Determining Minimum Loading Margin of Power System Using Boundary Equations for Saddle Node Bifurcation

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Abstract—Obtaining the minimum loading margin is the critical factor to determine the boundary of the voltage stability of power system. The paper formulates the electric power network equation with the status variables being nodal voltages and branch currents. The boundary characteristic equation for the saddle node bifurcation, which represents the boundary of voltage stability, is built based on the derived expression of power flow equilibrium curve in branch currents. The steady state boundary conditions for voltage on critical voltage circle are given. The equation for the distance between the loading point and the bifurcation boundary is found by geometrical analysis. This equation represents the shortest distance of the loading variation. The distance equation along with the boundary characteristic equation serves as an additional set of equations for analyzing the minimum loading margin. These equations are solved by Newton method which replaces the critical points to avoid the sigularity around saddle node bifurcation points. Simulation results demonstrate the method's correctness and effectiveness.

Keywords—Saddle node bifurcation, loading margin, critical point, power flow, power system

I. Introduction

The voltage stability is facing new problems with the deepening of the reform of electric power system. The uncertainty brings larger impact to power grid along with massive renewable energy accessing to the network and the scale of the power system's size became increasingly large as the uhv power grid is built. The power systems' operation has become more and more complex which leads to the difficulties in control, so the voltage stability problem has become increasingly prominent. Furthermore, economic and environmental considerations require the existed equipments to be fully exploited, which drives the system operating much more closer to its limit, especially in the heavily-loaded case. It is easier for failures caused by the disturbation to extend in a broader scale, even resulting in voltage collapse. Therefore, it is desperately necessary to analyze voltage stability, figuring out the weakest point of the system's operation and taking a pointed preventing action.

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Researchers have proposed many ways to evaluate voltage stability. The minimum loading margin, whose value is widely taken as the effective measure for the steady-state voltage stability[1,2], refers to the difference of the load power in current condition to that at the operational boundary. It can provide the operators a quantitative evaluation for security of the current state. At present, the approaches to determine the minimum loading margin may be categorized as:

1) Direct method[3,4]. Given a certain direction in which the load increases, the critical point along this direction is obtained by continuation power flow or zero eigenvalue method. The new direction in which the load increases is determined by the left eigenvector corresponding to the zero eigenvalue of Jacobian matrix at the critical point. The problem is iteratively solved until the direction in which the load increases is close enough to that of the left eigenvector at the critical point.

2) Methods based on optimization[5]. Taking advantage of the fact that the vector of the minimal load increasing direction is perpendicular to the tangent plane of the corresponding critical point, and incorporating this geometric constrain into the optimization formulation, the requirements of the critical point is transformed to an optimized load problem which is solved by Kuhn-Tucker optimal conditions.

3) Evolutionary methods. The loading margin thus the boundary of the voltage stability is determined by either Evolutionary Method[6], or Genetic Algorithm or its combination with Artificial Neural Network transforming the original problem to optimization problem[7,8], or Regression Tree together with Artificial Neural Network constructing intelligent system and calculating saddle node bifurcation point (SNBP)[9].

Minimum load margin research is often associated with increased transmission capability closely. The decision-trees method is used to determine the identification of the critical transmission lines and their proper compensation rate, thereby increasing the transmission capacity[10]; the maximum active power transmission capacity and the critical value are obtained by calculating the Jacobian matrix systems and P-V curve, and then get the system weak areas through the singular vector and V-Q curve analysis[11]; improving grid transmission capacity can also use static reactive power compensation device and flexible ac transmission systems, and establish multi-objective optimization function to improve the minimum load margin[12].

The methods of calculating the minimum load margin mentioned above are all based on the numerical solution, rather than the analytical method, analytical method can expound the essence of voltage instability more clearly on the basis of theory. And the existing load power margin is calculated for a given load growth direction, get the local optimal solution, if you want to calculate the minimum load margin in the global scope, also need to study new methods.

Among the methods above-mentioned, the first one is easily trapped into the local minimums; whereas the second and the third one are attractive in formulation in determining the minimum load increasing direction. However, the solution to algorithms themselves is difficult with the high computational cost and poor practicability. The minimum static loading margin is the shortest distance between current system operating point and the boundary. It can be found by comparing the values of the loading margin at each node and it represents the voltage stability margin of the overall system. The key to obtaining the minimum loading margin is to find the critical loading point whereas the latter is closely related to the calculation of SNBP.

Bifurcation theory, which studies the relationship between the solution of the non-linear system and the parameters, has been widely applied to the analysis of power system static voltage stability[13,14]. When a power system structure becomes unstable, the typical situation is that with the parameters variation the equilibrium point and unstable points overlap with each other; the Jacobian matrix of the electric power network equation is singular, and SNBP appears. Therefore, SNBP is a kind of critical status and represents the boundary of power system static voltage stability.

Based on Bifurcation theory, the paper proposes a model for solving the minimum static loading margin. Starting from the electric power network equations in status of the branch current and nodal voltage, the model formulates the explicit expression of the equilibrium curve and critical equation for saddle node bifurcation which represents the boundary condition for voltage stability. On the basis of these, the paper derives the distance equation for the minimum loading margin and demonstrates that it expresses the shortest distance in terms of geometry.

II. Network Representatives Based on Nodal Voltage-Branch Equations

Electric power components, such as line or transformer, can be modeled as π-equivalent circuits which is shown in Fig. 1. Each circuit is composed of 3 branches: an impedance branch and two ground-branches.

In Fig. 1, $\dot{s}_i$ and $\dot{s}_j$ are nodal injected powers with $\dot{s}_i = p_i + \mathrm{j}q_i$, $\dot{s}_j = p_j + \mathrm{j}q_j$. Setting the nodal voltages as $\dot{u}_i = e_i + \mathrm{j}f_i$, $\dot{u}_j = e_j + \mathrm{j}f_j$ with $i, j \in N$, where N is the number of buses. The current in the impedance branch is $\dot{i}_l = i_l^a + \mathrm{j}i_l^r$ with $l \in L$, where L is the number of branches.

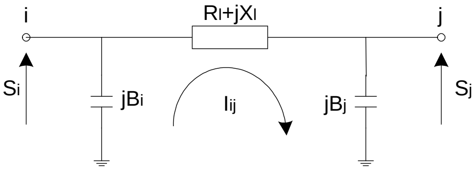

Fig. 1. π-Equivalent Circuit.

For ground-branches, taken the i-th node as an example and shown in Fig. 2, the shunt conductance is neglected for simplicity.

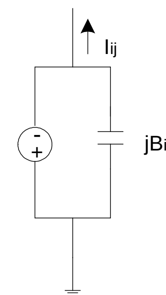

Fig. 2. Diagram of Grounding Branch

For each ground-branch, the current divides into two parts: one through the branch with earth capacity; the other one through the loading branch which holds not only the current of this circuit but also the one of the neighboring circuit. Same as the basic idea of nodal voltage method, the voltage of the equivalent voltage source can be calculated as (e.g., the i-th node)

$$\overset{*}{u}_i = \frac{p_i - \mathrm{j}q_i}{\sum_{l\in i} i_l - \mathrm{j}u_i \sum_{l\in i} B_l} \tag{1}$$

where $p_i - \mathrm{j}q_i$ is the load power at bus i; $\sum_{l\in i} i_l$ is the sum of currents injecting to bus i; $\sum_{l\in i} B_l$ is the sum of shunt capacity at bus i; $\mathrm{j}u_i \sum_{l\in i} B_l$ is the sum of capacitive current from bus i to the ground; $\sum_{l\in i} i_{li} - \mathrm{j}u_i \sum_{l\in i} B_l$ is the current in loading branch connected to bus i. From (1), we have

$$\begin{aligned} e_i \sum_{l\in i} i_l^a + f_i \sum_{l\in i} i_l^r &= p_i \\ e_i \sum_{l\in i} i_l^r - f_i \sum_{l\in i} i_l^a - (e_i^2 + f_i^2) \sum_{l\in i} B_l &= -q_i \end{aligned} \tag{2}$$

Similar equations can be derived for node j. The impedance branch is

$$\dot{i}_l(R_{ij} + \mathrm{j}X_{ij}) = -(\dot{u}_i - \dot{u}_j) \tag{3}$$

which can be extended to

$$\begin{aligned} i_l^a R_{ij} - i_l^r X_{ij} + e_i - e_j = 0 \\ i_l^a X_{ij} + i_l^r R_{ij} + f_i - f_j = 0 \end{aligned} \tag{4}$$

from which we have

$$\begin{aligned} i_l^a &= \frac{-X_{ij}(f_i - f_j) - (e_i - e_j)R_{ij}}{R_{ij}^2 + X_{ij}^2} \\ i_l^r &= \frac{X_{ij}(e_i - e_j) - R_{ij}(f_i - f_j)}{R_{ij}^2 + X_{ij}^2} \end{aligned} \tag{5}$$

Equation (2) and (4) formulate the augmented network equations in rectangular coordination, where the status parameters are composed of the branch currents and nodal voltages.

III. Analysis of Boundary Conditions for Saddle Node Bifurcation

Letting $x_i = \sum_{l \in i} i_l^a$, $y_i = \sum_{l \in i} i_l^r$, and $B_{i0} = \sum_{l \in i} B_l$, we have following equations for PQ nodes according to (2):

$$\begin{aligned} e_i &= \frac{\left\{ \begin{aligned} &\left[2B_{i0} p_i x_i - y_i (x_i^2 + y_i^2)\right] \mp \\ &y_i \sqrt{(x_i^2 + y_i^2)^2 - 4B_{i0} q_i (x_i^2 + y_i^2) - 4B_{i0}^2 p_i^2} \end{aligned} \right\}}{2B_{i0}(x_i^2 + y_i^2)} \\ f_i &= \frac{\left\{ \begin{aligned} &\left[2B_{i0} p_i y_i + x_i (x_i^2 + y_i^2)\right] \pm \\ &x_i \sqrt{(x_i^2 + y_i^2)^2 - 4B_{i0} q_i (x_i^2 + y_i^2) - 4B_{i0}^2 p_i^2} \end{aligned} \right\}}{2B_{i0}(x_i^2 + y_i^2)} \end{aligned} \tag{6}$$

These are explicit expressions for the nodal voltage in terms of the branch currents. From (6), when the condition

$$x_i^2 + y_i^2 \geq 2B_{i0} q_i + 2B_{i0} \sqrt{p_i^2 + q_i^2} \tag{7}$$

is satisfied, the real solution to power flow equations exists. As demonstrated in Fig. 3 where the horizontal axis represents the real part of the current through loading branch whereas the vertical axis represents the imaginary part, we can conclude that 1) when the squared magnitude of the nodal injected current lies outside the circle with the origin being the center and the radius being $\sqrt{2B_{i0}(q_i + \sqrt{p_i^2 + q_i^2})}$, e.g., point b, only the symbol ">" in (7) holds. In this case there is only high voltage solution and the system is stable. 2) When only the symbol "<" holds in (7), i.e., the squared magnitude of the nodal injected current lies within the circle with the origin being the center and the radius being $\sqrt{2B_{i0}(q_i + \sqrt{p_i^2 + q_i^2})}$, e.g., point k, no real solution exists and the system is unstable. 3) When the symbol "=" holds in (7), there is a unique solution on the circle, e.g., point a; the boundary of the voltage stability is achieved. The critical point of voltage stability can be found by solving the operating status for the system.

Many points (e.g., points a and a') on the critical circle correspond to bus b, but there is only one which is closest to b. This point represents the minimum loading margin at bus b. If the loading margin at bus b is the minimum of all buses, then this point represents the minimum loading margin of the system. Therefore, one of the major purposes of this paper is to find the point on the critical circle which is closest to point b.

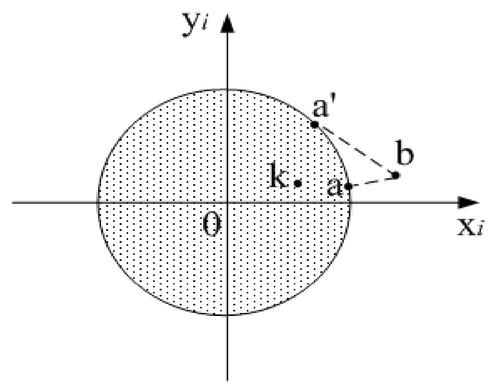

Fig. 3. Critical circle for nodal voltage

In power system analysis, buses type incorporates PU-node and slack bus as well. The voltage at the slack bus is known, while for PU-node, the reactive equation in (2) can be replaced by

$$e_i^2 + f_i^2 = U_i^2 \tag{8}$$

where U_i is the magnitude of voltage at bus i. Thus

$$\begin{cases} e_i = \dfrac{p_i x_i \mp y_i \sqrt{(x_i^2 + y_i^2)V_i^2 - p_i^2}}{x_i^2 + y_i^2} \\ f_i = \dfrac{p_i y_i \pm x_i \sqrt{(x_i^2 + y_i^2)V_i^2 - p_i^2}}{x_i^2 + y_i^2} \end{cases} \tag{9}$$

Similarly, only when

$$x_i^2 + y_i^2 \geq \frac{p_i^2}{V_i^2} \tag{10}$$

is satisfied, do the solution to network equations exist. In this case, there is a circle with radius p_i / V_i and center at the origin as well. The system is unstable within this circle therefore this circle is termed as "voltage instability circle". When the symbol "=" in (10) holds, the equation has a unique solution and the system reaches the boundary of voltage stability.

When the symbol "=" in (7) or (10) holds, the two solution curves of high and low voltage overlap, i.e., saddle node bifurcation occurs. Suppose the number of PQ-node is N_L, the number of PU-node is N_G, and the number of slack bus is N_S, we have $N_L + N_G = N - N_S$. The condition for the occurrence of saddle node bifurcation at PQ-node is

$$x_i^2 + y_i^2 = 2B_{i0}(q_i + \sqrt{p_i^2 + q_i^2}) \quad i \in N_L \tag{11}$$

while for PU-node is

$$x_i^2 + y_i^2 = \frac{p_i^2}{V_i^2} \quad i \in N_G \tag{12}$$

Here, (11) together with (12) is named as characteristic equation for saddle node bifurcation. Therefore, either (11) or (12) at any bus holds, saddle node bifurcation occurs. That is to say, the occurrence of saddle node bifurcation corresponds to the critical condition for the existence of the solution to electric power network equations.

IV. Distance Equation Representing Minimum Loading Margin of System

It can be observed from Fig. 3 that if the line connected by any point b outside the voltage instability circle and the center intersects the circle boundary at points a and c, then these points are SNBPs on the critical circle corresponding to node b. In this case, the tangent lines passing through points a and c, $l1$ and $l2$ (which are parallel to each other), are perpendicular to the normal passing through the center and points a, b, and c, $l3$. It can be proved from the geometric perspective that $\overline{ab}$ is the shortest distance from point b to the circle whereas $\overline{cb}$ is the longest one, as shown in Fig. 4. Therefore, if we know the representative functions of $l1$, $l2$, and $l3$, we can derive the function representing the distance between loading point b and point a taking advantage of the perpendicular relationship.

The function for the tangent lines passing through points a and c, $l1$ and $l2$, can be derived from (11):

$$\tan\alpha_a = \frac{\partial y_a}{\partial x_a} = \frac{-x_a}{\sqrt{2B_{a0}(q_a + \sqrt{p_a^2 + q_a^2}) - x_a^2}} \quad a \in N_L \tag{13}$$

where α_a is the angle of the tangent lines passing through points a and c. For PU-node, we have

$$\tan\alpha_a = \frac{\partial y_a}{\partial x_a} = \frac{-x_a}{\sqrt{\frac{p_a^2}{V_a^2} - x_a^2}} \quad a \in N_G \tag{14}$$

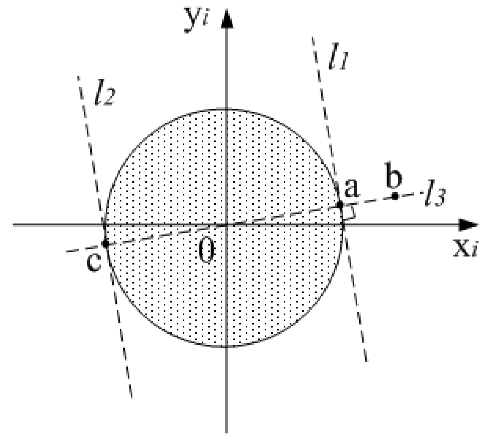

Fig. 4. Minimum Loading Distance

The function for the normal $l3$ is

$$\tan\beta_b = \frac{y_b}{x_b} \tag{15}$$

where β_b is the angle of the normal passing through the center and points a, b, and c. Since $\alpha_a = \beta_b + 90^\circ$, the distance equation for PQ-node can be formulated as

$$\frac{\sqrt{2B_{a0}(q_a + \sqrt{p_a^2 + q_a^2}) - x_a^2}}{x_a} = \frac{y_b}{x_b} \qquad a, b \in N_L \tag{16}$$

Similarly, the distance equation for PU-node is

$$\frac{\sqrt{\frac{p_a^2}{V_a^2} - x_a^2}}{x_a} = \frac{y_b}{x_b} \qquad a, b \in N_G \tag{17}$$

where y_b and x_b can be derived from (5).

Because the initial state of the given system is stable the power flow calculation is convergence at point b, i.e., the right side of the (16) and (17) is known through the calculation of (15) at the initial conditions. The system reaches unstable on the critical circle of node voltage and the power flow calculation is not convergent, so the points on the critical circle is exactly what we're going to get. The geometric meaning of distance (16) and (17) expresses the recent distance between node b and the critical circle, also specifies the direction that the node b achieves to the closest point a on the critical circle, so the $\overline{ab}$ represents the minimum loading margin of the power system.

V. Solution Technique

It is infeasible to use the classic Newton method to solve the power for the SNBP a on the critical circle corresponding to node b shown in Fig. 4 because the Jacobian matrix in this case is singular and therefore the power flow does not converge. By removing node b which makes the Jacobian matrix singular from the nodal voltage equation, we have the nodal voltage equations with one dimension reduced.

$$\begin{cases} e_i x_i + f_i y_i = p_i \\ e_i y_i - f_i x_i + (e_i^2 + f_i^2) B_{i0} = -q_i \quad or \quad e_i^2 + f_i^2 = V_i^2 \end{cases}$$
$$\begin{cases} i_l^a R_{ij} - i_l^r X_{ij} - e_i + e_j = 0 \\ i_l^a X_{ij} + i_l^r R_{ij} - f_i + f_j = 0 \end{cases} \tag{18}$$

where $i \in (N_G + N_L)$ and $i \neq b$ with $l \in L$. For point a, we have

$$\begin{cases} e_a = \dfrac{p_a x_a - y_a (q_a + \sqrt{p_a^2 + q_a^2})}{2B_{a0}(q_a + \sqrt{p_a^2 + q_a^2})} \\ f_a = \dfrac{p_a y_a + x_a (q_a + \sqrt{p_a^2 + q_a^2})}{2B_{a0}(q_a + \sqrt{p_a^2 + q_a^2})} \\ x_a^2 + y_a^2 = 2B_{a0}(q_a + \sqrt{p_a^2 + q_a^2}) \\ \dfrac{\sqrt{2B_{a0}(q_a + \sqrt{p_a^2 + q_a^2}) - x_a^2}}{x_a} = \dfrac{y_b}{x_b} \end{cases} \quad \text{or}$$

$$\begin{cases} e_a = V_a x_a \\ f_a = V_a y_a \\ x_a^2 + y_a^2 = \dfrac{p_a^2}{V_a^2} \\ \dfrac{\sqrt{\dfrac{p_a^2}{V_a^2} - x_a^2}}{x_a} = \dfrac{y_b}{x_b} \end{cases} \tag{19}$$

where (18) are electric power network equations. Equation (19) is the boundary characteristic equations for SNBP and distance equations. When calculating, we plug the critical nodal voltage of (19) in branch current (18), combine the boundary characteristic equation of the node and distance, and formulate the Jacobian matrix of the nodal voltage equation, whose dimension is reduced by one comparing with the original one.

From the calculation we have p_a, q_a, therefore

$$S_a = \sqrt{p_a^2 + q_a^2} \quad or \quad P_a = p_a \tag{20}$$

S_a or P_a represents the parametric conditions for saddle node bifurcation boundary, i.e., the boundary of the static voltage stability at bus b. After obtaining p_a and q_a at point a, we can determine the direction in which the minimum loading margin at bus b varies as follows:

$$\delta_b = \arctan \frac{Q_a - Q_b}{P_a - P_b} \tag{21}$$

where δ_b is the angle with which the loading at bus b changes. The minimum loading margin is determined by Euclidean distance as follows:

$$\eta_b = \left\| S_b - S_a \right\| = \sqrt{(P_b - P_a)^2 + (Q_b - Q_a)^2} \tag{22}$$

There are two solutions, representing the nearest point a and the farthest point c, resp. η_b is used to judge which one is minimal.

For the system shown in Fig. 4, when using equation sets (18) and (19) to calculate the minimum loading margin at all buses of a power system, we take the following steps:

1)Arbitrarily given bus b in the system, y_b and x_b are calculated from (5) (for PU-node, q_b is calculated from the reactive equations in (2)) taking advantage of the power flow results given by conventional Newton method or the real-time data measured by automatic systems. They will be served as the basic data for determining the normal $l3$ afterwards.

2)Given initial values of nodal voltage $\dot{u}_i$ and branch current $\dot{i}_l$.

3)To determine the critical point a, the iterative equations with nodal voltage and branch current being the status variables are formulated according to (18) or (19), the Jacobian matrix reduced by one dimension is constructed, and the problem is iteratively solved by Newton method.

4)Using the calculated value of critical point a, the direction and value of the system's minimum loading margin can be calculated from (21) and (22), meanwhile the boundary conditions under which the system bifurcates can be determined by (20).

VI. Case Studies

The proposed method is validated by IEEE 118-bus test system with slack bus #69 and bus #118 exchanged. Tab. 1 presents the results of the minimum loading margin at some nodes with respect to the saddle node bifurcation boundary.

Tab. 2 presents the loading margin on the critical circle for different power factor angles at the same bus. Only the representative results are shown in Table 1 and Tab. 2.

TABLE I. RESULTS OF MINIMUM LOADING MARGIN FOR IEEE 118-BUS TEST SYSTEM(P.U.)

No.	p_b	q_b	p_a	q_a	δ_b	η_b
4	0.700	0.710	1.982	1.098	16.84	1.339
8	0.400	-0.445	2.221	0.966	37.77	2.304
11	0.700	0.230	2.090	0.838	23.63	1.517
14	0.140	0.010	2.471	1.128	25.62	2.585
15	0.780	0.273	1.892	1.011	33.57	1.335
21	0.140	0.080	2.360	1.033	23.23	2.416
27	0.700	-0.228	1.994	1.110	45.96	1.861
32	0.390	0.570	2.226	1.094	15.93	1.909
42	0.980	-0.170	1.559	0.689	56.02	1.036
44	0.160	0.080	1.871	0.774	22.08	1.846
52	0.180	0.050	1.911	0.779	22.84	1.878
59	1.220	0.361	1.613	0.883	53.02	0.653
90	1.630	-0.271	2.219	1.187	68.00	1.572
105	0.210	0.416	2.006	1.163	22.58	1.945
108	0.020	0.010	2.490	1.058	22.99	2.683
112	0.680	-0.285	2.488	1.212	39.62	2.347

TABLE II. RESULTS OF DIFFERENT DIRECTION IN WHICH LOAD INCREASES(P.U.)

No.	δ_b	p_a	q_a	η_b
a	56.00	1.559	0.689	1.036
c	32.00	3.887	1.669	3.440
d	37.00	2.590	1.060	2.026
e	48.00	1.782	0.733	1.208
f	33.00	3.221	1.320	2.691
n	48.00	1.844	0.779	1.283
g	39.00	2.872	1.333	2.416
h	51.00	2.382	1.554	2.222

We can observe from Tab. 1 that SNBPs, and therefore the value and direction of the minimum loading margin, can be determined for different buses in the system. However, the loading margins of different buses are different from each other. For instance, bus #59 has a relatively smaller minimum loading margin whereas bus #108 has a larger one. The reason is that #59 initially undertakes a heavier load and it locates in a heavily-loaded area, which leads to a relatively smaller power of the critical point at saddle node bifurcation. Therefore the distance between bus #59 and the critical voltage circle is closer and the loading margin is smaller. On the contrary, the loading margin at bus #108 is larger.

Taking bus #42 as an example, Tab. 2 shows the results for different points on nodal critical voltage circle under different δ_b and the situation that the system's operating condition satisfies the boundary characteristic equation for saddle node bifurcation. In this case, equation sets (18) and (19) do not contain boundary distance (16) or (17). Initial load undertaken by the bus is $p_b = 0.980$ and $q_b = -0.170$.

When the value of δ_b is given, the direction in which the load at bus b varies and the location where bus b is at the critical voltage circle can be determined. The observation can be made from Tab. 2 and Fig. 5 that if the load at the same bus increases in different directions, then different SNBPs will be achieved. The intersection points of the circle and the line passing through bus b and the center form the shortest distance $\overline{ab}$ and the longest distance $\overline{cb}$. The closest SNBP a can be taken as the criteria for evaluating the minimum loading margin of the system whereas the farthest SNBP c can be taken as the criteria for planning generation or managing demand-side load.

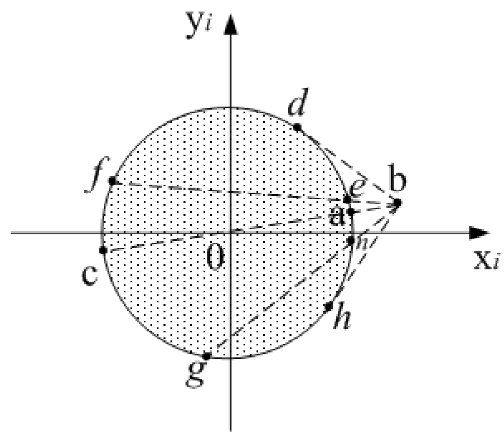

Fig. 5. Loading Margin in Different Directions in Which Load Increases

Tab. 3 compares the method proposed in this paper, Continuation Power Flow, and Genetic Algorithm for determining the minimum loading margin, taking bus #21 as an example. From the results we can see that the proposed method gets a higher precision.

TABLE III. COMPARISON OF DIFFERENT METHODS(P.U.)

	Proposed Method	**Continuation Power Flow**	**Genetic Algorithm**
δ_b	23.23	24.51	24.39
η_b	2.416	2.431	2.423

VII. CONCLUSIONS

The value of a power system's minimum loading margin and the direction in which it changes are determined by the equation sets composed of the boundary characteristic equation for the saddle node bifurcation and the distance equation for minimal load. The simulation results demonstrate that:

1) The proposed method can be applied to determining the minimum loading margin and analyzing the boundary margin for static voltage stability.

2) A power system's loading margin is affected by the initial loads at each bus and the load density within the area which the system covers. Therefore, various situations should be comprehensive considered while regulating the load.

3) The derived expression for equilibrium solution and boundary equation for SNBP give an insight to the critical

conditions for voltage stability. They can be applied to other research areas in power system analysis as well.

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