3D Modelling of the Earth Crustal and Upper Mantle by Geometrical Features of Bi-Characteristic Curves

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Abstract**—** The aim of this paper is the introduction of a new approach to the 3D modelling of elastic piecewise homogeneous media, for instance Earth crust and upper mantle. The method is based on the principle of tomography with a point-force (singlet) as a source of the signal and a set of observations (records) at the surface. For the sake of simplicity of the exposition in this article we consider solid media only.

The wave propagation in solid media is described by a system of three strongly coupled hyperbolic equations with piecewise constant coefficients. The characteristic set and bi-characteristic curves of this system are computed in a piecewise homogeneous half-space with free boundary and the formulae of reflection and diffraction of the bi-characteristics on the internal boundaries of the media. Applications of the characteristic set and bi-characteristic curves for the inverse problem in geophysics and Earth modelling are given.

Keywords**—3D modelling of Lithosphere, bi-characteristics, strongly coupled linear systems of PDE**.

I. Introduction

In this paper is presented a new, geometrical method to generate 3D mathematical models of elastic piecewise homogeneous media. For the sake of simplicity the media is supposed entirely solid one. In the case it has some cavities the method is applicable as well with reasonable improvement. If the cavity contains liquid, which is the case of oil and gas deposits, then the equations for wave propagation in liquids is applied for this area. If the cavity is empty, it defines a part of the boundary of the domain.

The method is based on the tomography concept. Classical tomography method is applicable to finite objects and consists of two major steps – collection of information and its processing. In the first step – collection of information - a source emits energy in a particular form, which we call "signal", the energy distributes in time within the media, and sensors on the surface of the media record the signal. In the second step we imply various techniques in order to "restore" the structure of the media using the recorded signal from the sensors as input data. One way is to "construct" a virtual model of the media and then to "verify" the model against the real data. This step can be implemented (a) by solving the inverse problem with input (boundary) data – the recorded signal – and comparing the solution with the emitted energy; (b) another way is to solve the forward problem with initial data – emitted energy - and to compare the solution on the boundary with recorded signal.

Therefore in setting our tomography model we have to choose a proper source of energy, the way we model the media, etc.

As for the source of energy, in modelling the Earth structure are employed some sources of indirect information like gravity or magnetic filed deformation and seismic waves. The last ones are perhaps the most unpredictable source of information due to irregularity of earthquakes, both in space and time. Nevertheless the nature of seismic waves and the density of the seismic stations turn these waves into one of the most popular geophysical tools to study the Earth interior. The seismic features of the uppermost ground are a matter of deep interest to civil engineers as well.

The distribution of the seismic waves is described in many books and papers, for instance in [1] and [5]. For geophysical purposes it is sufficient if the Earth is considered as an elastic body that is a continuum. In other words, the matter is continuously distributed in space. Furthermore, for local to regional studies, the planet can be approximated with no loss of generality by the half-space Ω. If the elastic parameters depend only on the vertical coordinate z then the wave propagating in solid media satisfy system (1), where (ux, uy, uz) is the displacement function. The mathematical model of wave propagation in piecewise continuous body follows:

Let $\Omega = \{(x,y,z)\in R3 : z\geq 0\}$ be a half-space with free surface boundary $\partial\Omega = \{z=0\}$ and the z axis be positive downward. Wave propagation in solid media, in particular in solid half-space (see [1] and [5]) is described by the following system (1) of strongly coupled linear hyperbolic equations with piecewise continuous coefficients

$$\rho\frac{\partial^2 u_x}{\partial t^2} = X + (\lambda+2\mu)\frac{\partial^2 u_x}{\partial x^2} + \mu\frac{\partial^2 u_x}{\partial y^2} + \mu\frac{\partial^2 u_x}{\partial z^2} +$$

$$+(\lambda+\mu)\frac{\partial^2 u_y}{\partial x\partial y} + (\lambda+\mu)\frac{\partial^2 u_z}{\partial x\partial z} + \frac{\partial\mu}{\partial z}\frac{\partial u_x}{\partial z} + \frac{\partial\mu}{\partial z}\frac{\partial u_z}{\partial x}$$

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$$\rho\frac{\partial^2 u_y}{\partial t^2} = Y + \mu\frac{\partial^2 u_y}{\partial x^2} + (\lambda+2\mu)\frac{\partial^2 u_y}{\partial y^2} + \mu\frac{\partial^2 u_y}{\partial z^2} +$$

$$+(\lambda+\mu)\frac{\partial^2 u_x}{\partial x\partial y} + (\lambda+\mu)\frac{\partial^2 u_z}{\partial y\partial z} + \frac{\partial\mu}{\partial z}\frac{\partial u_y}{\partial z} + \frac{\partial\mu}{\partial z}\frac{\partial u_z}{\partial y}$$

$$\rho\frac{\partial^2 u_z}{\partial t^2} = Z + \mu\frac{\partial^2 u_z}{\partial x^2} + \mu\frac{\partial^2 u_z}{\partial y^2} + (\lambda+2\mu)\frac{\partial^2 u_z}{\partial z^2} +$$

$$+(\lambda+\mu)\frac{\partial^2 u_x}{\partial x\partial z} + (\lambda+\mu)\frac{\partial^2 u_y}{\partial y\partial z} + \frac{\partial\lambda}{\partial z}\left(\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} + \frac{\partial u_z}{\partial z}\right) + 2\frac{\partial\mu}{\partial z}\frac{\partial u_z}{\partial z}$$

with boundary conditions at the free surface z=0 are as follows:

$$\sigma_{zz} = (\lambda+2\mu)\frac{\partial u_z}{\partial z} + \lambda\left(\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y}\right) = 0$$

$$\sigma_{zx} = \mu\left(\frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x}\right) = 0 \qquad (2)$$

$$\sigma_{zy} = \mu\left(\frac{\partial u_y}{\partial z} + \frac{\partial u_z}{\partial y}\right) = 0$$

Here $grad(u) = \left(\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} + \frac{\partial u_z}{\partial z}\right)$, coefficients λ, μ and ρ are piecewise continuous functions and ux, uy, uz, σzz, σzx, σzy ∈C(Ω). Furthermore, at the point S∈Ω initial data are defined as an impulse alongside a given vector ξ0 = (ξ10, ξ20, ξ30). As for the source of the signal S, there are different models corresponding to very different sources of seismic waves. In this paper we consider point source that produce an impulse in a certain direction, namely alongside vector ξ0 = (ξ10, ξ20, ξ30). If the source is more complicated it can be represented as a vector field on given curve, that is the fault. In this case the method that is described below for a point source can be easily adapted.

II. Media modelling

Coefficients λ, μ and ρ depend on the geological properties of the rock. Though little information we have about exact ground structure, geological surveys near the surface show that the Earth crust is heterogeneous and consists of piece-wise homogeneous material. Therefore in a realistic model the coefficients λ, μ and ρ can be considered piecewise continuous functions. Unfortunately, in this case the results for the wave front set (Theorem 8.3.1, Hormander, v. I, p.271) are not applicable. On the other hand a reasonable approximation of the real Earth is a 3-dimensional structure of homogeneous blocks in welded contact {Bi,j,k: i, j, k ∈N}. It is not necessary that the blocks Bi,j,k have rectangular faces parallel to the coordinate system. Without loss of generality we assume the boundary of blocks Bi,j,k to be piecewise smooth: $\partial B_{i,j,k} = \cup\{F_{i,j,k,l}(x,y,z) = 0, l = 1,.....N_m\}$ where Fi,j,k,l(x,y,z)=0 is a smooth surface in R3 and Nm is a finite number. It is convenient as well to assume that the point source S of the seismic signal belongs to the block B0,0,0. In this way, in Ω ={Bi,j,k: i, j, k ∈N} the system (1) with constant coefficients in every block Bi,j,k is a realistic approximation to the wave propagation in the real medium (e.g. the Lithosphere).

III. Bi-characteristics of system (1), (2) and 3D modelling of the Lithosphere

There are various ways to solve system (1), (2), for instance numerical methods, computing the fundamental solution of the system, etc. Solving system (1), (2) numerically is limited by some natural constrains such as the size the domain Ω. If Ω is not sufficiently small (as is the general case) the grid is too large and the computational time is too large or the approximation error - too high. The same is the problem when we use another approach - the fundamental solution of (1). The fundamental solution of (1) can be explicitly written in integral form The numerical computation of the integral faces the same problems as pure numerical methods solving (1), (2) directly – high computation time or big error.

In another standard analytical approach widely used in geophysics, if the body forces are neglected, the solutions of (1) are considered as plane harmonic waves propagating along the positive x axis u(x,t) =F(z).ei(ωt-kx), where ω and k are constants - the angular frequency and the wavenumber (see for instance [5]). The main disadvantage of this approach is that the plane wave is a two - dimensional one, living in the plane y=0 only, and all information on the y coordinate is lost Therefore it is impossible to build a reasonable 3D model using plane waves of the type mentioned above.

This is the motivation to propound in this paper a new approach for 3-D modelling of solid body. Since earthquake generates a singularity at point S, the method suggested is built on the propagation of singularities of system (1) itself.

In view of the fact that the system (1) has constant coefficients in every block Bi,j,k we use the so called “train solutions” construction in our model. If we have initial data at the point source S the solution of system (1) in block B0,0,0 determines the boundary conditions in the neighbouring block and so on. In this way, instead of system (1) with piecewise constant coefficients we consider a series of related problems (1) with constant coefficients, which is a much easier task.

This method is based on the features of the bi-characteristic curves of system (1). As the principal part is real with constant coefficients, the wave front set is invariant under the bi-characteristic flow. Having in mind the source model described above - a point source with seismic impulse in some direction - actually the singularities of the solution carry all the information about the wave. On the other hand, the singularities propagate over bi-characteristic curves within every homogeneous block. At the boundary between two blocks bi-characteristics could reflect or refract. According to geometrical optics and micro-local analysis, if bi-characteristic curve reflects off the sides of every block the angle of incidence to the surface is equal to the angle of reflection. The refraction at the surface is computed in the usual way, more details and exact computations are given in the next Chapter. Therefore, if we know the position of the source S, the direction ξ0 of the seismic impulse and the media structure Ω ={Bi,j,k: i, j, k ∈N} we can compute the point s0 where bi-characteristic curve has contact with the surface z=0. The point s0 is in fact the centre of the surface waves in the plane z=0 generated by the section of the wave front and the plane z=0. When actual measurement of the seismic waves is done, the coordinates of the point s0 can be triangulated using the data from several stations. Exact coordinates of the epicentre S of an earthquake can be computed by P and S waves arrival time based on the data from seismic stations. Exact coordinates of the centre of the surface waves r0 is a matter of triangulation of the arrival time of surface waves taking into account the geography of the region. Then the verification of the media model Ω ={Bi,j,k: i, j, k ∈N} could be done. Given a certain 3-D media model Ω ={Bi,j,k: i, j, k ∈N}, we can compute the point s0. If the points s0 and r0 coincide within the error of the computations, then the media model is plausible.

For practical purposes 3-D models Ω ={Bi,j,k: i, j, k ∈N} can be generated using Monte Carlo type methods. Of course, like any other inverse problem, this algorithm has multiple solutions in the sense that many models can cover the requirement s0≡r0. Unfortunately, this is the best result we can hope for, given the complexity of the object to study and the information we have from the seismograms.

IV. Characteristic set and bi-characteristic strip in homogeneous block Bi,j,k

As it is well known, the characteristic set of a linear scalar operator $\mathrm{L}(\mathrm{x},\mathrm{D})=\sum_{|\alpha|\le 2}\mathrm{a}_\alpha(x)D^\alpha$ is given by the zeroes of its principal symbol $\mathrm{p}_\mathrm{L}(\mathrm{x},\xi)=\sum_{|\alpha|=2}\mathrm{a}_\alpha(x)\xi^\alpha$. In the case of linear strongly coupled system the characteristic set contains the zeroes of the determinant of the characteristic matrix of the system (see [6], p.40). Each element of the characteristic matrix is the principal symbol of the corresponding equation with respect to the corresponding argument. For instance, if the system is Li(u1, u2, ...un)=0, i=1,....n, then the element (k,m) of the characteristic matrix is the principal part of Lk with respect to um. Then the characteristic set of system (1) in every block Bi,j,k is given by the equation

$$p(x,\xi)=\beta^2(\beta+\xi_1^2+\xi_2^2+\xi_3^2)$$

Where we denote by

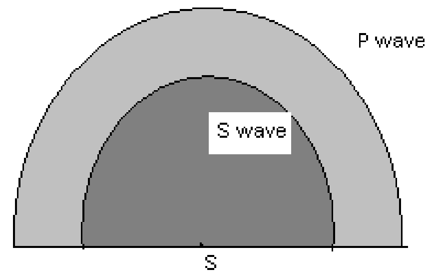
$$\beta=-(\lambda+\mu)^{-1}\left[\rho\tau^2-\mu(\xi_1^2+\xi_2^2+\xi_3^2)\right]$$

Therefore the characteristic set of system (1) decomposes into two subsets given by

$$p_1(x,\xi)=\rho\tau^2-\mu(\xi_1^2+\xi_2^2+\xi_3^2)=0$$
$$p_2(x,\xi)=\rho\tau^2-(\lambda+2\mu)(\xi_1^2+\xi_2^2+\xi_3^2)=0 \quad (3)$$

since (λ+μ)>0. This result corresponds to the theory of P (primary) and S (secondary) body waves in physics. P wave corresponds to the set defined by p2(x,ξ) = 0, and S wave - to the one defined by p1(x,ξ) = 0.

Note: If we consider z axis be positive upward, we derive the same equations (3).



Pic.1 Propagation of wave in solid media. Te wave decomposes to P and S wave.

Then the following theorem holds:

Theorem: Body wave propagating in homogeneous media is the composition of two waves - P wave and S wave. There are no other components of the body wave.

The bi-characteristic strip of the linear strongly coupled system (1) is another important object for our study, since the characteristic set of a operator with real principal part p(x,ξ) and constant coefficients is invariant under the bi-characteristic flow (see [2], vol.I Chapter 8). More important to our study is the fact that the singularities of (1) travel on the bi-characteristic curves. By definition if p(x0, ξ0)=0 then the bi-characteristic strip at point (x0, ξ0) is defined by the Hamilton equations

$$\frac{dx}{dt}=\frac{\partial p(x,\xi)}{\partial\xi},\frac{d\xi}{dt}=\frac{\partial p(x,\xi)}{\partial x}$$

The bi-characteristic strip bi generated by pj(x, ξ), j =1,2, through point (x0, ξ0) is

$$\left|\begin{array}{l} x_i=2\xi_i^0.s+x_i^0 \\ t=2c\,|\,\xi^0\,|.s+t^0 \\ \xi_i=\xi_i^0 \\ \tau=\tau^0=c\,|\,\xi^0\,| \end{array}\right. \quad (4)$$

for i=1,2,3. Constant $c=\sqrt{\mu/\rho}$ for bi-characteristics generated by p1(x, ξ) and $c=\sqrt{(\lambda+2\mu)/\rho}$ for ones generated by p2(x, ξ).

The values of ξ10, ξ20 and ξ30 are determined by the features of the seismic source. Without loss of generality we can assume the source of the seismic wave to be a point one with direction of the impulse (ξ10, ξ20,ξ30).

The restriction of the bi-characteristic strip into R4 is named bi-characteristic curve. For computational purpose it is more convenient to write the bi-characteristic strip of (4) in the form

$$\left| \begin{array}{l} x_1 = c^{-1}.\xi_1^0.(t-t^0)+x_1^0 \\ x_2 = c^{-1}.\xi_2^0.(t-t^0)+x_2^0 \\ x_3 = c^{-1}.\xi_3^0.(t-t^0)+x_3^0 \end{array} \right. \quad (5)$$

since $t-t^0 = 2c\left|\xi^0\right|s$ and without loss of generality we may assume $\left|\xi^0\right| = 1$.

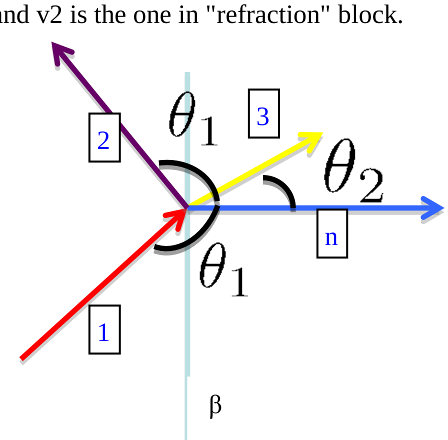
V. Reflection and Refraction

Equation (5) describes the bi-characteristic curves of (1) in each block Bi,j,k and their behaviour on the boundary ∂ Bi,j,k =∪{ Fi,j,k,l(x, y, z)=0} is studied by micro-local analysis and geometrical optics.

Let bin be a bi-characteristic curve in Bi,j,k and bin ∩{Fi,j,k,l(x, y, z)=0} = p0. At point p0 bin can be reflected or refracted. Let brr be the refracted curve and brl be the reflected one. Both brr and brl are bi-characteristics through point p0 – brr is in the next to Bi,j,k block (in the sense of propagation of the singularity generated in S) and brl is in Bi,j,k. The singularity at p0 propagates over the bi-characteristics as well and in this way the well known formula for reflection and refraction from geometrical optics are obtained.

If bi-characteristic curve bin is reflected the angle θin of incidence to the surface Fi,j,k,l(x, y, z)=0 is equal to the angle of reflection θrl, since in the same block the equation (5) has the same coefficients.

As for refraction at a surface, the match of the boundary conditions of the neighbouring blocks at the two sides of the boundary lead to the well known formula from geometric optics v1.sinθrr = v2.sinθin, where θrr is the angle of refraction, v1 is the speed of the wave in the "incidence" block and v2 is the one in "refraction" block.



Pic. 2 Incoming vector 1, the reflected 2 and the refracted one 3. β denotes the surface of reflection/refraction, n is the normal to β surface.

Computation of reflected and refracted bi-characteristic curve is simple. Let ξin = (ξ1in, ξ2in, ξ3in) be the unit vector along the incidental bi-characteristic curve bin in Bi,j,k; ξrr = (ξ1rr, ξ2rr, ξ3rr) be the unit vector along refracted bi-characteristic curve brr (in the neighbouring block), and ξrl = (ξ1rl, ξ2rl, ξ3rl) be the unit vector along reflected bi-characteristic curve brl (in Bi,j,k). Let $\vec{n} = (n_1, n_2, n_3)$ be the normal unit vector to the surface Fi,j,k,l=0 at the point of incidence p0. The speed of the wave is a physical feature of every material. For instance, the velocity of the P-wave in homogeneous isotropic media is $v_P = \sqrt{(\lambda+2\mu)/\rho}$, for S-wave it is $v_P = \sqrt{\mu/\rho}$.

Quantities θrl = θin and sinθrr are easy to compute using scalar, or dot product $\cos(\theta) = \xi \circ \vec{n}$, for instance $\sin^2(\theta_{in}) = 1 - \left(\xi_1^{in} n_1 + \xi_2^{in} n_2 + \xi_3^{in} n_3\right)^2$. Then equations of refraction and reflection from geometrical optics yield

$$\xi_1^{rr}.n_1 + \xi_2^{rr}.n_2 + \xi_3^{rr}.n_3 = \pm\left[1-\left(\frac{v_2}{v_1}\right)^2\left(1-\left(\xi_1^{in}.n_1+\xi_2^{in}.n_2+\xi_3^{in}.n_3\right)^2\right)\right]^{1/2} \quad (6)$$

$$\xi_1^{rl}.n_1 + \xi_2^{rl}.n_2 + \xi_3^{rl}.n_3 = \pm\left[1-\left(1-\left(\xi_1^{in}.n_1+\xi_2^{in}.n_2+\xi_3^{in}.n_3\right)^2\right)\right]^{1/2}$$

The right hand sides in (6) are known quantities.

In addition, the incident bi-characteristic curve, the refracted one and the normal to the surface vector lie on the same plane and give us the relation

$$\begin{vmatrix} n_1 & n_2 & n_3 \\ \xi_1^{in} & \xi_2^{in} & \xi_3^{in} \\ \xi_1^{rr} & \xi_2^{rr} & \xi_3^{rr} \end{vmatrix} = 0 \quad (7)$$

The same relation is valid for vector ξrl.

Finally, since we consider vectors ξin, ξrr and ξrl be unit ones, we obtain

$$(\xi_1^{rr})^2 + (\xi_2^{rr})^2 + (\xi_3^{rr})^2 = 1$$
$$(\xi_1^{rl})^2 + (\xi_2^{rl})^2 + (\xi_3^{rl})^2 = 1 \quad (8)$$

Equations (6), (7) and (8) define uniquely vectors of refraction ξrr and reflection ξrl . The sign in the right-hand side of (6) is "+" or "-" and depends on the orientation of vector □

VI. 3-D Modelling of Earth Crust and Upper Mantle

Earthquake at point S actually is a singularity of the solution u=(ux,uy,uz) of system (1). In our model the wave source is a point one, i.e. the impulse has direction (ξ10, ξ20,ξ30). Then the singularity that is generated by an earthquake in block B0,0,0 propagates over the bi-characteristic curve (5) in B0,0,0 until it intersects at point (x1, y1, z1) the boundary to the neighbouring block, B1,0,0 for instance. Continuous boundary conditions mean that at point (x1, y1, z1) system (1) in the block B1,0,0 has singularity, that propagates over the bi-characteristic curves in B1,0,0, and (6), (7) and (8) give us the reflected (in B0,0,0) and refracted (in B1,0,0) bicharacteristics. Therefore the following criterion for the 3D model of the Earth crust and upper mantle is defined:

Definition: Let {Bi,j,k} be a set of blocks and the source of seismic wave be a point one at S with pulse direction alongside vector ξ0. Let P be the point of the Earth surface belonging to the bi-characteristic curves generated by system (1), set of blocks {Bi,j,k} and source S. Given set of blocks {Bi,j,k} is plausible if the point P coincides with the epicentre E of the body waves at surface z=0, generated by the earthquake.

Since seismic stations record body waves at the surcease as well, point E is a subject of triangulation if there are enough sensors in the region. More details are given in the next chapter.

The computation of the bi-characteristic curves in the set Bi,j,k rises an important question. At the boundaries between two blocks - surfaces Fi,j,k,l(x,y,z)=0 - is the bi-characteristic curve reflected, refracted, or both? The answer comes from the so called reflection and refraction index. It is a physical feature of the constitutive material of the block.

The procedure for the computation of refraction and reflection index is well described in [1], [3] or in [4]

Furthermore, the body waves records are useful to determine the block structure of the closest to the seismic stations blocks. Wave front in a homogeneous block is a subset of the characteristic set of system (1), therefore it has constant speed by (4).

Using bi-characteristic curves and the characteristic set we can compute arrival time for P - and S - waves. In combination with the criteria from the Definition, we can generate and test plausible 3-D models of the Earth crust and upper mantle.

VII. Location of the point P. Body waves at the surface z=0

For the determination of the location of point P on surface z=0 we use the boundary conditions. The boundary conditions (2) of system (1) actually provide the information of what is the behaviour of the body wave on the boundary surface z=0. Therefore computing the characteristic set pb(x,ξ) for system (2) we obtain

$$\begin{vmatrix} \lambda\xi_1 & \lambda\xi_2 & (\lambda+2\mu)\xi_3 \\ \mu\xi_3 & 0 & \mu\xi_1 \\ 0 & \mu\xi_3 & \mu\xi_2 \end{vmatrix} =$$

$$\begin{vmatrix} \lambda\xi_1 & \lambda\xi_2 & \lambda\xi_3 \\ \mu\xi_3 & 0 & \mu\xi_1 \\ 0 & \mu\xi_3 & \mu\xi_2 \end{vmatrix} + 2\mu\xi_3 \begin{vmatrix} \mu\xi_3 & 0 \\ 0 & \mu\xi_3 \end{vmatrix} =$$

$$\lambda\mu^2 \begin{vmatrix} \xi_1 & \xi_2 & \xi_3 \\ \xi_3 & 0 & \xi_1 \\ 0 & \xi_3 & \xi_2 \end{vmatrix} + 2\mu^3\xi_3^{\,3} = 0$$

$$\lambda \begin{vmatrix} \xi_1 & \xi_2 & \xi_3 \\ \xi_3 & 0 & \xi_1 \\ 0 & \xi_3 & \xi_2 \end{vmatrix} + 2\mu\xi_3^{\,3} =$$

$$\xi_1 \begin{vmatrix} 0 & \xi_1 \\ \xi_3 & \xi_2 \end{vmatrix} - \xi_3 \begin{vmatrix} \xi_2 & \xi_3 \\ \xi_3 & \xi_2 \end{vmatrix} + 2\mu\xi_3^{\,3} =$$

$$\xi_3(\lambda\xi_1^2 + \lambda\xi_2^2 - (\lambda+2\mu)\xi_3^2) = 0$$

Since the singularities propagate over bi-characteristic curves, the initial point or the epicentre of the body waves on the surface z=0 is the point E where bi-characteristic curve contact the surface z=0. Therefore we can determine position of point E on the surface using data records from seismic stations and applying triangulation method.

Once having positions of points S and P we run a model generator (MG) that generates sets of blocks {Bi,j,k}. For each set we test the calculated position P and the observed one E. Using Definition 1 we adopt or reject the set {Bi,j,k}.

MG could be build on a various methods. For the algorithm and computer code, developed by the author are used Monte Carlo ones.

A key point for the speed and effectiveness of the MG is the choice of the starting sets of blocks {Bi,j,k}. Often it is suitable to use existing 2D models as a base of the set {Bi,j,k}.

VIII. Conclusion

The geometrical method proposed in this paper gives a quick and affective way to obtain a set of plausible blocks models {Bi,j,k}. Since we solve the inverse problem, it is natural to obtain multi-valued solution, i.e. more than one plausible set. For further refinement of this set could be applied crosscheck tools, or hybrid approach methods. For instance magnetic in-situ measurements could provide useful information for the media structure at a certain points. The flaw of the in-situ approach is the cost of surveys and the fact that we obtain information for the media structure only in one point.

The main disadvantage of the proposed in this paper 3D method is that we obtain reliable media structure in a relatively narrow strip between points S and P. In other words the method is effective in areas with enough sources of earthquakes, or regions with high seismic activity.

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