On the behavior of one stochastic automaton in a random environment

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Abstract - A construction (behavior algorithm) of a finite stochastic automaton functioning in a stationary random environment with three classes of reactions (encouragement, punishment, indifference) is proposed. Using the methods of the theory of random walks, formulas are obtained for the generating function of the probability of changing the action and for calculating the probability characteristics of the behavior of an automaton. The convergence of sequences of finite automata (when the memory of the automaton n → ∞) to the corresponding infinite automaton (with a countable number of states) of a similar structure is established and given a classification of his possible behavior in this stationary random environment.

Keywords - behavior algorithm, generating function, expediency of behavior, stationary random environment, stochastic automaton.

I. INTRODUCTION

The problem of finding the optimal choice from a finite set of alternatives with random reinforcement under conditions of a priori uncertainty of both the object itself and its environment was formulated by M.L. Tsetlin as a problem of the behavior of a finite automaton in a random environment [1]. The environment in the simplest case reacts to the actions of an automaton with two methods: either "punishes", or "encourages" the automaton with some probabilities. The automaton a priori information about the environment does not have and its structure must provide some property of symmetry: for an identical sequence of input signals coming in using different actions, the automaton must behave identically. The automaton, on the analysis of signals coming from the external environment, implements a certain learning algorithm, the result of which is the choice of the action for which the probability of receipt of a "punishes" is minimal (this action is considered optimal).

It should be noted that both in [1] and in the works of other authors [see, for example, 2,3], the study of the behavior of automata in stationary random environment is based on the study of the final probabilities chains of Markov describing the functioning of automata in these environment. This approach to the study of the behavior of automata in a random environment led to the fact that the individual behavior of automata was not fully and strictly studied, in particular, there was no complete classification of the possible asymptotic behavior of automata in stationary random environment. Such an analysis turned out to be possible due to the investigation of the behavior of infinite (with a countable number of states) stochastic automata the finding adequate to this behavior of a mathematical apparatus that is and the determination of the convergence (in a reasonable sense) of sequences of finite automata to their corresponding infinite automata [4]. In the implementation of this task, an essential role is played by such statistical characteristics of the behavior of the automaton as the probability of changing (ever) the action and the average time before the change of action [4]. With the help of these characteristics, a complete classification of the possible behavior of automata in a random environment is possible. However, the construction of the automaton, which is the best for some feature in all environments, practically does not exist. Therefore, it is important to construct new designs and develop analytical and numerical methods for finding statistical characteristics of the behavior of wide classes of automata that can be used to solve various practical problems.

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In this paper a construction (algorithm of behavior) is proposed of a stochastic automaton functioning in a stationary random environment with three classes of reactions (encouragement, punishment, indifference). Formulas for the generating function of the probability of a change in the action are obtained and for calculating the probability characteristics of the behavior of the automaton. Is establish the convergence of sequences of finite automata (when the memory of the automaton n → ∞) to the corresponding infinite automaton (with a countable number of states) of a similar structure is established and given a classification of his possible behavior in this stationary random environment.

II. THE FUNCTIONING OF A FINITE STOCHASTIC AUTOMATON $T_{kn,k}(l,m;\varepsilon)$ IN A TERNARY STATIONARY RANDOM ENVIRONMENT

We consider the stochastic automaton $T_{kn,k}(l,m;\varepsilon)$, where $0 \le \varepsilon \le 1$, and l and m are positive integers. The automaton has kn $(n = l + m - 1)$ internal states $L^{(n)} = \bigcup_{i=1}^{k} L_i^{(n)}$, $L_\alpha^{(n)} \cap L_\beta^{(n)} = \emptyset$, $\alpha,\beta = \overline{1,k}$, $\alpha \neq \beta$ and can perform k actions $F_k = \{f_1, f_2, \dots, f_k\}$. All subsets (region) of states $L_\alpha^{(n)}$, $\alpha = \overline{1,k}$ are isomorphic and have the same number of elements n. In the states of the region $L_\alpha^{(n)}$, the automaton commits the action f_α and from any subset of states $L_\alpha^{(n)} \in L^{(n)}$ it is possible to go to any other subset of states $L_\beta^{(n)}, \alpha \neq \beta$. We will consider only cyclic transitions from the states of the subset $L_\alpha^{(n)}$ in the states of the subset $L_{\alpha+1}^{(n)}$ (symbolically $L_\alpha^{(n)} \to L_{\alpha+1}^{(n)}$, where$L_k^{(n)} \to L_1^{(n)}$). It is a deterministic search of all actions in a row.

Let the stochastic automaton $T_{kn,k}(l,m;\varepsilon)$ be placed in a stationary random environment $C(a_1,r_1;a_2,r_2;\dots;a_k,r_k)$. This means that the output signals (actions) f_α of the automaton $T_{kn,k}(l,m;\varepsilon)$ are input signals for some device C, which reacts to the actions of the automaton by the response reactions S, which in turn are input signals for the automaton. The automaton, so to speak, uses them to decide on further actions.

We assume that all possible reactions $S\epsilon\{s_1,s_2,\dots,s_g\}$ of the environment C, in contrast to [1-4], are perceived by the automaton as belonging to one of three classes-the class of favorable reactions (encouragement, $s=+1$), the class of adverse reactions (punishment, $s=-1$) and the class of neutral reactions (indifference, $s=0$). Within each of these classes of reactions of the medium C for the automaton are indistinguishable.

We define the functioning of the automaton $T_{kn,k}(l,m;\varepsilon)$ in the stationary random environment $C(a_1,r_1;a_2,r_2;\dots;a_k,r_k)$ as follows.

Definition1. We say that the automaton $T_{kn,k}(l,m;\varepsilon)$ functions in a ternary stationary random environment $C(a_1,r_1;a_2,r_2;\dots;a_k,r_k)$ if the actions of the automaton and the values of the input signal is connected as follows: if the automaton performs the action $f_\alpha\left(\alpha=\overline{1,k}\right)$, then the medium C forms the value of the signal $s=+1$ at the input of the automaton with the probability $q_\alpha=\frac{1-r_\alpha+a_\alpha}{2}$; $s=-1$ with the probability $p_\alpha=\frac{1-r_\alpha-a_\alpha}{2}$ and the value of the signal $s=0$ with the probability $r_\alpha=1-q_\alpha-p_\alpha$ $(\alpha=\overline{1,k})$.

Here the quantity $a_\alpha=q_\alpha-p_\alpha$ $(|a_\alpha|<1-r_\alpha)$ has the meaning of the mathematical expectation of the payoff for the action f_α in the environment $C(a_1,r_1;a_2,r_2;\dots;a_k,r_k)$. For definiteness, we assume that $a_1>a_2\geq\dots\geq a_k$, i.e. the action of the automaton f_1 with the average of the payoff a_1 in the environment $C(a_1,r_1;a_2,r_2;\dots;a_k,r_k)$ is optimal.

We define the tactic of behavior of the stochastic automaton $T_{kn,k}(l,m;\varepsilon)$ in the ternary stationary random environment $C(a_1,r_1;a_2,r_2;\dots;a_k,r_k)$ as follows.

Let the automaton be in the states $x=\{1,2,\dots,l-1,l,l+1,\dots l+m-1\}$ of the domain $L_\alpha^{(n)}$. At the signal $s=0$, the automaton passes from the state $x=i$ to the state $x=l+1$ $(i=\overline{1,l})$ and from the state $x=l+i$ to the state $x=l+i+1$ $(i=\overline{1,m-2})$; at the signal $s=-1$, the automaton passes from the state $x=l+i$ to the state $x=l-1$ $(i=\overline{1,m-1})$, and from the state $x=i$ to the state $x=i-1$ $(i=\overline{2,l})$; at a signal $s=+1$ all states with probability ε go into themselves or with probability $1-\varepsilon$ go to the state $x=l$. The change in the actions of the automaton occurs from the state $x=1$ and $x=l+m-1$. These states pass to the state $x=l$ of the region $L_{\alpha+1}^{(n)}$ at the signals $s=-1$ and $s=0$ respectively (Fig. 1).

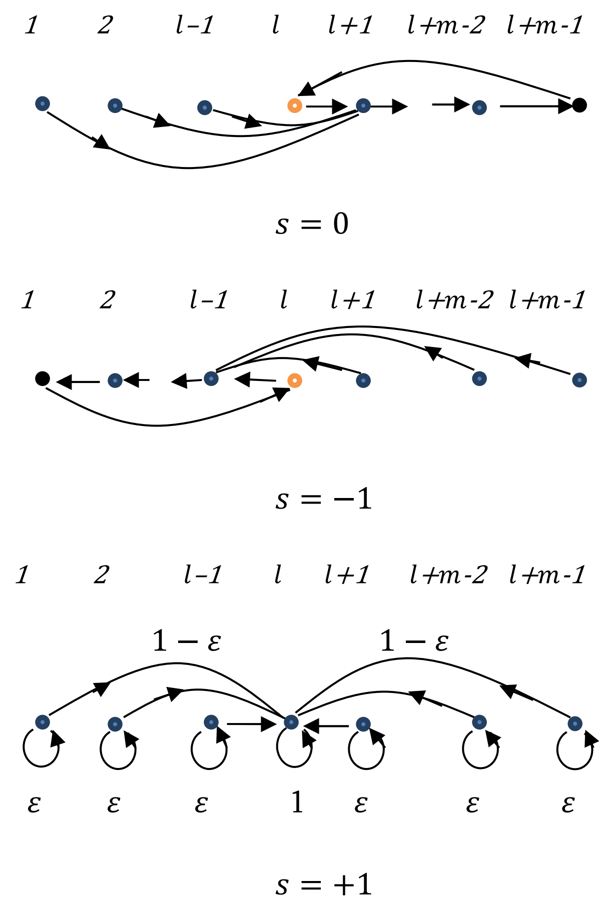

Fig.1. The graph of transitions between the states of the automaton $T_{kn,k}(l,m;\boldsymbol{\varepsilon})$ in the region $L_\alpha^{(n)}$, $\alpha=\overline{1,k}$ at the signal $s=0$, $s=-1$, $s=+1$.

Thus, the automaton $T_{kn,k}(l,m;\boldsymbol{\varepsilon})$ has one input and two outputs: the input state in the region $L_\alpha^{(n)}$, $\alpha=\overline{1,k}$ is the state with the number $x=l$ (light circle), and the output state is states with numbers $x=1$ and $x=l+m-1$ (black circles).

Note that the behavior of the deterministic automaton $T_{kn,k}(l,m;\varepsilon)$ at $\varepsilon=0$ was studied in [5] for a binary stationary random environment (encouragement, punishment), and in [6] - for a ternary stationary random environment (encouragement, punishment, indifference).

To study the possible behavior of an automaton in a stationary random environment $C(a_1,r_1;a_2,r_2;\dots;a_k,r_k)$, are the initial ones the following statistical characteristics of the behavior [4] : the probabilities $\sigma_{x,\alpha}^{(n)}$ change (ever) the action f_α and the mathematical expectations of the random time $\tau_{x,\alpha}^{(n)}$ before the change of the action f_α at the start from the state $x\in L_\alpha^{(n)}$, $\alpha=\overline{1;k}$.

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We denote by $u_{x,d}^{(n)}$ the probability that the automaton $T_{kn,k}(l,m;\boldsymbol{\varepsilon})$ at the instant d changes for the first time the action f_α, starting from any state numbered x of the domain $L_\alpha^{(n)}$.

In what follows we will consider the behavior of the automaton in some region $L_\alpha^{(n)}$ and the index α, for reduce the entries, omit.

Taking into account the behavior of the automaton $T_{kn,k}(l,m;\boldsymbol{\varepsilon})$ in a stationary random environment $C(a_1,r_1;a_2,r_2;\dots;a_k,r_k)$, with respect to the probabilities $u_{x,d}^{(n)}$, we obtain the following difference equation

$$u_{x,d+1}^{(n)} = pu_{x-1,d}^{(n)} + ru_{l+1,d}^{(n)} + (1-\varepsilon)qu_{l,d}^{(n)} + \varepsilon qu_{x,d}^{(n)}, \quad (1)$$
$$x = 1,2,\dots l,$$
$$u_{x,d+1}^{(n)} = pu_{l-1,d}^{(n)} + ru_{x+1,d}^{(n)} + (1-\varepsilon)qu_{l,d}^{(n)} + \varepsilon qu_{x,d}^{(n)}, \quad (2)$$
$$x = l+1,\dots,l+m-1,$$
$$d = 0,1,2,\ \ .\ \ .\ \ .$$

and the boundary conditions following from the probabilistic meaning of $u_{x,d}^{(n)}$

$$u_{0,0}^{(n)} = 1,\ u_{l+m,0}^{(n)} = 1,\ u_{x,0}^{(n)} = 0 \quad \forall x \neq 0, l+m. \quad (3)$$

Multiplying (1) and (2) by z^{d+1} and summing over all $d = 0,1,2,\ .\ .\ .\ .$, with respect to the generating function of the probability of changing the action

$$U_x^{(n)}(z) = \sum_{d=0}^{\infty} u_{x,d}^{(n)} z^d$$

from (1) - (3) we obtain the boundary value problem

$$U_x^{(n)}(z) = pzU_{x-1}^{(n)}(z) + rzU_{l+1}(z) + \quad (4)$$
$$+(1-\varepsilon)qzU_l^{(n)}(z) + \varepsilon qzU_x^{(n)}(z),$$
$$x = 1,2,\dots,l,$$
$$U_x^{(n)}(z) = rzU_{x+1}(z) + pzU_{l-1}^{(n)}(z) + \quad (5)$$
$$+(1-\varepsilon)qzU_l^{(n)}(z) + \varepsilon qzU_x^{(n)}(z),$$
$$x = l+1,\dots,l+m-1,$$
$$U_0^{(n)}(z) = 1,\ U_{l+m}^{(n)}(z) = 1. \quad (6)$$

From (4) - (6) we finally obtain that for the generating function $U_l^{(n)}(z)$ the actions change of the automaton $T_{kn,k}(l,m;\boldsymbol{\varepsilon})$

$$U_l^{(n)}(z) = \frac{f_1(z)+f_2(z)}{1-z+g_1(z)+g_2(z)-g_3(z)}, \quad (7)$$

where

$$f_1(z) = (1-p_\alpha z-\varepsilon q_\alpha z)P_\alpha^l(z)[1-R_\alpha^m(z)],$$
$$f_2(z) = (1-r_\alpha z-\varepsilon q_\alpha z)R_\alpha^m(z)[1-P_\alpha^l(z)],$$
$$g_1(z) = (1-p_\alpha-\varepsilon q_\alpha)z\,P_\alpha^l(z),$$
$$g_2(z) = (1-r_\alpha-\varepsilon q_\alpha)z\,R_\alpha^m(z),$$
$$g_3(z) = [1+(1-2\varepsilon)q_\alpha z]\,R_\alpha^m(z)P_\alpha^l(z),$$
$$P_\alpha(z) = \frac{p_\alpha z}{1-\varepsilon q_\alpha z},\quad R_\alpha(z) = \frac{r_\alpha z}{1-\varepsilon q_\alpha z},\quad \alpha = \overline{1;k}.$$

The functioning of the finite automaton $T_{kn,k}(l,m;\boldsymbol{\varepsilon})$ in the environment $C(a_1,r_1;a_2,r_2;\dots;a_k,r_k)$ is described by a homogeneous ergodic finite Markov chain. For finite automata the probabilities $\sigma_{x,\alpha}^{(n)}$ of the change of action f_α are equal to one, and the mean times $\tau_{x,\alpha}^{(n)}$ are finite in any non-degenerate $(|\ a_\alpha\ | \neq 1-r_\alpha)$ environment $C(a_1,r_1;a_2,r_2;\dots;a_k,r_k)$. Consequently, according to [4], the optimality of the behavior of a finite automaton is excluded and the quality of its behavior is determined by the degree of expediency of its functioning.

Definition2. Following [4], the automaton $T_{kn,k}(l,m;\boldsymbol{\varepsilon})$ has statistically expedient behavior in a stationary random environment $C(a_1,r_1;a_2,r_2;\dots;a_k,r_k)$ if $\sigma_{l,1} < \sigma_{l,\alpha}$ and for $\sigma_{l,1} = \sigma_{l,\alpha}$, $\tau_{l,1} > \tau_{l,\alpha}$, $\alpha = \overline{2,k}$. If $\sigma_{l,1} = \sigma_{l,\alpha}$, $\tau_{l,1} = \tau_{l,\alpha}$, $\forall x$, $\alpha = \overline{2,k}$, then the automaton is called indifferent, and if $\sigma_{l,1} > \sigma_{l,\alpha}$, then the behavior of the automaton is non-expedient.

We note that the statistical characteristics of the behavior of the automaton $T_{kn,k}(l,m;\boldsymbol{\varepsilon})$ - $\sigma_{l,\alpha}^{(n)}$ and $\tau_{l,\alpha}^{(n)}$ are calculated with the help of the generating functions (of course, under the corresponding conditions) by formulas

$$\sigma_{l,\alpha}^{(n)} = U_l^{(n)}(z)\,|_{z=1} = 1,$$

$$\tau_{l,\alpha}^{(n)} = \frac{dU_l^{(n)}(z)}{dz}\,|_{z=1} = \frac{[1-P_\alpha^l(1)][1-R_\alpha^l(1)]}{g_1(1)+g_2(1)-g_3(1)} < \infty, \quad (8)$$
$$\alpha = \overline{1;k}.$$

III. THE FUNCTIONING OF AN INFINITE STOCHASTIC AUTOMATON $T_k(l,m;\varepsilon)$ IN A TERNARY STATIONARY RANDOM ENVIRONMENT

Let us now consider the functioning of the infinite (with a countable number of states) counterparts $T_k(l,m;\boldsymbol{\varepsilon})$ of the automaton $T_{kn,k}(l,m;\boldsymbol{\varepsilon})$ in a stationary random environment $\boldsymbol{C}(a_1,r_1;a_2,r_2;\dots;a_k,r_k)$, the subsets of states L_α $(\alpha = \overline{1;k})$ which are equipotent.

Let $u_{x,d}$ be the probability that the automaton $T_k(l,m;\boldsymbol{\varepsilon})$ at the instant of time d first changes the action of f_α, starting from any state with the number x of the region L_α.

Assume that l is fixed and $m \to \infty$ $(n = l+m-1 \to \infty)$.

Then, taking into account the probabilistic meaning of the quantity $u_{x,d}$ and the construction of the infinite automaton $T_k(l,\infty;\boldsymbol{\varepsilon})$, with respect to the generating function of the probability of changing the action

$$U_x(z) = \sum_{d=0}^{\infty} u_{x,d}\, z^d$$

we have the boundary value problem

$$U_x(z) = pzU_{x-1}(z) + [rz+(1-\varepsilon)qz]U_l(z) + \varepsilon qzU_x(z),$$
$$x = 1,2,\dots,l, \quad (9)$$
$$U_0(z) = 1.$$

The solution to this problem is:

$$U_l(z) = \frac{(1-p_\alpha z-\varepsilon q_\alpha z)\left(\frac{p_\alpha z}{1-\varepsilon q_\alpha z}\right)^l}{1-z+(1-p_\alpha-\varepsilon q_\alpha)z\left(\frac{p_\alpha z}{1-\varepsilon q_\alpha z}\right)^l}.$$

With the help of (9), the probability characteristics $\sigma_{l,\alpha}$ and $\tau_{l,\alpha}$ are calculated:

$$\sigma_{l,\alpha} = U_l(1) = 1, \quad (10)$$

$$\tau_{l,\alpha} = \frac{dU_l(z)}{dz}\Big|_{z=1} = \frac{1-\left(\frac{p_\alpha}{1-\varepsilon q_\alpha}\right)^l}{(1-p_\alpha-\varepsilon q_\alpha)\left(\frac{p_\alpha}{1-\varepsilon q_\alpha}\right)^l} < \infty,$$
$$\alpha = \overline{1;k}.$$

Now let m be fixed and $l \to \infty$ $(n = l + m - 1 \to \infty)$. Then, renumbering the state of the automaton in the reverse order, it is easy to verify that the generating function of the probability of changing the action is a solution of the boundary value problem (9), if in it we replace l by m, p by r and r by p.

The solution obtained has the following form

$$U_m(z) = \frac{(1 - r_\alpha z - \varepsilon q_\alpha z)\left(\frac{r_\alpha z}{1-\varepsilon q_\alpha z}\right)^m}{1 - z + (1 - r_\alpha - \varepsilon q_\alpha) z \left(\frac{r_\alpha z}{1-\varepsilon q_\alpha z}\right)^m}$$

and

$$\sigma_{m,\alpha} = U_m(1) = 1, \qquad (11)$$

$$\tau_{m,\alpha} = \frac{dU_m(z)}{dz}\Big|_{z=1} = \frac{1-\left(\frac{p_\alpha}{1-\varepsilon q_\alpha}\right)^m}{(1-p_\alpha-\varepsilon q_\alpha)\left(\frac{p_\alpha}{1-\varepsilon q_\alpha}\right)^m} < \infty,$$
$$\alpha = \overline{1;k}.$$

If $l \to \infty$ and $m \to \infty$, then the infinite automaton remains forever in that subset of states in which it was at the initial instant of time. In this case $U_l(z) = 0$ and

$$\sigma_{l,\alpha} = 0, \qquad \tau_{l,\alpha} = \infty.$$

Passing to the limit in (7), we obtain that

$$\lim_{m\to\infty} U_l^{(n)}(z) = U_l(z), \qquad \lim_{l\to\infty} U_l^{(n)}(z) = U_m(z),$$
$$\lim_{\substack{l\to\infty \\ m\to\infty}} U_l^{(n)}(z) = U_l(z) = 0.$$

Thus, by the continuity theorem [7], the sequence of finite automata $\{T_{kn,k}(l,m;\boldsymbol{\varepsilon})\}_{l=1}^{\infty}$, $\{T_{kn,k}(l,m;\boldsymbol{\varepsilon})\}_{m=1}^{\infty}$ and $\{T_{kn,k}(l,m;\boldsymbol{\varepsilon})\}_{l,m=1}^{\infty}$ converges to the corresponding limit automata $T_k(\infty,m;\boldsymbol{\varepsilon})$, $T_k(l,\infty;\boldsymbol{\varepsilon})$ and $T_k(\infty,\infty;\boldsymbol{\varepsilon})$ of the same structure and, according to [4], the asymptotic behavior of the finite automaton $T_{kn,k}(l,m;\boldsymbol{\varepsilon})$ in a stationary random environment $C(a_1,r_1;a_2,r_2;\ldots;a_k,r_k)$ is determined by the behavior of the corresponding limit automaton $T_k(l,m;\boldsymbol{\varepsilon})$.

IV. Conclusion

Analyzing formulas (8), (10), (11) and taking into account Definition 2, with respect to the behavior of the automata $T_{kn,k}(l,m;\varepsilon)$ and $T_k(l,m;\varepsilon)$ in a stationary random environment $C(a_1,r_1;a_2,r_2;\ldots;a_k,r_k)$, we can draw the following conclusion.

□. The behavior of a finite stochastic automaton $T_{kn,k}(l,m;\varepsilon)$ in a stationary random environment $C(a_1,r_1;a_2,r_2;\ldots;a_k,r_k)$ is:

1. expedient if

$$\left\{\begin{matrix} p_1 - p_\alpha \le \varepsilon(q_\alpha p_1 - q_1 p_\alpha) \\ r_1 - r_\alpha \le \varepsilon(q_\alpha r_1 - q_1 r_\alpha) \end{matrix}\right\}, \quad \alpha = \overline{2;k}. \qquad (12)$$

at any finite integer values of the quantities l and m.

2. inexpedient if

$$\left\{\begin{matrix} p_1 - p_\alpha \ge \varepsilon(q_\alpha p_1 - q_1 p_\alpha) \\ r_1 - r_\alpha \ge \varepsilon(q_\alpha r_1 - q_1 r_\alpha) \end{matrix}\right\}, \quad \alpha = \overline{2;k}. \qquad (13)$$

at any finite integer values of the quantities l and m.

It should be noted that in expressions (13) and (14) both inequalities are not weak at the same time.

3. indifferent if

$$\left\{\begin{matrix} p_1 - p_\alpha = \varepsilon(q_\alpha p_1 - q_1 p_\alpha) \\ r_1 - r_\alpha = \varepsilon(q_\alpha r_1 - q_1 r_\alpha) \end{matrix}\right\}, \quad \alpha = \overline{2;k}. \qquad (14)$$

4. If (12) or (13) are not satisfied, then the behavior of the automaton $T_{kn,k}(l,m;\varepsilon)$ in a stationary random environment $C(a_1,r_1;a_2,r_2;\ldots;a_k,r_k)$ can be either expedient either inexpedient.

□. The behavior of an infinite stochastic automaton $T_k(l,\infty;\boldsymbol{\varepsilon})$ in a stationary random environment $C(a_1,r_1;a_2,r_2;\ldots;a_k,r_k)$ is expedient if

$$p_1 - p_\alpha < \varepsilon(q_\alpha p_1 - q_1 p_\alpha), \ \alpha = \overline{2;k} \qquad (15)$$

at any finite integer values of the quantity l.

□. The behavior of an infinite stochastic automaton $T_k(\infty,m;\boldsymbol{\varepsilon})$ in a stationary random environment $C(a_1,r_1;a_2,r_2;\ldots;a_k,r_k)$ is expedient if

$$r_1 - r_\alpha < \varepsilon(q_\alpha r_1 - q_1 r_\alpha), \ \alpha = \overline{2;k} \qquad (16)$$

at any finite integer values of the quantity m.

□V. The behavior of an infinite stochastic automaton $T_k(\infty,\infty;\boldsymbol{\varepsilon})$ in any stationary random environment $C(a_1,r_1;a_2,r_2;\ldots;a_k,r_k)$ is indifferent.

According to [4], the asymptotic behavior of the finite stochastic automaton $T_{kn,k}(l,m;\varepsilon)$ is completely determined by the behavior of the corresponding infinite automaton $T_k(l,m;\boldsymbol{\varepsilon})$.

In conclusion, we note that the stochastic automaton $T_{kn,k}(l,m;\varepsilon)$, at integer of the values of the parameter ε ($\varepsilon = 0$ or $\varepsilon = 1$) is a deterministic automaton. Consequently, from (12) - (16) we obtain the condition for the expedient behavior of these automata.

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