LQ Problem in Stabilization of Linear Metzlerian Continuous-time Systems

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***Abstract*—**The paper present a consistent set of linear matrix inequalities which guaranties asymptotic stability of the closed-loop system, warranties strictly Metzlerian system structure, and adjusts the state and output variables coincident with prescribed quadratic limits. To realize with a positive control law gain, the diagonal stabilizability of strictly Metzlerian linear continuous-time systems is approved, and the related closed-form expression of design conditions is provided. The results are illustrated using a particular LQ problem, for which numerical examples are given.

***Keywords*—**asymptotic stability, linear matrix inequalities, linear quadratic control, Metzlerian continuous-time systems, state feedback stabilization.

I. Introduction

Positive systems indicate the processes whose variables represent quantities that do not have meaning unless they are nonnegative [1]. Since, in the relevant continuous-time state-space description, the system matrix of a positive system is Metzler, theory of Metzler matrices is naturally applied to this kind of dynamical systems [2]. Additionally limited in the way that the system input and output matrices are at least nonnegative matrices [3], system stabilization means strictly defined task to design a positive gain matrix of control law so that the closed-loop system matrix is Metzler and Hurwitz [4]. Therefore, most of techniques applicable to ordinary linear systems can not be straightly nominated to positive linear systems [5], [6]. Mainly the books [7], [8] treat a considerable number of the approaches to positive system analysis, and include illustrative algorithms for many specific tasks, but there still remains a wide variety of related problems (controllability, observability, speed of response, robustness) which need to be solved and addressed to Metzlerian linear systems. A more detailed treatments of problems are given, e.g. in [9], [10].

The trend in synthesis of feedback control of Metzlerian systems tends to simplify and disambiguate the strictly defined design conditions. Supposing that the Metzlerian systems is represented by state-space equations, the synthesis of stabilizing state-feedback controllers, guaranteeing the closed-loop system is asymptotically stable and internally positive, is conditionally supported by linear programming to meet the closed-loop system positive structure [11], [12]. In order to reduce the number of constraints entering the solution in linear programming methods, an alternative synthesis procedure with is proposed in [13], where the system parameter boundaries are defined by *n* linear matrix inequalities (LMI), if the system is strictly Metzlerian. Because a solution of such defined base set of LMIs only assures that the closed-loop system matrix is strictly Metzler, the design conditions are complemented by another LMI that imposes a stable asymptotic solution. Since the applied LMI variables are of diagonal matrix structure, it can be refereed about diagonal stabilizability of the strictly Metzlerian continuous-time linear systems.

Constraining the class of controller matrix gains to be positive, it does not alleviate the complexity of the solutions for non strictly Metzlerian systems. Proceeding along the same lines, and pursuing the formal system analogy, some applicable extensions of the above formulations for strictly positive discrete-time linear systems can be found in [14], [15].

Analyzing the challenging problem of state-feedback stabilization of strictly Metzlerian linear continuous-time systems, the main motivation of this paper are design conditions formulated for infinite-time horizon control with linear quadratic cost functions. Since, at defined constraints on elements of a strictly Metzlerian system matrix structure, the task cannot be formulated using a Riccati equation form, the matrices of cost function are used to extend that one LMI, which reflects stability condition in overall completion of the LMIs set in design conditions. The configuration chosen corresponds a way exploiting the minimizing of the quadratic cost criterion subject to a closed-loop stability constraint, the framework used is standard and convenient because other additive constraints may be included into design formulation.

Used notations are conventional so that $\boldsymbol{x}^T$, $\boldsymbol{X}^T$ denote transpose of the vector $\boldsymbol{x}$, and matrix $\boldsymbol{X}$, respectively, $\boldsymbol{x}_+$, $\boldsymbol{X}_+$ indicate a nonnegative vector and a nonnegative matrix, $\boldsymbol{X} \succ 0$ means that $\boldsymbol{X}$ is a symmetric positive definite matrix, $\rho(\boldsymbol{X})$ reports the eigenvalue spectrum of the square matrix $\boldsymbol{X}$, the symbol $\boldsymbol{I}_n$ marks the *n*-th order unit matrix, $\mathrm{diag}[\bullet]$ enters up a diagonal matrix, $\mathbb{R}_+^n$, $\mathbb{R}_+^{n \times r}$ signify the set of all *n* dimensional real non-negative vectors and $n \times r$ real non-negative matrices, respectively.

The work presented in this paper was supported by VEGA, the Grant Agency of Ministry of Education and Academy of Science of Slovak Republic, under Grant No. 1/0608/17. This support is very gratefully acknowledged.

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II. Linear Continuous-time Positive Systems

To define the system positive structures, and to extend their formal stabilizability properties, it is preferred in the following the state-space system description defined in the standard way as

$$\dot{\boldsymbol{q}}(t) = \boldsymbol{A}\boldsymbol{q}(t) + \boldsymbol{B}\boldsymbol{u}(t) \tag{1}$$

$$\boldsymbol{y}(t) = \boldsymbol{C}\boldsymbol{q}(t) \tag{2}$$

where the equations (1), (2) belong to the Metzlerian class of positive systems if $\boldsymbol{q}(t) \in \mathbb{R}_+^n$, $\boldsymbol{u}(t) \in \mathbb{R}_+^r$, $\boldsymbol{y}(t) \in \mathbb{R}_+^m$ (all variables are nonnegative) for all $t \geq 0$.

In the general case, the matrix $\boldsymbol{A} \in \mathbb{R}^{n\times n}$ is restricted to being strictly Metzler (its diagonal elements are negative and its off-diagonal elements are positive) and the matrices $\boldsymbol{B} \in \mathbb{R}_+^{n\times r}$, $\boldsymbol{C} \in \mathbb{R}_+^{m\times n}$ are nonnegative (all its entries are nonnegative and at least one is positive). Satisfying these restrictions, the system (1), (2) is referred as a linear strictly Metzlerian system. Note, a strictly Metzler matrix is stable if it is Hurwitz.

Terminating the class of admissible controllers to be linear and considering, for simplicity, a SISO linear strictly Metzlerian system (1), (2) controlled by the dimensionally compatible control, constrained to use a linear function of the state measurements, and a strictly positive real vector $\boldsymbol{k}$ such that

$$u(t) = -\boldsymbol{k}^T\boldsymbol{q}(t), \quad \boldsymbol{k} \in \mathbb{R}^n \tag{3}$$

then the state-space enrollment of the closed-loop system is given as

$$\dot{\boldsymbol{q}}(t) = (\boldsymbol{A} - \boldsymbol{b}\boldsymbol{k}^T)\boldsymbol{q}(t) = \boldsymbol{A}_c\boldsymbol{q}(t) \tag{4}$$

$$\boldsymbol{y}(t) = \boldsymbol{C}\boldsymbol{q}(t) \tag{5}$$

where

$$\boldsymbol{A}_c = \boldsymbol{A} - \boldsymbol{b}\boldsymbol{k}^T \tag{6}$$

has to be a strictly Metzler matrix. Consequently, the closed-loop system matrix structure (5) prescribes the algebraic inequalities corresponding to the strictly Metzler matrix $\boldsymbol{A}_c$ as follows

$$a_{cii} = a_{ii} - b_i k_i < 0 \ \text{ for all } i = 1, 2, \dots, n \tag{7}$$

$$a_{cij} = a_{ij} - b_i k_j > 0 \ \text{ for all } i, j = 1, 2, \dots, n, \ i \neq j \tag{8}$$

where the detailed formats of the Metzler system matrix parameters, as well as the state controller gain vector structure are

$$\boldsymbol{A} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ & & \vdots & \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}, \quad \boldsymbol{b} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}, \quad \boldsymbol{k} = \begin{bmatrix} k_1 \\ k_2 \\ \vdots \\ k_n \end{bmatrix} \tag{9}$$

Although the structure of the state feedback control law (3) is simple, it should be noted that the positiveness constraint for the solvability of the gain $\boldsymbol{k}$ is extended by the set of n^2 scalar inequalities (6), (7).

Generalizing for MIMO (multiple input, multiple output) systems, the following lema yields.

Lemma1: [15] Within the basic notations as above and applying the vector input variable

$$\boldsymbol{u}(t) = -\boldsymbol{K}\boldsymbol{q}(t) = -\begin{bmatrix} \boldsymbol{k}_1^T \\ \vdots \\ \boldsymbol{k}_r^T \end{bmatrix}\boldsymbol{q}(t) \tag{10}$$

on the strictly Metzler MIMO system (1), (2), while the positive gain matrix $\boldsymbol{K} \in \mathbb{R}^{r\times n}$ is prescribed to force the closed-loop system matrix

$$\boldsymbol{A}_c = \boldsymbol{A} - \boldsymbol{B}\boldsymbol{K} = \boldsymbol{A} - \sum_{k=1}^{r} \boldsymbol{b}_k\boldsymbol{k}_k^T \tag{11}$$

then the matrix $\boldsymbol{A}_c$ is Metzler, if for given non-negative matrix $\boldsymbol{B} \in \mathbb{R}_+^{n\times r}$ and a strictly Metzler matrix $\boldsymbol{A} \in \mathbb{R}_+^{n\times n}$ there exist positive definite diagonal matrices $\boldsymbol{P}, \boldsymbol{R}_k \in \mathbb{R}_+^{n\times n}$ such that for $h = 1, 2, \dots n-1$, $k = 1, 2, \dots r$

$$\boldsymbol{P} = \boldsymbol{P}^T \succ 0 \tag{12}$$

$$\boldsymbol{R}_k = \boldsymbol{R}_k^T \succ 0 \tag{13}$$

$$\boldsymbol{A}(i,i)_{(1\leftrightarrow n)} - \sum_{k=1}^{r} \boldsymbol{B}_{dk}\boldsymbol{R}_k \prec 0 \tag{14}$$

$$\boldsymbol{T}^h\boldsymbol{A}(j, j+h)_{(1\leftrightarrow n)/n}\boldsymbol{T}^{hT}\boldsymbol{P} - \sum_{k=1}^{r} \boldsymbol{T}^h\boldsymbol{B}_{dk}\boldsymbol{T}^{hT}\boldsymbol{R}_k \succ 0 \tag{15}$$

subject to the notations

$$\boldsymbol{T} = \begin{bmatrix} 0 & 0 & \cdots & 0 & 1 \\ 1 & 0 & \cdots & 0 & 0 \\ & & \ddots & & \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix}, \quad \boldsymbol{T}^{-1} = \boldsymbol{T}^T \tag{16}$$

$$\boldsymbol{A}(j, j+h)_{(1\leftrightarrow n)/n} = \operatorname{diag}\left[a_{1,1+h} \ \cdots \ a_{n-h,n} \ a_{n-h+1,1} \ \cdots \ a_{n,h} \right] \tag{17}$$

$$\boldsymbol{B} = \left[\boldsymbol{b}_1 \ \boldsymbol{b}_2 \ \cdots \ \boldsymbol{b}_r \right] = \begin{bmatrix} b_{11} & \cdots & b_{1r} \\ b_{21} & \cdots & b_{2r} \\ & \vdots & \\ b_{n1} & \cdots & b_{nr} \end{bmatrix} \tag{18}$$

$$\boldsymbol{B}_{dk} = \operatorname{diag}\left[b_{1k} \ b_{2k} \ \cdots \ b_{nk} \right] \tag{19}$$

Then, if there are satisfied the above conditions for prescribed set of variables, the control gain is

$$\boldsymbol{K}_{dk} = \boldsymbol{R}_k\boldsymbol{P}^{-1}, \quad \boldsymbol{k}_k^T = \boldsymbol{l}^T\boldsymbol{K}_{dk}, \quad \boldsymbol{l}^T = [\,1\ 1\ \cdots\ 1\,] \tag{20}$$

Remark 1: Since the rows and columns of an $n \times n$ square matrix are indexed from 1 to *n*, the addition modulo $n+1$ on the set of residues *S* is considered in the following as $(j+h)_{mod\ n+1} = r+1$, where *r* is the element of *S* to which the result of the usual sum of integers *j* and *k* is congruent modulo $n+1$. The used shorthand symbolical notation is $(j+h)_{(1\leftrightarrow n)/n} = r+1$.

Comment 1: As it is seen from *Lemma 1*, the resulting conditions prescribe the Metzlerian structure of $\boldsymbol{A}_c \in \mathbb{R}_+^{n\times n}$ but do not guarantee that $\boldsymbol{A}_c$ is Hurwitz matrix [14]. To solve the stabilization problem for a strictly Metzlerian system (1), (2) with a diagonally stabilizable pair $(\boldsymbol{A}, \boldsymbol{B})$ the following theorem is proposed.

Theorem 1: [16] The control law (3) stabilizes the linear strictly Metzlerian system (1), (2) if for given positive definite diagonal matrices $\boldsymbol{Q} \in \mathbb{R}^{n \times n}, \boldsymbol{U} \in \mathbb{R}^{r \times r}$ there exist positive definite diagonal matrices $\boldsymbol{P}, \boldsymbol{R}_k \in \mathbb{R}^{n \times n}$ such that for $h = 1, 2, \dots n-1,\ k = 1, 2, \dots r$,

$$\boldsymbol{P} = \boldsymbol{P}^T \succ 0 \tag{21}$$

$$\boldsymbol{R}_k = \boldsymbol{R}_k^T \succ 0 \tag{22}$$

$$\begin{bmatrix} \boldsymbol{AP} + \boldsymbol{PA}^T - \sum_{k=1}^{r} \boldsymbol{B}_{dk} \boldsymbol{l} \boldsymbol{l}^T \boldsymbol{R}_k - \sum_{k=1}^{r} \boldsymbol{R}_k \boldsymbol{l} \boldsymbol{l}^T \boldsymbol{B}_{dk} & * & * \\ \sum_{k=1}^{r} \boldsymbol{h}_k \boldsymbol{l}^T \boldsymbol{R}_k & -\boldsymbol{U}^{-1} & * \\ \boldsymbol{P} & \boldsymbol{0} & -\boldsymbol{Q}^{-1} \end{bmatrix} \prec 0 \tag{23}$$

$$\boldsymbol{A}(i,i)_{(1 \leftrightarrow n)} - \sum_{k=1}^{r} \boldsymbol{B}_{dk} \boldsymbol{R}_k \prec 0 \tag{24}$$

$$\boldsymbol{T}^h \boldsymbol{A}(j, j+h)_{(1 \leftrightarrow n)/n} \boldsymbol{T}^{hT} \boldsymbol{P} - \sum_{k=1}^{r} \boldsymbol{T}^h \boldsymbol{B}_{dk} \boldsymbol{T}^{hT} \boldsymbol{R}_k \succ 0 \tag{25}$$

where $\boldsymbol{T}$, $\boldsymbol{A}(j, j+h)_{(1 \leftrightarrow n)/n}$, $\boldsymbol{B}_{dk}$, $\boldsymbol{b}_k$, $\boldsymbol{R}_k$, $\boldsymbol{l}^T$ are introduced in (16)-(19) and

$$\boldsymbol{h}_k^T = \begin{bmatrix} 0 \cdots 0\, 1_k\ 0 \cdots 0 \end{bmatrix} \tag{26}$$

is the vector with the value 1 on the k-th position.

When the above conditions hold, the positive control gain $\boldsymbol{K} \in \mathbb{R}^{r \times n}$ is given in (9), where

$$\boldsymbol{K}_{dk} = \boldsymbol{R}_k \boldsymbol{P}^{-1}, \quad \boldsymbol{k}_k^T = \boldsymbol{l}^T \boldsymbol{K}_{dk} \tag{27}$$

This encompass the design conditions with connection to LQ problem defined in Theorem 1. In this viewpoint, the condition unifies the design with quadratic constraints and feedback full state control for a class of strictly Metzlerian linear dynamical systems with performance appraisals of infinite time horizon and quadratic costs, focusing perfect state-feedback measurements and addressing the benefits of feedback in multi-input/multi-output Metzlerian linear dynamical systems.

Applying the above given positive definite diagonal matrices $\boldsymbol{P}, \boldsymbol{R}_k \in \mathbb{R}^{n \times n}, k = 1, 2, \dots r$, corroborated in theorem formulation, different matrix inequalities can be used instead of the complex inequality structure (23) to ensure stability in Lyapunov sense while, if the set of inequalities is affirmative, an asymptotically stable closed-loop system is obtained.

The simplest applicable matrix inequality which can replace inequality (23), but with no constraint on the system state and input variables, takes the form

$$\boldsymbol{AP} + \boldsymbol{PA}^T - \sum_{k=1}^{r} \boldsymbol{B}_{dk} \boldsymbol{l} \boldsymbol{l}^T \boldsymbol{R}_k - \sum_{k=1}^{r} \boldsymbol{R}_k \boldsymbol{l} \boldsymbol{l}^T \boldsymbol{B}_{dk} \prec 0 \tag{28}$$

while the control input is generated by the closed-loop control policy (10). Evidently, (28) can be simply derived from (23) prescribing the zero matrices $\boldsymbol{Q} \in \mathbb{R}^{n \times n}$ and $\boldsymbol{U} \in \mathbb{R}^{r \times r}$.

III. Enhanced Control Design

The following theorem gives a more general version of the design conditions, implying from the slack matrix decoupling principle.

Theorem 2: The control law (3) stabilizes the linear strictly Metzlerian system (1), (2) if for given positive definite diagonal matrices $\boldsymbol{Q} \in \mathbb{R}^{n \times n}, \boldsymbol{U} \in \mathbb{R}^{r \times r}$ and given positive $\delta \in \mathbb{R}$ there exist positive definite diagonal matrices $\boldsymbol{P}, \boldsymbol{V}, \boldsymbol{R}_k \in \mathbb{R}^{n \times n}$ such that for $h = 1, 2, \dots n-1,\ k = 1, 2, \dots r$,

$$\boldsymbol{P} = \boldsymbol{P}^T \succ 0 \tag{29}$$

$$\boldsymbol{V} = \boldsymbol{V}^T \succ 0 \tag{30}$$

$$\boldsymbol{R}_k = \boldsymbol{R}_k^T \succ 0 \tag{31}$$

$$\begin{bmatrix} \boldsymbol{AP} + \boldsymbol{PA}^T - \sum_{k=1}^{r} \boldsymbol{B}_{dk} \boldsymbol{l} \boldsymbol{l}^T \boldsymbol{R}_k - \sum_{k=1}^{r} \boldsymbol{R}_k \boldsymbol{l} \boldsymbol{l}^T \boldsymbol{B}_{dk} & * & * & * \\ \boldsymbol{V} - \boldsymbol{P} + \delta \boldsymbol{AP} - \delta \sum_{k=1}^{r} \boldsymbol{B}_{dk} \boldsymbol{l} \boldsymbol{l}^T \boldsymbol{R}_k & -2\delta \boldsymbol{P} & * & * \\ \sum_{k=1}^{r} \boldsymbol{h}_k \boldsymbol{l}^T \boldsymbol{R}_k & \boldsymbol{0} & -\boldsymbol{U}^{-1} & * \\ \boldsymbol{P} & \boldsymbol{0} & \boldsymbol{0} & -\boldsymbol{Q}^{-1} \end{bmatrix} \prec 0 \tag{32}$$

$$\boldsymbol{A}(i,i)_{(1 \leftrightarrow n)} - \sum_{k=1}^{r} \boldsymbol{B}_{dk} \boldsymbol{R}_k \prec 0 \tag{33}$$

$$\boldsymbol{T}^h \boldsymbol{A}(j, j+h)_{(1 \leftrightarrow n)/n} \boldsymbol{T}^{hT} \boldsymbol{P} - \sum_{k=1}^{r} \boldsymbol{T}^h \boldsymbol{B}_{dk} \boldsymbol{T}^{hT} \boldsymbol{R}_k \succ 0 \tag{34}$$

where $\boldsymbol{T}$, $\boldsymbol{A}(j, j+h)_{(1 \leftrightarrow n)/n}$, $\boldsymbol{B}_{dk}$, $\boldsymbol{b}_k$, $\boldsymbol{R}_k$, $\boldsymbol{l}^T$ are introduced in (16)-(19) and $\boldsymbol{h}_k^T$ in (26).

When the above conditions hold, the positive control gain $\boldsymbol{K} \in \mathbb{R}^{r \times n}$ is given in (9), where

$$\boldsymbol{K}_{dk} = \boldsymbol{R}_k \boldsymbol{P}^{-1}, \quad \boldsymbol{k}_k^T = \boldsymbol{l}^T \boldsymbol{K}_{dk} \tag{35}$$

Proof: Reflecting the fact that the system (1), (2) is linear, the Lyapunov function is chosen in the form

$$v(\boldsymbol{q}(t)) = \boldsymbol{q}^T(t) \boldsymbol{S} \boldsymbol{q}(t) \tag{36}$$

with $\boldsymbol{S} \in \mathbb{R}^{n \times n}$ taking the diagonal positive definite structure.

Considering positive definite diagonal matrices $\boldsymbol{Q} \in \mathbb{R}^{n \times n}$, $\boldsymbol{U} \in \mathbb{R}^{r \times r}$, the Lyapunov function derivative in the sense of the Krasovskii theorem [17] is predefined as

$$\begin{aligned} \dot{v}(\boldsymbol{q}(t)) &= \dot{\boldsymbol{q}}^T(t) \boldsymbol{S} \boldsymbol{q}(t) + \boldsymbol{q}^T(t) \boldsymbol{S} \dot{\boldsymbol{q}}(t) \\ &\le -(\boldsymbol{q}^T(t) \boldsymbol{Q} \boldsymbol{q}(t) + \boldsymbol{u}^T(t) \boldsymbol{U} \boldsymbol{u}(t)) < 0 \end{aligned} \tag{37}$$

and substituting (10) it yields

$$\dot{v}(\boldsymbol{q}(t)) = \dot{\boldsymbol{q}}^T(t) \boldsymbol{S} \boldsymbol{q}(t) + \boldsymbol{q}^T(t) \boldsymbol{S} \dot{\boldsymbol{q}}(t) + \boldsymbol{q}^T(t)(\boldsymbol{Q} + \boldsymbol{K}^T \boldsymbol{U} \boldsymbol{K}) \boldsymbol{q}(t) < 0 \tag{38}$$

Writing (4) with (11) as

$$\boldsymbol{A}_c \boldsymbol{q}(t) - \dot{\boldsymbol{q}}(t) = 0 \tag{39}$$

then with an arbitrary positive definite diagonal matrix $\boldsymbol{M} \in \mathbb{R}_+^{n \times n}$ and with a positive scalar $\delta \in \mathbb{R}_+$ it yields

$$(\boldsymbol{q}^T(\mathrm{t}) \boldsymbol{M} + \dot{\boldsymbol{q}}^T(\mathrm{t}) \delta \boldsymbol{M})(\boldsymbol{A}_c \boldsymbol{q}(\mathrm{t}) - \dot{\boldsymbol{q}}(\mathrm{t})) = 0 \tag{40}$$

Therefore, adding (36) and its transposition to (34) it is obtained

$$\begin{aligned}\dot{v}(\mathbf{q}(t)) = & \,\dot{\mathbf{q}}^T(t)\mathbf{S}\mathbf{q}(t) + \mathbf{q}^T(t)\mathbf{S}\dot{\mathbf{q}}(t) \\ & + \mathbf{q}^T(t)(\mathbf{Q} + \mathbf{K}^T\mathbf{U}\mathbf{K})\mathbf{q}(t) \\ & + (\mathbf{q}^T(t)\mathbf{M} + \dot{\mathbf{q}}^T(t)\delta\mathbf{M})(\mathbf{A}_c\mathbf{q}(t) - \dot{\mathbf{q}}(t)) \\ & + (\mathbf{q}^T(t)\mathbf{A}_c^T - \dot{\mathbf{q}}^T(t))(\mathbf{M}\mathbf{q}(t) + \delta\mathbf{M}\dot{\mathbf{q}}(t)) < 0\end{aligned} \tag{41}$$

and with

$$\boldsymbol{q}_v^T(t) = \begin{bmatrix} \boldsymbol{q}^T(t) & \dot{\boldsymbol{q}}^T(t) \end{bmatrix} \tag{42}$$

then (41) can be rewritten as

$$\dot{v}(\boldsymbol{q}_v(t)) = \boldsymbol{q}_v^T(t)\boldsymbol{T}_v\boldsymbol{q}_v(t) < 0 \tag{43}$$

where

$$\boldsymbol{T}_v = \begin{bmatrix} \boldsymbol{M}\boldsymbol{A}_c + \boldsymbol{A}_c^T\boldsymbol{M} + \boldsymbol{Q} + \boldsymbol{K}^T\boldsymbol{U}\boldsymbol{K} & * \\ \boldsymbol{S} - \boldsymbol{M} + \delta\boldsymbol{M}\boldsymbol{A}_c & -2\delta\boldsymbol{M} \end{bmatrix} \prec 0 \tag{44}$$

Because of the strictly structure of $\boldsymbol{A}_c$, the inequality (44) is a bilinear inequality, it is necessary to define the transformation matrix $\boldsymbol{T}_c$ for transformation (44) into the linear form so that

$$\boldsymbol{T}_c = \operatorname{diag}\begin{bmatrix} \boldsymbol{P} & \boldsymbol{P} \end{bmatrix}, \quad \boldsymbol{P} = \boldsymbol{M}^{-1} \tag{45}$$

Pre-multiplying from the left side and post-multiplying from the right side by $\boldsymbol{T}_c$ then (44) implies

$$\begin{bmatrix} \boldsymbol{A}_c\boldsymbol{P} + \boldsymbol{P}\boldsymbol{A}_c^T + \boldsymbol{P}\boldsymbol{Q}\boldsymbol{P} + \boldsymbol{P}\boldsymbol{K}^T\boldsymbol{U}\boldsymbol{K}\boldsymbol{P} & * \\ \boldsymbol{P}\boldsymbol{S}\boldsymbol{P} - \boldsymbol{P} + \delta\boldsymbol{A}_c\boldsymbol{P} & -2\delta\boldsymbol{P} \end{bmatrix} \prec 0 \tag{46}$$

and using the Schur complement property, it can write

$$\begin{bmatrix} \boldsymbol{A}_c\boldsymbol{P} + \boldsymbol{P}\boldsymbol{A}_c^T & * & * & * \\ \boldsymbol{V} - \boldsymbol{P} + \delta\boldsymbol{A}_c\boldsymbol{P} & -2\delta\boldsymbol{P} & * & * \\ \boldsymbol{K}\boldsymbol{P} & \boldsymbol{0} & -\boldsymbol{U}^{-1} & * \\ \boldsymbol{P} & \boldsymbol{0} & \boldsymbol{0} & -\boldsymbol{Q}^{-1} \end{bmatrix} \prec 0 \tag{47}$$

where a positive definite diagonal matrix $\boldsymbol{V} \in \mathbb{R}^{n\times n}$ is denoted as follows

$$\boldsymbol{V} = \boldsymbol{P}\boldsymbol{S}\boldsymbol{P} \tag{48}$$

Hence, with (27), (26) and with the $\boldsymbol{A}_c$ structure given in (11), it can write

$$\begin{aligned}\boldsymbol{A}_c\boldsymbol{P} &= \boldsymbol{A}\boldsymbol{P} - \sum_{k=1}^{r}\boldsymbol{b}_k\boldsymbol{k}_k^T\boldsymbol{P} \\ &= \boldsymbol{A}\boldsymbol{P} - \sum_{k=1}^{r}\boldsymbol{b}_k\boldsymbol{r}_k^T \\ &= \boldsymbol{A}\boldsymbol{P} - \sum_{k=1}^{r}\boldsymbol{B}_{dk}\boldsymbol{l}\boldsymbol{l}^T\boldsymbol{R}_k\end{aligned} \tag{49}$$

$$\boldsymbol{K}\boldsymbol{P} = \sum_{k=1}^{r}\boldsymbol{h}_k\boldsymbol{k}_k^T\boldsymbol{P} = \sum_{k=1}^{r}\boldsymbol{h}_k\boldsymbol{r}_k^T = \sum_{k=1}^{r}\boldsymbol{h}_k\boldsymbol{l}^T\boldsymbol{R}_k \tag{50}$$

where

$$\boldsymbol{r}_k^T = \boldsymbol{k}_k^T\boldsymbol{P}, \quad \boldsymbol{b}_k = \boldsymbol{B}_{dk}\boldsymbol{l}, \quad \boldsymbol{r}_k^T = \boldsymbol{l}^T\boldsymbol{R}_k \tag{51}$$

and the relations (49), (50) are used to modify (47) as (32). Then, combining (32) with (21), (22), (24), (25) concludes the proof. ■

Remark 2: Theorem 2 solves the state-feedback control problem for linear strictly Metzlerian system (1), (2), given by the diagonally stabilizable pair $(\boldsymbol{A}, \boldsymbol{B})$ with quadratic constraints represented by the couple of positive definite diagonal weighting matrices $(\boldsymbol{Q}, \boldsymbol{U})$ of appropriate dimensions. Introducing the symmetric slack matrix variable $\boldsymbol{P}$, the system parameter matrices $(\boldsymbol{A}, \boldsymbol{B})$ are strictly decoupled in the LMIs (30)-(32) from the Lyapunov matrix $\boldsymbol{V}$, while the matrix $\boldsymbol{V}$ verifying the closed-loop stability remains symmetric positive definite and diagonal. By this procedure, the control problem is parameterized in such LMIs structure, which admits more freedom in the controller design for Metzlerian systems since except free defining weighting matrices $(\boldsymbol{Q}, \boldsymbol{U})$ there is a free tuning parameter $\delta \in \mathbb{R}_+$.

Corollary 1: Defining the transformation matrix $\boldsymbol{T}_p$ as follows

$$\boldsymbol{T}_p = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{I}_n & & \\ \boldsymbol{I}_n & \boldsymbol{0} & & \\ & & \boldsymbol{I}_r & \\ & & & \boldsymbol{I}_n \end{bmatrix}, \quad \boldsymbol{T}_p^{-1} = \boldsymbol{T}_p \tag{52}$$

and premultiplying the left side and postmultiplying the right side, then (47) implies

$$\boldsymbol{H} = \begin{bmatrix} -2\delta\boldsymbol{P} & \boldsymbol{V} - \boldsymbol{P} + \delta\boldsymbol{A}_c\boldsymbol{P} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{V} - \boldsymbol{P} + \delta\boldsymbol{P}\boldsymbol{A}_c^T & \boldsymbol{A}_c\boldsymbol{P} + \boldsymbol{P}\boldsymbol{A}_c^T & \boldsymbol{P}\boldsymbol{K}^T & \boldsymbol{P} \\ \boldsymbol{0} & \boldsymbol{K}\boldsymbol{P} & -\boldsymbol{U}^{-1} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{P} & \boldsymbol{0} & -\boldsymbol{Q}^{-1} \end{bmatrix} \prec 0 \tag{53}$$

The inequality (53) can be factorized as

$$\begin{aligned}\boldsymbol{H} = & \begin{bmatrix} -2\delta\boldsymbol{P} & \boldsymbol{V} - \boldsymbol{P} + \delta\boldsymbol{A}_c\boldsymbol{P} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{V} - \boldsymbol{P} + \delta\boldsymbol{P}\boldsymbol{A}_c^T & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & 0 \end{bmatrix} \\ & + \begin{bmatrix} \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{A}_c\boldsymbol{P} + \boldsymbol{P}\boldsymbol{A}_c^T & \boldsymbol{P}\boldsymbol{K}^T & \boldsymbol{P} \\ \boldsymbol{0} & \boldsymbol{K}\boldsymbol{P} & -\boldsymbol{U}^{-1} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{P} & \boldsymbol{0} & -\boldsymbol{Q}^{-1} \end{bmatrix} \prec 0\end{aligned} \tag{54}$$

Setting $\delta = 0$, $\boldsymbol{V} = \boldsymbol{P}$ then, evidently,

$$\boldsymbol{H} = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{A}_c\boldsymbol{P} + \boldsymbol{P}\boldsymbol{A}_c^T & \boldsymbol{P}\boldsymbol{K}^T & \boldsymbol{P} \\ \boldsymbol{0} & \boldsymbol{K}\boldsymbol{P} & -\boldsymbol{U}^{-1} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{P} & \boldsymbol{0} & -\boldsymbol{Q}^{-1} \end{bmatrix} \leq 0 \tag{55}$$

and it is obvious, to be satisfied (55), the following inequality has to yield

$$\begin{bmatrix} \boldsymbol{A}_c\boldsymbol{P} + \boldsymbol{P}\boldsymbol{A}_c^T & * & * \\ \boldsymbol{K}\boldsymbol{P} & -\boldsymbol{U}^{-1} & * \\ \boldsymbol{P} & \boldsymbol{0} & -\boldsymbol{Q}^{-1} \end{bmatrix} \prec 0 \tag{56}$$

Thus, it can be finally observed using (49)-(51) that (56) provides componentwise interlinking with (23).

Note, the enhanced formulation gives substantiation for different solutions obtaining in dependency on linear matrix inequalities (23) or (30), (32), respectively, when combining them with (12)-(15) to construct the control design conditions with connection to LQ task for strictly Metzler linear systems.

IV. ILLUSTRATIVE EXAMPLE

The strictly Metzlerian system (1), (2) is concretized for the system parameters [13]

$$\boldsymbol{A}=\begin{bmatrix} -3.3800 & 0.2080 & 6.7150 & 5.6760 \\ 0.5810 & -4.2900 & 2.0500 & 0.6750 \\ 1.0670 & 4.2730 & -6.6540 & 5.8930 \\ 0.0480 & 2.2730 & 1.3430 & -2.1040 \end{bmatrix}$$

$$\boldsymbol{B}=\begin{bmatrix} 0.0400 & 0.0189 \\ 0.0568 & 0.0203 \\ 0.0114 & 0.0315 \\ 0.0114 & 0.0170 \end{bmatrix}, \quad \boldsymbol{C}=\begin{bmatrix} 4 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Since the algebraic manipulation provides that the eigenvalue spectrum of the strictly Metzler matrix $\boldsymbol{A}$ is

$$\rho(\boldsymbol{A})=\{1.9761,\ -9.4392,\ -4.4824\pm 1.2499\mathrm{i}\}$$

such defined Metzler system is unstable.

From the system parameter expressions the auxiliary design frameworks follow that

$$\boldsymbol{A}(i,i)_{(1\leftrightarrow 4)}=\operatorname{diag}\left[-3.3800\ -4.2900\ -6.6540\ -2.1040\right]$$

$$\boldsymbol{A}(i,i+1)_{(1\leftrightarrow 4)/4}=\operatorname{diag}\left[0.2080\ \ 2.0500\ \ 5.8930\ \ 0.0480\right]$$

$$\boldsymbol{A}(i,i+2)_{(1\leftrightarrow 4)/4}=\operatorname{diag}\left[6.7150\ \ 0.6750\ \ 1.0670\ \ 2.2730\right]$$

$$\boldsymbol{A}(i,i+3)_{(1\leftrightarrow 4)/4}=\operatorname{diag}\left[5.6760\ \ 0.5810\ \ 4.2730\ \ 1.3430\right]$$

$$\boldsymbol{B}_{d1}=\operatorname{diag}\left[0.0400\ \ 0.0568\ \ 0.0114\ \ 0.0114\right]$$

$$\boldsymbol{B}_{d2}=\operatorname{diag}\left[0.0189\ \ 0.0203\ \ 0.0315\ \ 0.0170\right]$$

while the permutation matrix $\boldsymbol{T}$ of the forth length and the vectors $\boldsymbol{h}_1$, $\boldsymbol{h}_2$ are

$$\boldsymbol{T}=\begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad \boldsymbol{h}_1=\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad \boldsymbol{h}_2=\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Theorem 1 gives the possibility to synthesize state feedback gains by solving this diagonal LMI problem. Within defined design framework, prescribing the weight matrices $\boldsymbol{Q}=10\boldsymbol{I}_4$, $\boldsymbol{U}=0.01\boldsymbol{I}_2$ and solving (21)-(25) rewritten for the SeDuMi package [18], all LMI conditions of the above theorem are satisfied and asymptotical stability of the closed-loop system is guaranteed by a stable strictly Metzler matrix $\boldsymbol{A}_c$, if the values of the LMI variables are

$$\boldsymbol{P}=\operatorname{diag}\left[0.2083\ \ 0.0142\ \ 0.0742\ \ 0.0113\right]$$

$$\boldsymbol{R}_1=\operatorname{diag}\left[0.7011\ \ 0.0267\ \ 0.8732\ \ 0.0273\right]$$

$$\boldsymbol{R}_2=\operatorname{diag}\left[0.0555\ \ 0.0595\ \ 4.8489\ \ 0.2536\right]$$

This gives, prescribed by (10), (27), the control law matrix gain $\boldsymbol{K}$ that

$$\boldsymbol{K}=\begin{bmatrix} 3.3661 & 1.8754 & 11.7690 & 2.4101 \\ 0.2667 & 4.1775 & 65.3565 & 22.4240 \end{bmatrix}$$

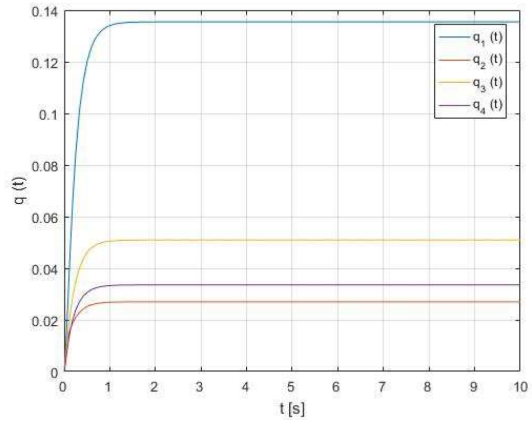

Fig. 1: Closed-loop system state response

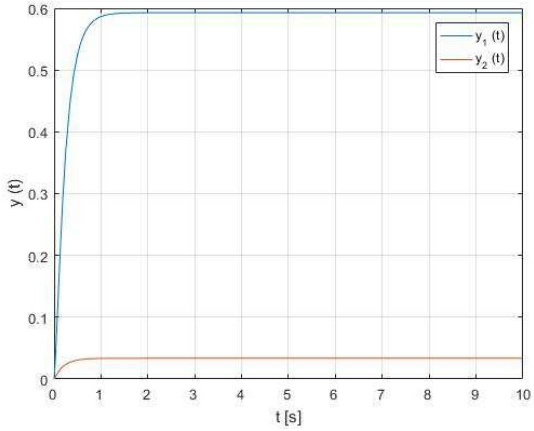

Fig. 2: Closed-loop system output response

Evidently, the control law gain matrix $\boldsymbol{K}$ is a strictly positive matrix.

It can simple show that the solution is the stable Metzler matrix $\boldsymbol{A}_c$ of the form

$$\boldsymbol{A}_c=\begin{bmatrix} -3.5197 & 0.0541 & 5.0103 & 5.1583 \\ 0.3844 & -4.4813 & 0.0549 & 0.0829 \\ 1.0204 & 4.1203 & -8.8438 & 5.1602 \\ 0.0052 & 2.1806 & 0.0976 & -2.5128 \end{bmatrix}$$

such that its eigenvalue spectrum is

$$\rho(\boldsymbol{A}_c)=\{-0.9580\ \ -9.6020\ \ -4.3988\pm 1.1567\mathrm{i}\}$$

Defining the system forced mod by the control policy

$$\boldsymbol{u}(t)=-\boldsymbol{K}\boldsymbol{q}(t)+\boldsymbol{w}(t), \quad \boldsymbol{w}^T=\left[1.36\ \ 0.68\right]$$

and setting $\boldsymbol{q}(0)=\boldsymbol{0}$, the simulation results for control of the system are shown in Fig. 1 and Fig. 2, where the state variables vector $\boldsymbol{q}(t)$, as well as the output variables vector $\boldsymbol{y}(t)$ are positive, when the parameters of the control input are set up as mentioned above. Because $\boldsymbol{A}_c$ is stable, then $\boldsymbol{q}(t)$, $\boldsymbol{y}(t)$ tend to constant values as $t\rightarrow\infty$.

Note, closed loop is asymptotically stable and externally positive if $\boldsymbol{A}_c$ is stable Metzler matrix and $\boldsymbol{C}\in\mathbb{R}_+^{m\times n}$ is a non-negative matrix.

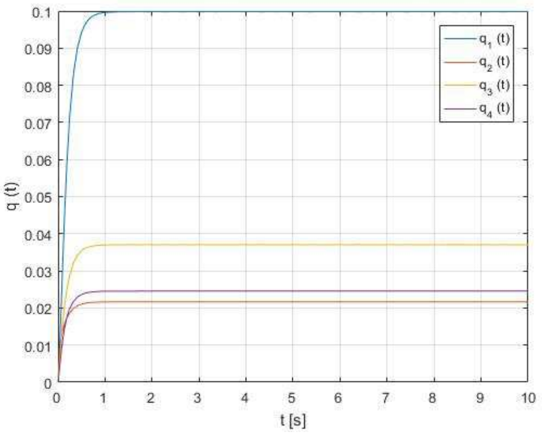

Fig. 3: Closed-loop system state enhanced response

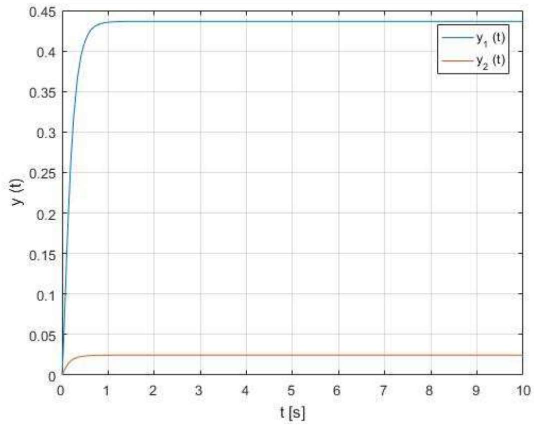

Fig. 4: Closed-loop system output enhanced response

Solving the set of LMIs (29)--(34) with the above prescribed $\boldsymbol{Q}$, $\boldsymbol{U}$ and the tuning parameter $\delta = 1.8$ the result is

$$\boldsymbol{P} = \text{diag}\begin{bmatrix}0.0216 & 0.0006 & 0.0125 & 0.0005\end{bmatrix}$$

$$\boldsymbol{V} = \text{diag}\begin{bmatrix}0.1152 & 0.0037 & 0.1757 & 0.0018\end{bmatrix}$$

$$\boldsymbol{R}_1 = \text{diag}\begin{bmatrix}0.0906 & 0.0001 & 0.1308 & 0.0001\end{bmatrix}$$

$$\boldsymbol{R}_2 = \text{diag}\begin{bmatrix}0.0002 & 0.0066 & 0.8966 & 0.0160\end{bmatrix}$$

In this case the gain $\boldsymbol{K}$ and the matrix $\boldsymbol{A}_c$ are such that

$$\boldsymbol{K} = \begin{bmatrix}4.1996 & 0.1699 & 10.4438 & 0.1346\\ 0.0104 & 10.4867 & 71.6190 & 32.7143\end{bmatrix}$$

$$\boldsymbol{A}_c = \begin{bmatrix}-3.5482 & 0.0032 & 4.9451 & 5.0530\\ 0.3423 & -4.5125 & 0.0030 & 0.0033\\ 1.0190 & 3.9412 & -9.0258 & 4.8623\\ 0.0001 & 2.0927 & 0.0061 & -2.6620\end{bmatrix}$$

while the stable eigenvalue spectrum of $\boldsymbol{A}_c$ is

$$\rho(\boldsymbol{A}_c) = \{\text{-}1.2896 \;\; \text{-}9.7062 \;\; \text{-}4.3764 \pm 1.1173\text{i}\}$$

It is evident from the eigenvalues spectrum that the closed-loop system dynamics is in this case faster. Running under the same simulation conditions as are given above, the closed-loop system responses are presented in Fig. 3 and Fig 4.

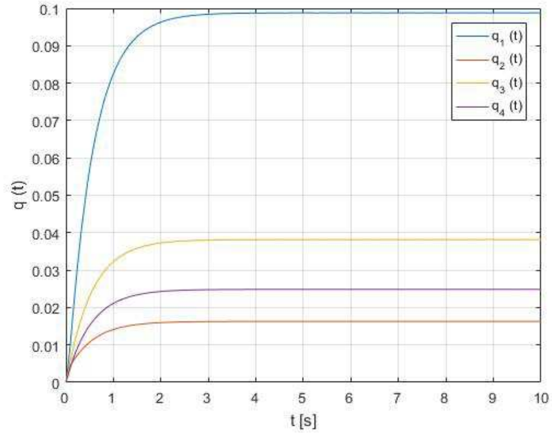

Fig. 5: Closed-loop system state standard response

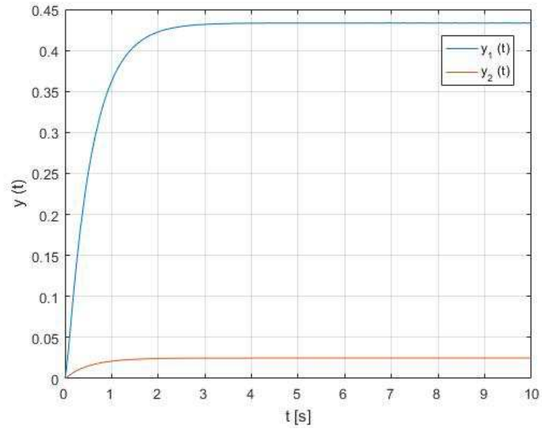

Fig. 6: Closed-loop system output standard response

Using the standard algorithms [13], where in the set of (21)-(25) is inequality (23) replaced by inequality (28), then

$$\boldsymbol{P} = \text{diag}\begin{bmatrix}0.6618 & 0.0590 & 0.1336 & 0.0532\end{bmatrix}$$

$$\boldsymbol{R}_1 = \text{diag}\begin{bmatrix}1.2310 & 0.0903 & 2.8440 & 0.0716\end{bmatrix}$$

$$\boldsymbol{R}_2 = \text{diag}\begin{bmatrix}0.5664 & 0.3015 & 4.2142 & 1.4121\end{bmatrix}$$

and the closed-loop system parameters are

$$\boldsymbol{K} = \begin{bmatrix}1.8601 & 1.5308 & 21.2869 & 1.3463\\ 0.8559 & 5.1131 & 31.5423 & 26.5640\end{bmatrix}$$

$$\boldsymbol{A}_c = \begin{bmatrix}-3.4706 & 0.0502 & 5.2680 & 5.1206\\ 0.4580 & -4.4807 & 0.2008 & 0.0593\\ 1.0189 & 4.0948 & -7.8881 & 5.0420\\ 0.0123 & 2.1686 & 0.5646 & -2.5711\end{bmatrix}$$

Since the eigenvalue spectrum of $\boldsymbol{A}_c$ is

$$\rho(\boldsymbol{A}_c) = \{\text{-}0.3771 \;\; \text{-}9.0806 \;\; \text{-}4.4764 \pm 1.2684\text{i}\}$$

it is evident that the closed-loop system dynamics is substantially slower.

To compare all responses in terms of steady-state variable values, the setting point vector of the control policy in simulation is changed as $\boldsymbol{w}^T = \begin{bmatrix}0.4 & 0.2\end{bmatrix}$. The closed-loop system responses are presented in Fig. 5 and Fig 6.

Note, as can be seen [16], [19], the direct use of the principle of static decoupling [19] can lead to that the closed-loop system being not internally positive. Since this means that not for every non-negative initial system state vector the output of the system will be positive, when setting the operating point of the system in the simulations, the principle of static decoupling was not used.

V. Concluding Remarks

The paper is concentrated specifically on effective design of the full state feedback control for strictly Metzlerian continuous-time systems, to accomplish that the closed-loop system matrix be strictly Metzler and Hurwitz and the control gain be a positive matrix.

The principle combines the algebraic constraints, implying from the predefined closed-loop system strictly Metzler matrix structure, and defined as a basic set of LMIs with an additive stabilizing matrix inequality, containing moreover the prescribed state and input signal amplitude quadratic constrains, to guarantee closed-loop system asymptotic stability and dynamics.

Whilst a solution can be obtained only via positive definite and diagonal matrix variables entering this set of inequalities, progress is made in incorporating an LMI structure to realize the diagonal stabilizability of strictly Metzlerian linear continuous-time systems. As it is illustrated by the numerical example, hopefully of interest to researchers, the proposed fulfilment provides numerically effective computational frameworks with potential adaptation to not strictly Metzlerian systems.

Many problems concerning the non unique solutions, such as design of the non-negative gain matrix for not strictly Metzlerian linear systems, or for discrete-time not strictly positive linear systems, are still open. These problems are challenging and interesting research works in the future, while the proposed LMI structure variant seems promising in further aspects to be subjects to future work.

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