Designing of wind farm layout by using of multi-objective optimization

Daniela I. Borissova and Ivan C. Mustakerov

Abstract—The article deals with designing of wind farm layout using multi-objective combinatorial optimization modeling approach. This approach is implemented in a proposed algorithm for design and assessment of wind farm layout design. The described multi-objective model and algorithm are numerically tested for a real-life problem. The testing results show their applicability for designing of wind farm layout (determination of Pareto-optimal layouts of turbines) taking into account wind site area, wind conditions, wake effect and decision-maker preferences.

Keywords—Wind farm layout, multi-objective optimization, design and assessment algorithm.

I. INTRODUCTION

The market of wind energy is growing rapidly and among many challenges in this industry costs reduction is important. One of the main considerations to be taken into account when designing a wind farm is the choice of turbines and their locations i.e. the wind farm layout design. The wind farm layout is an important wind farm design factor that influences essentially its profitability.

The design of wind farm layout is investigated by many authors and most of them rely on single criterion heuristic approaches as genetic algorithms [1, 2]; evolutive algorithm [3, 4]; particle swarm optimization algorithm and multiple adaptive methods [5]; Monte Carlo simulation [6], etc. Due the fact, that heuristic approaches does not guarantee finding of an optimal solution, some authors’ try to find the optimal wind farm layout using exact approaches as mixed-integer quadratic optimization [7], exact gradient information [8], combinatorial optimization [9, 10, 11], mathematical programming techniques [12], mixed integer programming [13], etc. Another group of publications for wind farm layout design deals with design of wind farm layout considering multiple objectives at the same time and emphasizing on multi-objective evolutionary algorithms hyper-heuristic and genetic approaches [14-17]. The design of wind farm layout could be realized also by using of multi-objective optimization and group decision making [18], or using new variation operators for the wind turbine placement problem [19]. Most of the above approaches consider predefined type and number of wind turbines that have to be positioned within given wind farm area.

In this article, the type of turbines and their number are considered as variables. At the design stage of wind farm, the expected wind energy output could be estimated as annual energy production depending on the operational hours over the year of overall installed turbines. The profitability of the designed wind farm could be defined as multi-objective problem, which simultaneously reflects the requirements for maximization of the output energy and minimization of the costs. For the goal, in the present work is proposed a mixed integer modeling approach for wind farm layout design taking into account the constraints for wind farm area and wake effect. The resulting model is used in a proposed algorithm for design and assessment of wind farm layout. The formulated multi-objective optimization problem is solved by means of two methods – weighed sum method and lexicographic method [20, 21]. The results of numerical testing demonstrate the applicability of the proposed modeling approach and algorithm for designing of wind farm layout. The rest of this article is organized as follows: Section 2 contains a description of the problem, Section 3 presents the proposed optimization approach, Section 4 evaluates its performance on the basis of real-life problem, Section 5 contains results analysis and discussion, and conclusions are given in Section 6.

II. PROBLEM FORMULATION

The investigated problem is to determine an effective design of wind farm layout using the following input data: 1) predetermined wind farm area dimensions, 2) known wind conditions for wind farm site, 3) set of available wind turbines to choose. There is also a requirement to select turbines of the same type to facilitate maintenance of the turbines. The goal of design of wind farm layout is to maximize the energy output while minimizing overall costs. The maximization of the output energy requires installing as much as possible wind turbines, but some separation distances are needed to overcome the so called wake effect [22]. This effect influences negatively the turbines’ effectiveness and requires properly defined separation distances between turbines depending of the wind direction and rotor diameter of turbines.

This work was supported by the project for ICT applications for optimization of manufacturing process management via models, algorithms, and multi-objective decision making methods (2017-2019).

D. I. Borissova is with the Institute of Information and Communication Technologies at Bulgarian Academy of Sciences, Sofia – 1113, Bulgaria, Department of Information Processes and Decision Support Systems (phone: 3952 9792055; e-mail: dborissova@iit.bas.bg).

I. C. Mustakerov is with the Institute of Information and Communication Technology at the Bulgarian Academy of Sciences, Sofia – 1113, Bulgaria, Department of Information Processes and Decision Support Systems (phone: 3952 9793241; e-mail: mustakerov@iit.bas.bg).
III. MULTI-OBJECTIVE APPROACH FOR DESIGNING OF WIND FARM LAYOUT

A. Multi-objective model for designing of wind farm

The multi-objective model for designing of wind farm layout takes into account two of the most important objectives – maximization of the energy output and minimization of the costs. Based on the assumptions used in authors’ previous works [9, 18] the following multi-objective model is formulated:

\[
\begin{align*}
\text{max } \text{Energy} \\
\text{min } \text{Costs}
\end{align*}
\]

subject to

\[
\begin{align*}
\text{Energy} &= NP_w h_i \eta \\
\text{Costs} &= N \left( \frac{2}{3} + \frac{1}{3} e^{-0.0174N^2} \right) \\
SD_x &= k_x D_x \\
SD_y &= k_y D_y \\
N &= N_x N_y \\
N_x &= (L_x/SD_x) + 1, \ N_x - \text{integer} \\
N_y &= (L_y/SD_y) + 1, \ N_y - \text{integer} \\
k_{y,\min} &\leq k_y &\leq k_{y,\max}, \ k_y > 0 \\
k_{x,\min} &\leq k_x &\leq k_{x,\max}, \ k_x > 0 \\
P_{wt} &= \sum_{i=0}^{m} x_i P_i^{\text{wt}} \\
D_{wt} &= \sum_{i=0}^{m} x_i D_i^{\text{wt}} \\
\sum_{i=0}^{m} x_i &= 1, \ x_i \epsilon \{0,1\}
\end{align*}
\]

where \( N \) is the number of installed wind turbines, \( N_x \) is the number of turbines in columns, \( N_y \) is the number of turbines in rows, \( P_{wt} \) is the rated power of particular turbine and \( D_{wt} \) is its diameter, \( L_x \) and \( L_y \) are dimensions of wind farm area, \( x_i \) are binary integer variables, \( SD_x \), \( SD_y \) are separation distances between turbines needed to overcome wake effect, which are determined by using of coefficients \( k_x, k_y \). The coefficients \( k_x, k_y \) are limited by proper upper and lower boundaries depending on wind conditions [1, 6, 23]. The expected energy output depends on number of turbines \( N \) and their rated power \( P_{wt} \), hours over the year \( h_i \) and nominal wind power utilization coefficient \( \eta \) [24]. The used costs objective is widely accepted non-dimensional relation that expresses the costs per year as a function of only the total number of turbines [1, 22].

B. Algorithm for using of multi-objective approach for design and assessment of wind farm layout

A distinguished feature of multi-objective optimization (MOO) is that it determines Pareto-optimal solution [20, 21]. This solution depends on decision-maker (DM) preferences about importance of each objectives considered in multi-objective model. In general, if these preferences are changed the Pareto-optimal solution is changed too. This feature of MOO is used in proposed algorithm for wind farm layout design and assessment. The generalized flowchart of this algorithm is shown in Fig. 1.

IV. NUMERICAL TESTING

The multi-objective model (1) – (13) and described algorithm are used for wind farm layout design within site with dimensions \( L_x = 4 \text{ km} \) and \( L_y = 1 \text{ km} \) (Fig.2).

The location of turbines depends on the distances \( SD_x \) and \( SD_y \), which are determined by the rotor diameter of the selected turbine, multiplied by the corresponding coefficients \( k_x \) and \( k_y \). The annual energy output from wind farm is calculated as number of hours over the year \( h_i = 8760 \) multiplied by utilization coefficient \( \eta \) for particular wind conditions of the site. In current example it is assumed \( \eta = 0.3 \).
The parameters of wind turbines used as input data are shown in Table I.

<table>
<thead>
<tr>
<th>#</th>
<th>WT</th>
<th>Rated power, kW</th>
<th>WT rotor diameter, m</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Vestas V80</td>
<td>2000</td>
<td>80</td>
</tr>
<tr>
<td>2</td>
<td>Enercon E-82</td>
<td>2000</td>
<td>82</td>
</tr>
<tr>
<td>3</td>
<td>Enercon E-70</td>
<td>2300</td>
<td>71</td>
</tr>
<tr>
<td>4</td>
<td>SWT-2.3-82 VS</td>
<td>2300</td>
<td>82.4</td>
</tr>
<tr>
<td>5</td>
<td>Enercon E-82 E2</td>
<td>2300</td>
<td>82</td>
</tr>
<tr>
<td>6</td>
<td>SWT-2.3-113</td>
<td>2300</td>
<td>113</td>
</tr>
<tr>
<td>7</td>
<td>SWT-2.3-108</td>
<td>2300</td>
<td>108</td>
</tr>
<tr>
<td>8</td>
<td>SWT-2.3-93</td>
<td>2300</td>
<td>93</td>
</tr>
<tr>
<td>9</td>
<td>Enercon E-92</td>
<td>2350</td>
<td>92</td>
</tr>
<tr>
<td>10</td>
<td>Clipper Windpower</td>
<td>2500</td>
<td>96</td>
</tr>
<tr>
<td>11</td>
<td>Vestas V100</td>
<td>2600</td>
<td>100</td>
</tr>
<tr>
<td>12</td>
<td>Enercon E-82</td>
<td>3000</td>
<td>82</td>
</tr>
<tr>
<td>13</td>
<td>Enercon -101</td>
<td>3050</td>
<td>101</td>
</tr>
<tr>
<td>14</td>
<td>Vestas V90</td>
<td>3000</td>
<td>90</td>
</tr>
<tr>
<td>15</td>
<td>Vestas V112</td>
<td>3000</td>
<td>112</td>
</tr>
<tr>
<td>16</td>
<td>SWT-3.6-120</td>
<td>3600</td>
<td>120</td>
</tr>
<tr>
<td>17</td>
<td>SWT-3.6-107</td>
<td>3600</td>
<td>107</td>
</tr>
<tr>
<td>18</td>
<td>SWT-6.0-154</td>
<td>6000</td>
<td>154</td>
</tr>
<tr>
<td>19</td>
<td>Vestas V164-7.0</td>
<td>7000</td>
<td>164</td>
</tr>
<tr>
<td>20</td>
<td>Enercon E-126</td>
<td>7580</td>
<td>127</td>
</tr>
</tbody>
</table>

Three general cases are tested: for uniform wind directions where \( k_x^{\min} = k_y^{\min} = 4.5 \) and \( k_x^{\max} = k_y^{\max} = 5.5 \) and for predominant wind directions as shown in Fig. 2, where \( k_x^{\min} = 1.5 \) and \( k_x^{\max} = 3 \) and \( k_y^{\min} = 8 \) and \( k_y^{\max} = 12 \) and vice versa in regard to wind farm orientation.

These input data are used to formulate multi-objective optimization tasks solved by weighted sum method and lexicographic method. These methods are used due their easy and intuitive establishment of DM preferences toward the objectives. Both methods are based on priori articulation of the DM preferences, but provided information from their solution can be also used for posterior preferences definition.

### A. Using of Weighted sum method

The weighted sum method is the most common multi-objective optimization method based on scalarization techniques that aggregates different objectives as a weighted linear sum of their normalization [20, 21]. The DM preferences about importance of objectives are expressed by assigning corresponding weight coefficients \( w_j \), which results to transformed optimization task:

\[
\max \left\{ w_1 \left( \frac{\text{Energy} - \text{Energy}_{\min}}{\text{Energy}_{\max} - \text{Energy}_{\min}} \right) + \cdots + w_n \left( \frac{\text{Costs} - \text{Costs}_{\min}}{\text{Costs}_{\max} - \text{Costs}_{\min}} \right) \right\}
\]

subject to (2) – (13) with additional restriction:

\[
w_1 + \cdots + w_n = 1
\]

where \( \text{Energy}_{\max}, \text{Energy}_{\min}, \text{Costs}_{\max}, \text{Costs}_{\min} \) are the minimal and maximal values for Energy and Costs obtained by solution of (1) – (13) as single objective optimization problems.

To investigate the possibility of using multi-objective optimization as a tool for design and assessment of wind farm layout, the transformed task is solved for different preferences of DM toward importance of objectives. In the first case more important is the output energy \((w_e = 0.9\) and \(w_c = 0.1\)), while the second case illustrate the prevailing importance of costs \((w_e = 0.1\) and \(w_c = 0.9\)). The solution results for both cases are shown in Table II.

### Table II: Results of using of the weighted sum method

<table>
<thead>
<tr>
<th>Wind direction</th>
<th>Weighted coefficients</th>
<th>Overall expected energy output</th>
<th>Costs</th>
<th>Separation coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uniform</td>
<td>( w_e = 0.9 ), ( w_c = 0.1 )</td>
<td># 3</td>
<td>52</td>
<td>314309</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>38.424</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>4.695</td>
</tr>
<tr>
<td>Predominant</td>
<td>( w_e = 0.9 ), ( w_c = 0.1 )</td>
<td># 17</td>
<td>27</td>
<td>255442</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>20.531</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>4.673</td>
</tr>
</tbody>
</table>

Putting more weight on energy output then on costs \((w_e = 0.9, w_c = 0.1)\), for three possible wind direction cases (uniform and two different prevailing wind directions) results in wind farm layout graphically illustrated in Fig. 3a, while Fig. 3b illustrates the other case where more important are costs than energy output. The corresponding wind farm layouts are marked as L-1 to L-6.

As can be seen from Fig. 3, the different weights lead to the determination of different turbines placement. In the case where \( w_e = 0.1 \) and \( w_c = 0.9 \), the solution defines the turbines with smaller rotor diameter, which leads to usage of larger number of turbines and respectively to a higher installed power capacity. In the opposite case, where \( w_e = 0.9 \) and \( w_c = 0.1 \), the solution determines more powerful turbines with larger rotor diameter, that reflects in a smaller number of turbines, respectively to more significant reduction in costs.
subject to restrictions (2) – (13) and additional restriction about the energy output:

\[ \text{Energy} \]

It should be noted that the orientation of wind direction toward the wind site area influences to the choice of turbines type, number and placement.

**B. Using of Lexicographic method**

This method requires ranking of objectives according to the DM preferences and optimization them in order one at a time. The solution of each single objective problem gives a limiting measure used to define a proper restriction on the next step when the next objective is optimized and so on [20, 21].

Two lexicographic orders were used to investigate their impact on wind farm layout. In the first case, the energy output has been chosen as more important than the costs. The corresponding optimization procedure is as follows:

**Step 1:** Solving of the optimization task:

\[ \text{max } \text{Energy} \]

subject to restrictions (2) – (13)

**Step 2:** Solving of the optimization task:

\[ \text{min } \text{Costs} \]

subject to restrictions (2) – (13) and additional restriction about the energy output:

\[ \text{Energy} \geq \varepsilon \text{Energy}^{\max} \]

where the coefficient \( \varepsilon \) determines the degree of proximity to the optimal value of \( \text{Energy}^{\max} \) determined in the first step. Two values of this coefficient are used (\( \varepsilon = 0.7 \) and \( \varepsilon = 0.9 \)) to express the preferences of DM toward the importance of energy objective and to show their influence on the final solution.

In the second case, the costs objective is chosen as more important, which leads to the following computational procedure:

**Step 1:** Solving of the optimization task:

\[ \min \left\{ N \left( \frac{2}{3} + \frac{1}{3} e^{-0.00174 N^2} \right) \right\} \]

subject to restrictions (2) – (13).

**Step 2:** Solving the optimization task:

\[ \text{max } \text{Energy} \]

subject to restrictions (2) – (13) and additional restriction about the costs:

\[ \text{Costs} \leq \varepsilon \text{Costs}_{\text{min}} \]

Here, the coefficient \( \varepsilon \) determines the degree of proximity to the optimum value \( \text{Costs}_{\text{min}} \) from the first step. Again two values of this coefficient are used (\( \varepsilon = 1.3 \) and \( \varepsilon = 1.1 \)) to express the preferences of DM toward the importance of costs objective and to show their influence on the final solution.

The solution results of lexicographic method implementation are shown in Table III.

<table>
<thead>
<tr>
<th>Wind direction</th>
<th>Chosen wind turbine # (Table I)</th>
<th>Wind turbines number</th>
<th>Expected energy output MWh/year</th>
<th>Costs Separation coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) lexicographic order: Energy, Costs</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Uniform</td>
<td>0.7</td>
<td># 17</td>
<td>27</td>
<td>255 442</td>
</tr>
<tr>
<td></td>
<td>0.9</td>
<td># 3</td>
<td>52</td>
<td>314 309</td>
</tr>
<tr>
<td>Predominant</td>
<td>0.7</td>
<td># 16</td>
<td>36</td>
<td>340 589</td>
</tr>
<tr>
<td></td>
<td>0.9</td>
<td># 16</td>
<td>46</td>
<td>435 197</td>
</tr>
<tr>
<td>Predominant</td>
<td>0.7</td>
<td># 20</td>
<td>24</td>
<td>478 086</td>
</tr>
<tr>
<td></td>
<td>0.9</td>
<td># 20</td>
<td>24</td>
<td>478 086</td>
</tr>
<tr>
<td>b) lexicographic order: Costs, Energy</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Uniform</td>
<td>1.3</td>
<td># 17</td>
<td>27</td>
<td>255 442</td>
</tr>
<tr>
<td></td>
<td>1.1</td>
<td># 17</td>
<td>27</td>
<td>255 442</td>
</tr>
<tr>
<td>Predominant</td>
<td>1.3</td>
<td># 17</td>
<td>36</td>
<td>340 589</td>
</tr>
<tr>
<td></td>
<td>1.1</td>
<td># 17</td>
<td>28</td>
<td>264 902</td>
</tr>
<tr>
<td>Predominant</td>
<td>1.3</td>
<td># 20</td>
<td>24</td>
<td>478 086</td>
</tr>
<tr>
<td></td>
<td>1.1</td>
<td># 20</td>
<td>16</td>
<td>318 724</td>
</tr>
</tbody>
</table>

The type and number of wind turbines and their placement within wind farm using the lexicographic order Energy, Costs for the various wind directions and two DM preferences are shown in Fig. 4.

Fig. 5 shows the type, number and corresponding locations of wind turbines within wind farm using the lexicographic order Costs, Energy for the various wind directions for two set of DM preferences.

The determined layouts of designed wind farm by lexicographic method are designated as L-7 to L-18 as shown in corresponding figures.
of DM. Several loops of the proposed algorithm are executed to get 18 wind farm layouts. Some of them are used to change the DM preferences, while the rest are used to select another solution method. The stored results of different multi-objective problems solutions can be compared toward expected energy output and corresponding investments costs. The results are illustrated graphically in Fig. 6 for easy visual assessment of different wind farm layouts.

Fig. 6. Investment costs and expected energy output for different layouts obtained from solution of multi-objective problems

The testing results show that the described in the article multi-objective modeling approach implemented in the proposed algorithm supports the DM to get the most acceptable wind farm layout design.

The formulated optimization tasks are solved by means of LINGO solver (ver. 12) on desktop PC with Intel® Celeron® 2.93 GHz processor, 2 GB of RAM and MS Windows 7 OS. The solution time’s information is rarely found in publications on the wind farm layout multi-objective design topic. The described numerical examples are very close to real-life problems and the computational efforts are far less (in order of seconds) in comparison to other proposed approaches. For example, using multi-objective stochastic evolutionary algorithm for wind farm layout design in 3×3 km area for fixed number of identical turbines: 30, 50, 70, the solution
times are about 15 minutes [17] and for 5, 10, 15 turbines and same area it takes 19.75, 70.87, 149.75 hours [16].

VI. CONCLUSION

The multi-objective combinatorial modeling approach for designing of wind farm layout described in this work has an important advantage – essentially short computational times. The solution of formulated optimization tasks define simultaneously Pareto optimal type, number and placement of turbines considering wind site area, wind conditions, wake effect and DM preferences. This modeling approach can be used with other types and number of objectives formulations or other considerations for wake effect influence accordingly to the specifics of particular wind farm site.

The proposed algorithm for design and assessment takes advantage of the essential property of multi-objective optimization to provide different Pareto optimal solutions corresponding to different DM points of view about importance of objectives. Combined with smaller computational efforts for solution of described combinatorial optimization tasks, the algorithm can be used as an effective tool to support getting the most acceptable design decision. The practical usability of the combinatorial multi-objective optimization was confirmed by numerical testing on examples that are close to real-life problems.

The described algorithm is not limited to the used modeling approach or multi-objective solution methods. Other mathematical models with different number and types of objectives as well as multi-objective solution methods can be used to explore the potential of the algorithm. That direction is planned for future investigations to determine the algorithm applicability and possible limitations for design and assessment of wind farm layout.

REFERENCES


D.Sc. Daniela I. Borissova is Associated Professor in Institute of Information and Communication Technologies at Bulgarian Academy of Sciences, and University of Library Studies and Information Technologies.

Major fields of scientific research: modeling and decision support systems, operations research, and Web-based applications.

Dr. Ivan C. Mustakerov is Professor in Institute of Information and Communication Technologies at Bulgarian Academy of Sciences, and University of Library Studies and Information Technologies.

Major fields of scientific research: operations research, systems modelling and optimization, decision support systems, engineering systems design, software systems for information processing, e-learning and Web applications.