

# A Calculation Method of Power System Static Stability Margin

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**Abstract:** - This paper establishes a hybrid power network equation with node voltage and branch current as state variables. The characteristic equation representing the stable boundary of the system is derived. Then the boundary conditions of static stability of the power system on the critical circle of voltage static stability are formed. Find the closest distance equation between the load point and the corresponding stable boundary through geometric analysis. The introduction of the distance equation avoids artificially setting the direction of load growth. The distance equation and boundary characteristic equation are added to the hybrid power network equations as additional equations for analyzing the minimum load boundary. The equations are solved by Newton's iterative algorithm. In order to avoid the Jacobian matrix singularity near the load boundary, the algorithm adopts the method of replacing the critical point in the Jacobian matrix to calculate. The simulation results show that the method is correct and effective.

**Key-Words:** - static stability; load margin; Newton method; power system; power flow; branch current

## I. INTRODUCTION

With the large-scale integration of renewable energy into the grid, its uncertainty has brought considerable impact to the grid. As the grid voltage level continues to increase, the scale of the power system has also become huge. Therefore, the operating state of the power system becomes more and more complicated. At the same time, due to insufficient reserve capacity and environmental protection, the grid operating state is getting closer and closer to the boundary of the stability limit.

Especially under heavy load conditions, grid faults caused by disturbances are more likely to spread in a wide range, and even lead to voltage collapse accidents. Therefore, it is urgent to analyze the static stability of the power system, find the weak links, and take targeted prevention.

In order to evaluate the static stability of the power system, scholars have proposed a variety of analysis methods. Finding the nearest load boundary of the power system and then obtaining the minimum load margin is generally considered to be an effective measure of static stability [1, 2].

At present, there are roughly the following methods for solving the nearest load boundary: 1) the method of direct solving. A new concept for finding saddle-node bifurcation points in voltage stability analysis of power systems by applying the extended functional method is proposed, and the main theoretical result establishes the saddle-node bifurcation point applicability for finding the maximum loading capacity of power systems[3]. A multi-period optimal power flow method is proposed, which uses demand response load to improve the steady-state voltage stability and is measured by the smallest singular value of the Jacobian matrix of the power flow[4].

2) Based on optimized methods. The paper proposes a real-time wide-area approach to estimate voltage stability margin of power systems with different possible load change scenarios under normal and contingency operating conditions[5]. Using interval numbers to describe the uncertain wind form output fluctuation and based on the interval optimisation theory, a new static voltage stability margin interval calculation method is

proposed. The established model is solved by the dual theory of convex programming[6].

3) Evolutionary algorithm. The artificial neural network is used to detect the voltage stability of the power system, and a Z-score based bad or missing data processing algorithm is implemented to make the methodologies robust[7]. Genetic algorithm or a combination of genetic algorithm and artificial neural network transforms the problem into an optimization problem[8]. The limit-induced bifurcations has used to provide a high quality approximation of load margin and fast assessment of voltage stability[9].

4) Other methods. A global sensitivity analysis method is proposed to perform a priority ranking of renewable energy variabilities, and the overall procedure is presented to identify critical variables that can affect the variability of load margins in voltage stability analyses[10]. A methodology to assess power system voltage stability margins from time-series data is presented, and the proposed method obtains time-series from dynamic simulations subject to sequential load changes that are then used as an input to a voltage stability estimation method[11].

Among the above analysis methods, the first one is easy to fall into the local optimal solution, and the latter two are formally attractive for solving the problem of the direction of minimum load growth, but the algorithm itself is a difficult point. And there is a problem that it is difficult to be practical. At the same time, most of the above methods for calculating the nearest load boundary of the power system are based on numerical solutions rather than analytical methods. The analytical method can explain the nature of voltage instability more clearly from the theoretical basis.

This paper proposes a solution model for the nearest load boundary of the power system. The model starts from the power network equation represented by the branch current-node voltage as the state variable, and defines the characteristic equation of the nearest load boundary of the power system. On this basis, the distance equation of the nearest load boundary is derived, and it is proved from the geometric sense that the distance equation is the expression of the nearest distance. The equations are solved by the reduced-dimensional Jacobian matrix, and the nearest load boundary of the system is obtained.

## II. POWER NETWORK EQUATION WITH NODE VOLTAGE AND BRANCH CURRENT AS VARIABLES

In the Cartesian coordinate system, when the branch conductance to the ground is ignored, the power network can be described as a mixed form of branch current and node voltage equations, as shown in equations (1) and (2) [12]. For branch  $l$ :

$$\begin{aligned} i_l^a R_{ij} - i_l^r X_{ij} - e_i + e_j &= 0 \\ i_l^a X_{ij} + i_l^r R_{ij} - f_i + f_j &= 0 \end{aligned} \quad (1)$$

For node  $i$ :

$$\begin{aligned} e_i \sum_{l \in i} i_l^a + f_i \sum_{l \in i} i_l^r &= p_i \\ e_i \sum_{l \in i} i_l^r - f_i \sum_{l \in i} i_l^a + (e_i^2 + f_i^2) \sum_{l \in i} B_l &= -q_i \end{aligned} \quad (2)$$

There is an equation of the same form for node  $j$ . Wherein:  $l=1,2,\dots,L$  is branch collection,  $i,j=1,2,\dots,N$  is node collection,  $i_l^a, i_l^r$  are the real part and imaginary part of the current of branch  $l$  respectively,  $e_i, f_i$  are the real part and imaginary part of the voltage of node  $i$  respectively,  $R_{ij}, X_{ij}$  are the real part and imaginary part of the impedance voltage of branch  $l$  respectively,  $B_l$  is the 1/2 susceptance to ground of branch  $l$ ,  $p_i, q_i$  are the active and reactive power injected into the node. Assuming  $x_i = \sum_{l \in i} i_{li}^a$  and  $y_i = \sum_{l \in i} i_{li}^r$  respectively represent the sum of the real and imaginary parts of the node  $i$  injected current (excluding the branch current to the ground),  $B_{i0} = \sum_{l \in i} B_l$  is the sum of the ground susceptance of the branch  $l$  connected to the node  $i$ .

It can be seen from the literature [12] that through the derivation of equations (1) and (2), the PQ node voltage expression can be obtained:

$$\begin{aligned} e_i &= \frac{\left\{ \left[ 2b_{il} p_i x_i - y_i (x_i^2 + y_i^2) \right] \mp \sqrt{\left[ y_i \sqrt{(x_i^2 + y_i^2)^2 - 4b_{il} q_i (x_i^2 + y_i^2)} - 4b_{il}^2 p_i^2 \right]} \right\}}{2b_{il} (x_i^2 + y_i^2)} \\ f_i &= \frac{\left\{ \left[ 2b_{il} p_i y_i + x_i (x_i^2 + y_i^2) \right] \pm \sqrt{\left[ x_i \sqrt{(x_i^2 + y_i^2)^2 - 4b_{il} q_i (x_i^2 + y_i^2)} - 4b_{il}^2 p_i^2 \right]} \right\}}{2b_{il} (x_i^2 + y_i^2)} \end{aligned} \quad (3)$$

In the same way, for PV nodes, the nodal reactive power equation in formula (2) is replaced

by  $e_i^2 + f_i^2 = V_i^2$ , the PV node voltage expression can also be derived:

$$\begin{cases} e_i = \frac{p_i x_i \mp y_i \sqrt{(x_i^2 + y_i^2)V_i^2 - p_i^2}}{x_i^2 + y_i^2} \\ f_i = \frac{p_i y_i \pm x_i \sqrt{(x_i^2 + y_i^2)V_i^2 - p_i^2}}{x_i^2 + y_i^2} \end{cases} \quad (4)$$

The expression of node voltage is expressed by branch current and node injection power, and whether there is a real number solution is determined by the expression in the root of the equation. Therefore, for PQ nodes, the boundary characteristic equation with solvable equations is obtained:

$$(x_i^2 + y_i^2) = 2B_{i0}(q_i + \sqrt{p_i^2 + q_i^2}) \quad (5)$$

And PV node:

$$(x_i^2 + y_i^2)V_i^2 - p_i^2 = 0 \quad (6)$$

Equations (5) and (6) can be expressed as a node voltage critical circle with  $x_i^2$  as the horizontal axis and  $y_i^2$  as the vertical axis. The PQ node is a circle with 0 as the center and  $\sqrt{2B_{i0}(q_i + \sqrt{p_i^2 + q_i^2})}$  as the radius. The PV node is a circle with 0 as the center and  $p_i / V_i$  as the radius. As shown in Figure 1, the point  $b$  represents any PQ or PV node in the system.

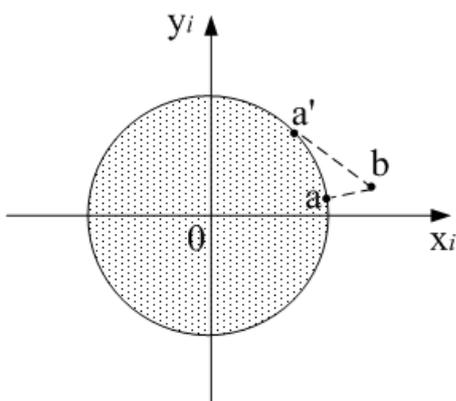


Fig. 1 Node voltage critical round

When the square of the magnitude of the node injected current is on the circle, such as point  $a$ , it reaches the load boundary of the system. Solving the operating state of the system at this time will also find the critical point of the static stability of the power system. There are multiple points on the circle (such as point  $a$  and point  $a'$ ) corresponding to node  $b$ , but there is only one point closest to

point  $b$ , and this closest point represents the minimum load margin. It is also the minimum load margin of node  $b$ . If the load margin of node  $b$  is the smallest among all nodes, then its load margin represents the minimum load margin of the system. Therefore, finding the point  $i$  on the critical circle closest to the point  $a$  is a main goal of this paper.

When conditions (5) or (6) are met, that is, at point  $a$  in Figure 1, the PQ node voltage becomes:

$$\begin{cases} e_i = \frac{2b_{il} p_i x_i - y_i (x_i^2 + y_i^2)}{2b_{il} (x_i^2 + y_i^2)} \\ f_i = \frac{2b_{il} p_i y_i + x_i (x_i^2 + y_i^2)}{2b_{il} (x_i^2 + y_i^2)} \end{cases} \quad (7)$$

The PV node voltage becomes:

$$\begin{cases} e_i = \frac{p_i x_i}{x_i^2 + y_i^2} \\ f_i = \frac{p_i y_i}{x_i^2 + y_i^2} \end{cases} \quad (8)$$

### III. THE DISTANCE EQUATION REPRESENTING THE MINIMUM LOAD MARGIN OF THE SYSTEM

It can be seen from Figure 2 that when any point  $b$  outside the voltage unstable circle is connected to the center of the circle, it intersects at two points  $a$  and  $c$  on the circle boundary.  $a$  and  $c$  are the critical points on the circle corresponding to node  $b$ . And the tangent lines  $l1$  and  $l2$  of the circle passing through points  $a$  and  $c$  are perpendicular to the normal lines passing through the origin and points  $a$ ,  $b$  and  $c$ , and  $l1$  and  $l2$  are parallel. From the perspective of geometric analysis, it can be proved that the closest distance between the  $b$  point and the circle is the  $a$  point, and the farthest distance is the  $c$  point. Therefore, if the equations of  $l1$ ,  $l2$ , and  $l3$  are known, through the mutual perpendicular relationship, the distance equation between the load point  $b$  and the point  $a$  can be obtained, which represents the closest distance between the node  $b$  outside the circle and the boundary of the circle.

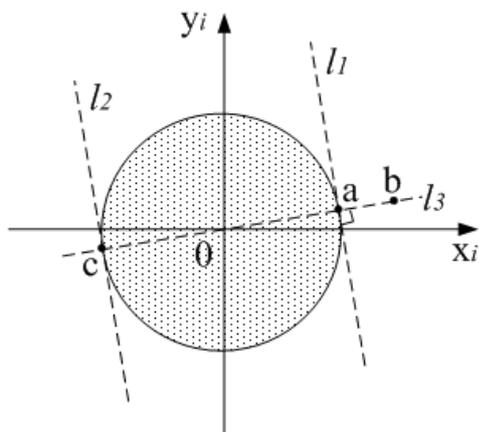


Fig. 2 The minimum load distance

For PQ nodes, from equation (7), the equations of the tangent lines  $l_1$  and  $l_2$  passing through points  $a$  and  $c$  on the circle are:

$$\tan \alpha_a = \frac{\partial y_a}{\partial x_a} = \frac{-x_a}{\sqrt{2B_{a0}(q_a + \sqrt{p_a^2 + q_a^2}) - x_a^2}} \quad a \in N_L \quad (9)$$

Wherein:  $\alpha_a$  is the angle of the tangent to point  $a$  or  $c$  on the circle. Suppose  $N_L$  is the number of PQ nodes and  $N_G$  is the number of PV nodes.

And for PV nodes:

$$\tan \alpha_a = \frac{\partial y_a}{\partial x_a} = \frac{-x_a}{\sqrt{(p_a/V_a)^2 - x_a^2}} \quad a \in N_G \quad (10)$$

And the normal  $l_3$  equation is:

$$\tan \beta_b = \frac{y_b}{x_b} \quad (11)$$

Wherein:  $\beta_b$  is the normal angle between points  $a$ ,  $b$ ,  $c$  and the origin. Because of  $\alpha_a = \beta_b + 90^\circ$ , the distance equation can be obtained for the PQ node:

$$\frac{\sqrt{2B_{a0}(q_a + \sqrt{p_a^2 + q_a^2}) - x_a^2}}{x_a} = \frac{y_b}{x_b} \quad a, b \in N_L \quad (12)$$

The distance equation of the PV node can also be obtained:

$$\frac{\sqrt{(p_a/V_a)^2 - x_a^2}}{x_a} = \frac{y_b}{x_b} \quad a, b \in N_G \quad (13)$$

$y_b$  and  $x_b$  in the equation can be determined by equation (1).

Because the initial state of the power system is stable, the power flow calculation at the location of the initial point  $b$  converges. That is, through the calculation of equation (11) under the initial conditions of the system, the right side of equations (12) and (13) are known. However, the power flow calculation does not converge on the critical circle of node voltage, and the point on the critical circle is exactly what we want to get. But there are many points on the voltage critical circle, which one represents the minimum load margin of the system? The geometric meaning of distance equations (12) (13) expresses the closest distance between node  $b$  and the critical circle of the node. It also specifies the direction in which node  $b$  reaches its closest critical point on the critical circle. The node  $b$  that changes in this direction reaches the system stability critical point  $a$  under the smallest power change, and  $\overline{ab}$  also represents the minimum load margin of the system.

#### IV. ALGORITHM SOLUTION

Solving the power value of the load boundary point  $a$  corresponding to the node  $b$  on the critical circle in Figure 4 is not feasible using the traditional Newton algorithm, because the Jacobian matrix is singular, which causes the power flow calculation to fail to converge. Therefore, the node  $b$  that makes the Jacobian matrix singular in the node voltage equation can be removed, and the node voltage expression (7) or (8) on the load boundary can be directly used, and the node voltage equation is reduced by one dimension.

$$\begin{cases} e_i x_i + f_i y_i = p_i \\ e_i y_i - f_i x_i + (e_i^2 + f_i^2) B_{i0} = -q_i \quad \text{or} \quad e_i^2 + f_i^2 = V_i^2 \\ i_l^a R_{ij} - i_l^r X_{ij} - e_i + e_j = 0 \\ i_l^a X_{ij} + i_l^r R_{ij} - f_i + f_j = 0 \end{cases} \quad (14)$$

Wherein:  $i \in (N_G + N_L)$  and  $i \neq b$ ,  $l \in L$ . Then point  $a$  is derived:

$$\left\{ \begin{aligned} e_a &= \frac{p_a x_a - y_a (q_a + \sqrt{p_a^2 + q_a^2})}{2B_{a0} (q_a + \sqrt{p_a^2 + q_a^2})} \\ f_a &= \frac{p_a y_a + x_a (q_a + \sqrt{p_a^2 + q_a^2})}{2B_{a0} (q_a + \sqrt{p_a^2 + q_a^2})} \\ x_a^2 + y_a^2 &= 2B_{a0} (q_a + \sqrt{p_a^2 + q_a^2}) \\ \frac{\sqrt{2B_{a0} (q_a + \sqrt{p_a^2 + q_a^2})} - x_a^2}{x_a} &= \frac{y_b}{x_b} \end{aligned} \right. \quad \text{or}$$

$$\left\{ \begin{aligned} e_a &= V_a x_a \\ f_a &= V_a y_a \\ x_a^2 + y_a^2 &= \frac{P_a^2}{V_a^2} \\ \frac{\sqrt{(P_a / V_a)^2 - x_a^2}}{x_a} &= \frac{y_b}{x_b} \end{aligned} \right. \quad (15)$$

Among them, equation (14) represents the power network equation, and equation (15) represents the boundary characteristic equation and distance equation of the load boundary point. In the actual calculation, the critical node voltage in equation (15) is substituted into the branch current equation of (14), and the boundary characteristic equation of the node and the distance equation are combined to form the Jacobian matrix of the node voltage equation, and the dimension is reduced by one dimension compared with the original Jacobian matrix.

According to the calculated  $p_a, q_a$ , then:

$$S_a = \sqrt{p_a^2 + q_a^2} \quad \text{or} \quad P_a = p_a \quad (16)$$

Wherein,  $S_a$  or  $P_a$  represents the parameter condition of the load boundary, and represents the boundary of the static voltage stability corresponding to the node  $b$ . After solving the  $p_a$  and  $q_a$  of the point  $a$ , the direction of change of the minimum load of the node  $b$  is determined by the following equation:

$$\delta_b = \arctan \frac{Q_a - Q_b}{P_a - P_b} \quad (17)$$

Where  $\delta_b$  is the angle of node  $b$ 's change. The minimum load boundary distance is determined by Euclidean distance:

$$\eta_b = \|S_b - S_a\| = \sqrt{(P_b - P_a)^2 + (Q_b - Q_a)^2} \quad (18)$$

The calculation result has two solutions, represent the closest distance point  $a$  and the farthest distance point  $c$  respectively, and  $\eta_b$  judges which one is the smallest solution.

For the system shown in Figure 2, when using equations (14) and (15) to find the minimum load margin of the power system nodes, the calculation steps used are as follows:

1) For any given node  $b$  in power network, use the conventional Newton iteration method to calculate the system power flow or the real-time measurement data of the automation system, and calculate  $y_b$  and  $x_b$  from equation (1) as the basic data for calculating the normal line  $l3$  later. If it is a PV node,  $q_b$  is calculated from the reactive power equation in equation (2);

2) Given the initial value of node voltage  $u_i$  and

branch current  $i_l$ ;

3) For the critical point  $a$  to be solved, equation (14) or (15) is used to form an iterative equation with node voltage and branch current as variables to form a reduced one-dimensional Jacobian matrix, and Newton's method is used for iterative solution;

According to the calculated value of critical point  $a$ , the direction and value of the minimum load margin of the power system can be calculated by equations (17) and (18), and the system load boundary conditions can be calculated according to equation (16).

## V. CALCULATION EXAMPLE

Take the IEEE118 node system as an example for verification. The balance node No. 69 and the node No. 118 are swapped, and the calculation is carried out using the per-unit value. Table 1 is the calculation result of the minimum load margin of the system node for the load boundary, and Table 2 is the load margin of the same node on the critical circle for different set power factor angles. Table 1 and Table 2 have selected representative calculation results.

Table 1 The Calculation results of IEEE118 system

No.	$P_b/p$ .u.	$q_b/p$ u.	$P_a/p$ .u.	$q_a/p$ .u.	$\delta_b/p$ u.	$\eta_b/p$ u.
4	0.700	0.710	1.982	1.098	16.84	1.339
8	0.400	-	2.221	0.966	37.77	2.304

		0.445				
11	0.700	0.230	2.090	0.838	23.63	1.517
14	0.140	0.010	2.471	1.128	25.62	2.585
15	0.780	0.273	1.892	1.011	33.57	1.335
21	0.140	0.080	2.360	1.033	23.23	2.416
27	0.700	- 0.228	1.994	1.110	45.96	1.861
32	0.390	0.570	2.226	1.094	15.93	1.909
42	0.980	- 0.170	1.559	0.689	56.02	1.036
44	0.160	0.080	1.871	0.774	22.08	1.846
52	0.180	0.050	1.911	0.779	22.84	1.878
59	1.220	0.361	1.613	0.883	53.02	0.653
90	1.630	- 0.271	2.219	1.187	68.00	1.572
105	0.210	0.416	2.006	1.163	22.58	1.945
108	0.020	0.010	2.490	1.058	22.99	2.683
112	0.680	- 0.285	2.488	1.212	39.62	2.347

It can be seen from Table 1 that for different nodes in the system, the load boundary points can be calculated, and then the value and direction of the minimum load margin of the nodes can be calculated. But the load margins among nodes are still different. For example, the minimum load margin of 59 nodes is relatively small, while the minimum load margin of 108 nodes is larger. This is mainly because the initial load value of the 59 node is relatively large, and it is located in the heavy load area, resulting in a relatively small power value at the critical point of its load. Therefore, the distance between node 59 and the voltage critical circle is relatively close, so the load margin is small. On the contrary, the load margin of 108 nodes is larger.

Table 2 uses node 42 as an example for calculation. Set different  $\delta_b$  values, and when the system satisfies the load boundary characteristic equation, the calculation results of different points on the critical circle of the node voltage are obtained. Equations (14) and (15) do not include boundary distance equations (12) or (13). The initial load value of the node is  $p_b = 0.980, q_b = -0.170$ .

Table 2 The Calculation results of different load direction

No.	$\delta_b$ /p.u.	$p_a$ /p.u.	$q_a$ /p.u.	$\eta_b$
1	56.00	1.559	0.689	1.036
2	32.00	3.887	1.669	3.440
3	37.00	2.590	1.060	2.026
4	48.00	1.782	0.733	1.208
5	33.00	3.221	1.320	2.691

6	48.00	1.844	0.779	1.283
7	39.00	2.872	1.333	2.416
8	51.00	2.382	1.554	2.222

When the value of  $\delta_b$  is given, that is, the direction of load change of the load node  $b$  is given, and the position of the load boundary point on the voltage critical circle is also determined. It can be seen from Table 2 that the load boundary points reached by the load of the same node in different directions are different. The nearest load boundary point 1 can be used as the basis for judging the minimum load margin of the system. The farthest load boundary point 2 can be used as a basis for making power generation plans or demand side load management.

In Table 3, the method proposed in this paper is compared with the continuous power flow method and genetic algorithm in the process of obtaining the minimum load boundary distance. Taking node 21 as an example, it can be seen from the calculation results that the calculation accuracy of the method in this paper is higher.

Table 3 The Comparison with other algorithms

	Method of this article	Continuous flow method	Genetic algorithm
$\delta_b$	23.23	24.51	24.39
$\eta_b$	2.416	2.431	2.423

## VI. CONCLUSION

This paper introduces the branch current as a variable on the basis of the traditional nodal voltage equation to form a hybrid power network equation. On this basis, the load boundary characteristic equation and the minimum load distance equation are added to form a set of solving equations. Calculate the value and change direction of the minimum load margin of the power system. The following conclusions are obtained through simulation calculation:

1) The method proposed in this paper can be applied to the calculation of the minimum load boundary to analyze the static stability boundary margin of the power system.

2) The initial conditions of the node load and the load density of the area have an impact on the calculation results of the load boundary distance, and various factors should be considered in the calculation process.

3) The derived equilibrium solution expression and load boundary equation intuitively express the critical conditions of the static stability of the system, and can be applied to the analysis and calculation of other fields of the power system.

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