# Analysis on Influence of Bottom Sediment Types on Side-scan Sonar Imaging Properties

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Abstract— Side-scan sonar is widely used in the field of most kinds of detection, due to the imaging prooerties side-scan sonar generates corresponding noise. The different bottom sediment types causing different effects on seabed reverberation effects. After considering the seabed reverberation statistical model, four typical probability distribution models are compared by experiments to get the conclusion that Gamma distribution model is superior from the fitting effect and fastest calculation speed. On the basis of this, model can adjusts parameters of Gamma distribution with different types of bottom sediment to simulate the noise. Using the image gray scale mean and the gray level entropy, the image distribution parameters of the image are analyzed by multiple regression analysis, and the corresponding model is obtained. According to the model, it is possible to simulate noise caused by the reverberation of different bottom sediment types. This result provides an important basis for the separation of the subsurface image and the subsequent de-noising.

*Keywords*—side-scan sonar, multivariate regression analysis, probability distribution

# I. INTRODUCTION

W ith the continuous development of sonar technology and the growing demand for the marine resources, side-scan sonar has become widely used to detect the seabed. In the process of detecting and imaging with side-scan sonar on the side of the seabed, in addition to the targets that may be found, the reverberation of the seabed, the marine environment noise, and some self-noise interference [1]. The effect of reverberation is particularly significant, and it appears as a randomly distributed spot on the side-scan sonar image, which called speckle noise [2]. Due to the different background of the seabed with irregular ups and downs and different roughness, it is important to study and establish the submarine reverberation statistical model during the analysis and processing of the side-scan sonar images. Since the end of the last century, Middleton D proposes the reverberation statistics model [3], more researches have been done at home and abroad. Gensane

Zhang Kaihan is with the Key Laboratory of Underwater Acoustic Communication and Marine Information Technology Ministry of Education, Xiamen University, Xiamen, Fujian, China. (e-mail: zhangkaihan@stu.xmu.edu.cn) proposes a lognormal distribution model after analyzing the data of the echo [4]. Jakeman gives the physical interpretation of the K distribution and comes to the most efficient conclusion of the K distribution [5]. Cobb and Slatton proposes a gamma distribution model [6]. Tian and Tong of the National Naval Engineering University compares the distribution of Rayleigh and Weibull after the distribution of the Weibull distribution to a better degree [7]. However, their analyses are based on one or two distributions, and there is no relevant simulation analysis based on the results. In this study, four different probabilistic distribution models are used to compare the different effects of different bottom sediment types on seabed reverberation sonar images. On this basis, the optimal distributions are chosen to construct the models related to the eigenvalues of sonar images according to their parameters.

Considering the seabed reverberation statistical model, it provide a Rayleigh distribution. But since the imaging mechanism not only includes reverberation, there are other factors such as optoelectronics and other factors. Through a large number of experiments on different types of seabed reverberant sonar image test and comparison of four typical probability distribution model. In conclusion, different bottom sediment types causes different effects on seabed reverberation effects. Finally, the parameters of the distributed model are correlated with the characteristics of the reverberant sonar image, and the distribution parameters can be adjusted under different bottom sediment types to better simulate the noise caused by reverberation. The result of research can provide theoretical bases for the subsequent processing of side-scan sonar, and laid a certain foundation with innovation for the study the model-based side-scan sonar image denoising.

The paper is organized as follows. Section 2 and 3 describes satisfical model analysis. Fitting experiments results are presented in Section 3. Section 4 makes multivariate regression analysis of distibuted parameter and image feature values. The conclusion is given in Section 5.

#### II. STATISTICAL MODEL ANALYSIS OF SIDE-SCAN SONAR

# A. Sonar equation

The side-scan sonar is an active sonar that uses submarine backscatter to achieve acquisition of submarine information. The sonar equation describes the complete process of sound waves from transmission to reception. The form of active sonar is represented by the following equation :

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$$EL = SL - 2TL + BS - NL + DI \tag{1}$$

where SL is source level, DI is cirectivity index of transducer, TL is transmission loss. NL is noise level consists of ambient noise and reverberation, BS is backscatter strength, EN is echo-to-noise level of receiving transducer, EL is excess level.

From the sonar equation, it can be seen that the performance of any sonar system will be affected by the background noise. Background interference can be divided into the following: environmental noise, reverberation and self-noise, where the reverberation has the greatest effect.

After the sound source emits an acoustic pulse, it propagates in different directions and may encounter chaotic scatterers and undulating interfaces [8] which results in a scattering wave different from the original propagation direction. All this affects the collection of effective information and generate corresponding noise.

# B. Statistical model of reverberation

Reverberation as a background noise with a large amount of messy scatterer echoes is derived from the same excitation source. So these superimposed echoes have their own unique statistical laws. According to Middleton's proposed reverberation statistics model [9], reverberation in the ocean is divided into three categories: volume reverberation, surface reverberation and sea bottom reverberation.

First, the nonuniformity of the acoustic scattering caused by the ocean is abstracted into scatter bodies that are randomly "floating" with scatterers in the sea and seawater or "embedded" on the seafloor. The main consideration is sea bottom reverberation as for side-scan sonar.

For bottom reverberation, because scattering block will not move, so for the rules of sound pressure signal:

$$p_{b}(t) = r_{b}(t)e^{f[\omega_{0}t + \phi_{0}(t)]}$$
(2)

where *r* is amplitude,  $\psi$  is phase. Its two orthogonal classification is the formula (2) the real and imaginary parts of the envelope:

$$\begin{aligned} x_{b(t)} &= r_b(t) \cos \psi_b(t) \\ y_{b(t)} &= r_b(t) \sin \psi_b(t) \end{aligned} \tag{3}$$

According to the central limit theorem,  $x_b(t)$  and  $y_b(t)$  concentration are correspondent to Gaussian distribution when the number of scatterers is large enough. As a result, the amplitude of the reverberation  $r_b(t) = \sqrt{x_b^2(t) + y_b^2(t)}$  follows the Rayleigh distribution.

# III. PROBABILITY DISTRIBUTION MODEL AND ITS PARAMETER ESTIMATION

The reverberation statistical model proves that the amplitude of the reverberation follows the Rayleigh distribution, but many measured data show that the distribution of the seabed has a longer "trailing". In order to fit the different seabed subsets, four typical probability distribution models are compared.

# A. Weibull distribution model

Weibull Probability Density Function (PDF) is a distribution

with two parameters. Sonar image usually adopt the three-parameter Weibull distribution [10]. The PDF of Weibull distribution is shown as follow:

$$f_{Weibull}(x) = \frac{\beta}{\alpha} \left(\frac{x - min}{\alpha}\right)^{\beta - 1} \exp\left[-\frac{(x - min)^{\beta}}{\alpha^{\beta}}\right] \quad (4)$$

where  $\alpha$  represents scale parameter,  $\beta$  is shape parameter, *min* is side-play mount, and x>min,  $\alpha>0$ ,  $\beta>0$ . In practical application, the method of maximum likelihood estimation is used to solve the parameter. Let the parameters of the partial derivative is zero, then easily get the value of  $\hat{\alpha}_{ML}$ .

$$\hat{\alpha}_{ML} = \left(\frac{1}{N} \sum_{i=1}^{N} \hat{x}_{i}^{\hat{\beta}_{ML}}\right)^{\frac{1}{\hat{\beta}_{ML}}}$$
(5)

According to equation (5) can be obtained:

$$\frac{\sum_{i=1}^{N} \left(\hat{x}_{i}^{\hat{\beta}_{ML}} \cdot \ln x_{i}\right)}{\sum_{i=1}^{N} \hat{x}_{i}^{\hat{\beta}_{ML}}} - \frac{1}{N} \sum_{i=1}^{N} \ln \hat{x}_{i} = \frac{1}{\hat{\beta}_{ML}}$$
(6)

Equation (6) needs to solve-iterate. For a start, give an initial value to  $\hat{\beta}$  and bring it into Equation (6) on the left, and get the corresponding new value on the right. Then, put the new value into Equation (6) on the left to repeat iterative until convergence. In this pepar, we choose the initial value  $\hat{\beta} = 1$ , getting a convergence value when iterating about 200 times. We can get an approximate value quickly proved by experiments.

# B. Rayleigh distribution model

Practically Rayleigh distribution is a special case of Weibull distribution when  $\beta = 2$ . The PDF of Rayleigh distribution is shown as follow:

$$f_{Rayleigh}(x) = \frac{2(x - min)}{\alpha^2} \exp\left[-\frac{(x - min)^2}{\alpha^2}\right] \quad (7)$$

Maximum likelihood estimation is adopted to estimate the parameters as those described in [11].

$$\hat{mn_{ML} \approx \hat{x}_{\min} - 1}$$
 (8)

$$\hat{\alpha}_{ML}^{2} = \frac{1}{2N} \sum_{i=1}^{N} \left( \hat{x} - m \hat{i} n_{ML} \right)^{2}$$
(9)

### C.Lognormal distribution model

A positive random variable x is log-normally distributed if exists a random variable x and its function ln *x* obeys the normal distribution  $N(\mu, \sigma^2)$ , where  $\sigma^2 > 0$ , and  $\mu$  and  $\sigma^2$  are the mean and the variance. The PDF of Lognormal distribution is shown as follow:

$$f_{Lognormal}(x) = \frac{1}{x\sigma\sqrt{2\pi}} \exp\left[-\frac{\left(\ln x - \mu\right)^2}{2\sigma^2}\right], x > 0 \quad (10)$$

According to Zhang's method [12], the log-likelihood function can be written and the maximum likelihood estimate of parameters are shown as:

$$\hat{\mu} = \frac{1}{N} \sum_{i=1}^{N} (\ln x_i)$$
 (11)

$$\hat{\sigma}^2 = \frac{1}{N} \sum_{i=1}^{N} \left( \ln x_i - \frac{1}{N} \sum_{i=1}^{N} \ln x_i \right)^2$$
(12)

As in the actual fitting, the value of data can be equal to zero. Equation (10) shows x must be greater than zero, so here introduces a parameter on the original function. The added function is shown below:

$$f_{Lognormal}(x) = \frac{1}{(x+\varepsilon)\sigma\sqrt{2\pi}} e^{-\frac{(\ln(x+\varepsilon)-\mu)^2}{2\sigma^2}} x \ge 0 \quad (13)$$

where  $\varepsilon$  is a preset low constant, in this paper  $\varepsilon = 10^{-3}$ .

# D.Gamma distribution model

Gamma distribution is model with a wide range of matches and simple estimation of parameters. The PDF of Gamma distribution is shown as follow:

$$f_{Gamma}(x) = \frac{x^{\alpha - 1}}{\beta^{\alpha} \Gamma(\alpha)} \exp\left(-\frac{x}{\beta}\right), x \ge 0 \qquad (14)$$

where  $\alpha$  and  $\beta$  are shape parameter and scale parameter,  $\Gamma$  (·) represents Gamma function. When  $\alpha$  is increasing, Gamma distribution approaches the normal distribution. To estimate the parameters of Gamma distribution, we use method of moments [13].  $m_1$  is first moment and  $m_2$  is second moment, we can get moment estimation of two parameters :

$$\hat{\alpha} = \frac{m_1^2}{m_2 - m_1^2}, \quad \hat{\beta} = \frac{m_1}{m_2 - m_1^2}$$
 (15)

# IV. SIDE-SCAN SONAR IMAGE BACKGROUND STATISTICAL DISTRIBUTION AND FITTING

#### A. Classification of side scan sonar image background

Sonar images show the bottom of scattering energy. Due to seafloor reverberation amplitudes with Rayleigh distribution, the sonar image will be similar to the Rayleigh distribution. The statistical distribution of the sonar image background is carried out in [14] and [15], and the probability density function of the sonar image is proposed depending on the roughness of the seafloor [16]. With the increase of roughness, the reverberation is stronger and the probability density distribution is wider.

In [17], the main sediments of the seabed are divided into two types: sand and soil. As shown in the red box in Figure 1:

# A. Statistical fitting of side-scan sonar image background

The statistical distributions of the two kinds of background images are fitted using the four typical probability distributions above, as shown in Figure 2.



Fig.1 Different types of side-scan sonar image background (a) soil (b) sand



Fig.2 Distribution fitting comparison of each probability distribution model (a) soil (b) sand (c) distribution fitting contrast to the soil (d) Distribution fitting contrast to the sand

In order to increase the accuracy and reliability of the experiment, this experiment selects 100 of each bottom sediment types of sonar images as material library with  $128 \times 128$  grayscale. The Kolmogorov distance criterion and the chi-square criterion are used to quantitatively evaluate the fitting results. In Table 1, the mean values of the  $\chi^2$  criterion and the Kolmogorov distance for the two types of basal sonar images are calculated. It can be seen from the results, the Rayleigh distribution curve and the image obtained by the large difference between the gray histogram, it can be considered Rayleigh distribution is not suitable for approximate sonar image background gray probability distribution, while the other three The probability distribution model can fit the different types of sonar image background.

	Туре	Soil	Sand	
	Rayleigh	0.527	0.446	
chi-square	Lognormal	0.202	0.163	
criterion : dχ2	Weibull	0.222	0.145	
	Gamma	0.166	0.113	
	Rayleigh	0.237	0.184	
Kolmogorov	Lognormal	0.056	0.032	
distance : dk	Weibull	0.057	0.023	
	Gamma	0.033	0.017	

Table.1 Comparison of fitting results of different distributions

According to Table.1, it can be obciously seen that Gamma distribution achieves optima overall and has a slight edge from others in a between seebed with less seabed reverberation. Weibull distribution is a little bit worse, but it is very close to Gamma distribution. The lognormal distribution is slightly worse than the other distributions, because the fitting error is slightly larger when the 0 value of the image is larger. From the standpoint of computing time, the parameters estimation of Weibull distribution needs it with with maximum calculation time. In summary, Gamma ditribution has the best fitting effect.

# V.MULTIVARIATE REGRESSION ANALYSIS OF DISTRIBUTED PARAMETER AND IMAGE FEATURE VALUES

It can be seen from Figure. 3 that there are some differences in the Gamma distribution of different bottom sediment types which is associated with multiple factors. In order to further study the relationship between the original characteristic coefficients and the Gamma distribution parameters of the sonar background image, this paper chooses two image characteristic parameters to estimate the distribution parameters. So that we can be based on the need for different images of the background to build a different model in practical applications.

# A. Establishment of parameter model

# 1) Parameter selection

As can be seen from the previous section, the Gamma distribution has a good applicability to fit the image background. There are two parameters  $\alpha$  and  $\beta$  respectively for the shape parameters and scale parameters. From the statistical indicators, the mean and variance are shown as follows:

$$EX = \alpha\beta, Var(x) = \alpha\beta^2$$
 (16)

Histogram of the shape of the image shows the total distribution of the image area as a result of gray histogram. The shape parameter  $\alpha$  and the scale parameter  $\beta$  of the gamma distribution are obtained by the moment estimation of the gray value of the image. Therefore, the selection of image feature parameters is more consideration is related to the image gray scale parameters.

We choose image gray scale mean and gray level entropy to make multivariate regression analysis of the distribution parameters after comparative test. The grayscale entropy of the sonar image is the average number of bits of the image gray set, which can represent the degree of discretization of the grayscale distribution of the image pixel [18], and also describe the average information of the image source. The gray scale of the image describes the total information of the image gray scale distribution.

Suppose that the nonnegative matrix  $A = (a_{ij})$  is a grayscale image,  $a_{ij}$  is the pixel of the image, and  $a \in [0, 255]$ , image size is  $M \times N$ . Gray value of image is denoted by mean(A), then average gray level image is :

mean 
$$(A) = \frac{\sum_{i=1}^{M} \sum_{j=1}^{N} a_{ij}}{MN}, i \in (1, M), j \in (i, N)$$
 (17)

As shown in Figure. 3, the probability distributions histogram of gray-scale entropy of sonar images in material reservoirs are shown. It can be seen from the figure that the entropy of the sonar image distribution roughly distributs between 4 and 8.



Fig.1 Histogram of probability distribution of grayscale entropy. (a) grayscale entropy in soil (b) grayscale entropy in sand

#### 2) Model's establishment

We use shape parameter to  $\alpha$  make polynomial fitting and get an expression between a similar dependent variable  $\alpha$  and two independent image entropy *e* and image mean value *m* by polynomial expansion.

$$\alpha = b_1 + b_2 \cdot e^2 + b_3 \cdot e + b_4 \cdot m^2 + b_5 \cdot m + b_6 \cdot e \cdot m \quad (19)$$

For the scale parameter  $\beta$ , the  $b_4$  value is less than  $10^{-5}$  times can be ignored, so the expression of  $\beta$  is:

$$\beta = b_1 + b_2 \cdot e^2 + b_3 \cdot e + b_5 \cdot m + b_6 \cdot e \cdot m$$
 (20)

The number of each type of sonar images is 100, 80 of 100 pieces are randomly selected for fitting the remaining 20 pieces to verify and circulate 1 000 times to obtain the expression. The specific coefficients are shown in Table 2. The fitting curve is

shown in Figure. 4, where the X axis is gray scale entropy, the Y axis is gray scale average, and the Z axis is shape parameter or scale parameter.

			b1	b2	b3	b4	b5	b6
	G . 1	α	208.9	8.144	-84.61	0.002	3.290	-0.518
	5011	β	5.112	0.136	-1.674	0	0.031	-0.004
	G 1	α	455.2	13.08	-157.8	0.0004	3.037	-0.414
Sand	β	7.724	0.174	-2.331	0	0.019	-0.003	

Table.2 Polynomial fitting coefficients



Fig.4 Parameter fitting surface of soil and sand. (a)  $\alpha_{soil}(e,m)$  fitting surface (b)  $\beta_{soil}(e,m)$  fitting surface (c)  $\alpha_{sand}(e,m)$  fitting surface (d)  $\beta_{sand}(e,m)$  fitting surface

#### B. Model Validation

According to [19], two indicators of judgment are introduced, called Spearman rank correlation correlation coefficient (SROCC) and Pearson linear correlation coefficient (PLCC). We use two indicators to compare calculated values of two Gamma parameters with the estimated values of the feature model.

PLCC is a linear correlation coefficient, reflecting two variables linear correlation degree of statistics, the formula is:

PLCC = 
$$\frac{\sum_{i} (q_{i} - \overline{q}) \cdot (o_{i} - \overline{o})}{\sqrt{\sum_{i} (q_{i} - \overline{q})^{2} \cdot \sum_{i} (o_{i} - \overline{o})^{2}}}$$
(21)

where  $q_i$  is estimated value of the model,  $o_i$  is the calculated value of the corresponding parameter, and the closer the value is to 1, the higher the accuracy of the model

SROCC is a coefficient used to estimate the correlation between two variables, where the correlation between variables can be described as a monotonic function. The formula is:

SROCC = 
$$1 - \frac{6\sum_{i=1}^{N} v_i^2}{N(N^2 - 1)}$$
 (22)

where  $v_i$  is the difference between the estimated value and the calculated value, which is the same as the PLCC, and the higher the monotonicity of the model when the value is closer to 1.

Table.3 is the material library 100 images of the correlation coefficient calculation results of the average, the establishment of the model is established. And through the establishment of the model, we can better describe the side of the characteristics of the background image.

Table.3 Comparison of parameters fitting results of different	2
background parameters	

Туре	Parameter	PLCC	SROCC
Coll	α	0.980	0.961
5011	β	0.983	0.969
Sand	α	0.996	0.987
Sanu	β	0.996	0.987

In order to verify the effect of different types of side-scan sonar, we select two sonar images of different backgrounds outside the model library, and the background part is shown in Figure.5. The calculated values of  $\alpha$ gam and  $\beta$ gam are obtained by parameter estimation method of gamma distribution, and compared with the estimated values obtained by the model, as shown in Figure. 6, Table 4 and 5.



(b)

Fig. 5 Original images and selected validation parts. (a) soil background of sonar image and left view red box enlarge image (b) sand background of sonar image and left view red box enlarge image



Fig. 6 Probability distribution contrast. (a) probability distribution of soil background contrast (b) probability distribution of sand background contrast

Table.4 Comparison of calculated value and estimate of $\alpha$
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	$\alpha_{\rm gam}$	$\alpha_{\rm sand}$	$\alpha_{\rm soil}$	Relative error
Soil	20.308	27.285	20.924	0.0303
Sand	10.400	10.547	5.7661	0.0013

Table.	5 Com	parison	of	calcula	ated	value	and	estimate	of	ß
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	$\beta_{\rm gam}$	$\beta_{\rm sand}$	$\beta_{\rm soil}$	<b>Relative error</b>
Soil	0.4155	0.5332	0.4107	0.0113
Sand	0.0912	0.0894	0.0390	0.0068

We select 20 images out of material library and calculate values for the two gamma parameters are compared with the estimated values of the feature model, as shown in Figure 7. The image characteristics of the sonar images of different types of backgrounds are calculated by the model. The calculated error of the corresponding Gamma distribution parameters is not significant. Therefore, parameters of the gamma distribution can be adjusted by this model, and model responds to noise caused by reverberation under the conditions of substrate.



Fig.7 Comparison of calculated value and estimated values. (a)  $\alpha_{gam}$  and  $\alpha_{soil}$  (b)  $\beta_{gam}$  and  $\beta_{soil}$  (c)  $\alpha_{gam}$  and  $\alpha_{sand}$  (d)  $\beta_{gam}$  and  $\beta_{sand}$ 

# VI. CONCLUSION

In this pepar, we select two typical bottom sediment types to study of side-scan sonar image the seabed background and get the probability distribution of noise model. The experimental results show that the Gamma distribution has a good fitting property under the condition of two bottom sediment types. After obtaining this conclusion, the relationship between Gamma distribution parameters and image features of the two types is obtained by multiple regression analysis. After verification, it is proved that bottom sediment of the sonar image can be discriminated by the relational model. It is a really the way to adjust the distribution parameters under different bottom sediment types and better simulate noise caused by reverberation. The resulting model can also provide a basis for the subsequent de-noising of the sonar image.

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