Derivation of boundary conditions for homogeneous reservoir with fractures

Elena Andriyanova, Vladimir Tyan

***Abstract*—** In this paper the new boundary conditions are presented for the understanding of flow behavior in a reservoir with fractures. As can be seen from the current seismic researches, most of the reservoirs in the world have faults with different permeability. The derived boundary conditions allow us to model filtration process for cases of tectonic faults and for hydraulic fracturing. It is achieved throw the complex potential and mathematical analyze technique.

***Keywords*—** fluid flow, filtration, boundary conditions, hydraulic fracturing, impermeable boundary, Darcy flow.

I. Introduction

Based on the fact that most of oil fields are on the late stage of field development, from the economical point of view, it becomes necessary to produce hard-to-extract oil, which can be obtained only by use of enhance oil recovery methods. The effective oil and gas field development is a complicated problem, especially in conditions of low oil price and depleted reservoirs. Currently more often we need to produce hard-to-extract oil, which requires application of complicated enhance oil recovery methods and modern geophysical researches. For example, many low permeable or shale formations can be developed only with application of massive hydraulic fracturing technique. Therefore, for engineers, there is an acute problem of fast evaluation and selection of a suitable production system and a well completion.

In addition, modern geophysical researches show that mostly oil bearing formations are complicated with tectonic faults of different shape and permeability. These discontinuities exert essential influence on the field development process and on the well performance. For the modeling of fluid flow in the reservoir with some area of different permeability, we should determine the boundary conditions. In this article for the first time the boundary conditions for the problem of fluid filtration in the reservoir with some discontinuity are considered. This discontinuity represents thin but long area, which can be hydraulic fracturing of tectonic fault. The obtained boundary condition equations allow us to take into account pressure difference above and below the section and different values of permeability.

One of the most effective enhance oil recovery techniques is hydraulic fracturing [1]. Many scientists are involved into the modeling of fluid flow to the well with hydraulic fracturing [2-8]. But generally the highly permeable fracture is considered to cross the well symmetrically.

However, the modern geophysical researches allow getting a more detailed conception of the reservoir's tectonics. As a result, we can see that most formations are crossed with tectonic fractures and faults of different shape. Hence, it is necessary to model the influence of such discontinuities on the fluid flow process. Consequently the problem formulation becomes more complicated, because we need to take into account not only various locations of fracture and wells but also different values of fracture permeability.

In this paper the derivation of the boundary conditions is shown. Such boundary conditions allow us to model the fluid filtration process in the reservoir with discontinuities. Given discontinuity represents thin area with different permeability in comparison with the rock and it can be tectonic fault or hydraulic fracture. In the works [9-14] the number of solutions of boundary value problems of the theory of fluid flow to the producing wells in the presence of discontinuities in the oil reservoir were considered. The equations for the flow potential were defined both for the case of a single well (production) and for the case of two wells (production and injection), i.e. the element of water-flooding system. These solutions consider not only all possible values of fracture conductivity but also the symmetry or asymmetry of the problem statement, that is the value of the difference of pressures on the upper and lower banks of the cut. However, the derivation of additional boundary conditions for this kind of discontinuity in [9-14] was absent, which this work is devoted to.

II. The derivation of boundary conditions

Let us consider a plane problem of the theory of homogeneous liquid filtration in the presence of discontinuities in oil reservoir. In the case when the discontinuity was a hydraulic fracture [1-4], highly permeable area of the fracture was represented in the form of a confocal ellipse that was displayed using the Zhukovsky function on the exterior of the unit circle.

In this work, a discontinuity of oil layer is represented as some discontinuity line AB on plane (Fig. 1), the thickness of which is $\delta(s)$. The equation of this curve is as follows:

$$\begin{cases} x = x(s), \\ y = y(s); \end{cases} \Rightarrow z(s) = x(s) + iy(s). \tag{1}$$

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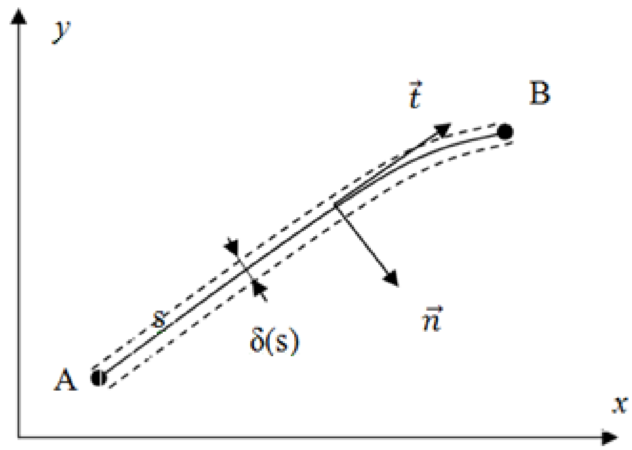

Fig. 1 The discontinuity line AB

Let the reservoir has a constant permeability k and the pressure $p(x,y)$. Suppose that k_f and p_f are the corresponding values of permeability and pressure in the fracture.

Let the motion of the fluid in the fracture and in the reservoir subject to linear Darcy's law. Then the velocity vector of a filtration in the reservoir in a Cartesian coordinate system $\mathbf{v}(v_x, v_y)$ is as follows:

$$v_x = -\frac{k\partial p}{\mu\partial x}; v_y = -\frac{k\partial p}{\mu\partial y} \tag{2}$$

Similarly, the filtration rate vector in the fracture in a curvilinear coordinate system associated with a crack (t, n) will have the form:

$$v^f{}_t(s) = -\frac{k_f\partial p_f}{\mu\partial t}; v^f{}_n(s) = -\frac{k_f\partial p_f}{\mu\partial n} \tag{3}$$

Further, the incompressibility condition of fluid in the reservoir and in the fracture can be written as

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0; \quad \frac{\partial v^f{}_t(s)}{\partial t} + \frac{\partial v^f{}_n(s)}{\partial n} = 0. \tag{4}$$

Equations (2) - (4) must be supplemented by the condition of continuity of the solution at the "reservoir-fracture", or condition of continuity of the pressure and the normal velocity component:

$$\begin{cases} p_f\left(s, \pm\frac{\delta(s)}{2}\right) = p\left(x(s) \pm \frac{\delta(s)}{2}\sin\alpha, y(s) \pm \frac{\delta(s)}{2}\cos\alpha,\right), \\ v_n^f\left(s, \pm\frac{\delta(s)}{2}\right) = v_n\left(x(s) \pm \frac{\delta(s)}{2}\sin\alpha, y(s) \pm \frac{\delta(s)}{2}\cos\alpha,\right). \end{cases} \tag{5}$$

Let us average incompressibility condition (5) on thickness of the crack $\delta\,(s)$:

$$\int_{-\delta/2}^{\delta/2}\left(\frac{\partial v_t^f}{\partial s} + \frac{\partial v_n^f}{\partial n}\right)dn =$$
$$= \int_{-\delta/2}^{\delta/2}\left(\frac{\partial v_t^f}{\partial s}\right)dn + v_n^f\left(s, \frac{\delta(s)}{2}\right) - v_n^f\left(s, -\frac{\delta(s)}{2}\right) = 0. \tag{6}$$

Let us define the total fluid flow along fracture through $q(s)$:

$$q(s) = \int_{-\delta(s)/2}^{\delta(s)/2} v_t^f(s,n)dn. \tag{7}$$

Let take into account that:

$$\frac{d}{ds}\int_{-\delta/2}^{\delta/2} v_t^f(s,n)dn =$$
$$\int_{-\delta/2}^{\delta/2}\left(\frac{\partial v_t^f}{\partial s}\right)dn + v_t^f\left(s, \frac{\delta(s)}{2}\right)\frac{\delta'(s)}{2} - v_t^f\left(s, -\frac{\delta(s)}{2}\right)\frac{\delta'(s)}{2},$$

As a result, we obtain the following equation:

$$\frac{dq(s)}{ds} + \left[v_n^f\left(s, \frac{\delta(s)}{2}\right) - v_t^f\left(s, \frac{\delta(s)}{2}\right)\frac{\delta'(s)}{2}\right] -$$
$$-\left[v_n^f\left(s, -\frac{\delta(s)}{2}\right) - v_t^f\left(s, -\frac{\delta(s)}{2}\right)\left(-\frac{\delta'(s)}{2}\right)\right] = 0. \tag{8}$$

Given that v_n^f and v_t^f are the projections of vector $\overrightarrow{v^f}$ at point P on the line L at vectors $\vec{n}$ and $\vec{t}$, and in (11) the values of these projections relate to the L_1 and L_2 lines (Fig. 2), placed from the line L along normal on distance $\pm\delta(s)/2$, equations for lines L_1 and L_2 can be written as follows:

$$\begin{cases} x_1(s) = x(s) + \frac{\delta(s)}{2}\sin\alpha, \\ y_1(s) = y(s) - \frac{\delta(s)}{2}\cos\alpha; \end{cases}$$

and

$$\begin{cases} x_2(s) = x(s) - \frac{\delta(s)}{2}\sin\alpha, \\ y_2(s) = y(s) + \frac{\delta(s)}{2}\cos\alpha; \end{cases}$$

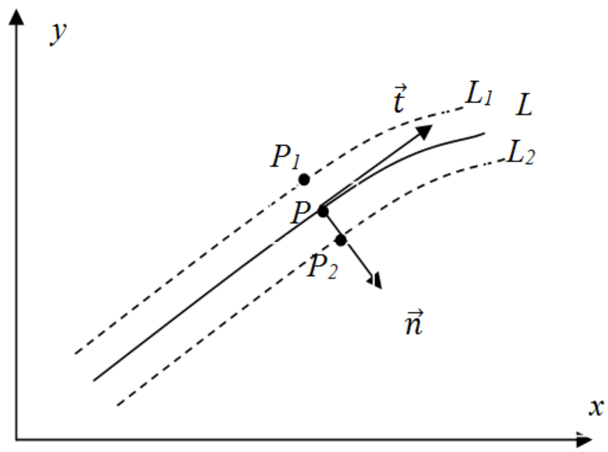

Fig. 2 Places of lines L, L_1, L_2.

Given that the normal components of the velocity vector along the crack as follows:

$$v_n^f(P_1) = v_n^f\left(s, \frac{\delta(s)}{2}\right) - v_t^f\left(s, \frac{\delta(s)}{2}\right)\frac{\delta'(s)}{2}, \tag{9}$$

$$v_n^f(P_2) = v_n^f\left(s, -\frac{\delta(s)}{2}\right) - v_t^f\left(s, -\frac{\delta(s)}{2}\right)\left(-\frac{\delta'(s)}{2}\right). \tag{10}$$

Substituting expressions (9) - (10) into equation (8), we get the following equation:

$$\frac{dq(s)}{ds} + v_n^f(P_1) - v_n^f(P_2) = 0. \tag{11}$$

Similarly, averaging Darcy equation (2) - (3) on thickness of the crack $\delta(s)$, we obtain:

$$\begin{gathered}\int_{-\delta/2}^{\delta/2}\left(v_t^f(s,n) + \frac{k_f}{\mu}\frac{\partial p_f}{\partial s}\right)dn = \\ q(s) + \frac{k_f}{\mu}\int_{-\delta/2}^{\delta/2}\frac{\partial p_f}{ds}dn = 0,\end{gathered} \tag{12}$$

$$\begin{gathered}\int_{-\delta/2}^{\delta/2}\left(v_n^f(s,n) + \frac{k_f}{\mu}\frac{\partial p_f}{\partial n}\right)dn = \\ \int_{-\delta/2}^{\delta/2} v_n^f(s,n)dn + \frac{k_f}{\mu}\left(p_f(P_1) - p_f(P_2)\right) = 0.\end{gathered} \tag{13}$$

Thus, the continuity equation (11) and Darcy equation in the fracture (12) - (13) take the following form:

$$\begin{cases}\dfrac{dq(s)}{ds} + v_n^f(P_1) - v_n^f(P_2) = 0, \\ q(s) = -\dfrac{k_f\delta(s)}{2\mu}\dfrac{d}{ds}\left(p_f(P_1) + p_f(P_2)\right), \\ \dfrac{\delta(s)}{2}\left(v_n^f(P_1) + v_n^f(P_2)\right) = -\dfrac{k_f}{\mu}\left(p_f(P_1) - p_f(P_2)\right).\end{cases} \tag{14}$$

If we take into account the condition of fluid flow continuance at the boundary "fracture-reservoir", or at the points P_1 and P_2 the relations $v_n^f(P_{1,2}) = v_n(P_{1,2})$ and $p_f(P_{1,2}) = p(P_{1,2})$ is performed, and if we let $P_1 \to P^+$ and $P_1 \to P^-$, we get the following additional boundary conditions on the discontinuity line L:

$$\begin{cases}\dfrac{dq(s)}{ds} + v_n^+(s) - v_n^-(s) = 0, \\ q(s) = -\dfrac{k_f\delta(s)}{2\mu}\dfrac{d}{ds}\left(p^+(s) + p^-(s)\right), \\ \dfrac{\delta(s)}{2}\left(v_n^+(s) + v_n^-(s)\right) = -\dfrac{k_f}{\mu}\left(p^+(s) - p^-(s)\right).\end{cases} \tag{15}$$

where $q(s)$ is the total flow of liquid along the fracture, $p(s)$ and $v_n(s)$ is the pressure and the inflow rate of fluid from the formation into the fracture through the upper and lower edges of the cut, reflected respectively by + and –.

Let assume, the well is located at the point z_0, flow rate Q, radios of external boundary Rc and well radios r_w. Let we define the complex flow potential for an incompressible fluid in the reservoir through $\varphi(z)$:

$$P(x,y) = Re\ \varphi(z),$$
$$v_n(P) = -\frac{k}{\mu} Im\ \varphi'(z(s)) = -\frac{k}{\mu} Im \frac{d\varphi}{ds} = -\frac{k}{\mu}\frac{d}{ds} Im\ \varphi(z(s)).$$

In this case, the system of equations (15) in the fracture will look like this:

$$\begin{cases}\dfrac{dq(s)}{ds} = \dfrac{k}{\mu}\dfrac{d}{ds}\mathrm{Im}\left(\varphi^+ - \varphi^-\right), \\ q(s) = -\dfrac{k_f\delta(s)}{2\mu}\dfrac{d}{ds}\mathrm{Re}\left(\varphi^+ + \varphi^-\right), \\ -\dfrac{k}{\mu}\dfrac{\delta(s)}{2}\dfrac{d}{ds}\mathrm{Im}\left(\varphi^+ + \varphi^-\right) = -\dfrac{k_f}{\mu}\mathrm{Re}\left(\varphi^+ - \varphi^-\right).\end{cases} \tag{16}$$

From the system (16), we obtain two equations for φ^+ and φ^-:

$$\begin{cases}\mathrm{Im}\left(\varphi^+ - \varphi^-\right) = -\dfrac{k_f\delta(s)}{2k}\dfrac{d}{ds}\mathrm{Re}\left(\varphi^+ + \varphi^-\right), \\ \dfrac{k}{k_f}\dfrac{\delta(s)}{2}\mathrm{Im}\left(\varphi^+ + \varphi^-\right) = \mathrm{Re}\left(\varphi^+ - \varphi^-\right).\end{cases} \tag{17}$$

The case when the fracture *(-l,l)* is axially symmetrical relative to the x-axis was considered in [15]. In this case, on conditions $Re\,\varphi^+ = Re\,\varphi^-$ and $Im\,\varphi^+ = -Im\,\varphi^-$,the boundary value problem (17) takes the form:

$$\operatorname{Im}\varphi = -Fcd\sqrt{1-\xi^2}\operatorname{Re}\varphi',$$

where $\xi = x/l$ is dimensionless coordinate along the fracture, $F_{cd} = k_f\delta_0/2kl$ is the dimensionless fracture conductivity.

III. Application

The boundary conditions, derived in this paper, can be widely applicate in various cases for fluid flow modelling in a reservoir. For instance in a case when a fracture intersects one of wells we have a hydraulic fracturing case. This is a symmetrical case, but if we consider a pair of wells (injection and production), we need take into account pressure difference above and below the fracture (boundary). This case was explained in [14]. The flow potential of that case:

$$\varphi(\nu) = \begin{cases} \ln\dfrac{(\nu - i\rho_0)}{(\nu + i\rho_0)} + 2i\sum\limits_{k=0}^{\infty}\dfrac{(-1)^k}{(2k+1)\rho_0^{2k+1}}\nu^{-(2k+1)}; & \beta_0 = 0, \\ \ln\dfrac{(\nu - i\rho_0)}{(\nu + i\rho_0)} - 2i\sum\limits_{k=0}^{\infty}\dfrac{(-1)^k}{(2k+1)\rho_0^{2k+1}}\nu^{-(2k+1)}; & \beta_0 = \infty. \end{cases} \tag{18}$$

These equations can be used for calculation of skin factor of the well and for the flow lines modelling (Fig. 3-4). Different colors in the pictures show the waterflooding stages, or the position of waterfront in a particular time. Red – the waterflooded area, from the beginning of the process to the half of the process; yellow – the waterflooded area, from the ½ of the process to the ¾ of the process; blue – the waterflooded area, from the ¾ of the process to the water breakthrow time; purple - the waterflooded area at the breakthrow time.

The equation (18) and boundary conditions (17) are suitable for cases when a fracture is located on some distance from wells, this case is described in [12], [13] and [15] (Fig. 4-5)

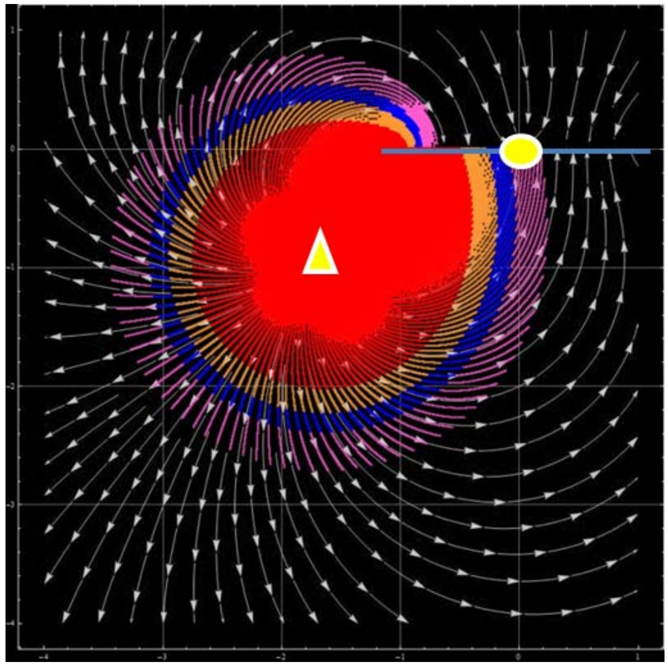

Fig. 3 Streamlines waterflooding process with tracer lines (angle 30). The injection well located - at the (-√3, -1), the hydraulically fractured production - at the point (0, 0)

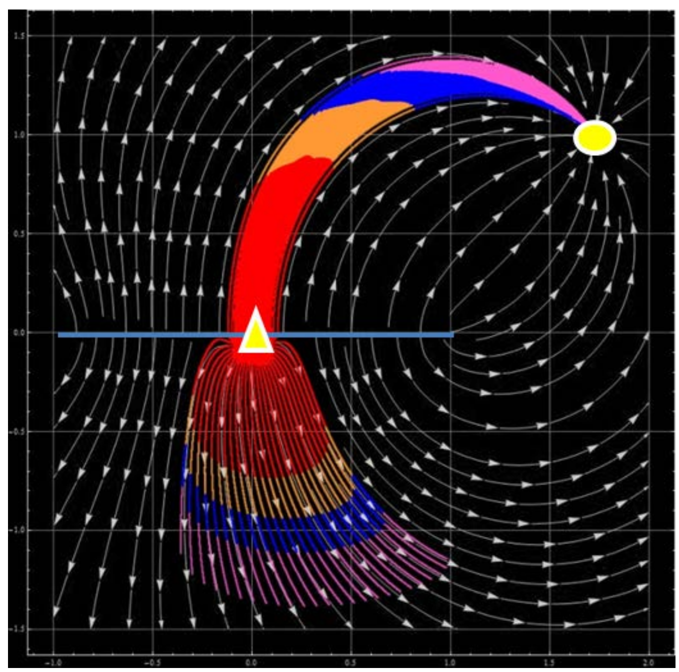

Fig. 4 Streamlines waterflooding process with tracer lines (angle 30). The hydraulically fractured injection well - at the (0, 0), the production well - at the point (1, √3)

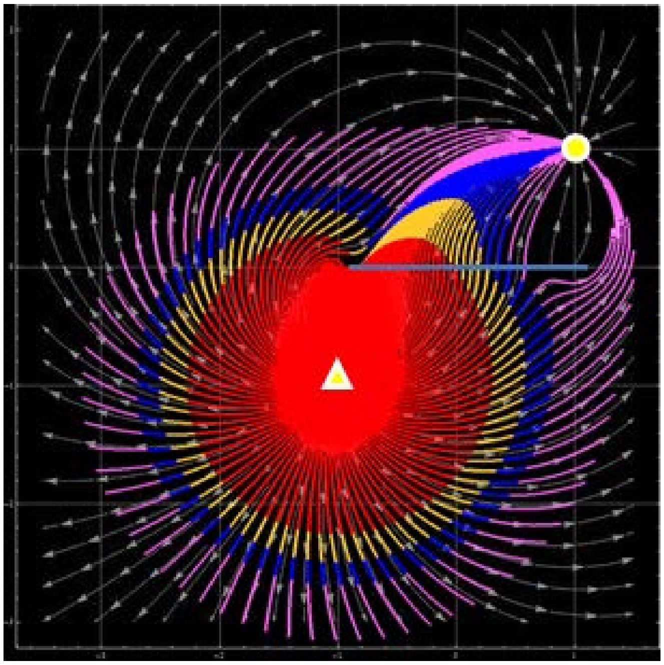

Fig. 5 Streamlines waterflooding process with tracer lines, the injection well located at the point (-1, -1), production well (1, 1), for the values of $\alpha_0=\infty$; $\beta_0=0$ (Upper) and $\alpha_0=0$; $\beta_0=\infty$ (Lower).

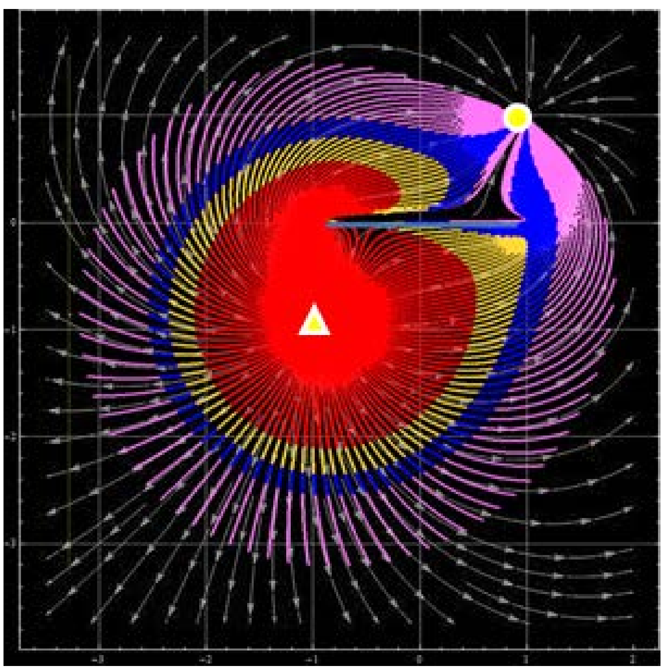

Fig. 6 Streamlines waterflooding process with tracer lines, the injection well located at the point (-1, -1), production well (1, 1), for the values of $\alpha_0=\infty$; $\beta_0=0$ (Upper) and $\alpha_0=0$; $\beta_0=\infty$ (Lower).

IV. Conclusion

In the present article derivation of boundary conditions are considered in depth for task of homogenous incompressible fluid in the reservoir complicated by narrow expanded heterogeneity by permeability. A boundary condition might be used for fracture with different conductivity, for instance hydraulic fracture, tectonic faults or impermeable boundary. At once received formulas allow deciding problems of filtration for all kinds of wells and discontinuity allocations. The results were already used for the modeling of fluid flow in the reservoir with fracture for the case of single production well [9-13] and for the case of production and injection wells or a waterflooding element in a production system [14-15]. The influence of discontinuity on the well productivity and equations for skin were defined using this boundary condition [13]. In addition the water breakthrough time and breakthrough sweep efficiency has been analyzed for different parameters of wells locations [14-15]. The further applications lie in modeling of fluid flow in the reservoir with massive hydraulic fracturing, it helps to optimize well spacing parameters.

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