Robust Carrier Tracking Approach for High Dynamic GNSS Signals based on Gaussian Particle Filtering

Pang Jing, Ni Shaojie, and Ou Gang

***Abstract*—**For high dynamic applications of GNSS receivers, the tracking sensitivity is heavily affected by the large unpredictable motion dynamics. The stability of the traditional tracking approaches is poor when outlier occurs among observations. In this paper, a novel Gaussian Particle Filtering-based carrier tracking algorithm is proposed for high dynamic GNSS signals. The proposed algorithm can work stable when the received signal is weak. To deal with the outliers in the observations, a robust carrier tracking algorithm based on Gaussian Particle Filtering is also proposed. Finally, the performance of the proposed algorithms is evaluated by simulation, compared with two typical approaches.

Keywords: Gaussian Particle Filtering, Nonlinear filtering, parameter estimation, GNSS.

I. INTRODUCTION

For high dynamic applications of Global Navigation Satellite System (GNSS) receiver, such as vehicle launching and low-earth-orbit (LEO) satellite positioning, a major problem is the difficulty in tracking GNSS signals. There are mainly two kinds of errors which affect signal tracking. One is additive thermal noise induced on the channel between GNSS satellite and receiver. The other is unpredictable dynamic state of the host vehicle. Traditional GNSS signal tracking approaches use frequency lock loop (FLL) or phase lock loop to estimate carrier phase and Doppler frequency[1]. However, such approach encounters performance tradeoffs when designing loop bandwidth in high dynamic applications: the loop bandwidth needs to be increased to tolerate higher dynamics of host vehicle, while the effects of thermal noise increase with increasing loop bandwidth. It's concluded that for signal-to-noise ratio (SNR) commonly encountered by GNSS receivers in the open sky, which is about 44 dB-Hz, reliable carrier tracking can only be achieved for acceleration not exceeding 5g and jerk not exceeding 5g/s when the traditional FLL/PLL tracking scheme is used [2].

The most straightforward approach to overcome the bandwidth tradeoff is using additional measurement units which can obtain vehicle dynamics independently, including inertial measurement units (IMU) or any other predictable resources such as ephemeris of LEO satellites [3]. In [4], the output of an accelerator is used to adjust Kalman Filter's process noise matrix. In [5], gyroscopic mounting is used to reduce frequency jitter in FLL. In [6], the high-dynamic performance is achieved by aiding from a strapdown inertial navigation system. However, trajectories of most receivers are unpredictable, and IMU based approaches are usually expensive and complex due to additional sensors.

Theoretically, maximum likelihood estimation (MLE) based approach yields the best performance of carrier tracking. A maximum likelihood estimator is used to track signal parameters simultaneously under high dynamic environments, including signal power, initial carrier phase, frequency and code delay [2]. The main drawback of MLE approach is lack of computational efficiency. The cost function is always high-dimensional and nonlinear, which needs iterative solution. Kalman filtering based approaches are also proposed for high dynamic signal tracking because they can provide larger carrier-to-noise ratio (CNR) gain in carrier phase and Doppler tracking compared to the traditional method [7-11]. Kalman filtering is optimal for parameter estimation of a linear system. When it's applied to GNSS signal tracking, the nonlinearity of measurement and state transition should be taken into consideration [12]. Unfortunately, the nonlinear effect plays an important role in carrier tracking performance limiting conditions under high dynamic environments [13]. In [9], second order Extended Kalman Filter (EKF) is used to suppress nonlinear effect, which makes use of higher orders in the Taylor series expansion. In [14], a maximum likelihood estimator is used as frequency discriminator in a Kalman filtering based Frequency Lock Loop. In [15], an FFT-based estimator is used to avoid nonlinearity in frequency estimation. A well-designed algorithm is used to detect Doppler-parameters in [16]. However, these approaches do not solve the essential problem that Kalman Filtering is not designed for a nonlinear system.

Particle filter, first proposed by Gordon [17], is a sequential Monte Carlo method based on Bayesian estimation principle. It uses a series of weighted random samples, which are also called particles, to approximate the posterior probability density, so that any kinds of statistical estimates like mean and variance can be computed easily, which makes it capable to deal with any nonlinear models. One of the key technologies for particle filter is importance sampling, which generates particles to approximate the posterior distribution. The performance of particle filter deteriorates when the statistical probability of generated particles deviates from the true probability distribution after several iterations [18-21]. Gaussian particle filter (GPF) uses Gaussian distribution as the posterior probability density to avoid particle distribution deviation [22].

In this paper, a novel high dynamic carrier tracking algorithm based on the GPF is proposed. Two optimization methods are also proposed to obtain better robustness.

II. PROBLEM DESCRIPTION

GNSS satellites transmit pseudorandom signals at L-band frequency. Due to the transmission delay and relative motion between satellite and receiver, the received signal can be expressed as

$$s_R(t) = AD(t)C(t)\cos\left[2\pi f_{IF}t + \theta_c(t)\right] + n_{0R} \tag{1}$$

where A is the received signal amplitude; $D(t)$ is the navigation message modulated on the signal with bit rate of 50 bps; $C(t)$ is the pseudorange code; f_{IF} is signal intermediate frequency after down-converted by the receiver front-end; θ_c is carrier phase at time t; n_{0R} is the thermal noise obeying Gaussian distribution. Note that in equation (1), Doppler effect caused by the relative motion is included in the time-varying carrier phase, so θ_c can be expressed as

$$\theta_c(t) = 2\pi\int_0^t \left(f_d - n_{dclk}\right)dt \tag{2}$$

where f_d is the Doppler frequency shift, and n_{dclk} is clock drift which can be modeled as random noise [23]. Although the code frequency $C(t)$ is also shifted by the Doppler effect, it's ignored in this paper because the code Doppler shift is a fixed ratio to the carrier Doppler shift [1]. Equation (2) can be expressed in discrete form,

$$\theta_c(k+1) = \theta_c(k) + 2\pi\left[f_d(k)T + \frac{1}{2}f_d'(k)T^2 + \frac{1}{6}f_d''(k)T^3\right] + n'_{dclk} \tag{3}$$

where T is the sampling period, n'_{dclk} is the combination of discrete clock drift noise and linear approximation error, which is assumed to follow Gaussian Distribution. The covariance matrix of n'_{dclk} is given by [24]

$$Q = Q_\theta\begin{bmatrix} T & O_{1\times3} \\ O_{3\times1} & O_{3\times3}\end{bmatrix} + Q_d\begin{bmatrix} T^3/3 & T^2/2 & 0 \\ T^2/2 & T & 0 \\ 0 & 0 & O_{2\times2}\end{bmatrix} + Q_a\begin{bmatrix} T^6/252 & T^5/72 & T^4/30 & T^3/24 \\ T^5/72 & T^4/20 & T^3/8 & T^2/6 \\ T^4/30 & T^3/8 & T^2/3 & T/2 \\ T^3/24 & T^2/6 & T/2 & 1\end{bmatrix} \tag{4}$$

where Q_θ denotes the spectral intensity of clock phase bias error, Q_d is the spectral intensity of clock frequency drift error, and Q_a is caused by the linear approximation error.

The received signal is then mixed with local generated I/Q carrier replicas, and the pseudorange code is removed through coherent integration with local generated pseudorange code replica. The output of the k-th coherent integration can be expressed as

$$r_k = AD_kR(\tau)\frac{\sin(\pi f_eT)}{\pi f_eT}\exp(j\theta_e) + n_0 \tag{5}$$

where $R(\tau)$ is the auto-correction of the pseudorange code due to code phase delay τ; f_e is the average frequency error during integration; θ_e is the average phase error during integration; n_0 is additive complex white Gaussian noise. Note that in equation (3), D_k is assumed to be constant during the integration because the integration time, typically 1ms for high dynamic applications, is much smaller than the bit duration. The covariance matrix of n_0 can be expressed by

$$R = E\left\{\begin{bmatrix}\mathrm{Re}(n_0) \\ \mathrm{Im}(n_0)\end{bmatrix}\begin{bmatrix}\mathrm{Re}(n_0) & \mathrm{Im}(n_0)\end{bmatrix}\right\} = \begin{bmatrix}\sigma_0^2 & 0 \\ 0 & \sigma_0^2\end{bmatrix} \tag{6}$$

III. GAUSSIAN PARTICLE FILTERING

The tracking error of carrier phase and Doppler frequency before each update epoch follows linear propagation model, according to equation (3),

$$X_k = \Phi X_{k-1} + BU_{k-1} + n'_{dclk} \tag{7}$$

where $X_k = \left[\theta_c(k), f_d(k), f_d'(k), f_d''(k)\right]^T$ is state vector; $U_k = \left[\theta_{nco}(k), f_{nco}(k)\right]^T$ represents the carrier phase and frequency of local generated carrier replicas; Φ and B denote the state transition matrix and control matrix, respectively,

$$\Phi = \begin{bmatrix} 1 & 2\pi T & 2\pi T^2/2 & 2\pi T^3/6 \\ 0 & 1 & T & T^2/2 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (8)$$

$$B = \begin{bmatrix} -1 & -2\pi T \\ O_{1\times 3} & O_{1\times 3} \end{bmatrix}$$

The measurement equation (5) can also be written as,

$$r_k = h(X_k) + n_0 \quad (9)$$

The basic idea of particle filtering is to use a series of weighted random particles to approximate posterior probability density. The approximated probability density is given by,

$$p(X_k \mid r_k) \approx \sum_{i=0}^{N-1} w_k^i \delta(X_k - X_K^i) \quad (10)$$

where $\{X_k^i\}$ is a set of the generated particles sampled at epoch k; $\{w_k^i\}$ are the corresponding normalized weights; N is the number of particles; $\delta(\cdot)$ is Dirac-Delta function. The un-normalized weights of particles at epoch k is derived by [21],

$$\overline{w}_k^i = w_{k-1}^i \frac{p(r_k \mid X_k^i)\, p(X_k^i \mid X_{k-1}^i)}{p(r_k \mid r_{k-1})} \quad (11)$$

The state vector at epoch k X_k is estimated by,

$$X_k = \sum_{i=0}^{N-1} w_k^i X_k^i \quad (12)$$

And then, the importance density is resampled and the particles are re-generated.

Gaussian Particle filter approximates the posterior probability density by Gaussian density,

$$p(X_k) = N(X_k; \mu_k, \Sigma_k) \quad (13)$$

where μ_k is mean value, and Σ_k is the covariance of X_k. So in each epoch when the importance density are resampled, only $\mu_{k|k-1}$ and $\Sigma_{k|k-1}$ need to be updated as,

$$\mu_{k|k-1} = \frac{1}{N} \sum_{i=0}^{N} X_{k|k-1}^i \quad (14)$$

$$\Sigma_{k|k-1} = \frac{1}{N} \sum_{i=0}^{N-1} \left(X_{k|k-1}^i - \mu_{k|k-1}\right)\left(X_{k|k-1}^i - \mu_{k|k-1}\right)^T \quad (15)$$

When a new measurement r_k is obtained, the posterior probability density is given by,

$$p(X_k \mid r_k) \approx \frac{p(r_k \mid X_k)\, N(X_k; \mu_{k|k-1}; \Sigma_{k|k-1})}{p(r_k \mid r_{k-1})} \quad (16)$$

So the proposed GPF-based carrier tracking algorithm can be concluded as follows,

1. Generate particles from $N(x_{k-1}; \mu_{k-1}, \Sigma_{k-1})$, denoted as $\{X_{k-1}^i; i = 1...N\}$;
2. For i = 1: N

    2.1 Generate particles from $p(X_{k|k-1} \mid X_{k-1} = X_{k-1}^i)$, denoted as $\{X_{k-1|k}^i\}$

    2.2 Compute $\mu_{k|k-1}, \Sigma_{k|k-1}$ due to equations (14) and (15);

    2.3 Compute un-normalized weights,

$$\overline{w}_k^i = w_{k-1}^i \frac{p(r_k \mid X_k)\, N(X_k; \mu_{k|k-1}; \Sigma_{k|k-1})}{p(r_k \mid r_{k-1})} \quad (17)$$

3. Normalize obtained weights,

$$w_k^i = \overline{w}_k^i \Big/ \sum_{j=0}^{N-1} \overline{w}_k^j \quad (18)$$

4. Compute the estimated state vector due to equation (12);
5. Update the mean and covariance of the Gaussian density used in step 1 for next epoch.

IV. ROBUST CARRIER TRACKING APPROACH

The proposed GPF-based tracking approach can eliminate nonlinear error effectively, since the transition of probability density is approximated directly as a finite number of particles. However, in most practical situations, random noise don't precisely obey the assumed probability model, especially Gaussian model used in GPF, which means outliers always appear sometime. The deviation to assumption models lead to performance degradation.

To improve robustness of GPF, the strong tracking filtering [25] concept is applied to GPF, denoted by STF-GPF. Since the prior probability model of the estimated state vector is decided by the posterior probability model of the previous epoch according to equation (7), STF-GPF computes $\mu_{k|k-1}$ and $\Sigma_{k|k-1}$ directly as follows, instead of steps 2.1 ~ 2.2 in Section 3,

$$\mu_{k|k-1} = \Phi\mu_{k-1}; \Sigma_{k|k-1} = \lambda_k \Phi \Sigma_{k-1} \Phi^T + Q_k \quad (19)$$

Notice that a fading factor λ_k is introduced in equation (19) so that the impact of old measurements is reduced automatically. It's proven that when the innovations $\{\gamma_k\}$, defined by $\gamma_k = r_k - h\left(X_{k|k-1}\right)$, are orthogonal at each epoch[25], all useful information in the observations is fully used during the filtering procedure. The orthogonality of innovations are defined by,

$$E\left(\gamma_{k+j}\gamma_k^T\right) = 0;\ j \neq 0 \tag{20}$$

According to equation (20), λ_k can be derived as follows,

$$\lambda_k = \begin{cases} \lambda_{0,k} = tr\left(N_k\right) / tr\left(M_k\right); \lambda_{0,k} \geq 1 \\ 1; \lambda_{0,k} < 1 \end{cases} \tag{21}$$

$$M_k = H\Phi\Sigma_{k-1}\Phi^T H^T; N_k = C_{0,k} - HQ_k H^T - l_k R_k \tag{22}$$

$$C_{0,k} = \frac{\rho C_{0,k-1} + \gamma_k \gamma_k^T}{1+\rho}; C_{0,0} = \gamma_0 \gamma_0^T \tag{23}$$

where $tr(\cdot)$ denotes the trace of a matrix; ρ is forgetting factor with typical value of 0.95; $l_k \geq 1$ is an adjustable softening factor; H_k denotes the Jacobian matrix of the measurement equation, which is given by

$$H_k = \left.\frac{dh\left(X\right)}{dX}\right|_{X = X_{k|k-1}} \tag{24}$$

V. SIMULATION RESULTS

The proposed algorithms are tested by simulation. The simulated signal uses real GPS PRN codes (PRN = 1), modulated by randomly generated navigation data bit with data transition probability of 50%. High dynamic model used in the simulation follows the dynamic model proposed by the Jet Propulsion Laboratory (JPL)[13]. The acceleration keeps at about -25 g/s for 3 seconds, and then a sudden jerk of 100 g/s2 happens, which lasts 0.5 second, and then constant acceleration for 3 seconds, jerk for 0.5 second and constant acceleration for 2 seconds consecutively.

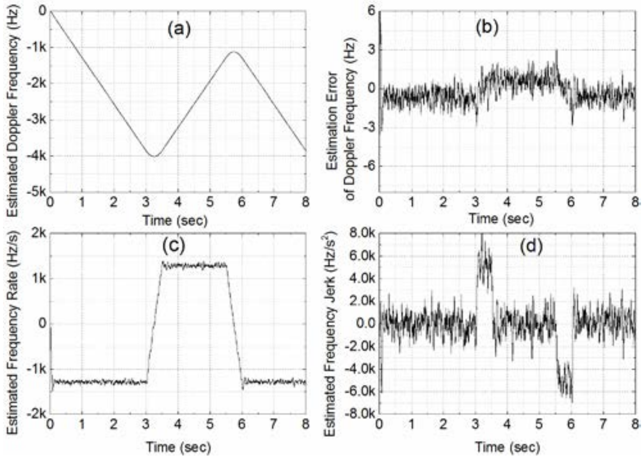

Figure 1 JPL test (CNR = 44dB-Hz)

The performance of proposed GPF-based tracking approach is evaluated under different CNRs. Figure 1 shows carrier tracking results of GPF-based approach when CNR is 44 dB-Hz. Figure 1(a) is the estimated Doppler frequency; Figure 1(b) is the estimation error of Doppler frequency; Figure 1(c) is the estimated frequency rate; Figure 1(d) is the estimated frequency jerk. It's obvious that the proposed GPF-based carrier tracking algorithm works well under extremely high dynamic situations.

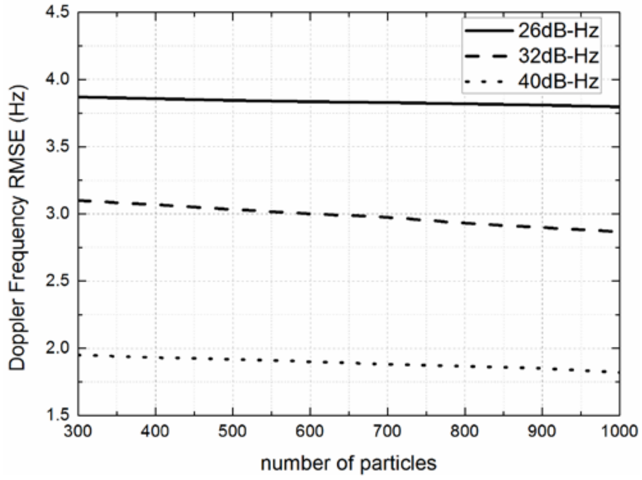

Figure 2 Tracking performance for different particle numbers and CNRs

The root mean square errors (RMSEs) of the estimated Doppler frequency under different CNRs are shown in Figure 2, with different number of particles used. The RMSE increases with the decreasing of CNR. It can be found that the estimation error can be reduced by increasing the number of particles used in GPF. As shown in Figure 2, the RMSE of Doppler frequency decreased about 0.2 Hz when the number of particles increases from 300 to 1000, with CNR = 32 dB-Hz.

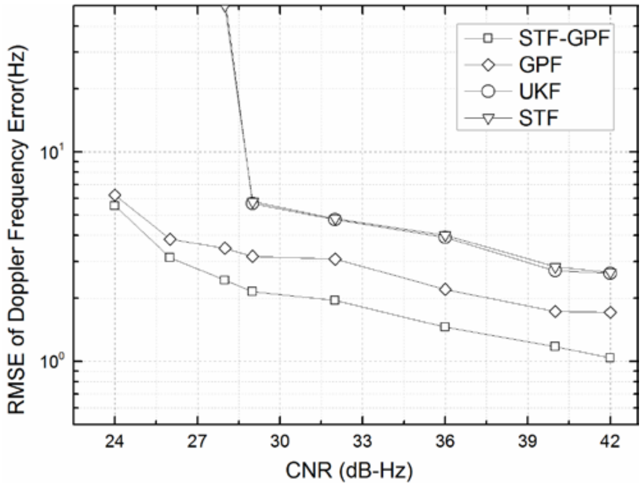

Figure 3 Performance comparison among different tracking algorithm

The performance of the proposed robust carrier tracking approach (STF-GPF) is also evaluated under different CNRs, compared with GPF-based approach and other 2 typical high dynamic tracking approaches, which are Unscented Kalman Filtering (UKF)[26] based approach and Strong Tracking Filtering[25] based approach. 1% (equally to 3sigma confidence interval) outliers are added to the simulated signal manually. The RMSEs of Doppler frequency are shown in Figure 3. UKF and STF based approaches loss lock when CNR is lower than 27dB-Hz, however STF-GPF and GPF can keep stable tracking when CNR is 24dB-Hz. The tracking error of both proposed STF-GPF and GPF is much smaller than UKF and STF. STF-GPF shows the best performance among all the four tracking approaches, especially when the signal quality is good, since STF-GPF shows better robustness to the outliers.

VI. CONCLUSION

In this paper, a novel carrier tracking algorithm based on GPF is proposed for high dynamic GNSS signals. The GPF-based algorithm uses Bayesian theorem to estimate carrier phase and Doppler frequency to eliminate nonlinear error. The posterior probability density is assumed to be Gaussian and approximated by randomly generated particles. To improve the robustness of GPF in case of model deviation, STF-GPF is proposed by applying fading factor to GPF. The performance of the proposed algorithms is evaluated by simulations, using JPL's high dynamic model. Two classical tracking algorithms are also tested for comparison. Simulation results show that the proposed algorithms are ideally suited for improving the performance of carrier tracking in high dynamic environments. The proposed STF-GPF shows obvious better performance when model deviation occurs.

ACKNOWLEDGEMENTS

This work was supported by "National Natural Science Foundation of China", No. 61403413.

REFERENCES

[1] E. Kaplan and C. Hegarty, Understanding GPS: principles and applications: Artech house, 2005.
[2] W. Hurd, J. Statman, and V. Vilnrotter, "High dynamic GPS receiver using maximum likelihood estimationand frequency tracking," Aerospace and Electronic Systems, IEEE Transactions on, pp. 425-437, 1987.
[3] Y. Xu, Q. Chang, and Z. Yu, "A tracking method for weak and high dynamic GNSS inter-satellite link signal," in Consumer Electronics, Communications and Networks (CECNet), 2012 2nd International Conference on, 2012, pp. 111-114.
[4] Y. Guo, W. Q. Wu, and K. H. Tang, "A New Inertial Aid Method for High Dynamic Compass Signal Tracking Based on a Nonlinear Tracking Differentiator," Sensors, vol. 12, pp. 7634-7647, Jun 2012.
[5] M. Abedi and T. Jin, "Improvement in tracking loop threshold of high dynamic GNSS receiver by installation of crystal oscillator on gyroscopic mounting," in Frequency Control Symposium & the European Frequency and Time Forum (FCS), 2015 Joint Conference of the IEEE International, 2015, pp. 522-527.
[6] X. L. Wang, X. C. Ji, S. J. Feng, and V. Calmettes, "A high-sensitivity GPS receiver carrier-tracking loop design for high-dynamic applications," Gps Solutions, vol. 19, pp. 225-236, Apr 2015.
[7] J. H. WON, D. Dötterböck, and B. Eissfeller, "Performance Comparison of Different Forms of Kalman Filter Approaches for a Vector‐Based GNSS Signal Tracking Loop," Navigation, vol. 57, pp. 185-199, 2010.
[8] K. H. Kim, G. I. Jee, and J. H. Song, "Carrier Tracking Loop using the Adaptive Two-Stage Kalman Filter for High Dynamic Situations," International Journal of Control Automation and Systems, vol. 6, pp. 948-953, Dec 2008.
[9] N. I. Ziedan and J. L. Garrison, "Extended Kalman filter-based tracking of weak GPS signals under high dynamic conditions," in Proceedings of ION GNSS, 2004, pp. 20-31.
[10] M. Lashley, D. M. Bevly, and J. Y. Hung, "Performance Analysis of Vector Tracking Algorithms for Weak GPS Signals in High Dynamics," Ieee Journal of Selected Topics in Signal Processing, vol. 3, pp. 661-673, Aug 2009.
[11] F. D. Nunes, J. M. N. Leitao, and F. M. G. Sousa, "Nonlinear Filtering in GNSS Pseudorange Dynamics Estimation Combining Code Delay and Carrier Phase," Ieee Journal of Selected Topics in Signal Processing, vol. 3, pp. 639-650, Aug 2009.
[12] R. G. Brown and P. Hwang, "Introduction to Random Signal Analysis and Applied Kalman

Filtering 3rd Ed. Pg 353," ed: John Wiley & Sons, 1997.

[13] S. Hinedi and J. Statman, "High-dynamic GPS tracking," NASA Publication, vol. 5, pp. 89-113, 1988.

[14] X. Yang and H. Xiulin, "Maximum likelihood estimation based high dynamic GPS carrier tracking loop," Acta Electronica Sinica, vol. 38, pp. 1563-1567, 2010.

[15] A. Peng, G. Ou, and M. Shi, "Frequency estimation of single tone signals with bit transition," IET Signal Processing, vol. 8, pp. 1025-1031, 2014.

[16] J. B. Zhang, S. F. Zhang, and Y. Jiang, "A Novel Doppler-Parameter Detection Algorithm in the High Dynamic Carrier Tracking Loop," Chinese Journal of Electronics, vol. 22, pp. 187-191, Jan 2013.

[17] N. J. Gordon, D. J. Salmond, and A. F. Smith, "Novel approach to nonlinear/non-Gaussian Bayesian state estimation," in IEE Proceedings F (Radar and Signal Processing), 1993, pp. 107-113.

[18] O. Hlinka, O. Slučiak, F. Hlawatsch, P. M. Djurić, and M. Rupp, "Distributed Gaussian particle filtering using likelihood consensus," in Acoustics, Speech and Signal Processing (ICASSP), 2011 IEEE International Conference on, 2011, pp. 3756-3759.

[19] R. Douc and O. Cappé, "Comparison of resampling schemes for particle filtering," in Image and Signal Processing and Analysis, 2005. ISPA 2005. Proceedings of the 4th International Symposium on, 2005, pp. 64-69.

[20] G. Grisetti, C. Stachniss, and W. Burgard, "Improving grid-based slam with rao-blackwellized particle filters by adaptive proposals and selective resampling," in Robotics and Automation, 2005. ICRA 2005. Proceedings of the 2005 IEEE International Conference on, 2005, pp. 2432-2437.

[21] M. S. Arulampalam, S. Maskell, N. Gordon, and T. Clapp, "A tutorial on particle filters for online nonlinear/non-Gaussian Bayesian tracking," Signal Processing, IEEE Transactions on, vol. 50, pp. 174-188, 2002.

[22] J. H. Kotecha and P. M. Djurić, "Gaussian sum particle filtering for dynamic state space models," in Acoustics, Speech, and Signal Processing, 2001. Proceedings.(ICASSP'01). 2001 IEEE International Conference on, 2001, pp. 3465-3468.

[23] M. Shin, C. Park, and S. Lee, "Atomic clock error modeling for GNSS software platform," in Position, Location and Navigation Symposium, 2008 IEEE/ION, 2008, pp. 71-76.

[24] M. L. Psiaki and H. Jung, "Extended Kalman filter methods for tracking weak GPS signals," in Proceedings of the 15th International Technical Meeting of the Satellite Division of The Institute of Navigation (ION GPS 2002), 2001, pp. 2539-2553.

[25] X. Xie, D. Zhou, and Y. Jin, "Strong tracking filter based adaptive generic model control," Journal of Process Control, vol. 9, pp. 337-350, 1999.

[26] J. Lopez-Salcedó, D. Peral-Rosado, and G. Seco-Granados, "Survey on robust carrier tracking techniques," Communications Surveys & Tutorials, IEEE, vol. 16, pp. 670-688, 2014.