Differential and average approaches to Rose and Mei dropwise condensation models

Solmaz Boroomandi Barati[*], Jean-Charls Pinoli[†], Stéphane Valette[‡], and Yann Gavet [§]

***Abstract*—Two well-known models for drop-size distribution function during dropwise condensation -called Rose model and Mei model- were examined in two different aspects, average and differential point of view. It has been proved that these two models are able to describe the relation between droplets size and distribution function at each time step. The goal of this research is to investigate how these models can predict the relation between average distribution function (N_{ave}) and average radius (r_{ave}) of droplets during a complete procedure of dropwise condensation and the relation between differential distribution function ($\frac{dN}{dr}$) and drops radius (r) at each time step. The empirical parameters are drop size distribution exponent (n) and fractal dimension (df) in Rose model and Mei model respectively. At first these two parameters were calculated based on the experimental data and then the validity of these calculations for our computer simulation was investigated. It was concluded that Rose method fits the results of differential distribution function with exponent n between 0.33 and 0.35, and average distribution function with n of around 0.38. The Mei model also can describe both differential and average results of simulation and experiments with fractal dimension of 1.79<df<1.99. Also it was observed that the value of both n and df vary with changing the ratio of radius of two following droplets generation (γ) in our computer simulation.**

***Index Terms*—dropwise condensation, fractal geometry, drop-size distribution function , Mei method, Rose method.**

I. Introduction

DROPWISE condensation has been in the center of concentration during last few decades due to its higher heat transfer coefficient with respect to filmwise condensation. It was said that heat transfer coefficient of filmwise condensation is about 5 to 7 times smaller than in dropwise condensation [1]. Generally dropwise condensation includes five main steps: nucleation of initial droplets, growth due to adsorption, growth due to coalescence, nucleation of new droplets, and sliding of very big drops (that was not considered here). The result of the last three stages is the change in number of droplets as well as their size. One can consider drop-size distribution function (N) as the cumulative number of droplets bigger than a specified value per unit area (or per unite area and size). It is obvious that both coalescence and nucleation of new droplets change N during dropwise condensation.

Changes of N with respect to droplets radius (r) attracted lots of attention specially, after the works of Rose and his colleagues during 1990s. Le Fevre and Rose [2] were the first ones who were able to describe N successfully and their model was used by a lot of scientists up to now. Then, Rose and Glicksman [3] derived a power law model to describe the relation between N and r as well. To investigate time-series features and the percentage of surface occupied by droplets, Tanaka [4] numerically solved two equations relating to the spatial distribution of droplets . Tanasawa and Ochiai [5] also derived an empirical distribution function and conducted a huge amount of experiments and numerical investigations on dropwise condensation. More recently Mei et al. [6], [7] developed a fractal model based on fractal geometry theory. They supposed that since the pictures taken at different scales from droplets during dropwise condensation are the same, the droplets will follow the law of fractal geometry theory. The importance of their work is due to describing N by parameters that have physical meanings, such as droplets surface fraction, fractal dimension of droplets pattern, and the maximum radius of droplets. Baojin and his colleagues [1] developed the Rose model for all contact angles considering contact angle hysteresis to be able to describe droplets growth in both hydrophobic and hydrophilic surfaces. Wu et al.[8] introduced an algorithm to generate droplets based on the Rose model and calculate the rate of heat transfer based on this algorithm. At last, Watanabe et al. [9] investigated the results of all these methods with their experimental data.

Although lots of works have been done up to now for describing N in dropwise condensation, two interesting aspects of this process remained still unclear: differential and average distribution function. Differential distribution function ($\frac{dN}{dr}$) represents the differential changes in drop-size distribution function and average distribution function (N_{ave}) deals with total number of droplets divided by total area and average radius of droplets (r_{ave}) at each time step. More precisely,

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almost all of the scientific publications deal with the relation between N and r at each time step, but the relationship between N_{ave} versus r_{ave} and $\frac{dN}{dr}$ versus r was not clearly discussed up to now.

In this research we are going to introduce these two different approaches towards dropwise condensation modeling and apply them to Rose and Mei models. In this regard at first a series of experiments were conducted and the images were taken from droplets at $\Delta T = 1s$. These images were binarized and used to measure experimental parameters. Then we used a fractal generating algorithm to produce different generations of droplets in computer simulation. The results of experiments and simulation are compared with theory according to the two mentioned aspects. Considering these two aspects for each of the theoretical formulas, we will represent four linear equations as below

1) Differential Rose method, describing evolution of $\frac{dN}{dr}$ with respect to r at each time step
2) Average Rose method, describing evolution of N_{ave} based on r_{ave} during the whole process time
3) Differential Mei method, describing evolution of $\frac{dN}{dr}$ with respect to r at each time step
4) Average Mei method, describing evolution of N_{ave} based on r_{ave} during the whole process time

A. Rose Method [3]

This method is cited as empirical or Rose method in the specialized literature and is based on the sequence of events occurring during the growth cycle. Rose [3] used a power law model to describe the total area (A) covered by droplets with radius greater than a specified value of r .

$$A = 1 - (\frac{r}{r_{max}})^n \tag{1}$$

where n is an empirical parameter known as the drop size distribution exponent and must be determined experimentally. Rose and Glicksman [3] reported $n = 0.382$ based on the theoretical evidences . Wu [8], [6] assumed $n = \frac{1}{3}$ based on the experimental works of Graham and Griffith [10] and Tanaka [11] that reported n laying in the real number range [0.313-0.350]. Mei[6] also used $n = \frac{1}{3}$ based on experimental results of other researchers . The most frequently reported value for n in literature is around $\frac{1}{3} = 0.33$. Maximum radius of each generation of droplets before sliding (r_{max}) can be derived by force balance between surface tension and droplet weight and for hemisphere droplets is [12]:

$$r_{max} = \sqrt{\frac{3\sigma}{(\rho_l - \rho_v)g}} \tag{2}$$

where σ is liquid surface tension, ρ_l and ρ_v are liquid and vapor density respectively, and g is earth acceleration. If we consider N as the number of drops per unit size which have radius greater than r, then the differential distribution function of droplets size will be as below:

$$f(r) = \frac{-dN}{dr} = \frac{-1}{\pi r^2}\frac{dA}{dr} = \frac{n}{\pi r_{max}^3}(\frac{r}{r_{max}})^{(n-3)} \tag{3}$$

Most of the time the logarithmic version of this equation is used to describe the spatial distribution pattern of droplets:

$$log(\frac{-dN}{dr}r_{max}^3) = log(\frac{n}{\pi}) + (n-3)log(\frac{r}{r_{max}}) \tag{4}$$

The logarithmic scale is more preferable because the terms are dimensionless and thus easier to compare. Moreover droplets nucleation function is normalized for size of droplets and area of fractal zone and thus the results will be independent of experimental situation. From equation 3, if we consider N as the total number of droplets of each generation per unit area and unit radius, the time average version of Rose method will be [6]:

$$log(N_{ave}) = log(\frac{n}{\pi}) - nlog(r_{max}) + (n-3)log(r_{ave}) \tag{5}$$

where r_{ave} here is the average radius of droplets of each generation. Equations 4 and 5 represent differential and average Rose method, respectively. It has been frequently said that if we take pictures of droplets at different scales over time or space, all of them are similar and follow the same spatial pattern. This is known as the rule of self-similarity. According to the concept of self-similarity, we can say that droplet size distribution obeys the same rule in all size ranges and this is apparent by comparing equations 4 and 5.

B. Mei Method [6], [7]

The second method applied to describe dropwise condensation based on fractal units comes from the fractal geometry theory. According to this theory, in a fractal zone if we assume the size of the biggest particle equal to l_{max} made up of smaller units with size of l, the number of fractal units bigger than l is [13]:

$$N = (\frac{l_{max}}{l})^{d_f} \tag{6}$$

$$d_f = \lim_{r \to 0} \frac{log(N)}{log(\frac{r_{max}}{r})} \tag{7}$$

where d_f is the fractal dimension and is different for each of fractal patterns. For 2-dimensional droplets growing on a flat surface d_f is smaller than 2, while for 3-dimensional ones d_f is smaller than 3 [7]. Based

on fractal geometry theory, the cumulative number of droplets bigger than a specified value of r on a flat surface during dropwise condensation is:

$$N = (\frac{r_{max}}{r})^{d_f}, \qquad for \quad r_{min} < r < r_{max} \tag{8}$$

By differentiating from equation 8 with respect to r, the differential distribution function of droplets size will be:

$$f(r) = \frac{-dN}{dr} = \frac{d_f}{r_{max}}(\frac{r_{max}}{r})^{d_f+1} \tag{9}$$

The negative sign indicates opposite relation between size and number of droplets. Equation 9 in log-log system is a straight line with slope and intercepts that are function of d_f:

$$log(r_{max}\frac{-dN}{dr}) = log(df) - (d_f+1)log(\frac{r}{r_{max}}) \tag{10}$$

To be able to derive an average model comparable with equation 5, this equation must be divided by the total area occupied by fractal particles. Total area of a fractal zone was calculated by Mei [6] by integrating equation 10 :

$$A = \int_{r_{min}}^{r_{max}} -dN\pi r^2 = \frac{\pi d_f(1-\phi)r_{max}^2}{(2-d_f)\phi} \tag{11}$$

where ϕ is the fraction of covered area by a fractal zone and is equal to:

$$\phi = (\frac{r_{min}}{r_{max}})^{2-d_f} \tag{12}$$

where r_{min} is the size of the smallest viable droplets and must be calculated based on physical evidences [14]:

$$r_{min} = \frac{(2\sigma T_s)}{H_{fg}\rho(\Delta T_t)} \tag{13}$$

It is obvious that r_{min} is just a function of the process conditions and physical properties of liquid. Dividing equation 9 by equation 11 and next taking logarithm, we will get the average distribution function of droplets for Mei model:

$$\begin{aligned} log(N_{ave}) = log(\frac{(2-df)\phi}{\pi(1-\phi)}) \\ +(d_f-2)log(r_{max}) - (d_f+1)log(r_{ave}) \end{aligned} \tag{14}$$

This equation represents a straight line with slop and intercept as functions of d_f and surface coverage(ϕ). Both of these parameters are depended on temperature difference between cold substrate and hot air. Equations 10 and 14 are differential and average Mei distribution of droplets during dropwise condensation. These two formulas are based on the fractal geometry theory and assume the droplets as fractal particles growing in a fractal zone with area equal to equation 11.

II. Experimental Apparatus

Experimental setup consists of a chamber containing hot air and cold substrate and a compressor to adjust relative humidity about 40 percent inside the chamber. Temperature of hot air is set to $86°F$, while substrate temperature is around $62°F$. Nucleation and growth of droplets are recorded by a high resolution CCD camera installed outside the chamber in time intervals of 1s. The images taken by CCD camera then are binarized and used to model droplets nucleation and growth.

III. Fractal Simulation Algorithm

Present simulation develops the model proposed by Wu et al. [8] both in time and space domains. According to this model the square substrate with length l is divided itteratively to γ^2 small squares and p percent of these small squares are chosen completely randomly as new generation droplets. The idea behind this simulation is indicated in figure1.

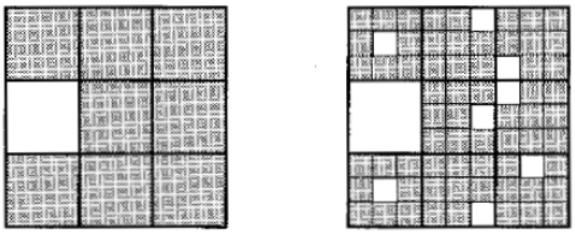

Figure 1. Illustration of how to generate droplets of each generation in fractal simulation algorithm ($\gamma = 3$)

In other words consider a square of length l. At the first step each side of this square is divided to γ, so now there are γ^2 small squares in the whole area. If we consider p as the fraction of available area for each generation of droplets, in the first step $p \times \gamma^2$ small squares must be randomly chosen as the first generation droplets. The choice of droplets was done based on Poisson point process, which generates completely random spatial distribution of points in the whole domain. In the second step each length of each small squares is divided by γ again and p percent of resulted squares are chosen as the second generation of droplets. This process continues till reaching the size of smallest viable droplets.

The fraction of available area p is the same for all generations and can be referred to as the probability of finding a droplet in a elementary square at each step. At each step, the chosen squares are assumed as hemispherical droplets so a correction factor equal to $p = p \times \pi/4$ must be considered in calculations. Procedure of generating random droplets of each generation is shown in figure 2. Although for generation

k available area is fewer than generation $k-1$, since there are more squares available to find droplets, there are more droplets in generation k with respect to $k-1$. In the other words, in each generation available area reduces, while number of droplets increase because droplets are smaller and need smaller amount of area to locate. This is the concept of linear relation between $\log(N)$ and $\log(r)$ during time. Analysis of droplets nucleation and growth was carried out based on each image at steady state time as well as based on average parameters during procedure.

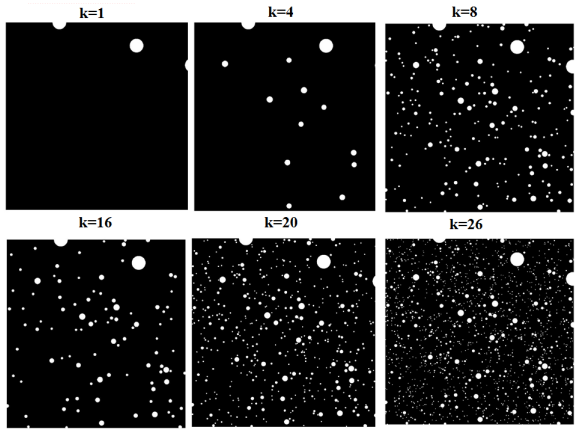

Figure 2. Illustration of generating random droplets, whose centers obey Poisson spatial point process in 6 different generations, $k = 1, 4, 8, 16, 20, 26$. At first the biggest droplets are formed by dividing substrate in to γ^2 squares and choosing p percent of them completely randomly. Then in each step smaller droplets are generated with the same pattern.

The droplets are located one by one to avoid any overlap between them. In reality, if a pair of droplets touch each other they will coalesce and form a bigger drop in their mass center. Thus, in the current method the droplets are added one by one and for each droplets there is a check to insure that the cell is not occupied by another droplets.

IV. Results and Discussion

A. Model Validation, Comparison With Experimental Results

Figures 3 and 4 compare the results of equations 4 and 10 with data obtained from experiments, at different time intervals. In these two figures Rose and Mei models are calculated for experiments of $t = 60s$ and $t = 100s$. The time intervals were chosen after starting coalescence in order to have both adsorbing -or small- and coalescence -or big- drops. The concept of small and big droplets comes from the critical radius (r_c), which was introduced as the half-spacing between active sites on substrate [15]. For square substrate with $A = L \times L$ it will be:

$$r_c = \sqrt{\frac{L^2}{4\hat{N}}} \qquad (15)$$

where $\hat{N}$ here is the number of droplets. Droplets smaller than r_c grow mainly due to adsorbing water molecules from humid air, while the main reason of growth of droplets bigger than r_c is coalescence. Figure 3 shows good agreement between experimental results and differential Rose model with n between 0.33 and 0.35, especially for bigger droplets. The same deviations near small drops was described before by Wu et al. [8] and Baojin et al. [1] while studying experimental results of dropwise condensation. Due to deviation around small droplets Rose method usually introduces as a method to describing evolution of coalescing droplets. Obtaining r_{max} from equation 2 and r_c from equation 15, we will have $-1.9 < log\frac{r_c}{r_{max}} < -2.9$ -depending on the experiment time- that is exactly the point after which the deviations from straight line start in figure 3. So, it can be claimed that equation 4 that has been used before, to describe N, has also a good understanding of $\frac{dN}{dr}$, especially for droplets bigger than r_c.

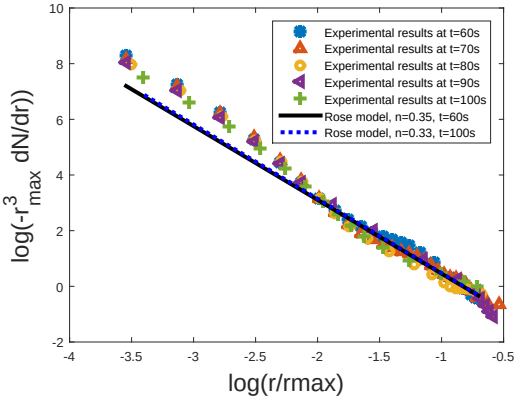

Figure 3. Comparison of the results of Rose model (equation 4) and experimental results at 5 different steps of $t = 60, 70, 80, 90, 100s$.

Figure 4 shows acceptable agreement between experimental results and equation 10 with df between 1.79 - 1.99 in all time intervals. The deviations around very small and very big droplets were observed by Mei himself [16]. For very small and very big droplets the number of droplets depends on the physical properties of process like contact angle hysteresis and starvation of substrate for growing small droplets in vacant area around bigger ones. So, these deviations maybe seen frequently.

The predicted results for N_{ave} by Rose method (equation 5) and Mei method (equation 14) is presented in figure 5 as well as experimental results. This figure highlights that experimental N_{ave}s are fitted by the two models very well with $n = 0.38$ and $df = 1.99$. Figure 5 shows that both Rose and Mei methods can describe droplets growth by adsorption and coalescence from minimum to maximum radius. Also, it can be said that according to this figure droplets nucleation and growth during the whole process from r_{min} to r_{max} obeys the same pattern in which fractal particles grow.

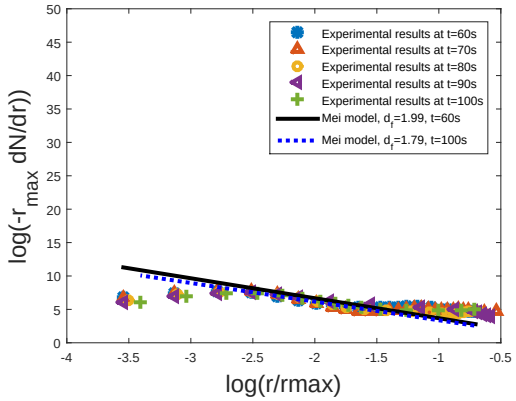

Figure 4. Comparison of the results of Mei model (equation 10) and experimental results at 5 different steps of $t = 60, 70, 80, 90, 100s$.

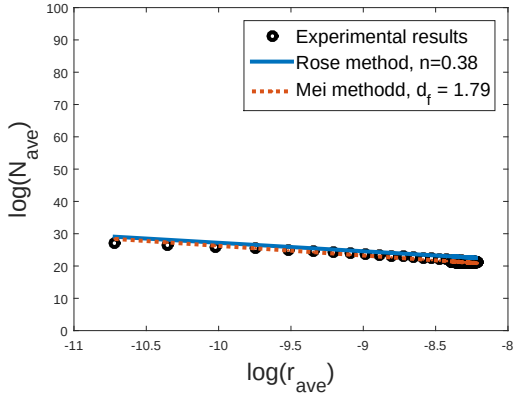

Figure 5. Comparison of the results of average Rose model (equation 5) and average Mei model (equation 14) with experimental results.

B. Examination Simulation Procedure With The Two Methods

Comparison of the results from simulation with the two models are presented in figure 6. It can be seen that predicted line of differential Mei model fits the simulation results with $df = 1.99$, while the line of differential Rose model fits them with $n = 0.35$. These values for n and df agree very well with experimental results discussed in section IV-A. Figure 7 also shows that both average Mei and Rose models fit simulation results with $n = 0.38$ and $df = 1.79$. The surprisingly good agreement between simulation and predicted values from both differential and average models with expected values of n and df validates the method that was used for simulating dropwise condensation. All of these graphs indicate that the process of dropwise condensation obeys the rule of fractals in both differential and average scales.

It is worth to point out here that the grid number is an important parameter in simulation that can affect the values of n and df. Grid number refers to the number of small squares in each iteration from which the number of droplets are chosen. So, it has a direct influence on the distribution function of each generation of

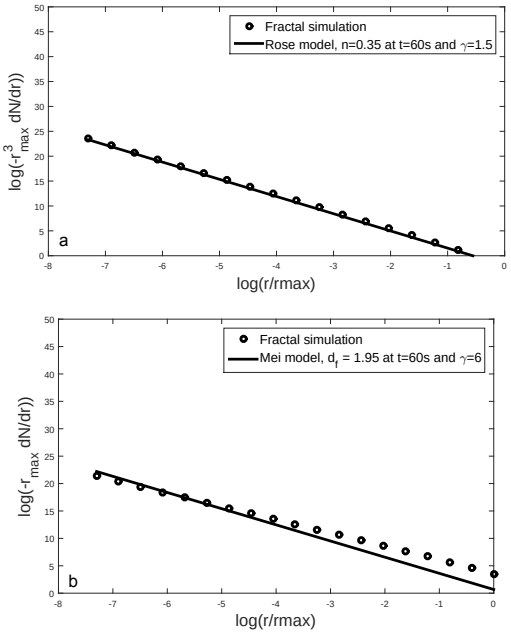

Figure 6. Comparison of the results of both models and results from fractal simulation (a) Rose model(equation 4) (b) Mei model (equation 10).

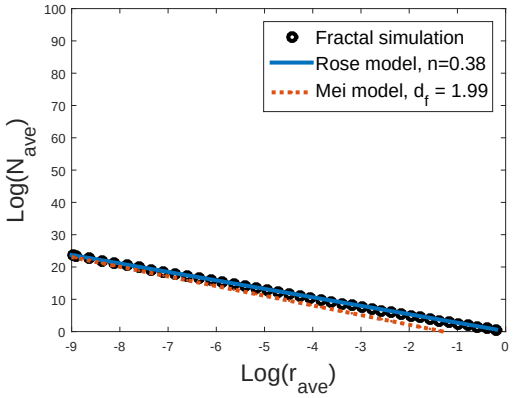

Figure 7. Comparison of the results of average Rose model (equation 5) and average Mei model (equation 14) with results from fractal simulation

droplets. For simplification, instead of grid number we can investigate the effect of γ that is the side length reduction coefficient in each step.

$$\frac{1}{\gamma} = \frac{r_{k+1}}{r_k} \tag{16}$$

Whit this definition the grid number will be γ^2. Choosing bigger values for γ will lead to smaller size but higher number of droplets in each generation (bigger γ, smaller r, bigger N). This will change the values of both right and left hand sides of equations 4 and 10 and changes in both parameters n and df. Figure 8 shows that by increasing γ the slope of both lines of equations 4 and 10 increase but the intercept does not

change significantly. This is because intercept of both lines are in log scale ($log(\frac{n}{\pi})$ and $log(df)$) and are not sensible enough to show the changes. From these two graphs, it is expected to see the sharp increase in both n and df.

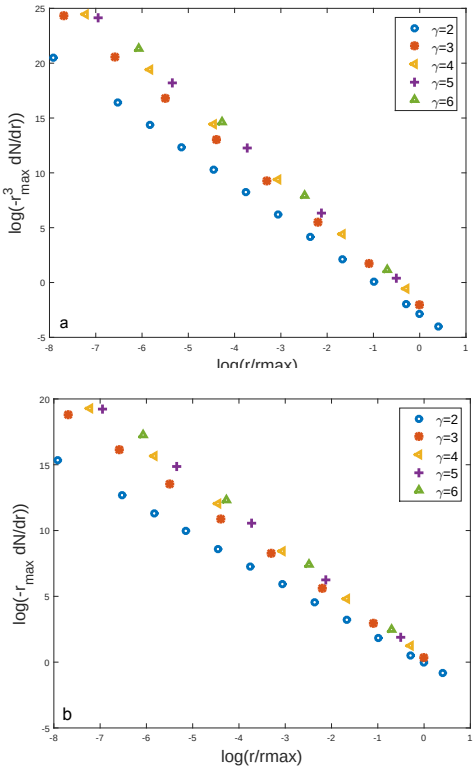

Figure 8. Results of both methods at different γ (a) Rose model (b) Mei model

The predictions for increase in n and df are supported by figure 9. According to this figure, increase in γ will lead to a jump in the value of n and then a constant trend around 0.75. The same pattern is apparent in figure 9 (b), for df that increases rapidly from 0.9 to 1.99 by changing γ from 2 to 6. These figures indicate that the accuracy of simulation proposed in section III depends on the value of γ. Depending on the value of γ n can vary from 0.04 to around 0.75 and the most reported value that is $n = 0.33$ can be obtained by $\gamma = 2.5$. In the case of differential Mei method df varies from 0.9 to 1.99 and the best value that is 1.99 is calculated with $\gamma = 6$.

V. Conclusion

The aim of this research was to investigate drop-size distribution function of droplets during dropwise condensation in both differential and average scales. In this regard, two sets of data were extracted from experimental set up and from our computer simulation algorithm. The models used here were Rose and Mei models that have powerful theoretical basis. It was concluded that both differential and average Rose models were able to describe coalescing droplets evolution with exponent (n) between 0.33 and 0.35 and 0.38

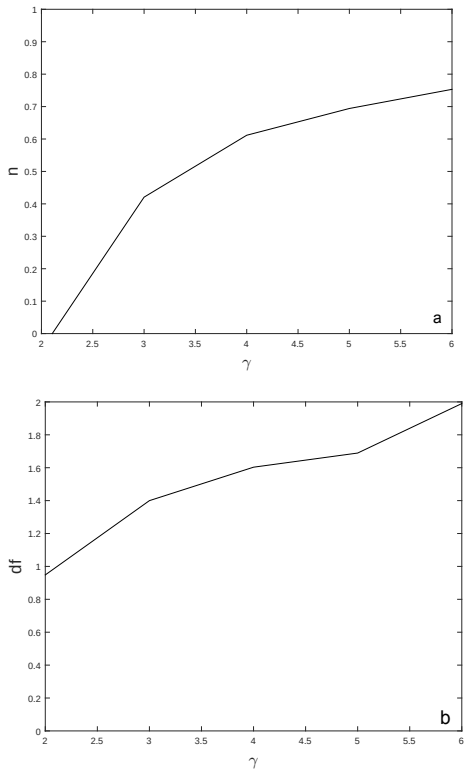

Figure 9. (a) n v.s γ (b) df v.s γ

respectively. While for differential and average Mei models the important parameter is fractal dimension (df) and was between 1.79-1.99 for both experimental and simulation data. Also the effect of side length reduction coefficient (γ) on n and df was investigated and it was found that the best γ for Rose model is around 2.5 and for Mei model is around 6.

Acknowledgments

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