Nonlinear Dynamics in Computer Design of Growth "Areas" for Regular and Irregular Schwartz's Cylinder

Eugeny I. Smirnov, Artem D. Uvarov, and Sergey A. Tikhomirov

Abstract—In this article problems of students' development of surface area concept by means of computer and mathematical modeling are discussed. The difficult mathematical concept is investigated on the example of lateral surface "area" of Schwartz's cylinder or Schwartz's "boot" with use of the Qt Creator environment. The directions and basic constructs of computer and mathematical modeling in the conditions of triangulations crushing of the cylinder lateral surface and the identification of dynamics growth of the areas of the corresponding many-sided complexes in case of regular (equal) and irregular (unequal) splitting of cylinder's height in research activity of students setting in small groups are revealed. Logistic mapping is reviewed as the basic instrument of regular triangulations crushing of lateral surface of the cylinder, which based on ideas of T. Malthus and generating compliance to the scenario of P. Verhulst of chaotic dynamics of nontrivial growth of many-sided complexes of the areas. Some of the new regularities similar to expansion of a tree of M. Feigenbaum via the cascade of bifurcation transitions of doubling of the period are received. It is concern of the synergy research of growth of the areas of many-sided of the operating parameters of the area's growth. Growth rates of area difference of regular and irregular Schwartz's cylinder is researched. The independence of areas growth dynamics at regular and irregular height splittings as functional variable at change of equal breakdowns of cylinder foundation is shown. The connection of research activity of students with increase of mathematical competence and creative activity processes of self-organization and educational motivation is established.

Keywords—***C***omputer and mathematical modeling, training in mathematics, Schwartz's cylinder, surface area, visual modeling.

I. Introduction

THE concept of surface area is one of the most difficult in mathematical education in secondary school. The surface triangulation by tangent planes, difficult limit processes by means of differential and integral calculus are required, as well as the consideration of features of surface area existence as necessary instruments for regular approach. There is also a possible approach to determination of surface area on G. Minkowski's method when the limit of private volumes of δ – neighborhoods of a surface is considered at $\delta \to 0$. However in this case a surface area is not always an additive function therefore the school practice is limited to empirical formulas of surface area of a cone, a sphere and the cylinder. At the same time the logic of comprehension of the essence of length of a curve γ is based on natural approximation of a curve by broken lines inscribed in it when the splitting are crushed. Thus length of a curve exists, for example, for graphics of continuously differentiable functions and a criterion of existence of curve length setting on a piece [a; b] decides by consideration of a class of functions on a limited variation V_a^b representing a complete metric space [1]. The desire to make the analogy construction (as T. Schwartz showed [2]) leads to emergence in "area" of a lateral surface of the cylinder of unusual properties. Therefore regular triangulations of a lateral surface of Schwartz's cylinder with crushing splitting bring to the infinite area. Research of this problem zone of mathematical education at secondary school in the form of resource lessons or laboratory and settlement occupations is possible with use of computer modeling means in synergetic procedures of approximation of a lateral surface of the Schwartz's cylinder. It allows bettering understanding the essence and the content of a concept of surface area as mathematical category, using means and methods of experimental mathematics and visual modeling [3].

II. Methodology, Methods and Technologies

"Area" pathological properties of a lateral surface of Schwartz's cylinder are well studied in a so-called "regular" case (see for example [4], [5]). It occurs when its height ***of H*** breaks to *m* equal parts (respectively – cylinder layers) and the circles lying in the basis are divided to n of equals parts with the subsequent shift φ on each layer on $\frac{\pi}{n}$. At such triangulation of lateral surface of the cylinder the formula for calculation of its "area" by means of the turned-out polyhedrons at $m, n \to \infty$ has an appearance:

$$S_q = 2\pi R\sqrt{R^2\frac{\pi^4}{4}q^2 + H^2}, \tag{1}$$

where

$$q = \lim_{m,n\to\infty}\frac{m}{n^2}. \tag{2}$$

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Thus, "area" of a lateral surface ***of Sq*** of the regular Schwartz's cylinder of height ***of H*** and radius ***of R*** (if this limit exists – final or infinite value) completely is defined by a limit (2). It is clear that true value of the area of a lateral surface (q=0) can be received by consideration of the tangent planes in points of a triangulation and the subsequent transition to a limit of the areas of external polyhedrons at unlimited of crushing splitting. Thus in view of independent nature of aspiration $m, n \to \infty$ the result of limit process becomes poorly predicted, multiple-valued, with lack of regularities in chaotic expansion of fractal structures of polyhedrons. B. Mandelbrot in [6] showed that at $m = n^k$ the area of a many-sided surface grows as n^k ($k \neq 2$).

It is simple to see that $\lim_{q\to\infty} S_q = \lim_{q\to\infty} cq$, where $c = R^2\pi^3$. Thus, for calculation of "area" of a lateral surface S_q of the regular Schwartz's cylinder of at rather great values q instead of a formula (1) it is possible to use a formula:

$$S_q = R^2\pi^3 q \qquad (3)$$

We will consider part of Schwartz's cylinder layer of radius ***R=1*** and height H=1.

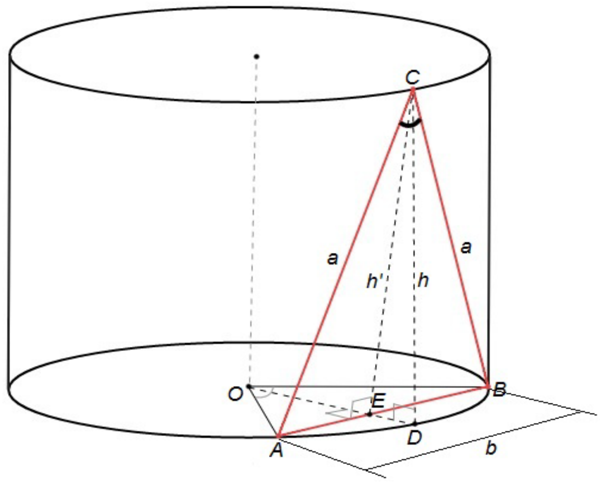


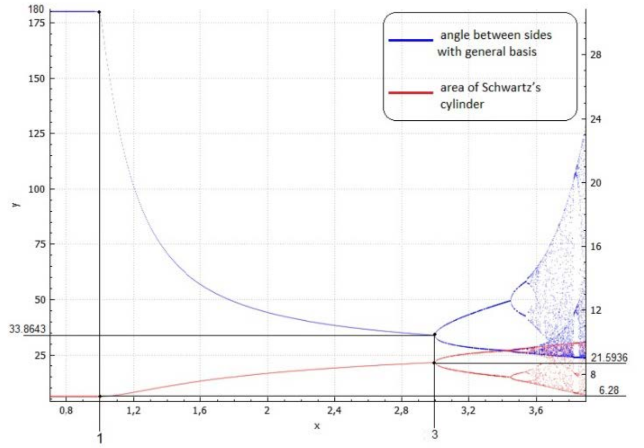
Fig. 1: One of the triangle in cylinder layer.

In article of E. I. Smirnov and A. D. Uvarov [7] the behavior of function (1) and a corner α between triangles with the general basis is investigated if

$$m = f^n(\mathrm{a}_0)\cdot \mathrm{n}^2 \text{ and } m, n \to \infty, \qquad (4)$$

where $f(\mathrm{a}_0) = xa_0(1-a_0)$ - the logistic mapping adequate to P. Verhulst's scenario [8]. Authors received the following bifurcation diagram (Fig. 2) with use of information technologies (Qt Creator environment).

In Fig.2 two bifurcation diagrams are represented for which $0.7 \le x \le 3.9$ and $a_0 = 0.2$. Thus on the left vertical axis the values of a corner α between sides with the general basis are fixed, which is calculated by formula



Blue – angle between sides with general basis
Red – area of Schwartz's cylinder

Fig. 2: The bifurcation diagram of "area" and angles of Schwartz's cylinder.

$$\alpha = 2\cdot \arctan\left(\frac{1}{1-\cos\frac{\pi}{n}}\cdot \mathrm{m}\right)$$

and on the right axis the areas values of the Schwartz's cylinder are calculated on a formula (1) taking into account that $R = H = 1$, at this *n* changes from 500 to 1000.

There are hierarchies of problems resolved by means of computer and mathematical modeling of research activity of students in small groups in the remote environment [9] or in the form of research of multi-stage mathematics-information tasks [10]. The main task of present article is updating of synergy manifestation in learning mathematics in small groups of students during an identification of the research directions of parameter m behavior from a formula (4) at n → ∞.

1. At first we will note that from Fig.2 becomes visible that in a limit process at n → ∞ and x <1 a corner α remains to a constant and equals 180 degrees. Thus the "area" of Schwartz's cylinder has to be equal to the area of the ordinary cylinder, that is 2π, however in this case Schwartz's cylinder can't be constructed; this results is appeared from the fact that *m=0* at x <1 and as a function

$$f(\mathrm{a}_0) = xa_0(1-a_0)$$

has two fixed points

$$a_0 = 0, a_0 = 1 - \frac{1}{x}$$

that the trivial point $a_0 = 0$ is steady.

Further research shows that at 1<x <3 the 2rd point

$$a_0 = 1 - \frac{1}{x}$$

is a steady fixed point, as in this case |2-x|<1 and its multiplicator

$$m = (1 - \frac{1}{x})\cdot n^2$$

is monotonously increasing function on this interval. So in this case the "area" of a lateral surface monotonously grows and the corner α monotonously decreases. Similar research at x> 3 shows that a point

$$a_0 = 1 - \frac{1}{x}$$

loses the stability and there is a bifurcation of limit areas values. Therefore as a point of bifurcation we fix the value of a corner

$$\alpha = 33.8643^0.$$

Thus the bifurcation point for "area" is equal $S = 21.5936.$

At 3 <x <3.57 (as shown in Fig. 2) students can look by means of computer and mathematical modeling the emergence of the subharmonic cascade of bifurcations (at each such bifurcation the period increases twice that corresponds to frequency "in half", i.e. emergence of a subharmonics in a spectra of fluctuations). Identification and visualization of integrative links of basic concepts and methods of school mathematics with new categories of manifestation with synergetic effects can be an important component of cognitive activity of students.

2. Other direction of researches for small groups of students can be realized if to consider the iterative sequence

$$y_n = [f^n(a) \cdot n^2],$$

where

$$f(\text{a}) = qa(1-a)$$

is logistic mapping. In Fig. 3 three graphics of sequence of y_n constructed for values

$$q = 3.67; q = 3.05; q = 2.2$$

at that 100 <n <100000, α =0.2. Points on graphics thus are connected by segments. In Fig. 4 the graphics fragment with increase in scale is presented. We will notice that at q=2.2 the iterative process possesses the only one point of an attraction therefore the sequence of y_n answering to this value of parameter q monotonously increases on all range of definition.

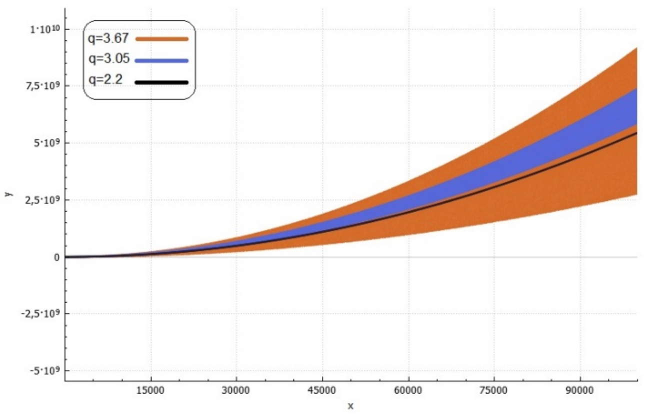


Fig. 3: Three graphics of sequence of y_n.

At q=3.5 the iterative process has two points of an attraction, so there is a bifurcation; thus the sequence of y_n stops being increasing though the trend of this sequence monotonously increases.

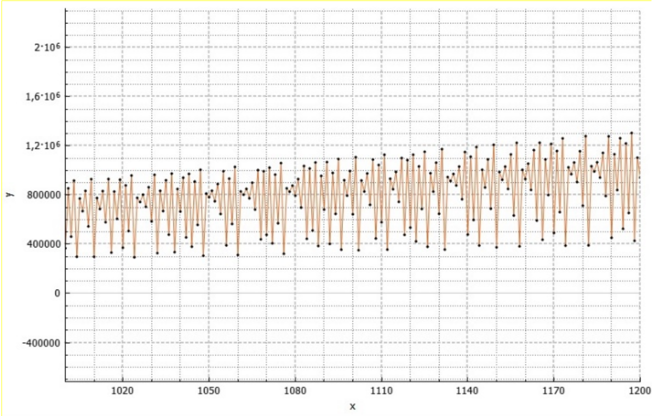


Fig. 4: Graphics fragments of iterative sequences for different q and a segment of value n.

Thereby, at a choice of any sequence

$$y_n = [f^n(\text{a}) \cdot n^2]$$

when q <3.57 arise the bifurcation transitions so that it is possible to allocate increasing subsequences if to believe that $n = 2^i \cdot k + r,$ where k=1,2,… and i – the period of iterative sequence of f^n(a). For example, for q=3.56 we have four increasing sequences

$$y0; y1; y2; y3$$

in Fig.5 (r = 0,1,2,3; i = 4).

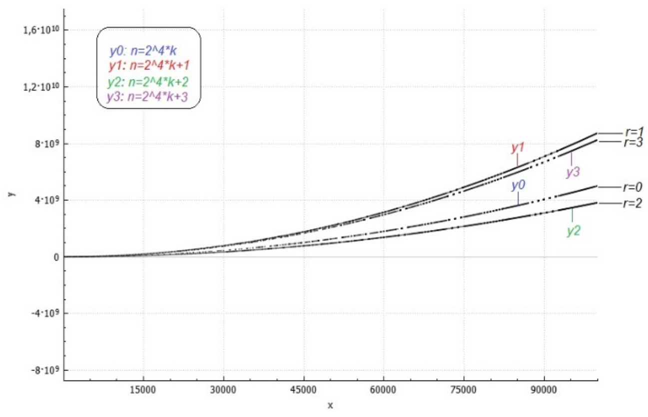


Fig. 5: Four increasing sequences.

As at q>3.57 the number of bifurcation transitions unlimitedly increases, the number of points of an attraction also aspires to ∞ (Fig. 6).

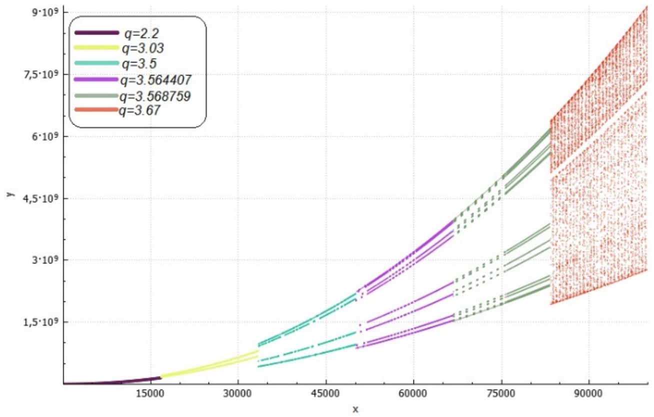


Fig. 6: Dynamics of bifurcation transitions for spectra of increasing subsequences for different q.

3. The following direction for research of Schwartz's cylinder by small groups of students can be connected with computer modeling and creation of figure 2, the block-scheme of the program and theoretical analysis of the entering parameters of logistic equation by means of mathematical modeling. Thus the block-scheme (Fig. 7) can be used.

4. In this section we discuss the actualization of the synergy attributes (bifurcation, attractors, fluctuations, basins of attraction) in the research learning process of irregular (unequal splitting height) Schwartz's cylinder in secondary school (adaptation of surface area to school mathematics while working in small groups with variations of the "area" of regular and irregular Schwartz's cylinder). While following the forms and means of mathematics teaching: distance learning project teams, laboratory and design classes, multistage mathematical and information sessions, conferences and seminars, networking and discussion forums [12-14]; resources: mathematical and computer modeling, QT Creator – a cross-platform free IDE for developing on C++, educational software products, graphic calculator ClassPad400, Web Quest as a means of integrating Web technologies with educational items, Wiki-sites, Messenger, Skype [15-17]. Under these conditions proved to be effective the following technologies: coordination graphs of mathematical knowledge and procedures, work in small groups, Web Quest – as the technology of self-organization in collective creativity, project methods, Wiki technology, visual modeling, foundation of personal experience [18-20].

Example 1. Consider a Schwartz's cylinder of which $H = R = 1$. At that the height of the cylinder is divided into layers through the sequence $\left\{\frac{a_i}{S_m}\right\}$, where $\{a_i\} = \{\frac{1}{\sqrt{i}}\}$ and $\sum_{i=1}^{\infty} a_i = \sum_{i=1}^{\infty}\frac{1}{\sqrt{i}}$ are a generalized harmonic series. Examine the dependence of the function distance between the "area" of regular cylinder (H is split into layers of equal height) and the "area" of irregular cylinder (H is split into layers of different height) of this example for sufficiently large of n and taking into account the parameter q defined by the equation (2). Thus the "area" of a regular cylinder will be calculated by the formula (1) and "area" of an irregular cylinder will be calculated according to the formula

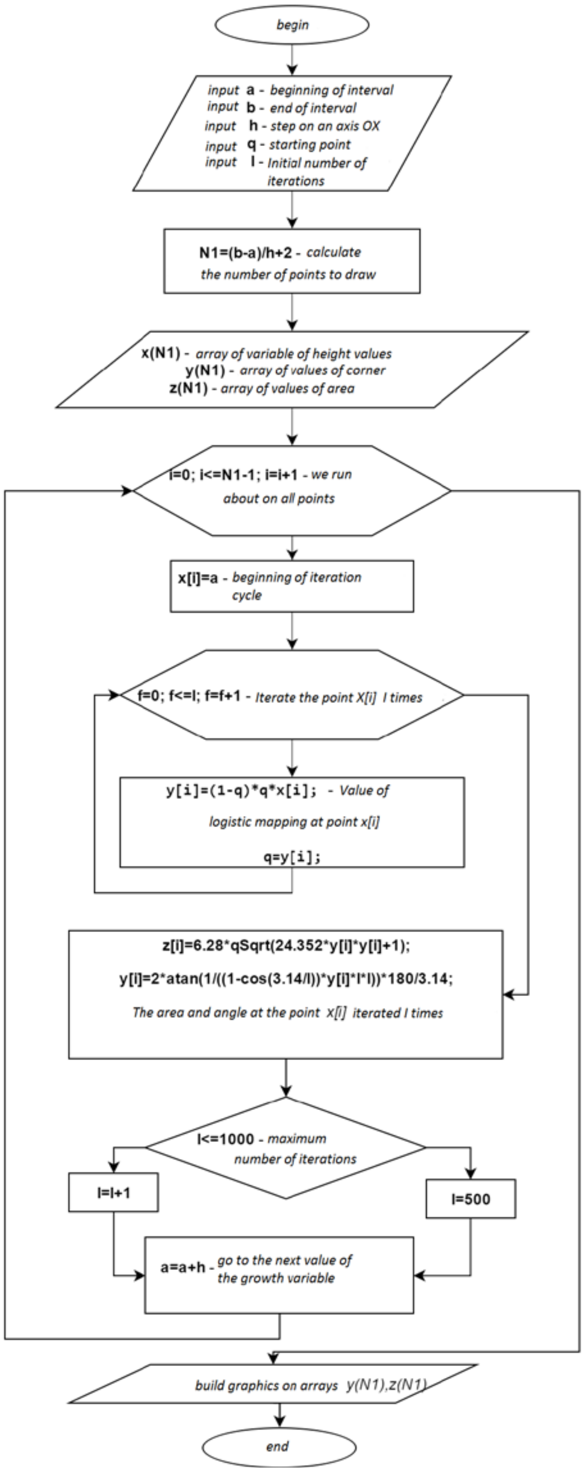


Fig. 7: The block-scheme of computer program.

$$S' = \lim_{m,n\to\infty}\left(2\pi R\sum_{i=1}^{m}\sqrt{R^2\frac{\pi^4}{4n^4}+H^2\frac{a_i^2}{S_m^2}}\right). \quad (5)$$

using the computer-environment of QT - Creator, version 4.8.5 and algorithmic language C++. For clarity we assume that n = 200, $m = 200^k$, where k changes from 0.8 to 3.2.

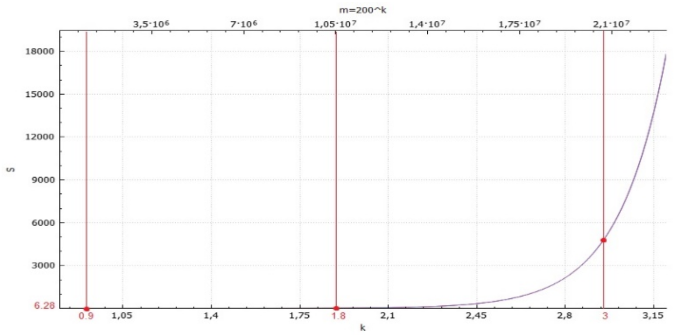


Fig. 8: Dependency graphs S' and S from parameter k.

However, both graphs are so close to each other, that in Fig. 8 they merge into a single chart, which may give the impression that $|S - S'| = 0$. More detailed resolution in the next Fig. 9 shows that this is not so and covers the range of the parameter, where k varies from 1.5 to 1.9.

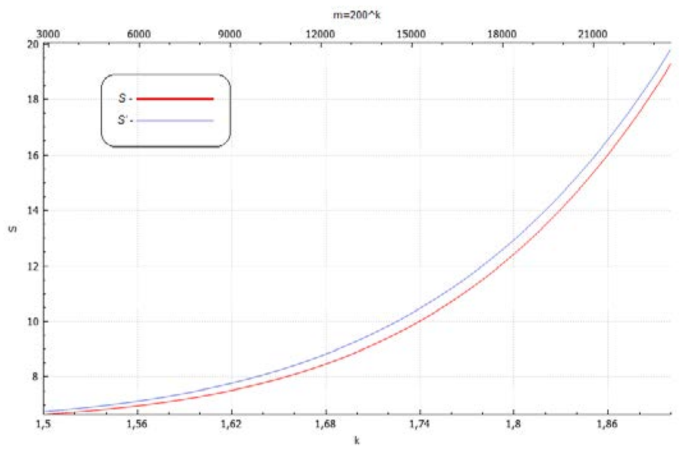


Fig. 9: Fragment of dependency graphics S' и S from parameter k.

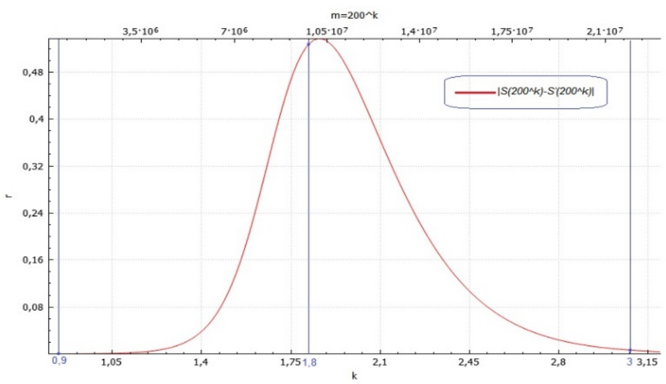


Fig. 10: Dependence graphic of module difference "area" of cylinder from k.

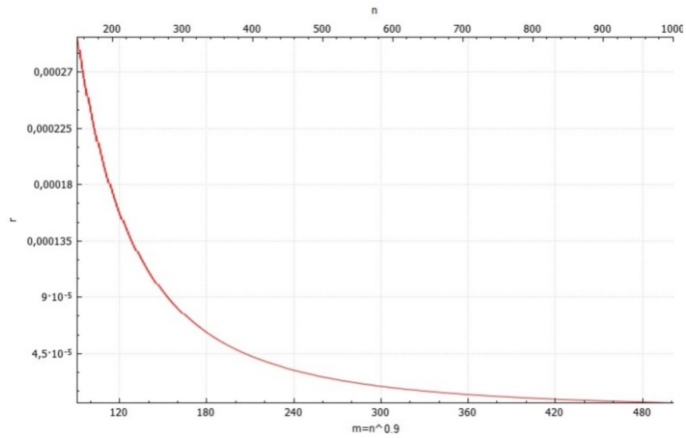


Fig. 11: Dependency graphic of module difference "area" of cylinder from k =0.9

In the following Fig. 11 we construct the dependency graph $\mathrm{r(k)} = |\mathrm{S}(200^{k-2}) - \mathrm{S'}(200^{k-2})|$ of "area" difference module of cylinder between $k \in [0.8; 3.2]$. The figure shows that the distance between the surfaces of the cylinders is not equal to zero for any $q = n^{k-2}$. We show that for $k \neq 2$ and at large, enough values of n this distance still tend to zero.

Denote by $S_1 = S_1(\mathrm{n})$ and $S_1' = S_1'(\mathrm{n})$ the functions of regular and irregular Schwartz's cylinders "area" at fixed parameter of value k. At the same time $S_1(\mathrm{n}) = S = S(\mathrm{q}) = \mathrm{S}(\mathrm{n}^{k-2})$ and $S_1'(\mathrm{n}) = S' = S'(\mathrm{q}) = \mathrm{S'}(\mathrm{n}^{k-2})$. Show based on computer modeling that the limit $\lim_{n\to\infty} |S_1(\mathrm{n}) - \mathrm{S}(\mathrm{n})| = 0$ at $k \in [0.8; 3.2] \setminus \{2\}$. Due to the limitation of computing power we will find the last limit at $k = 0.9$, $k = 1.8$ and $k = 3$ (vertical lines in Fig. 10). From Fig. 11 it follows that $\lim_{n\to\infty} |S_1(\mathrm{n}^{0.9-2}) - \mathrm{S}(\mathrm{n}^{0.9-2})| = 0$ and the limit converges to zero very quickly.

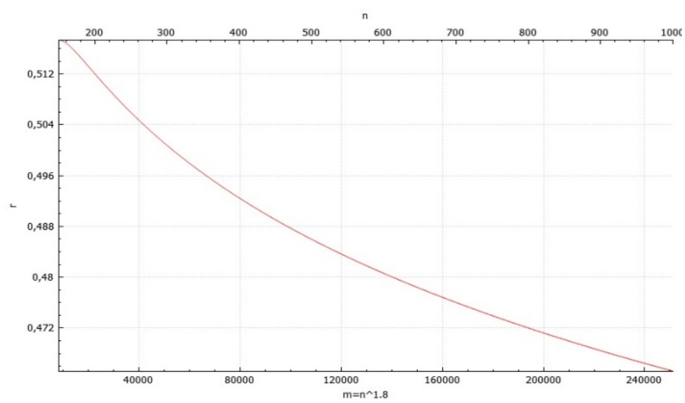


Fig. 12: Dependency graphic of module difference "area" of cylinder from k =1.8.

Fig. 12 also shows the convergence of the “area” to zero when k =1.8, namely, $\lim_{n\to\infty}|S_1(\mathrm{n}^{1.8-2})-\mathrm{S}(\mathrm{n}^{1.8-2})|=0$. However the distance between the “areas” decreases much slower than in Fig. 11, so when k goes to two the speed of “areas” convergence decreases (Fig. 13).

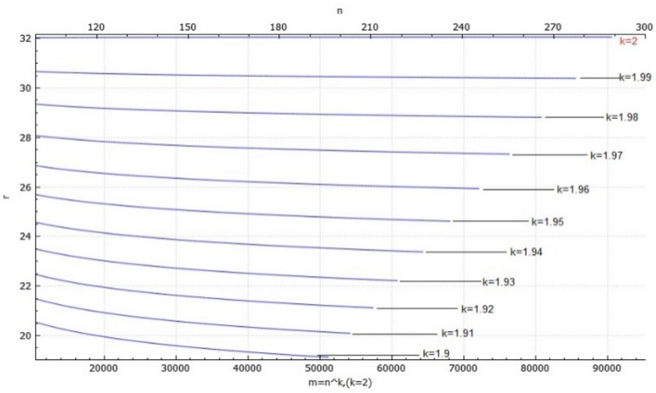


Fig. 13: Dependency graphics of module difference “area” of cylinder from n at different k.

Let $k\in(0;2]$. We introduce a function $S(\mathrm{k})=\lim_{n\to\infty}S(\mathrm{n}^{k-2})$ expressing the “area” of regular Schwartz’s cylinder in the assumption $q=n^{k-2}$. It is easy to see that $S(\mathrm{k})=\begin{cases}2\pi,k<2\\ 2\pi\sqrt{\dfrac{\pi^4}{4}+1},k=2\end{cases}$

that is the function $S(\mathrm{k})$ has a gap if $k=2$.

Denote by $S'(\mathrm{k})=\lim_{n\to\infty}S'(\mathrm{n}^{k-2})$ the function of irregular cylinder “area”. However $S'(2)\neq\mathrm{S}(2)$. The last inequality stems from the fact that $k\to2$ when the number l of irregular cylinder layers tends to n^2 the height of which is “fairly evenly” converge to zero. So the “area” of this part of the cylinder is committed to the “area” of regular cylinder, i.e. $\sum_{i=1}^{l}S'_i\to2\pi\sqrt{\dfrac{\pi^4}{4}+1}$, but the sum of the “areas” of remaining layers are very slowly committed to the sum of the “areas” at the same layers as for regular cylinder. Moreover, if $k=2$ so the equality $\sum_{j=1}^{n^2-l}(S'_j-S_j)=c\neq0$ and accordingly

$$\lim_{n\to\infty}S'(\mathrm{n}^2)=\lim_{n\to\infty}\mathrm{S}(\mathrm{n}^2)+\mathrm{c}. \tag{6}$$

The following Fig. 14 shows 4 graphics for $n=100, n=150, n=200$ and $n=250$ for $k\in[0.8;3.2]$.

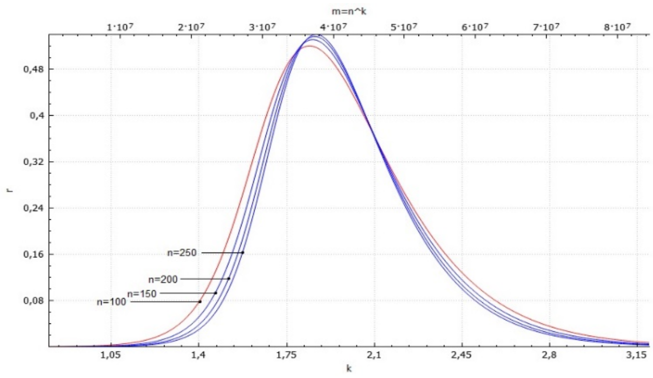


Fig. 14: Dependency graphics of module difference “area” of cylinder from k at different n.

Example 2. Let an irregular Schwartz’s cylinder be given for which $H=R=1$. In this case, the height of the cylinder is divided into layers by means of a sequence $\left\{\dfrac{a_i}{S_m}\right\}$, where $\{\mathrm{a}_i\}=\{\mathrm{i}\}$ and $\sum_{i=1}^{m}a_i=\dfrac{m(\mathrm{m}+1)}{2}$ is the sum of the first terms of arithmetic progression. As shown in the previous example we will study the modulus of distances difference between the “areas" of cylinders in regular and irregular cases. For this, as well as in Fig. 14, we will construct 4 graphics for $n=100, n=150, n=200$ and $k\in[0.8;3.2]$.

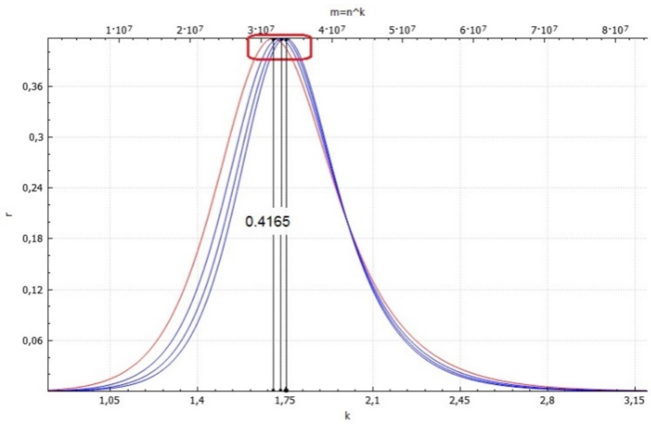


Fig. 15: Dependency graphics of module difference “area” of cylinder from k at different n.

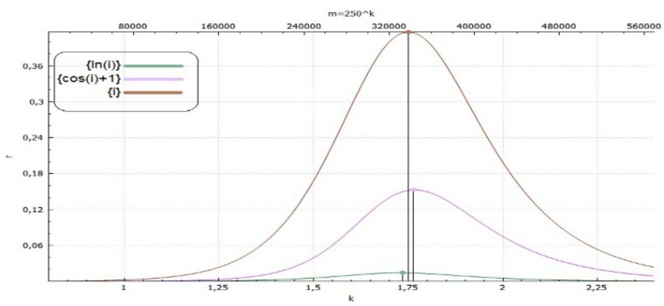


Fig. 16: Dependency graphics of module difference “area” of cylinder from k at different splitting for n =250.

Fig. 15 shows that in this case a situation arises similar to the situation in the previous example, while the height of the peak with increasing m also tends to two. A similar situation occurs in other cases, for example, when $\{a_i\} = \{\ln(i)\}$ or $\{a_i\} = \{\cos(i) + 1\}$, so in the last case the sequence $\{a_i\}$ is limited, but the series $\sum_{i=1}^{+\infty} a_i = +\infty$ is divergent (Fig. 16).

Previous experiments and computer design show that in the case when the heights of all layers of the irregular Schwartz's cylinder splitting tend to zero its "area" as a function of layers number tends to the "area" of regular cylinder wherever it is continuous. Similar studies, conducted by pupils on resource lessons or laboratory-calculating classes, using the multi-stage mathematical-information assignments during project activities or network interaction develop the intellectual operations of thinking and increase the educational motivation and the quality of mathematical abilities growth.

III. Results

Thus, means of computer and mathematical modeling revealed the new directions of research activity of students in profound comprehension of concept essence of surface area, using and analyzing category of "antithesis" in a specification of the Schwartz's cylinder or Schwartz's "boot". Thus, nonlinearity and visual modeling of many-sided designs of surface "area" of regular or irregular Schwartz's cylinder approximation, manifestation of synergetic attributes - points of bifurcation, P. Verhulst's scenarios, and pools of an attraction and fluctuations of the operating parameter is significantly analyzed. Means of mathematical and computer modeling revealed mechanisms of self-organization of the areas of many-sided complexes when crushing triangulations of a lateral surface of Schwartz's cylinder with use of dynamics of logistic function growth. Growth of educational motivation, communication and creative activity of students, qualities of mathematical knowledge and procedures development in the conditions of realization of the saturated information and education environment are noted.

IV. Discussions

Identification of complete models and regularities of expansion of many-sided complexes when crushing triangulations of a lateral surface of the Schwartz's cylinder represents the unresolved and difficult informational - mathematical task. Such statement of a problem can have educational value for practice of mathematical education at school and higher education in view of complexity and abstractness of concept of surface area. At the organization of research activity of students in the form of resource lessons or occupations for small groups or design activity with use of information technologies is possible to setting the following tasks specifying and concretizing statement of the main problem.

V. Problems

1. To consider the modified Schwartz's cylinder, namely to turn all ordinal sides on a corner $\frac{\pi}{n} + \beta$ concerning a cylinder axis, where $\beta = k \cdot \frac{\pi}{n}$ and $0 \le k \le 1$.
   1.1. To remove a formula for finding of its area.
   1.2. To formulate and solve the problems similar to article tasks.
2. To consider the irregular Schwartz's cylinder, namely ***k*** – a random number from zero to one where the coefficient **of *k*** is undertaken from the previous task. Thus height of each layer is too random number.
   2.1. By means of numerical methods to show that the area of this cylinder grows more slowly, than the area of the ordinary Schwartz's cylinder, in a case, when $m = \lambda n^2$ and $\lambda \to \infty$. To construct graphics and tables of area's growth.
   2.2. To solve task 2.1 analytically with a conclusion of formulas.
3. It is simple to see that for a situation of corners at top and between sides triangles such formulas take place:

$$\frac{r}{n} = \sqrt{\sin^2 \frac{\pi}{n} + \frac{1}{m^2} + (1 - \cos \frac{\pi}{n})^2} \quad (7)$$

and

$$m = \frac{\sqrt{c^2 - 1}}{1 - \cos \frac{\pi}{n}} . \quad (8)$$

As $m \in \mathrm{N}$, so equalities (5) and (6) can't be carried out for any $n \in \mathrm{N}$. Is there a subsequence of natural numbers for each member of which the equalities are carried out. To construct the table of values.

VI. Conclusion

Research of complex problems by integration tools of computer and natural science receptions and methods based on visual modeling in mathematical education of students is an effective tool and mechanism of development of thinking and personal qualities, which is trained up to manifestation of self-organization and self-development effects. Thus its shown that the synergy of mathematical education reflects nonlinearity and unpredictability of real world and activity of society that undoubtedly promotes better adaptation of the personality to world around. Integration of mathematical, information, humanitarian and natural science knowledge and methods, which accompany the decision and research of complex problems leads to dialogue of cultures and increases students' interest to mathematics and their development.

REFERENCES

[1] A. N. Kolmogorov and S. V. Fomin, *Elements of the theory of functions and functional analysis*, Moscow: Publishing house "Nauka", 1976.

[2] H. A. Schwarz, "Sur une définition erronée de l'aire d'une surface courbe", *Gesammelte Mathematische Abhandlungen*, no. 1, pp. 309–311, 1890.

[3] E. I. Smirnov, *Technology of evident and model training in mathematics*, Yaroslavl: Publishing house of YSPU, 1997 (in Russian).

[4] G. M. Fikhtengolts, *Course of differential and integral calculus. The textbook for higher education institutions*, Moscow: Publishing house "Fizmatlit", vol.1, 2001 (in Russian).

[5] V. N. Dubrovsky, "In search of determination of surface area", *Quantum*, no. 5, pp. 31–34, 1978 (in Russian).

[6] B. B. Mandelbrot, *Fractal geometry of the nature*, Moscow: Institute of computer researches, 2002 (in Russian).

[7] E. I. Smirnov, V. V. Bogun and A. D. Uvarov, *Sinergy of mathematical education of the teacher: introduction to the analysis*, Yaroslavl: Publishing house "Cancler", 2016 (in Russian).

[8] R. M. Kronover, *Fractals and chaos in dynamic systems. Theory bases*, Moscow: Publishing house "Post-market", 2000 (in Russian).

[9] E. I. Smirnov, *Founding of experience in vocational training and innovative activity of the teacher*, Yaroslavl: Publishing house "Cancler", 2012 (in Russian).

[10] V. S. Sekovanov, *Elements of the theory of discrete dynamic systems. Manual*, St.-Petersburg, Publishing house "Lan", 2016 (in Russian).

[11] E. I. Smirnov, N. E. Smirnov and A. D. Uvarov, "The stages of the technological support of the process of self-organization in the mathematical education of a future teacher", *Yaroslavl pedagogical Bulletin*, no. 3, pp. 102–111, 2017 (in Russian).

[12] E. I. Smirnov and S. F. Burukhin, "The complexity of tasks and the synergy of mathematics education" , *Tasks in the teaching of mathematics, physics and Informatics: theory, experience and innovation: materials of the international scientific–practical conference dedicated to the 125th anniversary of P. A. Larichev*, Vologda, Vologda State University, pp. 11–17, 2017 (in Russian).

[13] E. I. Smirnov, "The synergy of mathematical modeling in solving complex problems", *Harmonious personal development problem in relation to specificity of modern education and socialization processes. Peer-reviewed materials digest (collective monograph) published following the results of the CXXXI International Research and Practice Conference and III stage of the Championship in Psychology and Educational sciences*, London: IASHE, 2016.

[14] S. N. Dvoryadkina, E. I. Smirnov, "Evaluation of the synergistic effects of knowledge integration and activities on the basis of computer modeling", *Modern information technologies and IT-education*, Moscow, Moscow State University, pp. 35–42, 2016 (in Russian).

[15] V. N. Ostashkov, E. I. Smirnov and E. A. Belonogova, Education sinergy in research of attractors and basins of nonlinear mapping attraction, *Yaroslavl pedagogical Bulletin*, no. 6, pp. 146–155, 2016 (in Russian).

[16] S. N. Dvoryatkina, R. A. Melnikov and E. I. Smirnov, "Technology of synergy manifestation in the research of solution's stability of differential equations system", *European Journal of Contemporary Education*, vol. 6, no. 4, 2017 (accepted to publication).

[17] S. N. Dvoryatkina, E. I. Smirnov and A. D. Uvarov, "Technology of manifestation of nonlinear dynamics of Irregular Schwartz's Cylinder", *Modern information technologies and IT-education*, no. 12, pp. 12–13, 2017 (in Russian).

[18] S. N. Dvoryatkina, E. I. Smirnov and A. M. Lopukhin, "New Opportunities of Computer Assessment of Knowledge Based on Fractal Modeling", *US-China Foreign Language*, vol. 15, no. 7. pp. 452–459, 2017.

[19] S. N. Dvoryatkina and A. M. Lopukhin, "Practically-oriented mathematical objectives as a means of forming the cultural and professional competence of future lawyers", CONTINUUM. Mathematics. Informatics. Education, no. 3 (7), pp. 77–85, 2017 (in Russian).

[20] S. N. Dvoryatkina, "Active methods of training: positive and negative synergetic effects", CONTINUUM. Mathematics. Informatics. Education, no. 4 (8), pp. 67–80, 2017 (in Russian).

Eugeny I. Smirnov

PhD in Mathematics, Dr. Sc. in Pedagogics (Habilitation), Professor, Head of the Department of Mathematical Analysis and the Theory and Methods of Teaching Mathematics, Yaroslavl State Pedagogical University, Russian Federation, 150000, Yaroslavl, Respublikanskaya str., 108/1.
E-mail: e.smirnov@yspu.org

Artem D. Uvarov

PhD in Mathematics, Associate Professor, Yaroslavl State Pedagogical University, Russian Federation, 150000, Yaroslavl, Respublikanskaya str., 108/1.
E-mail: artiom_uvarov@inbox.ru

Sergey A. Tikhomirov

PhD in Mathematics, Associate Professor, Yaroslavl State Pedagogical University, Russian Federation, 150000, Yaroslavl, Respublikanskaya str., 108/1.
E-mail: satikhomirov@mail.ru