$IMSE = \frac{\sum_{i=1}^{N}(\theta_i - \theta)^2}{N}$$

- In the following, we present the values of the Pitman probabilities which allows us to compare the MLE with Bayesian estimators under the three loss function p= -0.5 and a=-0.5.

- As we can see in table 5, when the probability is greater than 0.5, the Bayesian estimator is better than the MLE estimator.

- Table 6 presents the values of integrated mean square error (IMSE) of the Bayesian estimators of the parameters under the three loss function and the maximum likelihood estimators.

n	m	Quadra tique		Entropy (p = −0.5)		Linex (a = −0.5)	
		θ	β	θ	β	θ	β
10	10	0.9162	0.8686	0.9378	0.9312	0.9928	0.8992
	8	0.9648	0.8448	0.958	0.838	0.9962	0.8458
	5	0.9654	0.9458	0.971	0.97	0.9962	0.9562
30	30	0.8838	0.851	0.8812	0.8444	0.9908	0.8504
	24	0.9186	0.757	0.9206	0.7488	0.9946	0.7556
	15	0.929	0.7854	0.9234	0.7706	0.9964	0.7816
100	100	0.672	0.6962	0.676	0.6706	0.9836	0.6894
	80	0.7932	0.7814	0.7922	0.7738	0.9906	0.778
	50	0.883	0.752	0.8824	0.724	0.9964	0.7422

Table5 : Comparison of the likelihood and Bayesian method approach with three function loss using criterion of pitman

n	m	MLE		Quad		Entropy −0.5		Linex −0.5	
		θ	β	θ	β	θ	β	θ	β
10	10	0.12169	0.02915	0.00306	0.00308	0.00185	0.00128	0.00214	0.00215
	8	0.16783	0.02054	0.00092	0.00090	0.00085	0.00090	0.00085	0.00083
	5	0.24237	0.07900	0.00146	0.00143	0.00123	0.00118	0.00127	0.00124
30	30	0.02317	0.01203	0.00220	0.00219	0.00176	0.00156	0.00186	0.00185
	24	0.04366	0.01693	0.00237	0.00231	0.00208	0.00195	0.00214	0.00209
	15	0.11187	0.01978	0.00281	0.00265	0.00288	0.00293	0.00284	0.00272
100	100	0.00427	0.00639	0.00244	0.00241	0.00227	0.00217	0.00231	0.00229
	80	0.01470	0.02127	0.00331	0.00308	0.00332	0.00326	0.00330	0.00313
	50	0.06556	0.01840	0.00470	0.00392	0.00474	0.00433	0.00472	0.00404

Table6: The IMSE of the estimators of θ,β.

d- Conclusions of the simulation study

The results of the simulation study of the maximum likelihood approach and the Bayesian approach (Metropolis-Hastings) by the three loss functions (Quadratic-Entropy-Linex) are presented respect in **(table1)** and (**tables 3,4,5)** with the following Conclusions:

(i) It is noted that the estimates for complete samples MLE and Bayes by three loss of function of θ, β are nearly unbiased. It is also observed for complete sample estimators of parameters MLE and Bayes θ, β are very good.

(ii) It is observed that for the parameter estimates with type II censored samples by both methods (MLE, Bayes) is a can off.

(iii) If we compare the two approach using criterion of pitman (table 5) notes that the Bayesian approach by the three loss functions is the best by contributing to the likelihood method and the linex loss function provides the best Bayesian estimator of θ, β. Also if we compare the two approaches. using the IMSE (table 6), we note that all the Bayesian estimators are better than the MLE estimators.

VI. APPLICATION

In this section, the LG distribution is applied to real data in order to illustrate the usefulness and applicability of the model. The data set repair times (hours) for an airborne communication transceiver discussed by Alven [1], Chhikara and Folks [3] and Dimitrakopoulou and al.[5].

It consists of the observation listed below:
0.2,0.3,0.5,0.5,0.5,0.5,0.6,0.6,0.7,0.7,0.7,0.8,0.8,1.0,1.0,1.0,1.0,1.1, 1.3,1.5,1.5,1.5,1.5,2.0,2.0,2.2,2.5,2.7,3.0,3.0,3.3,3.3,4.0,4.0,4.5,4.7, 5.0,5.4,5.4,7.0,7.5,8.8,9.0,10.3,22.0,24.5.

Estimates of the parameters of LG by MLE and Bayesian with three loss functions are given in the **table 8** for activity repair time data.

		MLE		Bay Quadratique		Bay Entropy		Bay Linex	
n	m	$\hat{\theta}$	$\hat{\beta}$	$\hat{\theta}$	$\hat{\beta}$	$\hat{\theta}$	$\hat{\beta}$	$\hat{\theta}$	$\hat{\beta}$
46	46	0.98905 0.01365	0.44648 0.01103	0.95042 0.00714	0.46658 0.02464	0.95865 0.00001	0.48777 0.00026	0.95869 0.00001	0.49206 0.00077
	38	0.95092 0.02454	0.46485 0.01284	0.91575 0.00683	0.41655 0.00638	0.91737 0.00003	0.42022 0.00001	0.91738 0.00001	0.42024 0.00002

Table7 : Bayesian and likelihood estimation of the parameters under three loss of functionn

n	m	Quadratique		Entropy(p=-0.5)		Linex(a=-0.5)	
		θ	β	θ	β	θ	β
46	46	0.794	0.797	0.759	0.815	0.984	0.811
	38	0.837	0.748	0.829	0.71	0.997	0.73

Table8: Comparison of the likelihood and Bayesian method

n	m	MLE		Quadratique		Entropy(p=-0.5)		Linex(a=-0.5)	
		θ	β	θ	β	θ	β	θ	β
46	46	0.02125	0.00803	0.00803	0.01109	0.00783	0.0079	0.0079	0.00965
	38	0.02598	0.01225	0.00238	0.00231	0.00215	0.00231	0.00231	0.00205

Table9 : The IMSE of the estimators of θ,β

- Conclusion of application

We use the data set repair times (hours) for an airborne communications to illustrate the use of estimation methods discussed in this paper. For complete sample case, the MLEs are $\hat{\theta} = 0.98905, \hat{\beta} = 0.44648$. One other censoring scheme a given in the second column. From these tables, we conclude that the Bayesian by the three loss function method is better than the likelihood method.

VII. CONCLUSION

In this paper, we propose a two parameters Lindley distribution for data modeling. The parameters estimation was performed by the maximum likelihood method and a Bayesian approach by three loss function (Quadratic, Entropy, Linex) with type II censored samples. These methods have been introduced using the simulation of different sample sizes. We compared between the maximum likelihood method and the Bayesian approach using a criterion of Pitman and the integrated mean square error (IMSE).Finally, we illustrate our Study by an example of real data.

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