# Numerical Study of the Turbulent Natural Convection in a Trapezoidal Cavity: Influence of the Inclination of the Lateral Walls

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**Abstract**— In this paper, we study the heat transfer in turbulent natural convection in a two- dimensional cavity with a trapezoidal section and isoscales filled out of air with as height H =2.5 m. In these conditions, the side walls are differentially heated while the horizontal walls are adiabatic. The k- $\varepsilon$  turbulence model with a small Reynolds number was integrated in our calculation code. The governing equations of the problem were solved numerically by the commercial CFD code Fluent; which is based on the finite volume method and the Boussinesq approximation. The elaborated model is validated from the experimental results in the case of the turbulent flow in a square cavity. Then, the study was related primarily to the influence of the slope of the side walls of the cavity on the dynamic behavior and the heat transfer within the cavity.

*Keywords*— turbulent natural convection, number of thermal Rayleigh, approximation of Boussinesq, isosceles trapezoidal cavity, slopes.

#### I. INTRODUCTION

THE studies of the natural convection in confined cavity have constituted for several years, the object of several research, because of its implication in many natural phenomena and industrialists applications. The majority of anterior works, interesting in the problem of the natural convection, concern the cases of regular enclosures form. Few studies were devoted to irregular forms, although the irregular geometry occurs in several applications to practical interest. Work concerning of the nonrectangular cavities was provided.

Salari et al. [1] studied numerical analysis of a 3D turbulent and transitional natural convection with different turbulence and transition models in a trapezoidal enclosure. The turbulent steady two-dimensional natural convection between inclined isothermal plates has been investigated numerically by Said et al. [2]. Their results indicated that the channel overall average Nusselt number is reduced, the rate of reduction increases as

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Z. D. Author is with Laboratory of Electromechanical Systems (LASEM), National School of Engineers of Sfax (ENIS), University of Sfax, Sfax, Tunisia. Department of chemical Engineering, Sfax, Tunisia. the inclination angle is increased and the overall average Nusselt number at different inclination angles can be presented by a single correlation. The relation between the Noll formulation of the principle of material frame indifference and the principle of turbulent frame indifference in large eddy simulation, is revised by Gallerano et al. [3]. A new rule for the formalization of turbulent closure relations is proposed. The generalized SGS turbulent stress tensor is related exclusively to the generalized SGS turbulent kinetic energy, which is calculated by means of its balance equation, and the modified Leonard tensor. Cannata et al. [4] proposed a numerical model for turbidity currents, based on two-phase flow motion equations. In particular, we proposed three different formalizations of the two-phase flow motion equations. The most general formalization presented is valid for high concentration values. A more simplified formalization introduces the hypothesis of diluted concentrations. The twophase flow motion equations are presented in an integral form in time-dependent curvilinear coordinates. The proposed numerical model has been compared with several experimental validation tests. Furthermore, the numerical model has been used to reproduce the case study of Pieve di Cadore reservoir, under several inflow conditions; the possibility of the formation a turbidity current during several different flood events, has been investigated...Salat et al. [5] investigated experimentally and numerically the turbulent natural convection flow developed in a differentially heated cavity, submitted to a t emperature difference between the active vertical walls equal to 15 K resulting in a characteristic Rayleigh number equal to 1.5 10<sup>9</sup>. Both 2D and 3D LES and 3D DNS are performed. Time-averaged quantities and turbulent statistics in the median vertical plane are presented and compared. Ridouane et al. [6] addressed the turbulent natural convection of air confined in an isosceles triangular enclosure representing conventional attic spaces of houses and buildings with pitched roofs and horizontally suspended ceilings. Turbulence is modeled by a low-Reynolds-number  $k-\varepsilon$  model. Sheremet et al. [7] investigated the natural convection flow and heat transfer in a triangular cavity filled with a micropolar fluid Thier work studied the effects of the vortex viscosity parameter and micro-gyration parameter in a triangular cavity filled with a micropolar fluid on the fluid flow and heat transfer. Varol et al. [8] analyzed numerically the natural convection heat transfer in a triangle enclosure with flush mounted heater on vertical wall. Finite difference

method was used in solution of governing equations in stream function-vorticity form and linear algebraic equations were solved via successive under relaxation. Walsh et al. [9] studied the performance of several commonly used numerical turbulence models such as k-ɛ, Renormalized Group k-ɛ and Reynolds stress model, in predicting heat transfer due to natural convection inside an air-filled cubic cavity. Results of the three turbulence models were compared to experimentally determined values or values from correlations. It was found that the standard k- $\varepsilon$  model was the most effective model in terms of accuracy and computational economy. Aberra et al. [10] employed the direct numerical simulation to investigate the two-dimensional boundary layer instability of a natural convection flow on a uniformly heated vertical plate submerged in a homogeneous quiescent environment. Lee [11, 12] and Peric [13] presented numerical results, up to a Rayleigh number of  $10^5$ , for laminar natural convection in trapezoidal enclosures of horizontal bottom and top walls that are insulated and inclined sidewalls that are maintained at different uniform temperatures.

Studies related to buoyancy-induced heat transfer in partially divided trapezoidal cavities were limited to the ones reported in [14-17]. In their work, Moukalled and Acharya [14,15] dealt with natural convection heat transfer in a partially divided trapezoidal cavity with the partial divider being attached to the lower horizontal base [14] or the upper inclined surface [15] of the cavity. However, two offset partial vertical dividers attached to the upper inclined surface and the lower horizontal base of the cavity, were employed by Moukalled and Acharya [16]. For all configurations, two boundary conditions representing summer-like conditions (upper surface heated) and winter-like conditions (upper surface cooled) were used. The presented Results showed that the presence of baffles decreases the heat transfer. The study reported in [17] differs from the previous ones in the geometry and boundary conditions. In [14-16], the cavity was symmetric in the x-direction and the upper inclined surface was either heated or cooled. (The computations were performed in the left half of the domain and the symmetry boundary condition was applied along the right vertical boundary) In these conditions, [17] the cavity was half the one studied in [14-16] and the left vertical was a wall boundary. In addition, the upper inclined surface was insulated. Similar to the work of Moukalled and Acharya [14], the baffle was attached to the lower horizontal base of the enclosure. The configuration in the present study is similar to the work of Moukalled and Darwish [17] with the baffle, as of Moukalled and Acharya [15], being mounted on the upper inclined surface of the cavity. Hossain et al. [18] investigated the numerical simulation of two-dimensional laminar steady-state on MHD free convection within trapezoidal cavity with non-uniformly heated bottom wall. The cavity consists of a non-uniform heated bottom wall, insulated in the top wall and isothermal side walls with inclination angles. Heat flow patterns in the presence of free convection within trapezoidal enclosures have been analyzed with heatlines concept. Gholizadeh et al. [19] studied numerically a double-diffusive natural convection in a

studied the numerically the effects of the magnetic field on double diffusive natural convection in a trapezoidal enclosure. Both inclined walls and bottom wall were kept at constant temperature and concentration where the bottom wall temperature and concentration are higher than those of the inclined walls. Top wall of the cavity is adiabatic and impermeable. The trapezoidal enclosure is subjected to a horizontal magnetic field. Hasanuzzaman et al. [21] studied the effects of the magnetic field on the natural convection for a trapezoidal enclosure. Both inclined walls and bottom wall have constant temperature where the bottom wall temperature is higher than the inclined walls. Top wall of the cavity is adiabatic. To investigate the effects, finite element method was used to solve the governing equations for different parameters such as Rayleigh number, Hartmann number and inclination angle of inclined wall of the enclosure. Papanicola et al. [22] studied numerically the natural convective heat and mass transfer in an asymmetric, trapezoidal enclosure. Such configuration was encountered in greenhouse-type solar stills, where natural convection in the enclosed humid air is due to vertical temperature and concentration gradients between the saline water and the transparent cover. Bhattacharya et al. [23] analyzed the flow structures and temperature patterns that may evolve during mixed convection within a lid-driven trapezoidal enclosure with cold top wall and hot bottom wall as the speed of moving lid varies with respect to the intensity of imposed temperature gradients. Silva et al. [24] investigated numerically the natural convection in trapezoidal cavities, especially those with two internal baffles in conjunction with an insulated floor, inclined top surface, and isothermal leftheated and isothermal right-cooled vertical walls. The effect of three inclination angles of the upper surface as well as the effect of the Rayleigh number, the Prandtl number, and the baffle's height on the stream functions, temperature profiles, and local and average Nusselt numbers has been investigated. Jannot [25] presented a theoretical study on the transition from laminar to turbulent natural convection flow of gas along a vertical isothermal plate. The analysis of the transition to unstable flows is based on an approximate solution, deduced from the integral formulation of the boundary layer equations for  $Pr \approx 1$ , and on the Rayleigh equation. An approximate disturbance profile was calculated. Lam et al. [26] studied experimentally and numerically the natural convection heat transfer for horizontal prismatic cavities of trapezoidal section having a hot horizontal base, a cool inclined top, and insulated vertical walls. Basak et al. [27] investigated the heat flow patterns in the presence of natural convection within trapezoidal enclosures and with heatlines concept. In their study, natural convection within a trapezoidal enclosure was studied for uniformly and non-uniformly heated bottom wall, insulated top wall and isothermal side walls with inclination angle. Lasfer et al. [28] studied the heat transfer of turbulent natural convection in a two-dimensional cavity to trapezoidal and isoscales section with a height H = 2.5 m filled out of air and whose side walls differentially heated while the horizontal

trapezoidal enclosure with a partial heated active right

sidewall using the finite difference method. Teamah et al. [20]

walls are adiabatic. The model of turbulence k-w with a small Reynolds number was integrated in this code.

The bibliographical study reveals that the majority of work refers to the rectangular or square cavity, differentially heated on the lateral sides or by bottom and cooled by the top. In addition, some relative studies with the triangular profile are published.

Very little cases concerning the trapezoidal cavity exist if it excludes the special case of the solar distiller. The analysis of this configuration leads us to propose a model to treat the three geometries quoted above, the triangle, the trapezoid and the rectangle, by simple change of the inclinations of the active walls. We start then by proposing a numerical model based on the finite volume method in 2D, to obtain fields of temperature, speed and rates of heat transfer similar with those given by some work. The standard k- $\epsilon$  turbulence model was considered and a test bench for the experimental investigations was used. The results obtained are coherent and comparable with those obtained in particular cavity. Turbulence characteristics in this type of cavity constitute original works, which are not meet in the literature.

Thus, the principal aim of the present study is to analyze this phenomenon in a trapezoidal enclosure differentially heated from the inclined wall. The analysis is performed to understand the influences of the different inclined of active sidewalls, the structure of the flow, and the heat transfer performance based on the average Nusselt in the trapezoidal enclosure.

### II. PROBLEM DESCRIPTION GOVERNING EQUATIONS

The physical model considered is shown schematically in Figure 1. The problem consists in studying the flow of air confined in a large, differentially heated cavity (H = 2.5 m), filled with air. The equations of the system are to be solved with the following conditions : The walls inclined relative to the horizontal, in the direction *x*, are maintained isotherms (at different temperatures to ensure a temperature gradient), The right wall is at a cold temperature T<sub>C</sub> and the left wall at a hot temperature T<sub>H</sub>, with T<sub>H</sub>> T<sub>C</sub>. The other two walls are adiabatic, and The adhesion conditions to all walls of the domain are imposed on the velocity fieldThe flow in this cavity is turbulent Ra<sub>t</sub> >10<sup>9</sup>.



Fig. 1 Problem description.



Fig. 2 Typical grid distribution with non-uniform and orthogonal distributions.

#### III. NUMERICAL MODEL

#### A. Governing equation

Using the Boussinesq approximation, and assuming the flow to be turbulent, steady, and two-dimensional with constant fluid properties, except for the induced variations in the body force term, the dimensionally governing transport equations of mass, momentum, and energy are, respectively, written as:

$$\frac{\partial u_i}{\partial X_i} = 0 \tag{1}$$

$$u_{j}\frac{\partial u_{i}}{\partial X_{j}} = -\frac{1}{\rho}\frac{\partial P}{\partial X_{i}} + \frac{\partial}{\partial X_{j}}\left[\left(v + v_{t}\right)\frac{\partial u_{i}}{\partial X_{j}}\right] + g_{i}\frac{\left(\rho_{réf} - \rho\right)}{\rho_{réf}}$$
(2)

$$u_j \frac{\partial T}{\partial X_j} = \frac{\partial}{\partial X_j} \left[ \left( \frac{v}{Pr} + \frac{v_t}{\sigma_t} \right) \frac{\partial T}{\partial X_j} \right]$$
(3)

No single turbulence model can be universally applied to all situations. Some consideration must be taken when choosing a turbulence model including level of accuracy and computation resources available. The k- $\epsilon$  model is one of the most widely used turbulence models as it provides robustness, economy and reasonable accuracy for a wide range of turbulent flows. The standard k- $\epsilon$  turbulence model presented in this section. The turbulent kinetic energy k and its rate of dissipation  $\epsilon$ , for this model are obtained by the following equations:

$$u_{j} \frac{\partial k}{\partial X_{j}} = \frac{\partial}{\partial X_{j}} \left[ \left( \nu + \frac{C_{\mu} \kappa^{2}}{\sigma_{k} \varepsilon} \right) \frac{\partial k}{\partial X_{j}} \right] + \frac{C_{\mu} k^{2}}{\sigma_{k}} \left( \frac{\partial u_{j}}{\partial X_{j}} + \frac{\partial u_{j}}{\partial X_{j}} \right) \frac{\partial u_{j}}{\partial X_{j}} - \varepsilon$$

$$(4)$$

$$u_{j} \frac{\partial \varepsilon}{\partial X_{j}} = \frac{\partial}{\partial X_{j}} \left[ \left( \nu + \frac{C_{\mu} \kappa^{2}}{\sigma_{k} \varepsilon} \right) \frac{\partial \varepsilon}{\partial X_{j}} \right] + C_{1\varepsilon} C_{\mu} k \left( \frac{\partial u_{j}}{\partial X_{j}} + \frac{\partial u_{j}}{\partial X_{j}} \right) \frac{\partial u_{j}}{\partial X_{j}} - C_{2\varepsilon} \frac{\varepsilon^{2}}{k}$$

$$(5)$$

Where  $C_{l\ell}$ ,  $C_{2\ell}$  and  $C\mu$  are the constants that have been determined experimentally and are taken to have the following values:

$$C\mu = 0.09, C_{1\ell} = 1.44, C_{2\ell} = 1.9$$
 (6)

 $\sigma k$  and  $\sigma_{\ell}$  are the turbulent Prandtl numbers for the turbulent kinetic energy and its dissipation rate. These values have also been derived experimentally and are defined as follows.

$$\sigma k = 1, \ \sigma_{\mathcal{E}} = 1.22$$
 (7)  
The turbulent viscosity at each point is related to the local

The turbulent viscosity at each point is related to the local values of turbulent kinetic energy and its dissipation rate by:

$$v_t = C_\mu \frac{k^2}{\varepsilon} \tag{8}$$

Where  $C_{\mu}$  is constant and defined above.

## B. Initial conditions and boundary conditions

The initial conditions and boundary conditions of the system can be defined.

(13)

Following the conditions of adhesion, the velocities are nul, on all the walls:

$$u = 0, v = 0$$
 (9)

$$x = 0 \Rightarrow u = v = 0, T = T_H = 293K$$
(10)

- The right inclined wall is cooled at constant temperature:  $x = L \Rightarrow u = v = 0, T = T_c = 283K$  (11)
- The horizontal walls are adiabatic. Thus, we can write:

$$y = 0, y = H \Rightarrow u = v = 0, \frac{\partial I}{\partial y} = 0$$
 (12)

- The turbulent kinetic energy is nul at the walls: k = 0

## C. Numerical approach

The considered equations are integrated, by the method of finite volumes. We have approximated the diffusive terms by a centered scheme and the convective terms by an upwind scheme. The resulting system of algebraic equations is solved by the Stone procedure [29]. The coupling pressure velocity is based on the SIMPLE algorithm [30]. An irregular grid tightened near to the walls was used for better precision. As it is illustrated in the figure 2.

#### IV. RESULTS AND DISCUSSIONS

The effect of the different inclined angle on the turbulent natural convection of the heat transfer in an enclosure trapezoidal filled of air with Pr = 0.71 was studied for five different inclined angles. Results are presented in the form of streamlines, isotherms, and average Nusselt numbers. The turbulent natural convection in square cavity experimentally by Ampofo et al. [31] and by Lasfer et al. [28] was chosen for the validation of this study.

## A. Comparison with anterior results

Figure 3 compares the experimental and numerical results obtained by [31] and [28] for dimensionless temperature and velocity at mid-height of the cavity, with the numerical results given by considered model. The result indicates an acceptable agreement with the anterior results.



Fig. 3 Comparison of our results with experimental and numerical results

#### B. Inclination angle effect

We performed the numerical study for various angle of inclination  $\alpha$  of the side walls of the cavity. The calculations are lead for a number of Rayleigh,  $Ra_t = 2.5 \ 10^{10}$  and for values of  $\alpha$  between 70° and 110°.

Figure 4 illustrates the distribution of the streamlines, the isotherms and the iso-lines of the turbulent kinetic energy. According to these results, it has been noted that the flow presents a parallel aspect and a p ronounced thermal stratification at the heart of the cavity. The major part of the flow moves on the level of the side walls. This face is translated by the formation of a boundary layer to the proximity of these walls. We can distinguish also that the streamlines become increasingly distorted in the upper part or the lower part of the cavity, respectively, if the inclination is increased or decreased from  $\alpha = 90^{\circ}$ . This structure is accompanied by zones of recirculation in the corner lower right and the left higher corner of the cavity. These zones of recirculation are found by Lasfer [28].



Fig. 4 (a) Streamlines, (b) Isotherms, (c) Iso-lines of the turbulent kinetic energy

## C. Profiles of the average temperature and the velocity

Figures 5 and 6 show the profiles of the average temperature and the vertical component velocity at mid-height of the cavity (Y=0.5). From these results, it has been noted that all the angles have the same average temperature of the air in the heart of the cavity. While going from the hot wall towards the cold wall, the profile of the average temperature shows an abrupt decrease in the temperature near the hot wall, then a stabilization and again a fast decrease near the cold wall. The falls of the temperature are made inside the boundary layers which are developed in the vicinity of the vertical walls. We can notice that the thermal thickness of boundary near to the hot and cold wall decreases with the increase inclined angle. Contrary to the velocity, we note a fast increase of the vertical velocity near the hot wall, and a sudden decrease of it near the cold wall. In the remainder of the cavity, the velocity is uniform. A maximum value of 0.15 m/s is reported. Also, it has been noted that the thickness of the dynamic boundary layer decreases when the inclined angle increases.



Fig. 6 Profile of the vertical velocity with Y=0.5

Figure 7 shows the profile of the horizontal velocity for different values of the inclined angle. From these results, it is clear that the highest velocities are located near the hot wall adjacent to the upper horizontal wall. However, the lowest velocities are located near the cold wall adjacent at the lower horizontal wall. Inside the cavity, and because of the stable thermal stratification, the velocity values are more or less comparable in the different angles. At the ceiling wall, the flow presents itself forms of a jet and the initial velocity is





Fig. 7 Profile of the horizontal velocity

#### D. Thickness of boundary layer

Figure 8 presents the evolution of the thickness of the boundary layer according to the inclined angle. According to these results, it is noticed that the thickness of the thermal boundary layer decreases in the turbulent region when the angle of the hot side wall is raised. The decrease of the maximum temperature and the thickness of boundary layer are due to the decrease in flow because of the appearance of the flow going down again in border of the hot turbulent boundary layer, which feeds the cold boundary layer.



Fig. 8 Evolution of thickness of the boundary layer according to the angle of slope

## E. Turbulent kinetic energy

Figure 9 shows the profile of the turbulent kinetic energy from the rms fluctuations of the horizontal and vertical components of the velocity. From these results, it has been noted that the turbulent kinetic energy is distributed in the boundary layers where it has important values in the zones near to the active walls. Beyond these zones, the turbulent kinetic energy decreases gradually.



Fig. 9 Profile of the turbulent kinetic energy with Y=0.5

### F. Average Nusselt number

The intensity of the heat transfer through the sidewalls is given by figure 10, which presents the variation of the average Nusselt number according to the inclination. For the hot wall, the variation of the intensity of the gained energy is low and has a maximum value for the case where the cavity is rectangular ( $\alpha = 90^\circ$ ). Indeed, the dominant factor that allows this pace is the driving term of the natural convection  $\beta \Delta T g$  $\cos(\alpha)$ . In this case, the intensity is more intense as the slope tends towards 90°. While for the cold wall, an increase in the slope, illustrates a remarkable decrease in the intensity of the lost energy. Indeed, the upper right corner becomes increasingly acute and the length of the upper base of the cavity increases. So, the fluid particles which leaving the hot wall lose enough heat before adhering to the cold wall. In addition, the quantity of fluid imprisoned in the upper right corner obliges a portion of the fluid particles to crawl on the upper basis before adhering partially to the wall.



## G. Maximum streamline

On figure, 11 are presented the values of the maximum intensity of the  $\Psi$ max current, relative to the different values of  $\alpha$ . The profile obtained is largely influenced by the opening angle of the inferior left and upper right corners. Indeed, it has been noted that the profile of the  $\Psi$ max function increases with the slope except for the values of  $\alpha$  ranging between 70°

and 90°. Indeed, if the slope increases, the left lower corner becomes increasingly obtuse and participates to easily transmit the flow to the hot wall in spite of the deceleration caused by the upper right corner. As for the values of  $\alpha$  from 70° to 90°, the maximum intensity of the current intensifies quickly. This face can be explained by the fact that the cold fluid seems to be guided by the stagnant region of the hot fluid in the left pointed corner. As a r esult, the streamline becomes increasingly inclined; indicating that part of the flow is creeping on the lower horizontal wall without passing through the lower left corner whose contribution is to slow down the flow. 0.015



Fig. 11 Profile of maximum streamline according to the slope

## V. CONCLUSION

We have been able to highlight the behavior of the air in turbulent mode at interior of a symmetrical trapezoidal cavity differentially heated. To do this, we used the standard k-E turbulence model. This paper shows a numerical characterization of the thermal and dynamic fields in the core of a large differentiated trapezoidal cavity which the number of Rayleigh characteristic is about 2.52 10<sup>10</sup>. This simulation enabled us to determine the evolution of the thermal stratification in the differentially heated cavity. It comes out from our simulation that the central part of this cavity is thermally stratified. In addition, we have found that the evolution of the thermal stratification at the heart of the cavity is stratified in temperature. This linearity disappears when approaches the ceiling or the floor. The parietal jet mainly explains this. The state of the boundary layers along the hot wall is turbulent. They are relatively thick because of the strengthening of the horizontal movement of the large swirls. The local heat flow was quantified by determining the Nusselt number. We noticed that the increase of the exchange surface, decreases the transfer of heat from  $\alpha = 90^{\circ}$ . In addition, we noticed that the heat transfer gained is greater than that lost, for the values of  $\alpha$  ranging between 90° and 110°.

These results will be used for the design of new solar distiller and air-conditioning system.

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