Adomian Decomposition Method to Generalized Thermoelastic Infinite Medium with Cylindrical Cavity under Three Theorems

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***Abstract*—** In this paper, a mathematical model of thermoelastic an infinite body with cylindrical cavity has been improved. A unified system of governing equations has been formulated in the context of three different models of thermoelasticity; Biot model, Lord-Shulman model, and Green-Lindsay model. Adomian decomposition method has been applied to get the solution of the model. The boundary surface of the cavity is subjected to harmonic thermal loading with zero heat flux and strain. The first components of the iteration have been calculated and used to get the rest components of the iteration formulas by using MAPLE 17 and by applying a certain Algorithm. The numerical results for the temperature, radial stress, strain, and displacement have been represented graphically. The angular thermal load and the relaxation times have significant effects on all the studied fields in the context of the three applied thermoelastic models. The results show that, Lord-Shulman model is much closed to Green-Lindsay model.

***Keywords*—** Adomian Decomposition Method; Generalized Thermoelasticity; Relaxation time; Iteration method; Algorithm

I. NTRODUCTION

Biot derived the coupled thermoelasticity (CTE) in which the heat conduction is parabolic type partial differential equation which generates thermal wave with infinite speed [1]. To fix this paradox, generalized thermoelasticity theory has been derived by Lord and Shulman (L-S) by using the definition of the second sound effects [2]. This definition leads to heat conduction of parabolic type partial differential equation which generates the finite velocity of the thermal wave. Green and Lindsay (G-L) theory suggest two relaxation times and both the energy equation and the equation of motion are modified [3]. Many mathematical models of the infinite body with a cylindrical cavity in context of different types of thermoelasticity models have been solved [4-7].

Recently, much attention has been devoted to the numerical methods in which do not require discretization of time-space variables or to the linearization of the nonlinear equations [8]. Adomian method is a decomposition method which solves linear and nonlinear partial and ordinary differential equations [9-11]. This method offers computable, accurate, convergent solutions to linear and nonlinear partial and ordinary differential equations. The solution can be applied to any degree of approximation. Recently, the Adomian decomposition technique has been used to get the formal solutions to many classes of partial and ordinary differential equations [12-23]. Adomian method solved different mathematical models of the mechanics interaction of immune with viruses, antigens, bacteria or tumor cells which had been modeled as a system of nonlinear partial differential equations by using the ADM [11, 24-26].

Adomian decomposition method (ADM) separates the differential equation into linear and nonlinear parts, invert the highest-order derivative in both sides, and obtain the successive terms of the solution by iteration relation [8, 20]. Many modifications to the method made to enhance the accuracy or to expand the applications of the original method by many authors [17, 19, 23]. Recently, the decomposition method has been used in fractional partial differential equations [27-29].

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II. BASIC EQUATIONS

The unified system of governing equations in the context of CTE, L-S and (G-L) has been constructed for a linear, and homogeneous, isotropic medium without any external heat source to be in the following form [7]:

$$\mu u_{i,jj} + (\lambda+\mu) u_{j,ji} + F_i - \gamma\left(1+\upsilon\frac{\partial}{\partial t}\right)T_{,i} = \rho \ddot{u}_i \tag{1}$$

$$K T_{,ii} = \rho C_E\left(\frac{\partial}{\partial t}+\tau_0\frac{\partial^2}{\partial t^2}\right)T + T_0\gamma\left(\frac{\partial}{\partial t}+n\tau_0\frac{\partial^2}{\partial t^2}\right)u_{j,j} \tag{2}$$

$$\sigma_{ij} = \mu\left(u_{i,j}+u_{j,i}\right) + \lambda u_{i,i}\delta_{ij} - \gamma\left(1+\upsilon\frac{\partial}{\partial t}\right)(T-T_0)\delta_{ij} \tag{3}$$

Putting $\tau_0 = \upsilon = 0$ for coupled thermoelasticity (Biot model).

Putting $n=1$, $\upsilon = 0$ and $\tau_0 \neq 0$, for generalized thermoelasticity with one relaxation time (Lord-Shulman model "L-S").

Putting $n=0$, $\tau_0 \neq 0$, $\upsilon \neq 0$ for generalized thermoelasticity with two relaxation times (Green-Lindsay model "G-L"), where $i, j = 1,2,3$ are the indicators of the coordinates system.

III. FORMULATION THE PROBLEM

Assume a thermoelastic perfectly conducting infinite body with cylindrical cavity fills the region $R \le r < \infty$. The cylindrical coordinates system (r,ψ,z) with the z-axis lying along the axis of the cylinder will be used. Due to symmetry of the medium, the problem is one-dimensional with all the considered functions and depending on the radial distance r only and the time t. It is considered there are no external body forces and heat sources in all parts of the medium even the surface of the cavity.

Thus the governing equations (1)-(3) in cylindrical one dimensional take the following forms

$$(\lambda+2\mu)\frac{\partial e}{\partial r}-\gamma\frac{\partial}{\partial r}\left(1+\upsilon\frac{\partial}{\partial t}\right)T=\rho\frac{\partial^2 u}{\partial t^2} \tag{4}$$

$$\nabla^2 T=\frac{\rho C_E}{K}\left(\frac{\partial}{\partial t}+\tau_0\frac{\partial^2}{\partial t^2}\right)T+\frac{T_o\gamma}{K}\left(\frac{\partial}{\partial t}+n\tau_0\frac{\partial^2}{\partial t^2}\right)e \tag{5}$$

$$\sigma_{rr}=2\mu\frac{\partial u}{\partial r}+\lambda e-\gamma\left(1+\upsilon\frac{\partial}{\partial t}\right)(T-T_0) \tag{6}$$

$$\sigma_{\psi\psi}=2\mu\frac{u}{r}+\lambda e-\gamma\left(1+\upsilon\frac{\partial}{\partial t}\right)(T-T_0) \tag{7}$$

$$\sigma_{zz}=\lambda e-\gamma\left(1+\upsilon\frac{\partial}{\partial t}\right)(T-T_0) \tag{8}$$

$$\sigma_{zr}=\sigma_{\psi r}=\sigma_{zz}=0 \tag{9}$$

$$e=\frac{1}{r}\frac{\partial\ (ru)}{\partial\ r} \tag{10}$$

where $\nabla^2=\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}$

For convenience, we shall use the following non-dimensional variables [7]:

$$(r',u')=c_o\eta(r,u),\ (t',t_0',\tau_0',\upsilon')=c_o^2\eta(t,t_0\tau_0,\upsilon),\ \theta=\frac{(T-T_0)}{T_o},$$

$$\sigma'=\frac{\sigma}{\mu}$$

where $c_o^2=\frac{\lambda+2\mu}{\rho}$ and $\eta=\frac{\rho C_E}{K}$.

Equations (4)-(8) take the form (where the primes are suppressed for simplicity)

$$\nabla^2 e-\alpha\left(1+\upsilon\frac{\partial}{\partial t}\right)\nabla^2\theta=\frac{\partial^2 e}{\partial t^2} \tag{11}$$

$$\nabla^2\theta=\left(\frac{\partial}{\partial t}+\tau_o\frac{\partial^2}{\partial t^2}\right)\theta+\varepsilon\left(\frac{\partial}{\partial t}+n\tau_0\frac{\partial^2}{\partial t^2}\right)e \tag{12}$$

$$\sigma_{rr}=\beta^2 e-2\frac{u}{r}-\alpha\beta^2\left(1+\upsilon\frac{\partial}{\partial t}\right)\theta \tag{13}$$

$$\sigma_{\psi\psi}=\beta^2 e-2\frac{\partial u}{\partial r}-\alpha\beta^2\left(1+\upsilon\frac{\partial}{\partial t}\right)\theta \tag{14}$$

$$\sigma_{zz}=(\beta^2-2)e-\alpha\beta^2\left(1+\upsilon\frac{\partial}{\partial t}\right)\theta \tag{15}$$

where $\alpha=\frac{\gamma T_o}{\lambda+2\mu}$, $\varepsilon=\frac{\gamma}{\rho C_E}$, $\beta^2=\frac{\lambda+2\mu}{\mu}$, and $\gamma=(3\lambda+2\mu)\alpha_T$.

III. ADOMIAN DECOMPOSITION METHOD (ADM)

Before we apply Adomian method we re-write the equation (11) and (12) to be in the forms:

$$\begin{aligned}\frac{\partial^2 e(r,t)}{\partial r^2}=&\frac{\partial^2 e(r,t)}{\partial t^2}+\alpha\left(1+\upsilon\frac{\partial}{\partial t}\right)\frac{\partial^2\theta(r,t)}{\partial r^2}\\&+\alpha\left(1+\upsilon\frac{\partial}{\partial t}\right)\frac{1}{r}\frac{\partial\theta(r,t)}{\partial r}-\frac{1}{r}\frac{\partial e(r,t)}{\partial r}\end{aligned} \tag{16}$$

and

$$\begin{aligned}\frac{\partial^2\theta(r,t)}{\partial r^2}=&\left(\frac{\partial}{\partial t}+\tau_0\frac{\partial^2}{\partial t^2}\right)\theta(r,t)+\\&\varepsilon\left(\frac{\partial}{\partial t}+n\tau_0\frac{\partial^2}{\partial t^2}\right)e(r,t)-\frac{1}{r}\frac{\partial\theta(r,t)}{\partial r}\end{aligned} \tag{17}$$

The Adomian decomposition method usually defines the equation in an operator form by considering the highest-ordered derivative in the problem. We define the differential operator "L" in terms of the two derivatives contained in the problem [9-11, 27].

Consider equations (16) and (17) in the operator form as following:

$$\begin{aligned}L_{rr}e(r,t)=&L_{tt}e(r,t)+\alpha(1+\upsilon L_t)L_{rr}\theta(r,t)+\\&\alpha(1+\upsilon L_t)\frac{1}{r}L_r\theta(r,t)-\frac{1}{r}L_r e(r,t)\end{aligned} \tag{18}$$

$$\begin{aligned}L_{rr}\theta(r,t)=&(L_t+\tau_0 L_{tt})\theta(r,t)+\\&\varepsilon_1(L_t+n\tau_0 L_{tt})e(r,t)-\frac{1}{r}L_r\theta(r,t)\end{aligned} \tag{19}$$

Where the operators which appeared in the above equations are defined as:

$$L_t=\frac{\partial}{\partial t},\ L_{tt}=\frac{\partial^2}{\partial t^2},\ L_r=\frac{\partial}{\partial r},\ L_{rr}=\frac{\partial^2}{\partial r^2} \tag{20}$$

Assuming that the inverse of the operator "L_{rr}^{-1}" exists and is taken as a definite integral with respect to "r" from "0" to "r" as following [9-11, 27]:

$$L_{rr}^{-1}f(r)=\int_0^r\int_0^r f(r)\,dr\,dr \tag{21}$$

Thus applying the inverse operator on both the sides of (18)-(19), we obtain

$$\begin{aligned}e(R,t)=&e(R,t)+\left.\frac{\partial e(r,t)}{\partial r}\right|_{r=R}+\\&L_{rr}^{-1}\begin{bmatrix}L_{tt}e(r,t)+\alpha(1+\upsilon L_t)L_{rr}\theta(r,t)+\\ \alpha(1+\upsilon L_t)\frac{1}{r}L_r\theta(r,t)-\frac{1}{r}L_r e(r,t)\end{bmatrix}\end{aligned} \tag{22}$$

$$\theta(r,t) = \theta(R,t) + \left.\frac{\partial\theta(r,t)}{\partial r}\right|_{r=R} + L_{rr}^{-1}\begin{bmatrix}(L_t + \tau_0 L_{tt})\theta(r,t) + \varepsilon_1(L_t + n\tau_0 L_{tt})e(r,t) \\ -\frac{1}{r}L_r\theta(r,t)\end{bmatrix} \tag{23}$$

Now, we will decompose the unknown functions $\theta(r,t)$ and $e(r,t)$ by a sum of components defined by the following series:

$$e(r,t) = \sum_{k=0}^{\infty} e_k(r,t) = e_0 + \sum_{k=1}^{\infty} e_k(r,t) \tag{24}$$

$$\theta(r,t) = \sum_{k=0}^{\infty} \theta_k(r,t) = \theta_0 + \sum_{k=1}^{\infty} \theta_k(r,t) \tag{25}$$

The zero-components are defined by the terms that arise from the boundary conditions on the surface of the cavity $r = R$, which give

$$e_0 = e(R,t) + \left.\frac{\partial e(r,t)}{\partial r}\right|_{r=R} \tag{26}$$

$$\theta_0 = \theta(R,t) + \left.\frac{\partial\theta(r,t)}{\partial r}\right|_{r=R} \tag{27}$$

Substituting from equations (24)-(27) in equations (22) and (23), we get

$$e(r,t) = \sum_{k=0}^{\infty} e_k(r,t) = e(R,t) + \left.\frac{\partial e(r,t)}{\partial r}\right|_{r=R} + L_{rr}^{-1}\begin{bmatrix}L_{tt}\sum_{k=0}^{\infty} e_k(r,t) + \alpha(1+\upsilon L_t)L_{rr}\sum_{k=0}^{\infty}\theta_k(r,t) + \\ \alpha(1+\upsilon L_t)\frac{1}{r}L_r\sum_{k=0}^{\infty}\theta_k(r,t) - \frac{1}{r}L_r\sum_{k=0}^{\infty} e_k(r,t)\end{bmatrix} \tag{28}$$

$$\theta(r,t) = \sum_{k=0}^{\infty}\theta_k(r,t) = \theta(R,t) + \left.\frac{\partial\theta(r,t)}{\partial r}\right|_{r=R} + L_{rr}^{-1}\begin{bmatrix}(L_t + \tau_0 L_{tt})\sum_{k=0}^{\infty}\theta_k(r,t) + \\ \varepsilon_1(L_t + n\tau_0 L_{tt})\sum_{k=0}^{\infty} e_k(r,t) \\ -\frac{1}{r}L_r\sum_{k=0}^{\infty}\theta_k(r,t)\end{bmatrix} \tag{29}$$

We obtain these components by $e_k(r,t)$ and $\theta_k(r,t)$ the recursive formulas [9-11, 27]:

$$e_{k+1}(r,t) = L_{rr}^{-1}\begin{bmatrix}L_{tt}e_k(r,t) + \alpha(1+\upsilon L_t)L_{rr}\theta_k(r,t) + \\ \alpha(1+\upsilon L_t)\frac{1}{r}L_r\theta_k(r,t) - \frac{1}{r}L_r e_k(r,t)\end{bmatrix} \tag{30}$$

$$\theta_{k+1}(r,t) = L_{rr}^{-1}\begin{bmatrix}(L_t + \tau_0 L_{tt})\theta_k(r,t) + \\ \varepsilon_1(L_t + n\tau_0 L_{tt})e_k(r,t) - \frac{1}{r}L_r\theta_k(r,t)\end{bmatrix} \tag{31}$$

We assume that the surface of the cavity $r = R$ is thermally loaded by harmonic heat with zero strain and heat flux. Hence, we have:

$$\theta(0,t) = \theta^0 \sin(\omega t), \quad \left.\frac{\partial\theta(r,t)}{\partial r}\right|_{r=R} = 0 \tag{32}$$

$$e(0,t) = 0, \qquad \left.\frac{\partial e(r,t)}{\partial r}\right|_{r=R} = 0 \tag{33}$$

where θ^0 is constant and ω is the angular thermal load and assumed to be constant. Thus, we have

$$\theta_0 = \theta^0 \sin(\omega t), \quad e_0 = 0 \tag{34}$$

Substituting from equations (36) into equations (30) and (31), we get the complete of the iteration formulas.
The first components of the iteration take the forms:

$$e_1(r,t) = 0 \tag{35}$$

$$\theta_1(r,t) = \frac{\omega}{2}\left(\cos(\omega t) - \omega\tau_0 \sin(\omega t)\right)(r-R)^2 \tag{36}$$

The rest components of the iteration formulas (30) and (31) have been calculated by using the MAPLE 17. Moreover, the decomposition iteration solutions (30) and (31) are convergent rapidly in any real physical problem and its convergence has been approved by several authors [17-20].

In an algorithmic form, the ADM can be expressed and implemented in linear generalized magneto-thermoelasticity models with the suitable value for the tolerance $Tol = 10^{-6}$ and k is the iteration index, as follows [17-20]:

ALGORITHM

1- Compute the initial approximations $\theta_0 = \theta(0,t)$ and $e_0 = e(0,t)$ given by (36).

2- Use the calculated values of $\theta_k(r,t)$ and $e_k(r,t)$ to compute $\theta_{k+1}(r,t)$ and $e_{k+1}(r,t)$ from (30) and (31).

3- If $max\left|\theta_{k+1}(r,t) - \theta_k(r,t)\right| < Tol$ and $max\left|e_{k+1}(r,t) - e_k(r,t)\right| < Tol$, stop and set $k+1 = m$, otherwise continue and go back to step 2.

4- Calculating $e(r,t) = \sum_{k=0}^{m} e_k(r,t)$ and $\theta(r,t) = \sum_{k=0}^{m}\theta_k(r,t)$.

5- Calculating the displacement from equations (10) and (28) as follows:

$$u(r,t) = \frac{1}{r}\int_R^r e(\xi,t)\,d\xi = \frac{1}{r}\int_R^r \sum_{k=0}^{m} e_k(\xi,t)\,d\xi \tag{37}$$

6- Calculating the stress from the equations (13), (28), (29), and (41) as follows:

$$\sigma(r,t) = \beta^2\sum_{k=0}^{m} e_k(r,t) - 2\frac{\partial}{\partial r}\left(\frac{1}{r}\int_R^r \sum_{k=0}^{m} e_k(\xi,t)\,d\xi\right) - \alpha\beta^2\left(1 + \upsilon\frac{\partial}{\partial t}\right)\sum_{k=0}^{m}\theta_k(r,t) \tag{38}$$

IV. THE NUMERICAL RESULTS AND DISCUSSION

For the numerical evaluations, the copper material has been chosen and the constants of the problem were taken as follows [4-7] :

$K = 386\ W/(mK)$, $\alpha_T = 1.78\times10^{-5}\ K^{-1}$, $C_E = 383.1\ J/(kg\ K)$, $\eta = 8886.73\ s/m^2$, $T_0 = 293\ K$, $\mu = 3.86\times10^{10}\ N/m^2$, $\lambda = 7.76\times10^{10}\ N/m^2$, $\rho = 8954\ kg/m^3$, $\tau_0 = 0.35\times10^{-14}$, $\upsilon = 0.33\times10^{-14}$.

Thus, the following non-dimensional parameters have been obtained;

$\varepsilon_1 = 1.618$, $\nu = 0.02$, $\tau_0 = 0.05$.

We calculate the numerical solutions when the non-dimensional value of the time is $t = 2.0$, the non-dimensional value of the distance is $1.0 \le R \le 2.0$, $\omega = \pi$, and $\theta^0 = 1.0$.

According to the above algorithm, we stopped the calculation on the 5th component $\theta_5(r,t)$ and $e_5(r,t)$.

Figures 1-4 show the temperature increment, the strain, the radial stress, and the displacement distribution with different values of angular thermal load parameter $\omega = (\pi, 1.1\pi)$ for the three models of thermoelasticity; Biot, L-S, and G-L. The numerical results of the L-S model and G-L model almost are identical particularly the temperature increment distribution and the stress distribution for the different values of ω, while the strain and the displacement distributions are not. The different result between the Biot's model and the other models tell us that the relaxation times have significant effects on all the studied functions. Moreover, the angular thermal load parameter has significant effects on the temperature increment, the strain, the radial stress, and the displacement distribution. The figures show also that the changing of the value of the angular thermal load parameter ω leads to large changing in all the studied functions. When the value of the parameter ω increases, then the value of all functions under consideration increase.

Figures 5 and 6 show the temperature increment, the strain, the radial stress, and the displacement distribution for L-S model with respect to the time *t* and the radial distance *r* when $\omega = \pi$ and $\omega = 2\pi$, respectively. Again, the angular thermal load parameter has significant effects on all the distributions. The number of the peak points of the increment temperature and the strain distribution increase when the value of the angular thermal load parameter increases. Finally, the temperature increment, the strain, the radial stress, and the displacement have higher values in the context of Biot model more than L-S and G-L models and the reason is coming back to the relaxation times.

V. CONCLUSION

A mathematical model of thermoelastic an infinite body with cylindrical cavity has been constructed. A unified system of governing equations has been formulated in the context of three different models of thermoelasticity; Biot model, Lord-Shulman model, and Green-Lindsay model. Adomian decomposition method has been used when the surface of the cavity is subjected to harmonic thermal loading with zero heat flux and strain.

The numerical results conclude that:

- The relaxation times and the angular thermal load have significant effects on all the studied fields.
- The results almost from the Lord and Shulman model to match the results obtained when applying the Green and Lindsay model.
- The temperature increment, the strain, the radial stress, and the displacement have higher values in the context of Biot model more than L-S and G-L models.
- Adomian decomposition method is a successful method to solve mathematical models of thermoelasticity based on cylindrical co-ordinates.

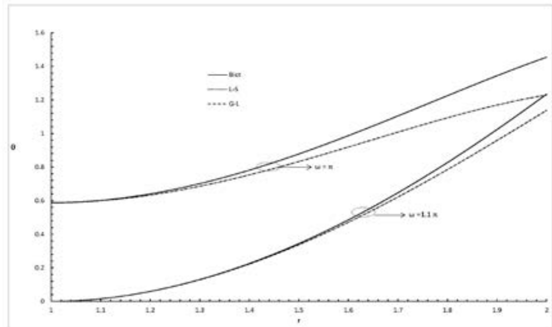

Figure 1: The temperature increment distribution with various values of angular thermal load

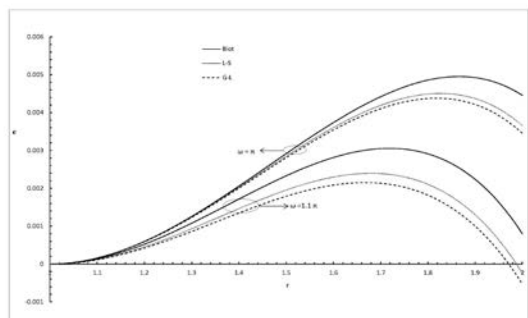

Figure 2: The strain distribution with various values of angular thermal load

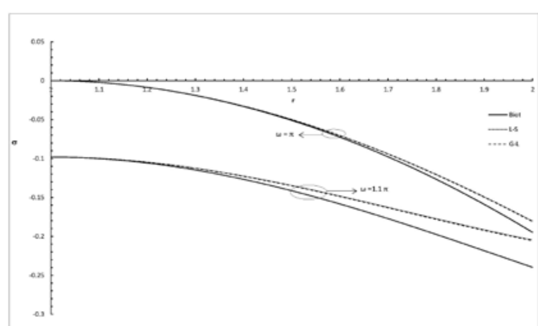

Figure 3: The stress distribution with various values of angular thermal load

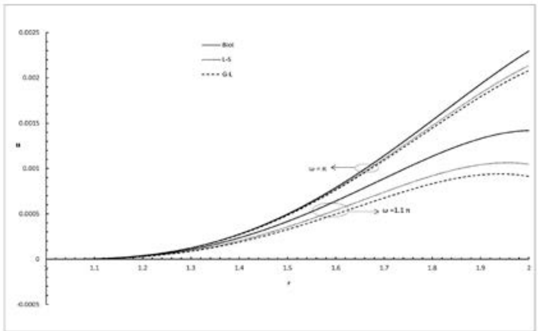

Figure 4: The displacement distribution with various values of angular thermal load

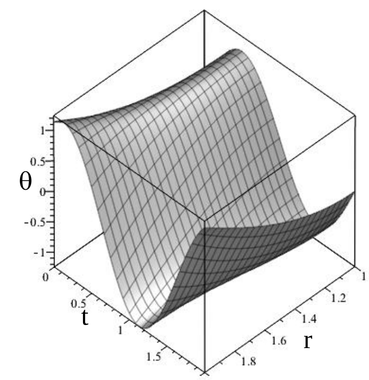

Figure 5-a: The temperature increment

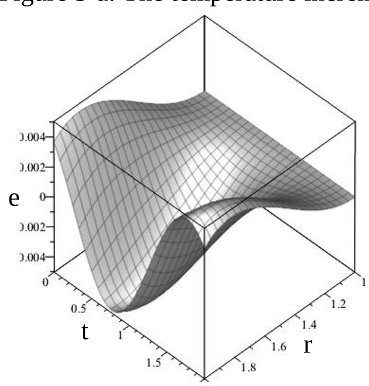

Figure 5-b: The strain

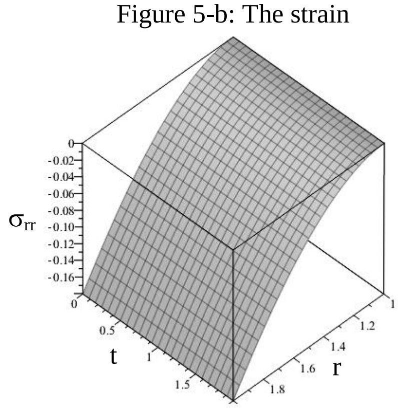

Figure 5-c: The stress

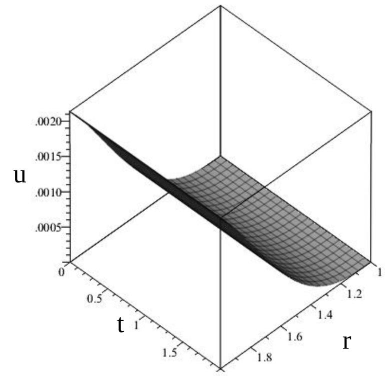

Figure 5-d: The displacement

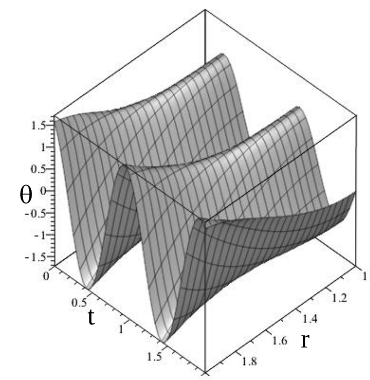

Figure 6-a: The temperature increment

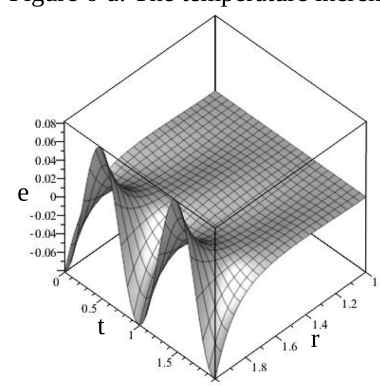

Figure 6-b: The strain

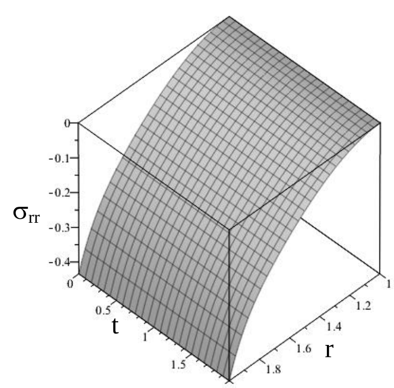

Figure 6-c: The radial stress

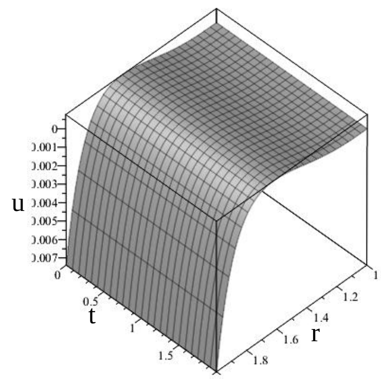

Figure 6-d: The displacement

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NOMENCLATURE

λ , μ Lame's constants

ρ Density

C_E Specific heat at constant strain

α_T Coefficient of linear thermal expansion

$\gamma = (3\lambda+2\mu)\,\alpha_T$

t Time

T Temperature

T_0 Reference temperature

$\theta = \left(T - T_0\right)$ Temperature increment such that $\left|\theta\right| / T_0 << 1$

σ_{ij} Components of stress tensor

e_{ij} Components of strain tensor

u_i Components of displacement vector

F_i Body force vector

K Thermal conductivity

τ_0, υ Relaxation times