# Optimization of Airport Passenger Throughput

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***Abstract*—Reducing the passenger waiting time at security checkpoints is key to improving passenger experience. It is known that the process of passengers queuing for identity checks corresponds to the M / M / 1 model in queuing theory, and the process of waiting for baggage and body scanning corresponds to the M / G / 1 model. In this model, the ratio of the common security channels to the number of pre-security channels is an important factor affecting the waiting time. The optimal values for a number of parameters are calculated, including the abovementioned ratio and passengers divestment time. When these important factors exceed the optimal value, the waiting time increase very quickly in the model. Whatmore,the model was optimized , in regards baggage and body security inspection process. We detail the optimized model and the time equation of the process. The Monte Carlo algorithm is used to simulate this process and the results are consistent with the variance. The amount of sensitivity analysis of the model, mainly described the regional cultural differences on the impact of the model. Finally, a concrete solution is put forward to solve the problems caused by such factors.**

***Index Terms*—Queue theory, Pre-check, Monte Carlo algorithm,Model**

## I. Introduction

With the development of the civil aviation industry and the diversification of the mode of passenger travel, the proportion of passengers choosing the mode of civil aviation transportation is increasing. The rapid growth of passenger flow and number of flights have brought great pressure on the service efficiency of airport terminals. Under the premise of meeting the needs of passengers through the security channel to the terminal quickly, improving the efficiency of passenger security services, reducing the waiting time of passengers, adjusting allocation and scheduling regulation security service process, enhancing the security system have certain practical significance on improving the efficiency of the terminal passengers stranded for a long time, reducing passenger waiting costs and travelling costs, reducing staff intensive degree ,saving the airport operating costs and so on.

In todays era of pursuing efficiency, time is money, with the change of time,the safety of airports passenger has reduced a lot of threats, so , It is still the most concerned problem of airline and our passengers that the airline should improve the safety and comfort of passengers.However, the the airport passenger checkpoint must screen passengers and their baggage in he presence of baggage and other dangerous items for preventing the aircraft from being destroyed and the safety of passengers during the their trip.Therefore, the airline need minimize the waiting time of passengers in the security checkpoints and maximize the convenience of the passengers.

## II. Assumption

Suppose ID and document check is zone A,unload bags and Personal items is Zone B

TABLE I: Assumptions

| |
|---|
| (1)Each passenger arrives at the security port individually |
| (2)The movement time between process stations is ignored |
| (3)Passengers arriving at the check-in ID and boarding documents window are Poisson |
| (4)Inspectors check identity information time obeys the negative exponential distribution |
| (5)Each luggage and personal belongings are in a single box lonely |
| (6)The instruments of Safety Checking work normally |
| (7)The efficiency of Safety inspectors are in the same way |

## III. Symbol Descriptions

The major symbols in model, as shown in Table1

Notes: we will explain other symbols when we use them.

TABLE II: Notation

| Symbol | Implication |
|---|---|
| $L_s$ | The expected number of customers in the system |
| $L_q$ | The number of customers queued for service in the system |
| $W_s$ | The length of time a passenger spends in the system |
| $W_q$ | The time that a passenger waits in the system |
| $\lambda_1$ | Distribution parameters for the time interval at which passengers arrive at the checkpoint |
| $\lambda_2$ | Distribution parameters for the time interval at which the passenger arrives at area B. |
| $\mu$ | Interval distribution parameters of the identity check |
| $\mu_2$ | The distribute the parameters of time when passengers pass the millimeter-wave scanning |
| $\tau_1$ | Time indicators of ordinary security throughout the process |
| $\tau_2$ | Time indicators of pre-screening of the entire process m Total passenger volur |
| m | Total passenger volume |
| C | The ratio of General security and pre-security channel |
| r | The Percentage of passengers with pre-security check |
| $\alpha$ | Box alarm rate |
| $\beta$ | Body scan alarm rate |

## IV. Model

The airport security process is divided into three parts, the first part is the passengers arrive at the security gate and wait in line to wait for the security card to check their identity documents and boarding documents.The second part is that the passengers move to the next open Check line, waiting for the staff to carry out physical security checks. The third part is the passengers baggage scanning and physical examination until the passengers take back their luggage and leave the security area.

### A. ID and document check(zone A)

As passengers arrived at the airport randomly , we can use queuing theory to build the model which can express the relations of the security time and other factors.

According to the assumptions, we can use the M / M / 1 model of queuing theory (M / M / 1 is the most basic model of queuing theory [?]) to express the passengers queueing up at the security entrance.As shown in Figure 1.

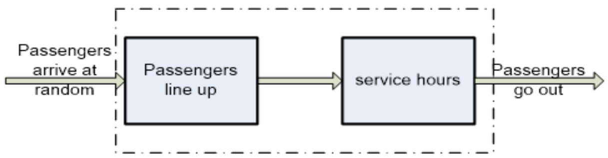



Fig. 1: Queuing theory

$\lambda_1$ is the number of customers arriving on average per unit time, $\mu$ is the number of the served customers per unit time ,$\rho = \lambda_1/\mu$, indicating the service intensity. In the circumstance of the system capacity and passenger source unlimited , we can get the following formula:

$$L_s = \frac{\rho}{1-\rho} \tag{1}$$

$$W_s = \frac{L_s}{\lambda} \tag{2}$$

$$L_q = L_s - \rho \tag{3}$$

The M / M / s model [?]) in queuing theory is similar to M / M / 1, except that the number of service stations in each queue is changed from 1 to s. The distribution of the service time is the same for each service desk in this model.

$$L_q = \sum_{n=s+1}^{\infty} (n-s) * P_n \tag{4}$$

$$W_s = \frac{L_s}{\lambda} \tag{5}$$

$$W_q = \frac{L_q}{\lambda} = W_s - \frac{1}{\mu} \tag{6}$$

To find the average time spent on the ID check process, we should get the distribution parameter $\lambda$ of the passenger arriving time and the distribution parameter $\mu$ of the inspection time. Based on the provided time sample data, the maximum likelihood estimate can be used to estimate the parameter and $\mu$. According to the assumption, the process of passenger arriving is Poisson process, so the interval distribution of the time is exponential distribution, service time is also exponential distribution.$f(x; \lambda_1) = \lambda_1 e^{-\lambda_1 x}$,The likelihood function of the exponential distribution is

$$L(\lambda_1) = \lambda_1^n e^{\lambda_1 \sum_{i=1}^{n} x_i} \tag{7}$$

The maximum likelihood estimate is $\frac{1}{x}$, which is the reciprocal of the sample mean. The estimated value of the parameter $\lambda_1$ is 0.0789 calculated from the sample data, and the estimated value of $\mu$ can be deduced by the same method as 0.0981, which is calculated from the above formula:$L_s = 4.109$ $L_q = L_s - \rho = 3.266$ $W_s = 52.083$ $W_q = 41.394$.It can be seen that the average waiting time for a passenger is about 52 seconds if there is only one window.

In order to compare the traffic, we assume that the total amount of passengers is a certain value. As long as we reduce the security time-consuming of the passengers, we indirectly increase the passengers traffic.Passengers are divided into two types, in addition to ordinary passengers, there is the pre-security passengers. The ordinary passengers and passengers are waiting in different pre-security queuing channel. Assuming that the ratio of the common channel to the pre-security channel is $c:1$, the total number of passengers is m, and the proportion of pre-security passengers is r.The average number of people in the identity check window per ordinary channel is $m * \frac{1-r}{c}$,The average number of pre-security channels is$m{*}r$,so,The time of the ordinary channel:$T_{r1} = \frac{m(1-r)}{c(\mu-\lambda_1)}$;The time of the pre-security channels:$T_{p1} = \frac{m*r}{\mu-\lambda_1}$.

### B. Unload bags and Personal items(Zone B)

After the checking of the ID and boarding documents, the passengers then move to a subsequent queue for an open screening line.The passengers need to queen again. Once the passengers reach the front of this queue, they prepare all of their belongings for X-ray screening. Passengers must remove shoes, belts,jackets, metal objects, electronics, and containers with liquids, placing them in a bin to be X-rayed separately; laptops and some medical equipment also need to be removed from their bags and placed in a separate bin. Assuming that the time taken by a passenger to travel from the identification gate (Zone A) to the baggage and body scan port (Zone B) is ignored, then in Zone B, the passenger arrival queue time may be measured by the time spent by the passenger at the ID checkpoint .Since each passenger is queued to Zone B immediately after the ID check, the time between arrival of the two adjacent passengers is the time spent at the ID checkpoint. Based on the above assumptions, the passenger arrival time interval distribution of the queuing model in region B is also exponentially distributed, where $\lambda_2 = \mu$ of the exponential distribution. But the data does not record each passenger take off shoes, solute belt, take out the laptop computer and the time of the other processes .We will process this as a queue in the service process, the corresponding distribution of service time is arbitrary. Then the queuing process in region B can be applied to the M / G / 1 model in queuing theory.

$$L_s = L_q + L_{se} \tag{8}$$

$$W_s = W_q + E[T] \tag{9}$$

Where the T is the service time, that is the time for the passenger to take off his shoes, solute belts, take out the laptop, etc. If T follows the exponential distribution,$E[T] = \frac{1}{\mu}$ is discussed above. In this model, the relationships of the queuing theory Little formula are often used:

$$L_s = \lambda * W_s \tag{10}$$

$$L_q = \lambda * W_q \tag{11}$$

In the seven indicators, as long as you know the three can find the rest of the indicators. For the M / G / 1 model, the distribution of service time T is general (but the expectation and variance should knowed), and the other conditions are the same as the standard M/M/1 model,where $\rho = \lambda * E[T]$.Under these conditions there are

$$\rho = \rho + \frac{\rho^2 + \lambda^2 Var[T]}{2(1-\rho)} \tag{12}$$

This is the Pollaczek-Khintchine(PK). If we know $\lambda$,E[T] and Var[T],we can get $L_s$, regardless of the distribution of the service time T. We can get the $L_q, W_s, W_q$.

$$W_s = \frac{L_s}{\lambda_2} = E[T_1] + \frac{\lambda^2 E[T_1]^2 + \lambda_2 Var[T_1]}{2(1-\rho)}\rho + \frac{\rho^2 + Var[T]}{2(1-\rho)} \tag{13}$$

From the above formula, we can see that in order to get passengers from queuing to luggage items placed on the conveyor belt on the average time, you must get the mean and variance of service time. Since the speed of the conveyor belt is generally constant, it is assumed that each passenger puts the bag on the conveyor immediately after the baggage has been tidied up, the time interval between the baggage exit X-ray scanning and the passengers luggage placement is the same. The service time of the model is the time interval for the baggage to exit the X-ray scanning, so the mean and variance of the service time can be estimated according to the data in the table. For pre-security passengers, the check-in process for the pre-checked passengers and the regular passengers is the same,except for some improvements in speeding up the design. They also need to remove electronics and medical equipment and fluids for inspection, but do not need to take off shoes, belts and thin jacket and do not need to take out the computer from the bag. The service time of the passenger who is pre-checked is $E[T_1] < E[T_2]$.The respective total time spent on queuing in Zone B for ordinary passengers and pre-security passengers is,

$$T_{r2} = (E[T_1] + \frac{\lambda E[T_1]^2 + \lambda_2 Var[T_1]}{2(1-\lambda_2 E[T_1])}) fracm(1-r)c \tag{14}$$

$$T_{p2} = (E[T_2] + \frac{\lambda E[T_2]^2 + \lambda_2 Var[T_2]}{2(1-\lambda_2 E[T_2])}) m \tag{15}$$

$T_1, T_2$ are the service time of ordinary passengers and pre-security passengers respectively, $E[T_1] < E[T_2]$. In order to achieve steady state, $\rho < 1$ this condition is necessary, that $\lambda_2 E[T_1] < 1$.

### *C. Baggage and body check*

In the third part, the passenger puts all his personal belongings on the conveyor belt, which is moved through the X-ray apparatus. After placing luggage and personal belongings, the passenger passes through a microwave scanner or a metal detector. After the luggage and physical examinations are completed, passengers get their luggage and leave the security area, the security process is complete. The following is a flowchart of the baggage and physical examination. In this process, since the baggage scanning and the body scanning are performed simultaneously and the baggage can be retrieved only after the body is scanned. The time from the putting bag to retrieving the baggage is bigger or equal to the body scanning time. The bag scanning time is used as the total time. The time of the interval between the arrival of the baggage by the passenger and the retrieval of the baggage by the passenger is given in the table. In model 1, the time data are used to estimate the residence time of the process. Through the data for the histogram, only roughly estimate the time interval is subject to normal distribution. Likewise, the maximum likelihood estimate of the parameter is obtained by using the maximum likelihood estimation.$\mu_1 = \overline{x}$ and the normal distribution of $E(x) = \mu_1$, so the expected time of the passengers putting baggage to retrieving can be obtained, the general security and presecurity procedures for the total time indicators were

$$\tau_1 = \frac{m(1-r)}{c}(T_{r1} + T_{r2} + \mu_1) \tag{16}$$

$$= (\frac{1}{\mu - \lambda_1} + E[T_1] + \frac{\lambda_2 E[T_1]^2 + \lambda_2 Var[T_1]}{2(1-\lambda_2 E[T_1])} + \mu_1)\frac{m(1-r)}{c} \tag{17}$$

$$\tau_2 = mr(T_{r1} + T_{r2} + \mu_1) \tag{18}$$

$$= mr(\frac{1}{\mu - \lambda_1} + E[T_2] + \frac{\lambda_2 E[T_2]^2 + \lambda_2 Var[T_2]}{2(1-\lambda_2 E[T_2])} + \mu_1) \tag{19}$$

Here $\tau_1$ and $\tau_2$ are not the total time that these passengers pass through the security check, but as a measure of pre-security and general security as an indicator In order to better reflect the ratio of the common channel to the pre-security channel, which is the relationship between c and the total time $\tau$. We consider a special case where Var [T] = 0, the service time is constant. There are formulas(18)(19) in this case.

### *D. Solution of the model*

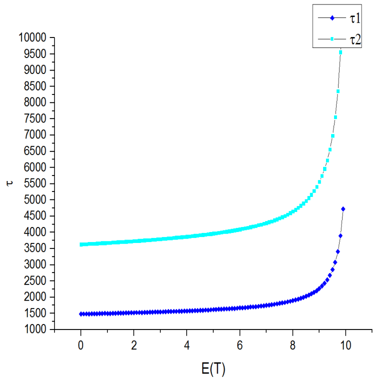
Suppose 100 passengers arriving, m = 100 and 45 percent passengers using the pre-security process, r = 0.45. According to the data provided , we can know $\lambda_1 = 0 : 0789$; $\lambda_2 = 0 : 0981$; $\mu = 28 : 62$, due to

$$\rho = \lambda_2 E[T] < 1$$

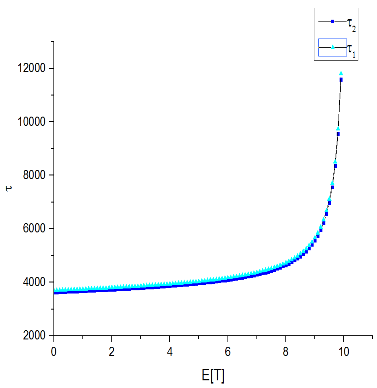
$$E[T] < 10.19$$

In Figure 2 (a), according to the different expectations of passengers stripping time, in most cases. The general time indicators for general security are lower than the pre-security time indicators. Only when the general security expectations are large, pre-security expectations are very small .That is pre-security undetected undeclared time is much smaller than ordinary security, pre-screening is better than ordinary security.

For the airport, this result is clearly unreasonable. According to the above equation, in addition to undress time expectations, the ratio of general security and pre-security check channels




(a)




(b)

Fig. 2: The time indicator changes as the divestment time changes

is also an important factor in the impact of time indicators. In Figure 2 (b), it is clear that the two time indices coincide well when c = 1.2. That is to say when the ratio of the number of pass-through and pre-security channels is 1.2: 1, the two methods have the same effect. When the number of pass-through and pre-security check is less than this ratio, pre-security is better than common security. For passengers, they are more concerned about the average waiting time for each passenger, that is

$$W_1 = \frac{1}{\mu - \lambda_1} + E[T_1] + \frac{\lambda_2 E[T_1]^2 + \lambda_2 Var[T_1]}{2(1 - \lambda_2 E[T_1])} + \mu_1 \quad (20)$$

$$W_2 = \frac{1}{\mu - \lambda_1} + E[T_2] + \frac{\lambda_2 E[T_2]^2 + \lambda_2 Var[T_2]}{2(1 - \lambda_2 E[T_2])} + \mu_1 \quad (21)$$

Here passenger traffic is the number of passengers per unit of time passed, the average waiting time for each passenger is smaller, the greater the traffic. As can be seen from the image, when $E[T] > 9$, the waiting time increases rapidly. So to increase traffic, it is necessary to ensure that divestment time expectations within 9s. Once more than 9s, the average waiting time for each passenger will increase dramatically, the corresponding traffic will be drastically reduced. The bottleneck affecting traffic is the divestment time expectation of 9s.

### *E. The Sensitivity Analysis of Model*

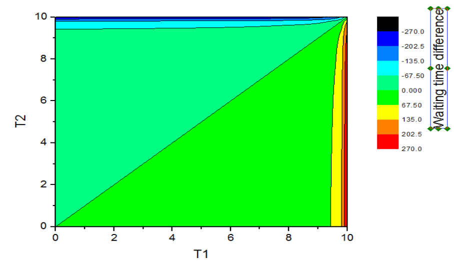



Fig. 3: The change of waiting time

In order to reflect the difference in waiting time between the average passenger and the pre-security passengers divestment time, we can adjust the difference in time to observe the total change in waiting time [?]). After setting some parameters, we can get the Figure 3 that waits for the time with the divestment time. The difference between the waiting times in the graph after about 9s sharply increased, also confirms the above conclusions.As is shown in Figure 3.

## V. Optimization of the Model

In the above model, the baggage scanning and the physical examination process are simplified. The provided data is small, some data are not representative which leading to the actual situation is too large. In order to make the results more accurate, this process needs to be more specific. Model 2 is based on the model 1 on the basis of precision, the first two parts of the model is still in accordance with the model 1. In the third part, security is divided into two parts: baggage scanning and body scanning. Figure 4 shows the specific steps of this process.

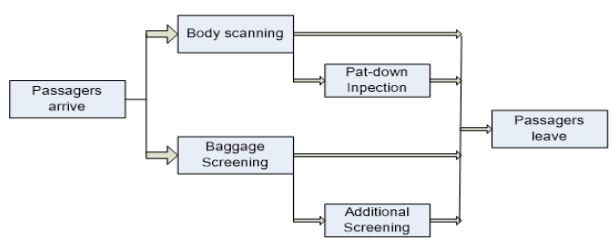



Fig. 4: The flow chart for optimization of the Model

### *A. Baggage scanning*

For the baggage scanning process, the speed of the conveyor belt is usually constant, so that the time for each box bag to travel from the conveyor belt to the end of the conveyor belt is the same. Use the d as the time. X-ray inspection may alarm because of the box has the prohibited .Assuming that the box alarm rate is ?s. After the alarm the staff check the package, assuming that the time distribution of they checking the package is index distribution. Another case is that the probability of baggage with 1 - directly through the X-ray inspection, at the end of the conveyor belt waiting for passengers to take away. The time of the progress is

$$T_b = c + \alpha \times E[t_1] \tag{22}$$

$E(t_1)$ represents the expected value of the baggage checking by the staff.

### B. Body scanning

For the body scanning process, the time interval sequence of the passengers spending on the scanner is obtained according to the schedule when the passenger leaves the millimeter-wave scanner. Through the data histogram, it can be estimated that the time interval is obeying the normal distribution. Through the maximum likelihood estimation of the sample data, the normal distribution parameters $\mu_2$ and $\sigma^2$ are obtained. According to the normal distribution properties,the expected value of the normal distribution is $\mu_2$. Similarly, the body scanning also has the corresponding alarm rate $\beta$, the staff doing pat-down inspection is also an index distribution. The total time of the body scanning is

$$T_p = \mu_2 + \beta \times E[t_3] \tag{23}$$

$E(t_3)$ is the desired value for the time of the pat-down inspection. Since these two processes are simultaneous, the total time is

$$y = \begin{cases} c + \alpha \times E[t_1] \\ \mu_2 + \beta \times E[t_3] \end{cases}$$

### C. Simulation of optimization of the Model

Due to the process of model two is more complex and random, Monte Carlo algorithm can be used to simulate the process. Suppose that the checking time of security staff check the baggage and passengers after the occurrence of the alarm obey the exponential distribution[?]), exponential distribution parameters were $r_1$ and $r_2$. The following is a simulation algorithm specific steps.

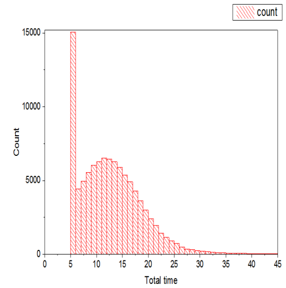
(1) Suppose bag scanning process time is $t1, t1 = 0$, body scanning process time is $t2, t2 = 0$;

(2) Supposing that the time of the baggage pass the X-ray inspection is $t1 = t1 + c$, while the passengers through the microwave scanner or metal detector: t2 increments for the parameter $\mu_2$ and $\sigma^2$ ,the two parameter follow the normal distribution of random numbers;

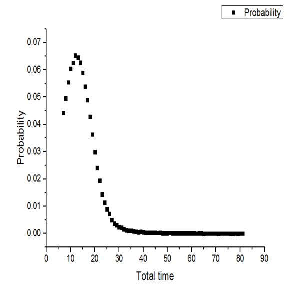
(3) R1 is the random number that obeys the uniform distribution in interval [0,1].If $0 <= R1 < \alpha$,we will get the alarm.$t1 = t1+$exp_rand(r1),exp_rand(r1) represents a random number subject to exponential distribution with parameter r1.

(4) Similarly, R2 is also a random number obeying a uniform distribution in interval [0,1], if $0 <= R2 < \beta$, $t2 = t2+$exp(r2).

(5) Comparing t1 and t2 s, output a larger time. Repeat (1) (2) (3) (4) (5) Using this algorithm, the total time distribution of the process can be obtained, as shown in Figure 5(a) and Figure 5(b).




(a)




(b)

Fig. 5: The total time distribution of the process

The parameters of the inspection time distribution of the baggage and the passengers after the alarm are r1 = 0.1 and r2 = 0.15 respectively. The parameter $\mu 2 = 11.57$,$\sigma = 5.8$ after the maximum likelihood estimation from the known data . In Figure 5(a), it can be seen that the probability of 5 s at the far left of the graph is large, since we assume that the conveyor belts carry baggage to the end at time c = 5 s in which case the time is constant , We will not analyze. In figure 5(b), the frequency distribution with total time of 5 is removed. It can be seen from the figure that the total time increases rapidly when time is less than 15 s, reaches a maximum at about 15 s, and then decreases rapidly. In optimization of the Model, this part of the security checking most of the time In optimization of the Model, most of these security periods are in the interval [5,30], and they are concentrated around 15s. In model, the mean residence time of this process is 28.62s, which is in accordance with the data provided in the dataset table. There are a number of factors that can lead to this gap. Such as the alarm rate, the staff of the baggage and passengers to check the time distribution parameters, etc., because these parameters are estimated by the inevitable and the actual situation, but still able to depict the probability of change of time[?]).

Now we analyze a special case, that is, C = 0, $\alpha = 0$, without considering baggage scanning process. This is often the case in real life, where we often have a longer physical examination time than baggage scanning, and Figure 6 is a frequency distribution that takes into account only the waiting time of the physical examination process. It can be seen from the figure that the time distribution follows a normal distribution. In Model, we also assume a normal distribution, which is consistent with the distribution in the Figure 6.

## THE RECOMMENDATIONS BASED ON THE MODEL

The model cannot completely simulate the reality of the security situation. For example, the service quality of security personnel, the plane arrangement of security environment, the queuing path and the psychology of passengers have great influence on the queuing efficiency and passenger satisfaction of the whole security inspection system. The above analysis can be seen, increasing the number of security checkpoints

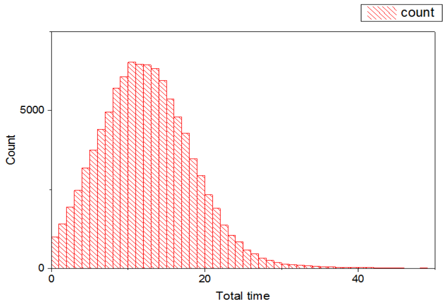



Fig. 6: A frequency distribution that takes into account only the waiting time of the physical examination process

is to a large extent able to ease the security system to deal with stress. Therefore,the airline not only to improve the security checkpoint hardware service capabilities, but also to enhance the stations soft service capabilities [?]).Through the survey found that the entire security inspection process exists many unreasonable phenomenon,such as lines too long ,Some of the following suggestions are given for these irrational phenomena.

(1) Security inspection staff is one of the main body of the service process, its service efficiency directly determines the length of service time and thus determine the size of service efficiency. In the service process, the staff should maintain a good working attitude and service quality.
(2) The airlines can be based on the flight number of passengers on the flight order to inform passengers of the flight in which the specific time period to conduct security checks, of course, which cannot be mandatory, after all, some passengers will have some uncontrollable factors cannot reach the security inspection points in time. So it is recommended.
(3) The airlines set up feedback information system and set the passenger safety evaluation and evaluation of the flight after the flight, the airline improvably based on the passenger experience continuously.
(4) The airlines can also be sub-divided into customer groups, different groups of customers for different services. They can set different security checkpoints such as pre-check in this model, which may increase the waiting time for some people, but the overall service time will improve greatly.
(5) While passengers wait for tickets, airport staff can complete ancillary tasks in order to shorten core service hours such as introducing flight schedules, safety details to be noted, and so on.

## SUMMARY

In this paper we present a queuing model which can predict the optimal number of security checkpoints to open according to the passenger volume. This has the potential to provide cost savings to countless airports and minimize waiting times for passengers. The model can also prescribe the optimal allocation of both Preliminary screening staff and ordinary security staff to reduce the length of the security process. The sensitivity analysis of this model reduces the existing risk of the model. The recommendations from our model are practical and implementable. Finally the model assumptions are modest in terms of the size of dataset and statistical distributions. As a results this model may be applied both to sparse data sets while the methods are easily scalable to much larger data sets. only has modest requirements on, of data distribution and sample size, the number of indicators without strict restrictions.

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