Asymptotic Stability by Lyapunov and Assessment of Areas of Attraction of Phase Systems

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***Abstract*—** In work are investigated asymptotic stability by Lyapunov and assessment of area of an attraction on the basis of the second method of Lyapunov by means of new Lyapunov's function. On the basis of theoretical results received in work computing experiments on concrete examples of electrical power systems, which have shown sufficient efficiency of the offered method for the studied electrical power system are made.

***Keywords*—** mathematical model, asymptotic stability, Lyapunov's function, power systems.

I. Introduction

Mathematical model of a modern electrical power complex, consisting of turbogenerators and complex multiply connected energy blocks, is a system of nonlinear ordinary differential equations. The task of optimizing the functioning of these complexes, and also creation of algorithms of stability of the movement for such systems and until now draw attention of many researchers and are relevant.

Industrial development of modern society leads to the constant growth of electricity consumption. To satisfy these constantly growing requirements, difficult electrical power complexes are created. At mathematical modeling of such complexes it is required to resolve a number of theoretical and practical issues. Ensuring stability of the movement is the major problem at a design stage and operation of the studied systems.

We will note that work is devoted to a research of asymptotic stability of movement of phase systems [1].

II. Formulation

Consider a system of the look

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$$\frac{d\delta_i}{dt} = S_i, \quad \frac{dS_i}{dt} = -D_i S_i - f_i(\delta_i) - \psi_i(\delta_{i^*}), \quad i = \overline{1,l} \tag{1}$$

where function $\psi_i(\delta_{i^*})$ is defined by a ratio

$$\psi_i(\delta_*) = \sum_{k=1,k\neq i}^{l} P_{ik}(\delta_{ik}), \delta_{ik} = \delta_i - \delta_k .$$

Owing to frequency of a phase portrait of system on δ_i coordinates it is enough to study of him, for example, in a strip of $\overline{G_{0i}}$ set by inequalities

$$\delta_{-1i} < \delta_i < \delta_{oi} \quad S_i \in R_i^1, i = \overline{1,\ell}$$

The set of the special points of system (1) which are in $\overline{G}_{0i}$ strip is defined by a set

$$\overline{\Delta} = \left\{ (\delta, S) \middle| S_i = 0, f_i(\delta_i) + \sum_{r=1,k\neq i}^{\ell} P_{ik}(\delta_{ik}) \right. \\ \left. (\delta_i, S_i) \in G_{0i}, i = \overline{1,\ell} \right\} = \{T_0, T_1, \ldots, T_N\}. \tag{2}$$

Notice, that point $T_0 = \{\delta_i = 0, S_i = 0, i = \overline{1,\ell}\}$ is also an element of a stationary set $\overline{\Delta}$. Forming the characteristic equation of first approximation system, it is possible to establish the nature of stability of special points (2). As is well-known, in order that special point system was unstable, negativity suffices, at least, one coefficient of the characteristic equation of systems of the first approach in the neighborhood of this special point.

Consider the function

$$V(\delta, S) = \sum_{i=1}^{\ell} v_{0i}(\delta_i, S_i) + \sum_{j=2}^{\ell} \sum_{i=1}^{j-1} \int_0^{\delta_{ij}} P_{ij}(\lambda) d\lambda, \tag{3}$$

where

$v_{0i}(\delta_i,S_i)=\frac{1}{2}(S_i+\alpha_i D_i\delta_i)^2+\frac{1}{2}\alpha_i D_i^2(1-\alpha_i)\cdot \delta_i^2+F_i(\delta_i)+2D_i\sqrt{\alpha_i(1-\alpha_i)}\widetilde{F}_i(\delta_i)=\frac{1}{2}(S_i+\alpha_i D_i\delta_i)^2+\int_0^{\delta_i}N_i(\delta_i)d\delta_i$

defined in band $\overline{G}_{0i}$.

Theorem 1. Let there are $D_i,\tau_i>0$ scalars such that

1) Phase system of second order $\frac{d\delta_i}{dt}=S_i$,

$$\frac{dS_i}{dt}=-D_iS_i-f_i(\delta_i). \quad (4)$$

globally asymptotically stable (that is $D_i>(D_i)_{kp}$),

2) Matrix $\widetilde{A_i}$ of Hurwitz,

3) $(\widetilde{A_i},G_i)$ – completely observed pair,

4) $(\widetilde{A_i},Q_i)$– completely controlled pair,

5) $\overline{\overline{\Gamma}}_i>0, det\left[2\overline{\overline{\Gamma}}_i-\chi_i^*h_i^*\widetilde{D}_i^{-1}h_i\chi_i\right]\neq 0\ (i=\overline{1,l})$ $\Gamma_i+Re\,W_i(j\omega)\geq 0(\forall\omega\in(-\infty,+\infty)),L_i\geq 0,\widetilde{D}_i>0.$

Then control

$$u_i=a_i^*x_i+\theta_iS_i+\overline{\varepsilon}_i\phi_i(\delta_i)+\frac{S_i\psi_i(\delta_{i^*})}{x_i^*H_ib_i}$$

at $z_i\in\sum_i$

$$u_i\neq-(b_i^*H_ib_i)^{-1}(b_i^*H_iA_ix_i+b_i^*H_iq_iS_i+b_i^*H_ie_i\phi_i(\sigma_i))$$

at $z_i\notin\sum_i, i=\overline{1,l}$

provides global asymptotic stability of system

$$\frac{d\delta}{dt}=S\ ,$$

$$\frac{dS}{dt}=w-KS-f(\delta)-\psi(\delta_*),w=C^*x,$$

$$\frac{dx}{dt}=Ax+qS+bu+e\phi(\sigma),\sigma=g^*x+\gamma S,$$

Theorem 2. Let the following conditions be satisfied:

1) function $f_i(\delta_i)$ meets a condition

$$f_i(\delta_i)=f_i(\delta_i+2\pi),\ (\forall\delta_i\in R_i^1),$$

$$\gamma_0=\frac{1}{2\pi}\int_0^{2\pi}f_i(\delta_i)d\delta_i\leq 0$$

$$f_i(0)=0,$$

$$\left.\frac{df_i(\delta_i)}{d\delta_i}\right|_{\delta_i=0}>0,\quad f_i(\delta_{0i})=0,\quad \left.\frac{df_i(\delta_i)}{d\delta_i}\right|_{\delta_i=\delta_{0i}}<0,$$

;

2) function $P_{ij}(\lambda)$ meets a condition

$$-P_{ij}(\lambda)=P_{ji}(\lambda),$$
$$P_{ij}(\lambda)=-P_{ij}(-\lambda), \quad (5)$$
$$P_{ij}(\delta_{ij})\delta_{ij}\geq 0;$$

3) constants $\alpha_i,D_i>0$ such that

a) $\alpha_i=\frac{K}{D_i},0<K<\min\{D_1,\ldots,D_\ell\},i=\overline{1,\ell}$

b) $f_i(0)=\alpha_iD_i^2(1-\alpha_i),i=\overline{1,\ell}$.

Then zero position of balance of T_0 asymptotically is steady across Lyapunov, and internal assessment of area of an attraction of a special point of T_0 is defined by area, limited surface $V(\delta,S)=T$, where $T=\min_{1\leq i\leq N}V(T_i)$, if $T_i,i=\overline{1,N}$ unstable special points of system (1).

Proof. On condition 3 b) theorems and, by Theorem 1, functions $v_{0i}(\delta_i,S_i),i=\overline{1,\ell}$ definitely - positive in band $\overline{G}_{0i}$, and a full derivative on time *t* owing to system (4), - sign negative and set $\dot{v}_{0i}=0$ doesn't contain the whole trajectories of system (4), except a special point T_0. Function $V(\delta,S)$ in band $\overline{G}_{0i}$ is also definitely - positive, owing to a condition (5).

On condition (5) and 3 a) theorems following equalities are fair

$$\sum_{i=1}^{\ell}S_i\sum_{j=1,j\neq i}^{\ell}P_{ij}(\delta_{ij})=\sum_{j=1}^{\ell}\sum_{i=1}^{j-1}P_{ij}(\delta_{ij})(S_i-S_j),$$

$$\sum_{i=1}^{\ell}\alpha_iD_i\delta_i\sum_{j=1,j\neq i}^{\ell}P_{ij}(\delta_{ij})= \quad (6)$$

$$=K\sum_{i=1}^{\ell}\sum_{j=1,j\neq i}^{\ell}P_{ij}(\delta_{ij})=K\sum_{j=2}^{\ell}\sum_{i=1}^{j-1}P_{ij}(\delta_{ij})\delta_{ij}.$$

Full derivative of function $V(\delta,S)$ (3) on time в owing to system (1), taking into account (6) will take a form

$$\dot{V}(\delta,S)=\sum_{i=1}^{\ell} D_i\left[\sqrt{\alpha_i(1-\alpha_i)}S_i-\sqrt{\alpha_i\delta_i f_i(\delta_i)}\right]^2-$$
$$-\sum_{i=1}^{\ell}\alpha_i D_i\delta_i\sum_{j=1,j\neq i}^{\ell}P_{ij}(\delta_{ij})= \tag{7}$$
$$=-\sum_{i=1}^{\ell} D_i\left[\sqrt{\alpha_i(1-\alpha_i)}S_i\sqrt{\alpha_i\delta_i f_i(\delta_i)}\right]^2$$
$$-K\sum_{j=2}^{\ell}\sum_{i=1}^{j-1}P_{ij}(\delta_{ij})\delta_{ij}.$$

Expression (7) of the sign negative and set $\dot{V}(\delta,S)=0$ doesn't contain the whole trajectories of system (1), except a special point T_0 . Then areas of an attraction of a special point T_0 it is possible to define by a limited surface $V(\delta,S)=T$, where $T=\min_{1\le i\le N}V(T_i)$, if T_i - unstable special points of system (1). The theorem is proved.

Notice, that areas of an attraction of a special point T_0 can be found by the method of work [2].

Consider now function $\upsilon_{0i}(\delta_i,S_i)=\frac{S_i^2}{2}+\alpha_i S_i\delta_i+\frac{\alpha_i D_i}{2}\delta_i^2+F_i(\delta_i)+2\beta_i(\delta_i)\tilde{F}_i(\delta_i)$ (8) and function $V(\delta,S)$ (3).

Theorem 3. Let there are scalars $\varepsilon_{1i},\varepsilon_{2i},\tau_i\ge 0,\tau_{1i}\ge 0,\ \theta_{5i}>0,D_i>0$ such that

1) Matrix $\tilde{A}_i$ of Hurwitz,
2) $(\tilde{A}_i,G_i)$ – completely observed pair,
3) $(\tilde{A}_i,Q_i)$– completely controlled pair,
$\Gamma_i+Re\,W_i(j\omega)\ge 0(\forall\omega\in(-\infty,+\infty))$,
4) $L_i\ge 0,\tilde{D}_i>0,\quad \bar{\bar{\Gamma}}_i>0$, $det[2\bar{\bar{\Gamma}}_i-\chi_i^*\tilde{D}_i^{-1}h_i\chi_i]\neq 0$.

Then control

$u_i=a_i^*x_i+\theta_i S_i+\bar{\varepsilon}_{1i}\phi_i(\sigma_i)+\varepsilon_{1i}\gamma_i(\delta_i)+\varepsilon_{2i}f_i(\delta_i)+\frac{(S_i+\theta_{1i}f_i(\delta_i))\psi_i(\delta_{i^*})}{x_i^*H_ib_i}$,

at $z_i\in\tilde{\Sigma}_i$,

$u_i\neq\bar{a}_i^*x_i+\theta_i S_i+\bar{\varepsilon}_{1i}\phi_i(\sigma_i)+\varepsilon_{1i}\gamma_i(\delta_i)+\varepsilon_{2i}f_i(\delta_i)$,

at $z_i\notin\tilde{\Sigma}_i$,

provides global asymptotic stability of system

$$\frac{dS_i}{dt}=w_i-K_iS_i-f_i(\delta_i)-\psi_i(\delta_{i^*}),\quad w_i=C_i^*x_i$$

$$\frac{dx_i}{dt}=A_ix_i+q_iS_i+b_iu_i+e_i\phi_i(\sigma_i),\quad i=\overline{1,l}$$

Theorem 4. Let exist a vector a_i, scalars, $\alpha_{1i},\alpha_{2i},\alpha_{3i},\varepsilon_{1i}>0$,
$\alpha_i\in(0,1),\ D_i>0$ such that

1) matrix $\tilde{A}_i$ of Hurwitz,
2) pair $(\tilde{A}_i,B_i)$ it is completely controllable,
3) пара $(\tilde{A}_i,g_i^*)$ it is completely observable,
4) $\Pi_i(j\omega)>0\ \ (\forall\omega\in(-\infty,+\infty))$,
5) $f_i'(0)\neq\alpha_iD_i^2(1-\alpha_i)$.

Then, under control

$u_i=a_i^*x_i+\alpha_{1i}S_i+\alpha_{2i}\delta_i+\alpha_{3i}f_i(\delta_i)$

area of an attraction of the beginning of coordinates in band G_{0i}, which can be estimated by means of Lyapunov's function

$$\tilde{v}_{0i}(\delta_i,S_i,x_i)=\frac{1}{2}x_i^*H_ix_i+\frac{\varepsilon_{1i}}{2}S_i^2+v_{0i}(\delta_i,S_i) \quad (9),$$

it is set by inequality of a look $\tilde{v}_{0i}(\delta_i,S_i,x_i)<\bar{\tilde{v}}_{0i}$, where criteria value $\bar{\tilde{v}}_{0i}$ is defined by following ratio

$$\dot{\bar{\tilde{v}}}_{0i}=\min\{\bar{\tilde{\rho}}_{0i},\bar{\tilde{\rho}}_{-1i}\},\quad \bar{\tilde{\rho}}_{0i}=\tilde{v}_{0i}\Big|_{\substack{\delta_i=\delta_{0i}\\ S_i=0,\ x_i=0}},$$
$$\bar{\tilde{\rho}}_{-1i}=\tilde{v}_{0i}\Big|_{\substack{\delta_i=\delta_{-1i}\\ S_i=0,\ x_i=0}},$$

Theorem 5. Let are carried out 1), 2) theorems 2 and 3) constants $\alpha_i,D_i>0$ such that:

a) $\alpha_i=K\in(0,D_i),i=\overline{1,l}$,

b)
$$\sqrt{\left.\frac{df_i(\delta_i)}{d\delta_i}\right|_{\delta_i=0}}\neq -sign\beta_i'\sqrt{(D_i-\alpha_i)\alpha_i},$$
$$\sqrt{\left.\frac{df_i(\delta_i)}{d\delta_i}\right|_{\delta_i=0}}\neq +sign\beta_i''\sqrt{(D_i-\alpha_i)\alpha_i}.$$

Then zero position of balance of T_0 asymptotically is steady across Lyapunov , and internal assessment of area of an attraction of a special point of T_0 is defined by area, a limited surface $V(\delta,S)=T$, where $T=\min_{1\le i\le N}V(T_i)$; if T_i, $i=\overline{1,N}$, unstable special points of system (1).

Proof. On condition of this theorem and according to the theorem 3 functions $\nu_{0i}(\delta_i,S_i),i=\overline{1,l},V(\delta,S)$ definitely - positive in band $\overline{G_{0i}}$. Full derivative on time t from function $\nu_{0i}(\delta_i,S_i)$ (8) owing to system (4) sign negative, and set $\nu_{0i}=0$ doesn't contain the whole trajectories, except a special point T_0. Derivative on time from function $V(\delta,S)$ owing to system (1) taking into account equalities (6), will take a form

$$\dot V(\delta,S)=\sum_{i=1}^{l}\dot v_i(\delta_i,S_i)\Big| - $$
$$-\sum_{i=1}^{l}\alpha_i\delta_i\sum_{j=1,j\neq i}^{l}P_{ij}(\delta_{ij})=\sum_{i=1}^{l}\dot v_i(\delta_i,S_i)\Big| - \quad (10)$$
$$-K\sum_{j=2}^{l}\sum_{i=1}^{j-1}P_{ij}(\delta_{ij})\delta_{ij},$$

where right part of equality of the sign negative and set $\dot V_i(\delta_i,S_i)=0$ doesn't contain the whole trajectories, except a special point T_0. From here as well as in theorem 2 it is easily possible to receive approvals of theorem. The theorem is proved.

2°). Consider system now

$$\frac{d\delta_i}{dt}=S_i,$$
$$\frac{dS_i}{dt}=w_i-D_iS_i-f_i(\delta_i)-\psi_i(\delta_{i^*}),w_i=C_i^*x_i, \quad (11)$$
$$\frac{dx_i}{dt}=A_ix_i+q_iS_i+b_iu_i, i=\overline{1,l}, \quad (12)$$

where function

$$\psi_i(\delta_*)=\sum_{k=1,k\neq i}^{l}P_{ik}(\delta_{ik}),\delta_{ik}=\delta_i-\delta_k \quad (13)$$

Owing to frequency of a phase portrait of system on coordinates δ_i - it is enough to study of him in band G_{0i}, set by inequalities

$$-\delta_{1i}<\delta_i<\delta_{0i} \qquad S_i\in R_i^1, x_i\in R_i^{n_i}, i=\overline{1,l}$$

Will consider a set of the special points of system (11), (12) which are in band G_{0i}, i.e. introduce the set of stationary points:

$$\overline{\Lambda}=\left\{\begin{array}{l}(\delta,S,x)\big|S_i=0,\ f_i(\delta_i)+\sum_{k=1,k\neq i}^{l}P_{ik}(\delta_{ik})=0,\\ x_i=0,(\delta_i,S_i,x_i)\in G_{0i}, i=\overline{1,l}\end{array}\right\}=$$
$$=\{\overline{T}_0,\overline{T}_1,...,\overline{T}_N\},$$

where $\overline{T}_0=\{\delta_i=0,S_i=0,x_i=0,i=\overline{1,l}\}\in\overline{\Lambda}$

Consider the function

$$\overline{V}(\delta,S,x)=\sum_{i=1}^{l}\tilde v_{0i}(\delta_i,S_i,x_i)+\sum_{j=2}^{l}\sum_{i=1}^{j-1}\int_0^{\delta_{ij}}P_{ij}(\lambda)d\lambda, \quad (14)$$

where function $\tilde v_{0i}(\delta_i,S_i,x_i)$ (9) defined in band G_{0i}. Then combining the results of theorems 4 and 2, it is not difficult to prove the following theorem.

Theorem 6. Let the parameters D_i,α_i such that $D_i>0,\ 0<\alpha_i<D_i$

$$\sqrt{\left.\frac{df_i(\delta_i)}{d\delta_i}\right|_{\delta_i=0}}\neq sign\beta_i'\sqrt{(D_i-\alpha_i)\alpha_i},$$
$$\sqrt{\left.\frac{df_i(\delta_i)}{d\delta_i}\right|_{\delta_i=0}}\neq sign\beta_i''\sqrt{(D_i-\alpha_i)\alpha_i}.$$

Then area of an attraction of steady equilibrium state O(0,0) of system (4) in band $\bar G_{0i}$, which can be estimated by means of Lyapunov (9) function, it is set by inequality of a look $\nu_{0i}(\delta_i,S_i)<\bar\nu_{0i}$, where the criteria value and parameters entering in $\nu_{0i}(\delta_i,S_i)$ are chosen as follows:

a) In case $(\nu_{0i}'')_{max}\leq(\nu_{0i}')_{max}$: $\bar\nu_{0i}=(\nu_{0i}'')_{max}$,

$$\alpha_i=\frac{D_i}{2}\left[1+\sqrt{\frac{D_i^2\delta_{0i}^4}{D_i^2\delta_{0i}^4+16F_i^2(\delta_{0i})_i}}\right];$$

if $(\nu_{0i}')_{min}\leq(\nu_{0i}'')_{max}$, $\beta_i''=\sqrt{(D_i-\alpha_i)\alpha_i}$,

$$\beta_i'=\frac{\alpha_iD_i(\delta_{0i}^2-\delta_{-1i}^2)+2[F_i(\delta_{0i})-F_i(\delta_{-1i})]+4\sqrt{(D_i-\alpha_i)\alpha_i}F_i(\delta_{0i})}{4F_i(\delta_{-1i})}$$

if $(\nu_{0i}')_{min}>(\nu_{0i}'')_{max}$, $\beta_i''=\sqrt{(D_i-\alpha_i)\alpha_i}$

$$\beta_i'=-\sqrt{(D_i-\alpha_i)\alpha_i}.$$

b) In case $(\nu_{0i}')_{max}\leq(\nu_{0i}'')_{max}$: $\bar\nu_{0i}=(\nu_{0i}')_{max}$,

$$\alpha_i=\frac{D_i}{2}\left[1+\sqrt{\frac{D_i^2\delta_{-1i}^4}{D_i^2\delta_{-1i}^4+16F_i^2(\delta_{-1i})_i}}\right];$$

if $(\nu_{0i}'')_{min}\leq(\nu_{0i}')_{max}$, $\beta_i'=\sqrt{(D_i-\alpha_i)\alpha_i}$,

$$\beta_i'=\frac{\alpha_iD_i(\delta_{-1i}^2-\delta_{0i}^2)+2[F_i(\delta_{-1i})-F_i(\delta_{0i})]+4\sqrt{(D_i-\alpha_i)\alpha_i}F_i(\delta_{-1i})}{4F_i(\delta_{0i})}$$

if $(\nu_{0i}'')_{min}>(\nu_{0i}')_{max}$, $\beta_i'=\sqrt{(D_i-\alpha_i)\alpha_i}$

$$\beta_i''=-\sqrt{(D_i-\alpha_i)\alpha_i}.$$

It is supposed that for the chosen α_i conditions are satisfied

$$\sqrt{\frac{df_i(\delta_i)}{d\delta_i}}\Bigg|_{\delta_i=0} \neq -sign\beta_i'\sqrt{(D_i-\alpha_i)\alpha_i},$$

$$\sqrt{\frac{df_i(\delta_i)}{d\delta_i}}\Bigg|_{\delta_i=0} \neq -sign\beta_i''\sqrt{(D_i-\alpha_i)\alpha_i},$$ otherwise it is possible to take other value $\alpha_i \in (0, D_i)$.

Theorem 7. Let exist a vector α_i, scalars $\alpha_{1i}, \alpha_{2i}, \alpha_{3i}, \varepsilon_{1i} > 0$, $\alpha_i \in (0,1)$, $D_i > 0$ such that

1) matrix $\tilde{A}_i$ of Hurwitz,
2) pair $(\tilde{A}_i, B_i)$ it is completely controllable,
3) пара $(\tilde{A}_i, g_i^*)$ it is completely observable,
4) $\Pi_i(j\omega) \geq 0$ $(\forall\omega \in (-\infty, +\infty))$,
5) conditions of the theorem 6 are satisfied.

Then at control

$$u_i = a_i^* x_i + \alpha_{1i}S_i + \alpha_{2i}\delta_i + \alpha_{3i}f_i(\delta_i)$$

area of an attraction of the beginning of coordinates in band G_{0i}, which can be estimated by means of Lyapunov's function (9), it is set by inequality of a look $\tilde{v}_{0i}(\delta_i, S_i, x_i) < \tilde{v}_{0i}$,

Theorem 8. Let conditions of the theorem 4 and theorems 2 be satisfied. Then at control $u_i = a_i^* x_i + \alpha_{1i}S_i + \alpha_{2i}\delta_i + \alpha_{3i}f_i(\delta_i)$ internal assessment of area of an attraction of beginning of coordinates of a special point T_0 is defined by area, a limited surface $\bar{V}(\delta, S, x) = \bar{T}$, where $\bar{T} = \min_{1\le i\le N}\bar{V}(T_i)$, if $T_i, i = \overline{1,N}$ unstable special points of system (11), (12).

Consider function (14) now, where $\tilde{v}_{0i}(\delta_i, S_i, x_i)$ is defined by a ratio (9) in band G_{0i}. Then, combining results of the theorem 7 and 5, it is easy to prove the following theorem.

Theorem 9. Let conditions of the theorem 7 and theorems 5 be satisfied. Then at control $u_i = a_i^* x_i + \alpha_{1i}S_i + \alpha_{2i}\delta_i + \alpha_{3i}f_i(\delta_i)$ internal assessment of area of an attraction of beginning of coordinates of a special point $\bar{T}_0$ определяется областью, is defined by area, a limited surface, $\bar{V}(\delta, S, x) = \bar{T}$, where $\bar{T} = \min_{1\le i\le N}\bar{T}_i$, if $T_i, i = \overline{1,N}$ unstable special points of system (11), (12).

III. Numerical example

Consider the system of the following look

$$\frac{d\delta_i}{dt} = S_i,$$

$$\frac{dS_i}{dt} = -D_iS_i - f_i(\delta_i) - \psi_i(\delta_{i*}) \qquad (15)$$

where function

$$\psi_i(\delta_*) = \sum_{k=1,k\neq i}^{l} P_{ik}(\delta_{ik}), \delta_{ik} = \delta_i - \delta_k$$

defines communication between subsystems and $P_{ik}(\cdot)$ – the set continuously differentiable periodic function.

For l=2 (15) the system has the following appearance:

$$\frac{d\delta_1}{dt} = S_1,$$
$$\frac{d\delta_2}{dt} = S_2,$$
$$\frac{dS_1}{dt} = -D_1S_1 - f_1(\delta_1) - \psi_1(\delta_{1*}), \qquad (16)$$
$$\frac{dS_2}{dt} = -D_2S_2 - f_2(\delta_2) - \psi_2(\delta_{2*}).$$

where

$$f_1(\delta_1) = f_{o1}[\sin(\delta_1 + \theta_{01}) - \sin\theta_{01}],$$
$$f_2(\delta_2) = f_{o2}[\sin(\delta_2 + \theta_{02}) - \sin\theta_{02}],$$
$$\psi_1(\delta_{1*}) = \frac{P_{12}}{T_1}[\sin(\delta_{120} + \delta_{12}) - \sin\delta_{120}],$$
$$\psi_2(\delta_{2*}) = \frac{P_{21}}{T_2}[\sin(\delta_{210} + \delta_{21}) - \sin\delta_{210}].$$

Numerical dates of the system (16):

$D_i = 50.5\cdot10^{-4}$, $H_1 = 2135$, $H_2 = 1256$,
$P_1 = 0.85$, $P_2 = 0.69$, $\theta_{0i} = 0.3562$,
$f_{oi} = 1.513\cdot10^{-4}$.

And the initial conditions

$\delta_1(0) = 0.18$, $\delta_2(0) = 0.1$,
$S_1(0) = 0.001$, $S_2(0) = 0.001$.

To study stability, consider the function

$$V(\delta, S) = \sum_{i=1}^{\ell} v_{0i}(\delta_i, S_i) + \sum_{j=2}^{\ell}\sum_{i=1}^{j-1}\int_0^{\delta_{ij}} P_{ij}(\lambda)d\lambda, \qquad (17)$$

where

$$v_{0i}(\delta_i, S_i) = \frac{1}{2}(S_i + \alpha_iD_i\delta_i)^2 + \frac{1}{2}\alpha_iD_i^2(1-\alpha_i)\cdot\delta_i^2 + F_i(\delta_i) + 2D_i\sqrt{\alpha_i(1-\alpha_i)}\tilde{F}_i(\delta_i) = \frac{1}{2}(S_i + \alpha_iD_i\delta_i)^2 + \int_0^{\delta_i} N_i(\delta_i)d\delta_i$$

defined in band $\bar{G}_{0i}$.

For the numerical solution of the considered task the program module the using interface of application creation Windows Forms written in the C# programming language has been created. In the program for numerical integration of system (16) are used a two-step method of Adams-Bashfort. The formula of this method has following appearance:

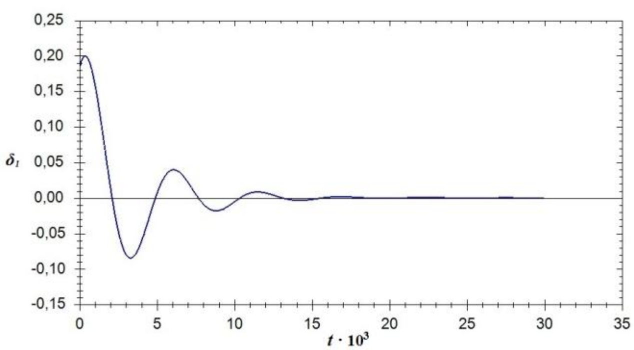
$$y_{i+1} = y_i + h\cdot\left(\frac{3}{2}\left(f\left(x_i, y_i\right)\right) - \frac{2}{1}\left(f\left(x_{i-1}, y_{i-1}\right)\right)\right). \qquad (18)$$

Using a formula (18) rewrite system (16) in a look:

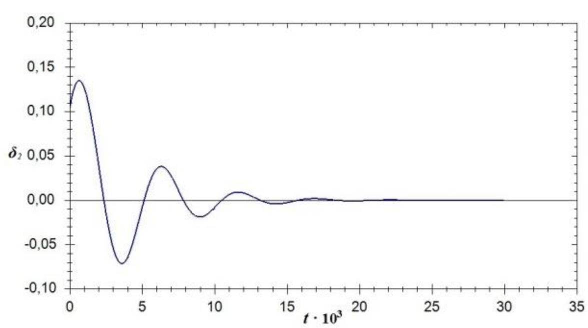
$$\delta_{i+1} = \delta_i + h\cdot\left(\frac{3}{2}S_i - \frac{1}{2}S_{i-1}\right);$$

$$S_{i+1} = S_i + h\cdot\left(\frac{3}{2}\left(-DS_i - f\left(\delta_i\right) - \psi_i(\delta_{i*})\right) - \frac{1}{2}\left(-DS_{i-1} - f\left(\delta_{i-1}\right) - \psi_{i-1}(\delta_{i*})\right)\right);$$

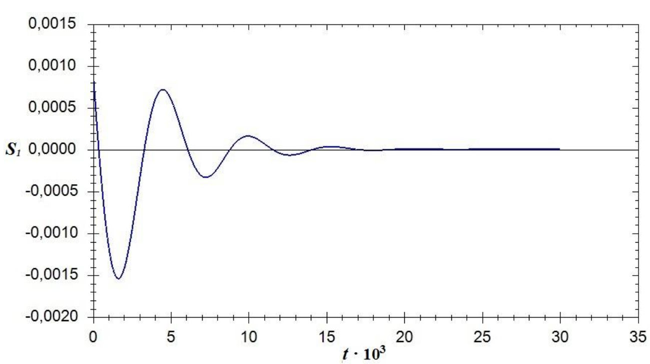
Results of numerical differentiation of system (17) are given below:



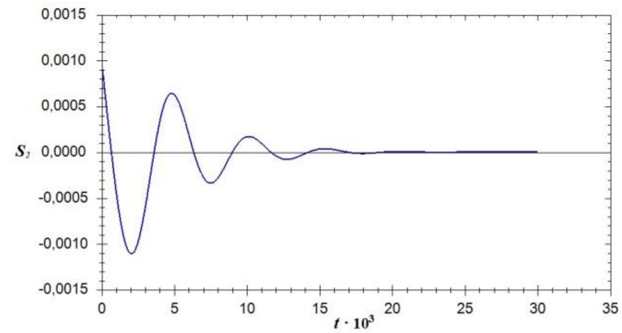
Pic. 1 Graph of parameter δ_1.



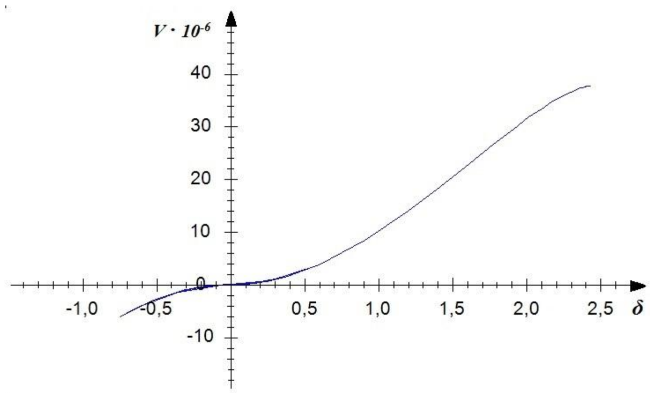
Pic. 2 Graph of parameter δ_2.



Pic. 3 Graph of parameter S_1.



Pic. 4 Graph of parameter S_2.



Pic. 5 Graph of Lyapunov function

IV. Conclusion

In work are investigated asymptotic stability by Lyapunov and assessment of area of an attraction on the basis of the second method of Lyapunov by means of new function of Lyapunov. On the basis of theoretical results received in work computing experiments on concrete examples of electrical power systems, which have shown sufficient efficiency of the offered method for the studied electrical power system are made.

Acknowledgment

This work is partially supported by Grant No3314/GF4 of a Science Committee of the Ministry of Education and Science of the Republic of Kazakhstan.

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