Comparison between available assessment methods of historical masonry arches

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***Abstract*—**The aim of this paper is to compare each other the main analytical and numerical methods for the assessment of masonry arch bridges, highlighting strengths and weaknesses. The methods are mainly three: i) the Thrust Line Analysis Method; ii) the Mechanism Method; iii) the Finite Element Method. In addition a particular closed-form approach has been recently developed, the Elastio-Plastic Method. The Thrust Line Analysis Method and the Mechanism Method are analytical methods and are based on two of the fundamental theorems of the Plastic Analysis, while the Finite Element Method is a numerical method that uses different strategies of discretization to analyze these structures.

***Keywords*—** Masonry arch bridges, Structural Models, Discrete limit analysis, Collapse Mechanism.

I. INTRODUCTION

AS reported in [1,2,3], numerous methods have been employed during the last decades for the Collapse Analysis of Masonry Arch Bridges. Structural analysis is a general term describing the operations to represent the real behavior of a construction. The analysis can be founded on mathematical models created on theoretical bases or on physical models tested in laboratory. In both cases, the models try to individuate the load carrying capacity of the structure, identifying the stress state, the strain and the internal forces distribution of the entire structure or of its parts. Besides, the models proposed for arch structures try to indicate the failure mode and the location of plastic hinges.

As previously seen, among the three fundamental structural criteria (strength, stiffness and stability), it is the stability that governs the life of the masonry arches because the average medium stresses are low and the strains are negligible. So the most important methods for the evaluation of masonry arch bridges are based on Heyman's theories and on the fundamental theorems of the Plastic Analysis. They are: i) the thrust line analysis method; ii) the mechanism method.

The Thrust Line Analysis Method is based on the lower bound theorem or "safe" theorem and defines the limits for the thrust line location. It uses a static approach and defines the limit load that ensures the equilibrium of the arch bridge analyzed. On the contrary, the Mechanism Method is based on the upper bound theorem and studies the number of plastic hinges needed to transform the arch in a mechanism. In this case, the stability of the arch is analyzed with regards to a kinematic approach. Both the methods are valuable: due to their different bases, the first one underestimates the structure strength, while the second overestimates it.

Another method frequently used to describe the structural behavior of the masonry arch bridges is the Finite Element Method. The Finite Element Method represents the most versatile tool for the numerical analysis of structural problems. However in the case of historic masonry, the peculiar nature of materials leads to pay particular attention to the application of this method (see e.g. [4]).

A particular closed-form approach has been recently developed in [5,6] .This method is based on the fundamental theorems of limit analysis and is used to determine the critical points with a relatively small modeling effort. To assure the stability of the masonry arch bridges, a model based on equilibrium equations and compatibility conditions is first developed. Next, the material properties are added to determine the formation of the hinges.

The aim of this paper is to give an overview of the four methods and to compare each other in terms of collapse load and the position of the four hinges.

A. Thrust Line Analysis Method

This general method analyzes the arch stability, evaluating the location of the thrust line inside the cross section. The thrust line represents the locus of points along the arch through which the resultant forces pass. If all the arch voussoirs have the same size, the line of thrust has almost the shape of an inverted catenary.

The thrust line analysis method defines the load carrying capacity by limiting the zone where the resultant force can be positioned. This method presents some variants which differ from each other by the size of the limits. The limits depend on the theory and the material model assumed.

Thrust line analysis together with Heyman's safe theorem can be used to elaborate computational strategies for the structural analysis of masonry arch bridges. For example, Philip Block [7] developed an interactive computational procedure that uses the thrust lines to clearly visualize the forces within the masonry and to predict possible collapse modes.

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The program lets the user to change the arch geometry, analyzing the different locations that can be assumed by the thrust line.

Between the specialized analysis programs based on this method, there is also Archie-M developed by Harvey and OBVIS Ltd12 in 2001 [8].

Although the aim of Archie-M is to demonstrate whether an arch bridge can withstand a given load or not, the collapse load can be estimate by varying the load value until a sufficient number of hinges is formed. The program provides also the internal forces and the thrust zone position for each arch segment. The live load is distributed through the fill with a sine shape. The backfill is modeled as a continuous body that spreads the load and provides both active and passive soil pressure.

B. Mechanism Method

The Mechanism Method is a kinematical method, based on the upper bound approach. This method belongs to the plasticity theory and was firstly used for steel structures. Later Heyman has applied it to masonry arch. The term mechanism refers to the possibility of structure to move in accordance to internal and external constraints. This Method assumes that a masonry arch becomes a mechanism when at least four plastic hinges open. Many experimental tests confirm this hypothesis. However position of hinges is unknown.

The two-dimensional rigid-plastic analysis has been inserted by Gilbert and Melbourne into a software called RING[9] , developed by the University of Sheffield spin-off company, LimitState Ltd. The program is able to analyze multi-span masonry arch bridges, built of arch barrels, supports and backfill. A particular feature of this software is the capacity to analyze multi-ring arches enabling separations between the various rings [10].

The program employs an efficient linear programming technique for the solution of virtual works equations. This mathematical optimization allows identifying the ultimate limit state, determining the percentage of live load that will lead to the collapse.

As a result of the analysis, the minimum adequacy factor for live load is obtained, together with a graphic representation of the thrust line and the failure mode. Exact location of hinges is indicated. The live load is distributed through a Boussinesq distribution with a maximum spread angle. The passive pressure is the only lateral pressure used.

C. Finite Element Method

Masonry arch bridges can be analyzed also using the Finite Element Method (FEM). Nowadays this method can be considered the most general instrument for numerical analysis in structural problems.

In the last twenty years, many researchers have developed different finite element models for materials with low tensile strength, such as masonry However the current knowledge of masonry mechanics is underdeveloped in comparison with other fields, as concrete and steel. So the Finite Element Method can be applied to the masonry analysis, but with particular attention due to the specific nature of the material.

The discretization of the structure is the first step of this method. While in the frame structures there is a univocal choice, in the masonry structures there are different strategies of discretization. The main reason is due to the particular characteristics of masonry, which is an anisotropic material composed by bricks and mortar. In particular the presence of the mortar is difficult to model.

The key point in the development of accurate stress analyses of masonry constructions is the definition and the use of suitable constitutive laws. Taking into account the heterogeneity of the masonry material, the models proposed in literature can be divided three different classes concerning their grade of definition: micro-modeling; multi-scale modeling; macromodelling.

Micro-models simulate each constituent of the masonry material with its own specific constitutive law and failure criteria. Micro-models can be detailed or simplified [11]. In the first case, the unit and the mortar are constituted by continuum elements, while the unit-mortar interface is represented by discontinuous elements. In the second case, mortar and brick/mortar interface are combined in a single discontinuous joint element, so it is possible to consider masonry as a set of elastic blocks bonded together by potential fracture line. The mechanical properties of elements that characterize the micro-model can be obtained through experimental tests conducted on the single material components.

The principal disadvantages of the micro-models are the highly refined mesh and the great computational effort. In fact both the unit blocks and the mortar beds have to be discretized, obtaining a high number of nodal unknowns. Nevertheless, this model is the most suitable to reproduce laboratory tests.

Multi-scale models consider firstly different constitutive laws for the units and the mortar joints; then, a homogenization procedure is performed obtaining a macro-model for masonry which is used to develop the structural analysis.

In this contest analysis, the model developed by Brasile et al.[12] is one the most significant. In this case, the strategy is based on an iterative scheme which uses simultaneously two different modeling of masonry. The first one is defined at the scale of the local brick and joint and describes the nonlinear mechanical interaction; the second one is defined at the global scale of the wall and looks like an approximation of the previous model. The passage from one scale to another is obtained through an operator that is able to define the global displacements starting by local ones. Also in this case, the mechanical properties of units and mortar joints are obtained through experimental tests.

The main advantage of this model is to derive the stress-strain relationship in a rational way, taking into account the mechanical properties of each material component. On the other side, the nonlinear homogenization procedure could induce some computational difficulties.

Macro-model is also called homogeneous or continuous model because it considers the masonry as a smeared continuum with no distinction between the units and mortar joints. The model treats the masonry as anisotropic composites and uses constitutive stress-strain relationship to define the behaviour of the masonry material.

Although the model is unable to describe in detail some damage mechanisms, it is very effective from a computational point of view when structural analyses are performed. As matter of fact it is the only method that can be used in presence of a large number of units and joints without a more expensive computational effort.

In general, the finite element method applied to masonry arch bridges is mainly based on global aspects rather than on local approaches. Macro-models use an isotropic homogenized failure surface similar to those developed for the analysis of concrete structure.

The most simplified idea can be given by Rankine criterion, but a more refined and appropriated criterion for concrete-like materials is the William-Warnke criterion[13].

It's a criterion that is conceived to describe the concrete, but can also be applied to other brittle materials, as masonry. It is a good criterion, but it is complicated because it uses five parameters. Cracking is modeled through an adjustment of the material properties and it is simulated through a "smeared band" of cracks, rather than discrete cracks.

The smeared crack model allows the crack opening in three orthogonal directions for every point of integration.

The complex behavior of masonry is assumed to be isotropic before cracking and orthotropic after cracking.

So cracking occurs when the tensile stress exceeds the limit value (Rankine criterion), while the crushing takes place when all of the principal stresses are compressive and exceed the limit value. Failure domain for biaxial stress state is represented in Fig.1. The meridians of tension and compression are respectively two parables. They are connected by an ellipsoidal surface, passing through the elliptical deviatoric curve as base section.

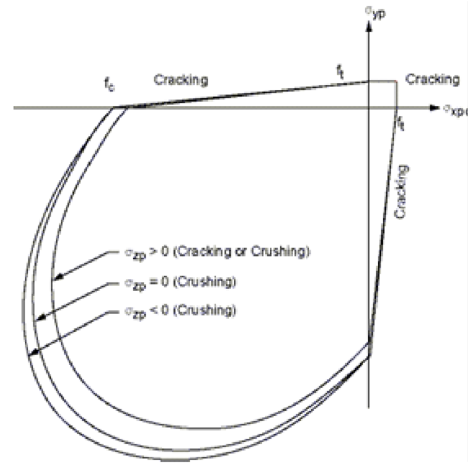

Fig.1 William-Warnke Criterion. Failure domain for plane stress states.

Also the backfill can be modeled through additional elements that allow the transfer of loads and passive reactions on the arch barrel. Different constitutive models have been proposed for soils modeling. The differences are based on the shape of the yield surface in the meridian plane, the shape of yield surfaces in the deviatoric stress plane and the use of flow rules. The material "soil" is considered usually nonlinear and is defined by Mohr-Coloumb or Drucker-Prager limit criteria.

Mohr-Coulomb Criterion is the best known failure criterion in soil mechanics. It is the first type of failure criterion that takes into consideration the effect of the hydrostatic pressure on the strength of granular materials.Mohr- Coulomb's failure surface is an irregular hexagonal pyramid in the principal stress space.

Drucker-Prager criterion represents the major advance in the extension of metal plasticity to soil plasticity. It is the approximate expression of the Mohr-Coulomb criterion. The aim is to overcome the problem of Mohr-Coulomb criterion, i.e. the gradient of the plasticization function was not defined in a univocal way on the pyramid corners. Drucker-Prager Criterion provides as failure surface a cone whose axis is the hydrostatic axis . Failure domains for biaxial stress state is represented in Fig.2.

In finite element method, it is more convenient to use Drucker-Prager criterion than Mohr-Coulomb criterion. In fact the Mohr-Coulomb hexagonal failure surface is mathematically convenient only in problems where it is obvious which one of six sides is to be used. If this information is not known in advance, the corners of the hexagon can cause considerable difficulties and give rise to complications in obtaining a numerical solution with the finite element models.

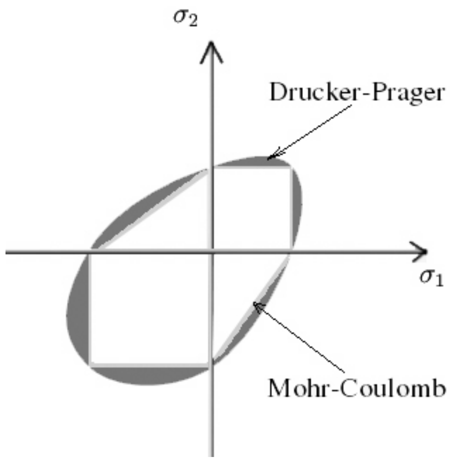

Fig.2 Mohr-Coulomb and Drucker-Prager Criteria. Failure domain for plane stress states.

Also the finite element method comprises computer-based representations. They are specialized ready-to-use computer programs that can be applied to masonry arches as to other type of structures.

Computer FEM systems frequently used to analyze the masonry structures are ABAQUS23 or DIANA24 often with self-implemented user codes to these applications. But there are other ones. For example, Ng et al.[14] used a FEM commercial package nonlinear, LUSAS26, with a two-dimensional model to analyze a series of arch bridges. In this case, masonry is modeled as Von Mises material with different strengths in tension and compression.

In 2001 Fanning et al. [15] generate three-dimensional non linear finite element models, using a commercially available finite element package, ANSYS. The masonry behavior is modeled using a solid element that may have its stiffness modified by the developments of cracks and crushing. The fill is modeled as a Drucker-Prager material. In this paper only two-dimensional finite element modeling will be employed, even if three-dimensional modeling is very accurate but requires a very high computational effort.

D. Elasto-Plastic Model(EPM)

A particular closed form solution for the structural stability of arch bridges has been proposed by Adenaert et al.[16].

Also this approach is based on the fundamental theorems of limit analysis and employs a simplified homogeneous material model [11] to determine the critical points with a relatively small modeling effort.

Firstly, a basic model is presented starting from the equilibrium equations. After solving the differential equilibrium equations, the analytical expressions for the internal forces are derived as a function of three constants of integration. To obtain an univocal solution, boundary conditions must be introduced.

These equations are used to determine the three constants of integration, starting from the value of the abutment displacements. In this way it is possible to determine also the displacements in every point of the arch.

Then the material properties can be added to allow the occurrence of cracks and the subsequent formation of the hinges. The elasto-plastic model assumes a hinge to behave in a perfect plastic manner. The load factor is increased until a hinge has been formed and the boundary conditions are changed so the moment in the hinge stays constant. The process is repeated until the formation of the fourth hinge.

II. APPLICATION TO A GENERIC MASONRY ARCH BRIDGE

In order to give a general overview on the use of these methods, the structural analysis of a generic fictitious arch bridge is performed (Fig.3). The material properties are reasonably hypothesized. The structure is statically determinate to the third degree and will collapse as soon the four hinges occur. A vertical concentrated point load P, applied at 0.75 (42.97°), and backfill load are imposed on the bridge.

The different methods and models are compared each other in terms of collapse load and the position of the four hinges.

The geometrical data are :

- Span = 2.80 m;
- Radius =1.4 m;
- Thickness of the Arch Barrel = 0.5 m;
- Height of the Backfill= 2 m;
- Width = 1 m.

The Masonry data are :

- Specific weight of the masonry arch = 21000 N/m3;
- Young's Modulus = 5000 MPa;
- Poisson's ratio = 0.3;
- Compressive Strength of Masonry = 8 MPa.

The Backfill data are:

- Specific weight of the backfill = 21600 N/m3;
- Young's Modulus = 15000 MPa;
- Poisson's ratio = 0.3;
- Angle of friction = 350;
- Cohesion = 0.001;
- Angle of dilatancy = 350.

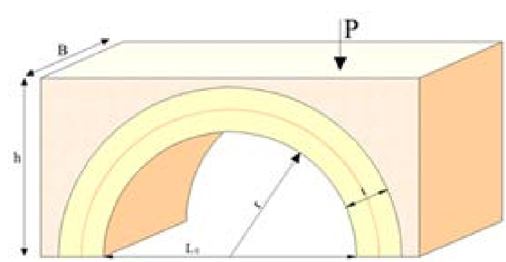

Fig. 3 Geometry of the Masonry arch bridge

Archie-M software related to the Thrust line method and

Ring software related to the Mechanism model are used. The Outputs are shown in Figs.4 end 5. It is easy to observe [2] that the hinges are located alternatively in the intrados and in the extrados, following a pattern comparable to that described by Heyman for the point load case (Fig.6).

In addition to the hinge positions, Ring also gives the failure mode as graphic output. Concerning the collapse load, Archie-M estimates a load smaller than Ring: the first one is equal to 165.2 KN, the second one is equal to 558 KN.

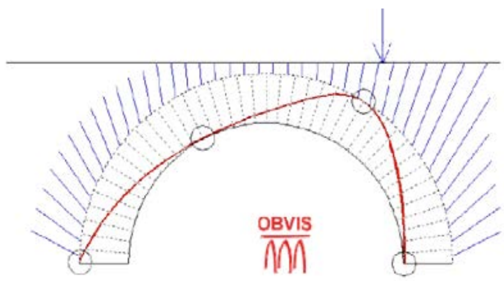

Fig. 4 Archie-M Output

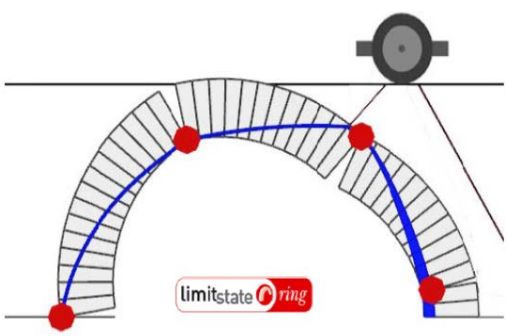

Fig. 5 Ring Output

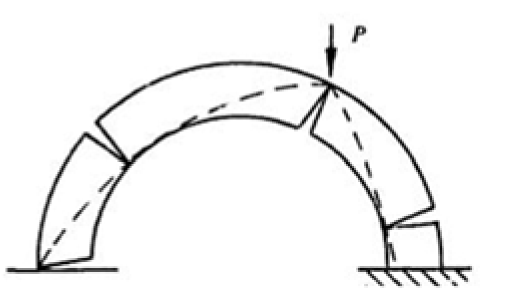

Fig. 8 Heyman pattern for the point load case

The software used to simulate the analysis of the generic masonry arch bridge with the Finite Element Method is Abaqus, which is able to solve a wide range of linear and non linear problems involving either static or dynamic response. The software is divided into modules that respect the logic of the organizational process.

Defining the material property is the most delicate step. Macro-modeling is applied trying to take advantage of constitutive laws already implemented in the software and using equivalent materials to model masonry.

The general description of a 2D nonlinear constitutive model of a concrete-like masonry consists of three elements: i) pre-failure behavior; ii) limit domain; iii) post-failure behavior. The pre-failure behavior is considered as linear elastic for both compression and tension. The data requested are the Young's Modulus and the Poisson's Ratio. There are various limit domains for concrete-like material. All of them are based on Von Mises domain in compression and assume a considerably limited tensile strength.

The material model used to define the properties of masonry outside the elastic range is the concrete smeared cracking model. This material model is based on the William-Warnke Criterion. The material behavior is characterized by the data derived from simple static tests.

The backfill can be modelled by means of 2D elements, which provide to transfer live loads and passive reaction on the arch barrel. The material "soil" is usually nonlinear defined by Mohr-Coloumb or Drucker-Prager limit criteria but also a crude approach involving linear elastic material is allowed.

In this case a Drucker-Prager domain is used. The data requested are: i) angle of friction; ii) Flow stress Ratio; iii) Dilatation Angle.

The analysis is non linear and requires an iterative solver. A Lower Bound Approach is used as criterion to determinate the maximum sustainable vertical load.

The maximum load coincides with the load required to form three hinges and initiate a fourth. The collapse load calculated is about 279 KN.

The principal stresses are reported in Fig.6. Under the point of application of the vertical load, a hinge occurs as expected. Two hinges open close to the two fixed supports. The location of these hinges is in good agreement with the experimental test. However the results indicate that the a priori assumption regarding the occurrence of two hinges in correspondence of two supports is approximately true.

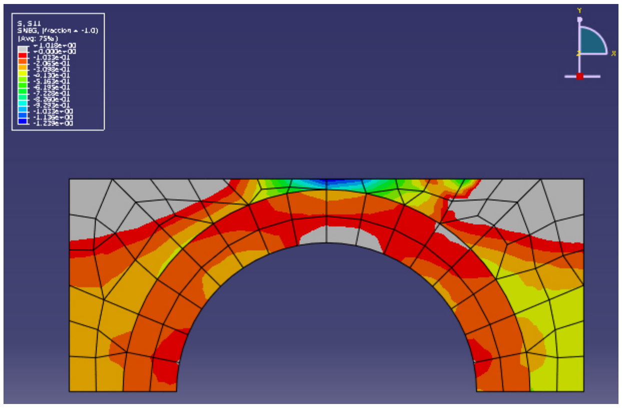

Fig.6 Principal Stresses

Finally, the loading capacity of the arch is calculated by applying the Elasto-Plasic Model.

Assuming that both supports are fixed, the structure is statically indeterminate to the third degree of freedom and the collapse will happen as soon as four hinges are formed.

The first hinge appears for the smallest value of P, which gives rise to a normal force and a moment able to satisfy the relationship above.

The corresponding angle θ identifies the position of the first hinge. The process is repeated until the fourth hinge is formed. The equilibrium equations are the same, only the boundary conditions change.

The collapse load is obtained by summing the Pmax1 and the increments ΔPmax1, ΔPmax2, ΔPmax3 calculated for the different hinges. Its value is equal to 134 KN.

Fig.7 shows the formation of the four hinges. Their location is in good agreement with the experimental tests and with Heyman's theory (Fig.8).

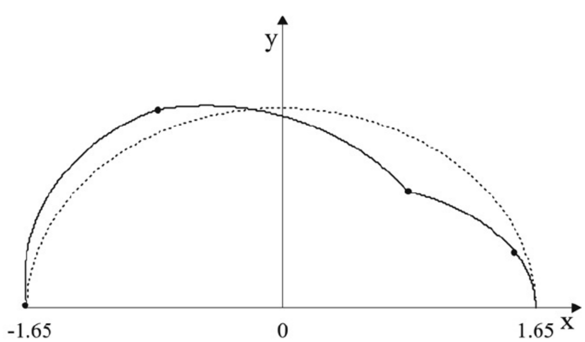

Fig.7 Location of the four hinges

III. COMPARISON

The different methods and models are compared with each other in terms of collapse load and the position of the four hinges.

All the comparisons are summarized in Table 1.

Table 1 Hinge position and Failure Load for Different Methods

	FIRST HINGE (rad)	SECOND HINGE (rad)	THIRD HINGE (rad)	FOURTH HINGE (rad)	Collapse Load (KN)
ARCHIE-M	0.60	-1.57	-0.47	1.57	165
LIMITSTATE RING	0.60	-1.57	-0.40	1.41	558
FINITE ELEMENT	0.67	-1.37	0	1.37	279
ELASTO-PLASTIC	0.72	-1.57	-0.40	1.33	138

Concerning the hinge positions, Ring, Archie-M and Elasto-Plastic Model give similar results. Small differences are found near the point of application of the concentrated load, probably due to the different distribution of live load in the various models. There are also little differences concerning the position of the fourth hinge near the right support. In particular, in Archie-M the hinge corresponds with the support, while in Ring and in the Elasto-Plastic Model the same hinge is positioned higher up of the line of the arch.

Concerning the hinge positions in the Finite Element Model, there is a good correspondence with the other models for the first hinge - that occur under the point of load application – and for the fourth hinge . The positions of the other hinges are different from those of other models. In fact the third hinge that is located near the arch center moves towards it, while the second hinge that occurs exactly at the left springing occurs at the intrados and not at the extrados as expected.

The differences between the Finite Element Model and the others three methods can be explained in this way : Archie-M, Ring and the Elasto-Plastic model derive from the principles of limit state analysis, while the Finite Element Model comes from a completely different approach.

However the results indicate that the a priori assumption regarding the occurrence of two hinges in correspondence of the two support points is only approximately true.

Concerning the collapse load, Elasto-Plastic Model and Archie-M give comparable results. The Ring collapse load is significantly higher than the others, probably due to two reasons.

The first one is that the other three models use a lower bound approach to determinate the maximum vertical load sustainable, while Ring uses an upper bound approach.

The second one is that the rigid-plastic model neglects the elasticity of the masonry. This factor is very important when the thickness of the arch is big as in the bridge in exam.

The comparison demonstrates that the elasticity of the material has a great influence on the determination of the collapse load. The differences in the collapse load can be summarized as follows: Elasto-Plastic collapse load ≤ Archie-M collapse load ≤ Finite Element collapse load ≤ Ring collapse load.

In this example, the position of the load has been assigned a priori to compare with each other the various methods.

Actually, for practical reasons it's very interesting to study the worst load position that gives rise to the smallest collapse load.

So the last analysis made on the generic arch bridge is of this type. The most critical position is founded at 2025 mm from the left abutment, about at a quarter of the span, as expected. The maximum load that can be applied at this point has been calculated with Ring and is equal to 250 KN.

IV. CONCLUSION

The methods for assessing historical masonry arches are mainly four: 1) the Thrust Line Analysis Method; 2) the Mechanism Method; 3) the Finite Element Methods; 4) the Elasto-plastic Model. The Thrust Line Analysis Method and the Mechanism Method are analytical methods and derive from two of the fundamental theorems of the Plastic Analysis, while the Finite Element Method is a numerical method, which uses different strategies of discretization to analyze the structures.

The Elasto-Plastic Model is a particular closed-form approach developed by some Belgian researchers in the last years and is based on the fundamental theorems of limit analysis. It can be employed with a relatively small modeling effort.

A comparison between the four methods has been made. All the models lead almost to the same collapse pattern even if the Limit Analysis Method is the most suitable to be applied to the arch masonry structures.

Significant difference is observed regarding the predicted collapse load in comparison between various models. The comparison demonstrates that the elasticity of the material has a great influence on the determination of the collapse load.

In the future, the next analysis step will be the comparison of the results obtained by all the four methods applied to a real case study.

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