Activity of a Neuron brought by Electro-Physical Dynamics

Atsushi Fukasawa and Yumi Takizawa

***Abstract*—**Activity of a neuron is presented and clarified by electro-physical dynamic analysis. A boundary is considered between areas of positive and negative ions. *p*- and *n*-zones are formed across a boundary without electric charges, which is assigned by a depletion layer. Two depletion layers are considered with three thin zones within a neuron. They are assigned as input, central, and output ports. Activity of a neuron is brought by interactions among the zones. By this analysis, it is shown that a neuron operates as an amplifier or as a pulse generator against input stimuli. Discussion is also done for a evaluation of the membrane model given by Hodgkin and Huxley, which was brought by passive process and elements.

***Keywords*—**Activity of neuron, electrical zones, depletion layer, interaction among zones, signal amplification, pulse generation.

I. Introduction

A neural systems of animals operate as fundamental systems to maintain lives and to drive motions of animals. A neural operates with bio-chemical components and electric charges of ions in electrolyte. A neuron behaves as an active element of an amplifier or a pulse generator for input stimuli. But the principle of the operation or internal structure for dynamic operation is little known.

The authors presented the principle of operation of a neuron and a neural group for neural systems. The theory was composed of two processes;

(1) To yield inner electrical structure inside a neuron

Three zones and two depletion layers are formed in electrolyte relating to positive and negative ions to carry signals.

(2) To yield electro-physical dynamics in a neuron

Dynamics of a neuron is brought as the results of interaction among three zones.

The first interaction is done at the first depletion layer to carry positive ions through the potential wall.

By the prier paper[1],[2] by the authors, these process are presented in description because of limited space in the paper.

In this paper, these processes are presented by electro-physical equations. The theory is given by the analysis of the Poisson's equation. The results of this analysis provide the proposed model with the dimensions, equivalent capacities, yield by the depletion layers. The electrical characteristics have already given in equation in the prior paper.

The formulation of operation by a neural group as a neural system is presented in the papers[1],[2], and signal processing of higher function with the synchronous neural system is presented in the other papers[3],[4],[5].

II. Electro-physical analysis in electrical medium

A. Motion of Charges in Electrolyte

When electric charges are injected into a zone in electrical medium, charge density at the zone becomes higher and the other zone remains lower. It is assumed that quantity of injected charges is little and velocity of charges is low in the medium. Special phenomena are induced at a boundary between two zones as shown in Fig. 1.

Phase 1

Injected *p*-charges diffuse to *n*-zone, and *n*-charges diffuse to *p*-zone by the force of gradient of density F_D.

Phase 2

Coulomb's force F_C (force by potential gradient) appears between diffused *p*- and *n*-ions. Directions of forces F_D and F_C are opposite. When they are balanced, diffusion is ceased.

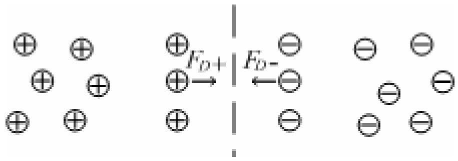
Phase 3

A pair of space charges appears at both sides of the boundary.

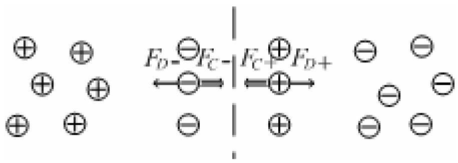
Potential difference appears in the boundary. And electric charges are driven outside the boundary, and two zones and a depletion layer formed at the boundary.

Atsushi Fukasawa was a professor, Chiba University, Yayoi-cho Inage-ku Chiba, 263-8522 Japan (e-mail: takizawa@ism.ac.jp)

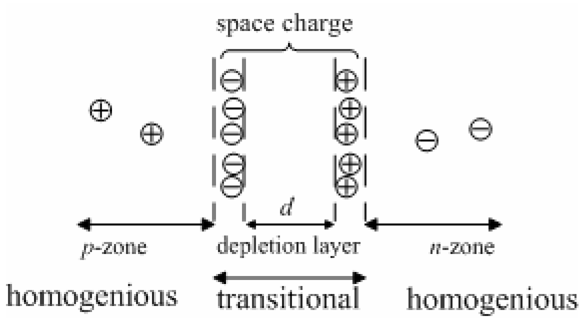
Yumi Takizawa is with the Institute of Statistical Mathematics, Tachikawa , Tokyo 190-8562, Japan (phone: 81-50-5533-8539; fax: 81-42-526-4335; e-mail: takizawa@ism.ac.jp).



Phase 1: Diffusion of charges by gradient of density F_D .

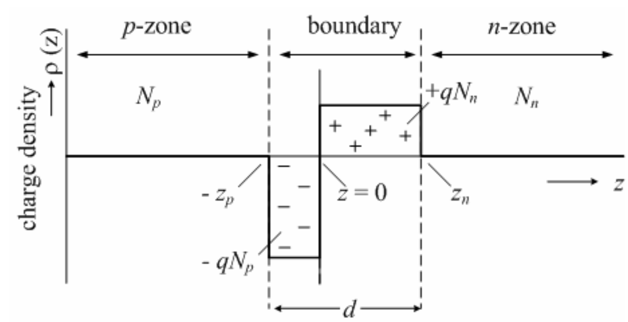


Phase 2: Balance of diffusion F_D and Coulomb's force F_C .

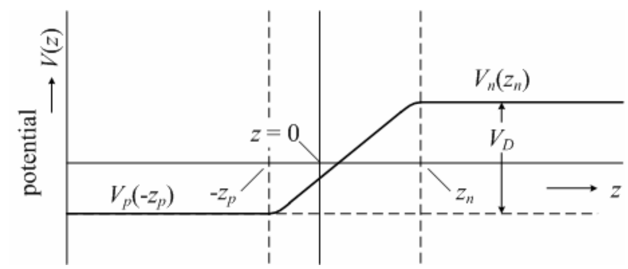


Phase 3: Cease of diffusion and formation of;
(a) p-zone and n-zone, and
(b) space charges and depletion layer with depth *d*.

Fig. 1 Formation of zones and a depletion layer of a boundary.



(a) Distribution of true electric charge density ρ(*z*).



(b) Potentials *Vp*, *Vn*, and diffusion potential V_D .

Fig. 2 Distribution of true electric charge and diffusion potential of a boundary.

B. Electro-physical Analysis of True Electric Charge Densities

Potential *V*(*z*) at a boundary is decided by true electric charge density $\rho(z)$ based on the Poison's equation.

$$\frac{d^2V(z)}{dz^2} = -\frac{\rho(z)}{\varepsilon_d} \qquad (1)$$

where, *z* is the longitudinal axis of a neuron, ε_d is permittivity of electrolyte solution.

True electric charge is defined as the charge unrestrained to any place. Then polarization charge at the membrane is removed from $\rho(z)$, because the polarization charge is restrained to the membrane in a neuron.

Distribution of true electric charge and diffusion potential of a boundary are shown in Fig.2.

True electric charge density $\rho(z)$ is given by the followings, and is shown in Fig.2 (a).

$$\left.\begin{aligned} \rho(z) &= -q\,N_p \quad ; \quad -z_p \le z \le 0 \\ \rho(z) &= +q\,N_n \quad ; \quad 0 \le z \le z_n \end{aligned}\right\} \qquad (2)$$

where *Np*, *Nn* are true electric charge densities at *p*- and *n*-side of the boundary. *q* is elementary electric charge.

The electric field dV(z)/dz is impressed at the boundary.

$$\frac{dV(z)}{dz} = 0 \quad ; \quad z = -z_p, \quad z = z_n \qquad (3)$$

$\rho(z)$ is approximately zero outside the boundary.

$$\rho(z) = 0 \quad ; \quad z < -z_p, \quad z > z_n \qquad (4)$$

Substituting Eq.s (2), (3) for Eq. (1),

$$\frac{d^2V_p(z)}{dz^2} = +\frac{q\,N_p}{\varepsilon_d} \qquad (5)$$

$$\frac{d^2V_n(z)}{dz^2} = -\frac{q\,N_n}{\varepsilon_d} \qquad (6)$$

where, Vp, Vn are the values of V(z) at p- and n-zone, Np, Na are the charge density diffused in p- and n-zone from each apposite side, q is the elementary electric charge of ions with balance 1.

By integration Eq.s (5) (6),

$$\frac{dV_p(z)}{dz} = \frac{q\,N_p z}{\varepsilon_d} + A_p \quad ; \quad -z_p \le z \le 0 \quad (7)$$

$$V_p(z) = \frac{q\,N_p z^2}{2\varepsilon_d} + A_p z + B_p \quad ; \quad -z_p \le z \le 0 \qquad (8)$$

$$\frac{dV_n(z)}{dz} = \frac{q\,N_n z}{\varepsilon_d} + A_n \quad ; \quad 0 \le z \le z_n \qquad (9)$$

$$Vn_p(z) = -\frac{q\,N_n z^2}{2\varepsilon_d} + A_n z + B_n \quad ; \quad 0 \le z \le z_n \qquad (10)$$

A, B are conditions of integration. At the boundary z = o, the potential and its gradient (electric field) are continuous, then

$$V_p = V_n \qquad (11)$$

$$\frac{dV_p(z)}{dz} = \frac{dV_n(z)}{dz} \quad ; \quad z = 0 \qquad (12)$$

then,

$$A_p = A_n = A \qquad (13)$$

$$B_p = B_n = B \qquad (14)$$

The potential V(z) is applied only in the boundary and all zero outside the boundary.

$$\frac{dV_p(z)}{dz} = 0 \quad ; \quad z = -z_p \qquad (15)$$

$$\frac{dV_n(z)}{dz} = 0 \quad ; \quad z = +z_n \qquad (16)$$

then,

$$A_p = A_n = \frac{qN_p z_p}{\varepsilon_d} = \frac{qN_n z_n}{\varepsilon_d} \qquad (17)$$

and

$$N_p z_p = N_n z_n \qquad (18)$$

The total amount of n-charges at a side is equal to the amount of p-charges at another side of the boundary.

If a condition $N_p >> N_n$ stands, $z_p << z_n$ as shown in Fig.2 (a).

Substituting Eq. (17) for Eq.s (7)~(9),

$$\frac{dV_p}{dz} = \frac{qN_p}{\varepsilon_d}(z + z_p) \qquad (19)$$

$$V_p = \frac{qN_p}{2\varepsilon_d} z^2 + \frac{qN_p z_p}{\varepsilon_d} z + B \qquad (20)$$

$$\frac{dV_n}{dz} = \frac{qN_n}{\varepsilon_d}(z_n - z) \qquad (21)$$

$$V_n = -\frac{qN_n}{2\varepsilon_d} z^2 + \frac{qN_n z_n}{\varepsilon_d} z + B \qquad (22)$$

B is an arbitrary value.

The diffusion potential V_D is given as follows.

$$V_D = V_n(z_n) - V_p(-z_p)$$
$$= \frac{q}{2\varepsilon_d}(N_p z_p{}^2 + N_n z_n{}^2 \quad (23)$$

The depth of depletion layer is given as,

$$d = z_p + z_n \quad (24)$$

z_p and z_n are calculated by Eq.(18) and (24).

$$N_p z_p = N_n z_n \quad (18)$$

$$z_p + z_n = d \quad (24)$$

$$N_p z_p = N_n(d - z_p)$$

$$(N_p + N_n) z_p = dN_n$$

$$z_p = \frac{N_n}{N_p + N_n} d \quad (25)$$

$$z_n = \frac{N_p}{N_p + N_n} d \quad (26)$$

Substituting Eq.s(25) and (26) for Eq. (23),

$$\frac{2\varepsilon_d}{q} V_D = N_p z_p{}^2 + N_n z_n{}^2$$
$$= N_p \left(\frac{N_n d}{N_p + N_n} \right)^2 + N_n \left(\frac{N_p d}{N_p + N_n} \right)^2$$
$$= \frac{N_p N_n}{N_p + N_n} d^2$$

$$d^2 = \frac{2\varepsilon_d}{q} \frac{N_p + N_n}{N_p N_n} V_D$$

$$d = \left(\frac{2\varepsilon_d (N_p + N_n)}{qN_p N_n} V_D \right)^{\frac{1}{2}} \quad (27)$$

When $N_p >> N_n$,

$$d = \left(\frac{2\varepsilon_d}{qN_n} V_D \right)^{\frac{1}{2}} \quad (28)$$

Now, bias *VB* is assumed applied to a boundary. When *VB* is reversely applied to *n*-zone, the followings are easily obtained.

$$d = \left(\frac{2\varepsilon_d (N_p + N_n)}{qN_p N_n} (V_D + V_B) \right)^{\frac{1}{2}} \quad (29)$$

$$Q = qN_n z_n = \left| -qN_p z_p \right|$$
$$= \left(\frac{2\varepsilon_d qN_p N_n)}{N_p + N_n} (V_D + V_B) \right)^{\frac{1}{2}} \quad (30)$$

Equal charges appear with opposite signs at the boundary. The structure is assumed as an equivalent capacitor.

$$c = \left| \frac{dQ}{dV} \right| = \left(\frac{\varepsilon_d}{2} \frac{qN_p N_n}{N_p + N_n} \frac{1}{V_D + V_B} \right)^{\frac{1}{2}} \quad (31)$$

If a condition $N_p >> N_n$ stands,

$$d = \left(\frac{2\varepsilon_d}{qN_n} (V_D + V_B) \right)^{\frac{1}{2}} \quad (32)$$

$$c = \left(\frac{\varepsilon_d}{2} \frac{qN_n}{V_D + V_B} \right)^{\frac{1}{2}} \quad (33)$$

When V_B is applied forward at the boundary, the height of the wall is reduced.

III. ELECTRO-PHYSICAL MODELING OF A NEURON

A. Whole Aspects of a Neuron

A neuron is exhibited as a three-port bio-electrical device with dendrite, central part, and axon terminal.

These ports are assigned as input, ground, and output ports. The ends of dendrite and axon terminal are composed of multiple branches which are connected to previous and post neurons with synapses. Biochemical and electrical couplings are formed by synapses. A bio-electrical modeling is given in Fig. 3. In the figure, an excitatory synapse is shown. An inhibitory synapse is presented later.

B. Ion Injection to a Resting Neuron

During a neuron is resting, inner potential is kept negative and uniform inside the neuron. When neurotransmitters are released from previous neurons and accepted by the neuron, *p*-charges of Na^+ are injected into the dendrite. Injected *p*-ions play as an excitatory signal into the neuron.

C. Motion of Signal p-ions at the First Depletion Layer

The first depletion layer is formed between the dendrite and the central parts.

The depth d_1 is given as;

$$d_1 = \left\{ \frac{2\varepsilon_d \ (N_d + N_{c1})}{q \ N_d \ N_{c1}} (V_{D1} + V_{B1}) \right\}^{\frac{1}{2}} \quad (34)$$

where, *Nd*, *Nc*1 are true electric charge density at the dendrite and input end of the central part. V_{D1} V_{B1} are diffusion potential and bias appeared at the first depletion layer.

The depth d_1 is lower as *p*-ion increases at the central part. Then signal *p*-ions pass over the first depletion layer easily.

D. Motion of Signal p-ions at the Second Depletion Layer

The second depletion layer is formed between the central part and the axon terminals.

The depth d_2 is given as;

$$d_2 = \left\{ \frac{2\varepsilon_d \ (N_{c2} + N_a)}{q \ N_{c2} \ N_a} (V_{D2} + V_{B2}) \right\}^{\frac{1}{2}} \quad (35)$$

where, *Nc*2 and *Na* are true electric charge density at the dendrite and input end of the central part. V_{D2} V_{B2} are diffusion potential and bias appeared at the second depletion layer.

The depth d_2 is higher than depth *d*1 depending on high and negative bias V_{B2} inside the original neuron.

Signal *p*-ions pass over the second depletion layer by the force of thermal motion of ions.

E. Motion of Signal n-ions

When signal *p*-ions arrive at the axon terminal, *n*-ions are injected into the axon terminal by Cl channels.

n-ions move from the right to the left passing over the second and then the first depletion layers. The motion of *n*-ions from right to left is forward, and from left to right is reverse.

These *n*-ions play also as the signal together with signal *p*-ions. The *p*- and *n*-ions carry signals to the same direction with the principle of duality.

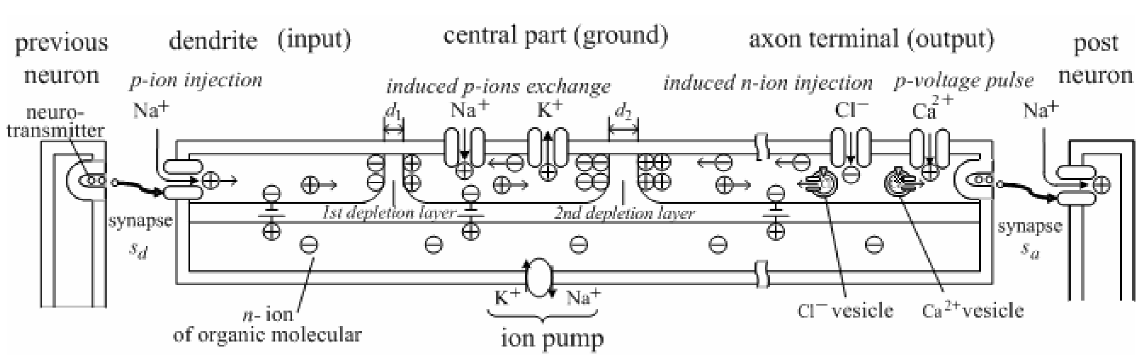


Fig. 3 Bio-electric modelling of an activated neuron. Leakage channels are abbreviated.

IV. Electrical analysis of activity of a neuron

A. Energy Diagram of Electrical Charges

Energy of *p*- and *n*-ions in a neuron are illustrated in Fig. 4. The energy of *p*- and *n*-ions are assumed with a small difference to Fermi level as shown in the figure.

Cl channels at the axon terminal inject *n*-ions to left at the second depletion layer passing over a slope shown in the figure.

The duality of motions of *p*- and *n*-ions is well informed by tracing the curve to right (*p*-ion) and to left (*n*-ion). The three port configuration is kept in spite with a slope at the axon terminal (refer to [1],[2]).

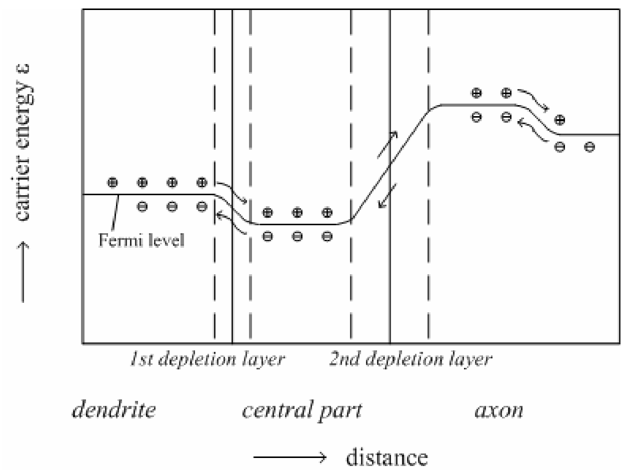


Fig. 4 Energy diagram of negative and positive ions with Cl channels at axon terminal.

B. Electrical Modeling of an Active Neuron

Electrical modeling of an operating neuron is shown in Fig.5.

i_d is the current of *p*-ions injected in the dendrite, i_a is the current of sum of arrived *p*-ions and *n*-ions injected by Cl-channels at the axon terminal. i_c is the current through resistance R_c of the central part to the outside of a neuron.

α is current multiplication factor and $\alpha \cdot i_d$ is equivalent current source for the axon terminal.

The points of d_0, a_0 are outside of membrane. c_0 is a virtual point taken in the central part. r_d and r_a are resistances of forward diode n_d and reverse diode n_a, r_c is the resistance at the central part to outside of a neuron. R_d and R_a are external resistances of synapses s_d and s_a..

$r_d << R_d$ and $r_a << R_a$. r_c is approximately zero.

The capacities C_d and C_a are caused by the first and second depletion layers respectively.

Input and output synapses s_d and s_a.are shown as forward diodes for excitatory synapses (*p*-ions). These synapses work as backward diodes for inhibitory synapses (*n*-ions).

C. Characteristics as an Active Element

An active neuron operates as an amplifier, when input potential height at the dendrite is lower than the potential height of barrier of the first depletion layer.

An active neuron operates as a pulse generator, when input potential height at the dendrite is higher than the potential height of barrier of the first depletion layer.

These electrical characteristics are shown in the paper by the authors[1],[2].

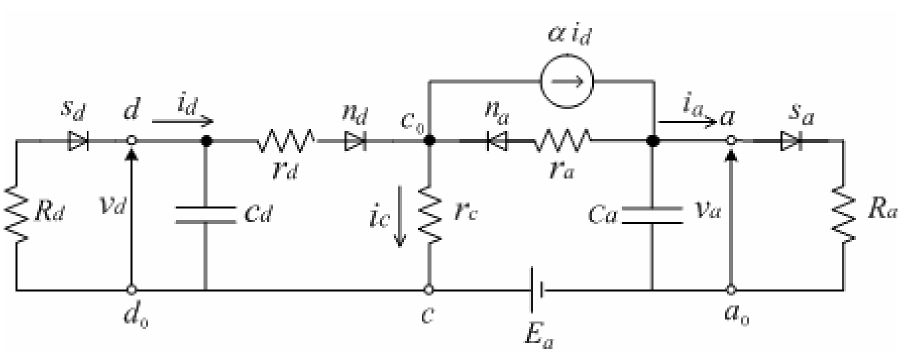


Fig. 5 Electrical modeling of an operating neuron.

V. Validity of the Membrane Model by Hodgkin-Huxley

The membrane model was given by Hodgkin and Huxley[6], which is widely accepted as a common knowledge. This model is based on the following ideas.

Activity of a neuron is brought by the motion of Na^{2+}- and K^{+}-charges across the membrane through their ion channels.

The membrane model is shown as an electric circuit with parallel connections of a membrane capacity C_M and three resistances R_{Na}, R_K, R_L with batteries E_{Na}, E_K, E_L for Na, K, and leakage channels (Fig. 1 in [6]).

This circuit is connected to outside and inside across the membrane of a neuron.

Now, three orthogonal axes *x*, *y*, *z*, and the input and the output ports are considered for a neuron. The *x* and *y* axes are for the transversal directions of a neuron. The *z* axis is assigned for the longitudinal direction of a neuron, and the direction of signal transmission.

The membrane model is shown along transversal plane (*x*, *y*) across a membrane. The current signal flows between outside and inside of a neuron. This model is a two-terminal or two-port circuit. It is known that stimulus at the dendrites is input signals and output pulse is given at the axon terminals. But the electric circuit in Figure 1 in [7] shows no difference between two ports.

Hodgkin and Huxley used this electrical circuit for generation of pulse waveforms by a neuron. Equations given by Hodgkin-Huxley are corresponding to charge-discharge equations of the circuit with a capacity, resistances, and batteries. Furthermore Membrane model was provided with wrong knowledge of electro-physics.

The followings are essential errors on electro-physical theories. The membrane model is shown in Fig. 6 also in another way to understand the process by Hodgkin-Huxley with the description by F. Delcomyn[8].

(1) Active element is not included in its electrical circuit.

This circuit is a certain circuit for charge – discharge operation. This circuit does not include any negative resistance nor other active elements. This circuit does not belong to any active circuit.

(2) Polarization at the membrane is not included in the true electric charge density $\rho(z)$ of the Poisson's equation (Chapter II).

The potential and the current to transmit signals are defined by the true electric charge density $\rho(z)$. The polarization charges cannot move freely, because they are almost fixed on the membrane. Polarization charges on the membrane must be deleted from the true electric charge density $\rho(z)$ as described in Chapter 2 in this paper.

In addition to the above problems, the first practical systems of telecommunication and computer were developed by invention of three-port active devices (thermal electron emission triode and PNP junction transistor[9]). These actual systems could not be composed by gas tube or semiconductor diodes (two ports element) including the Tunnel diode by L. Esaki.

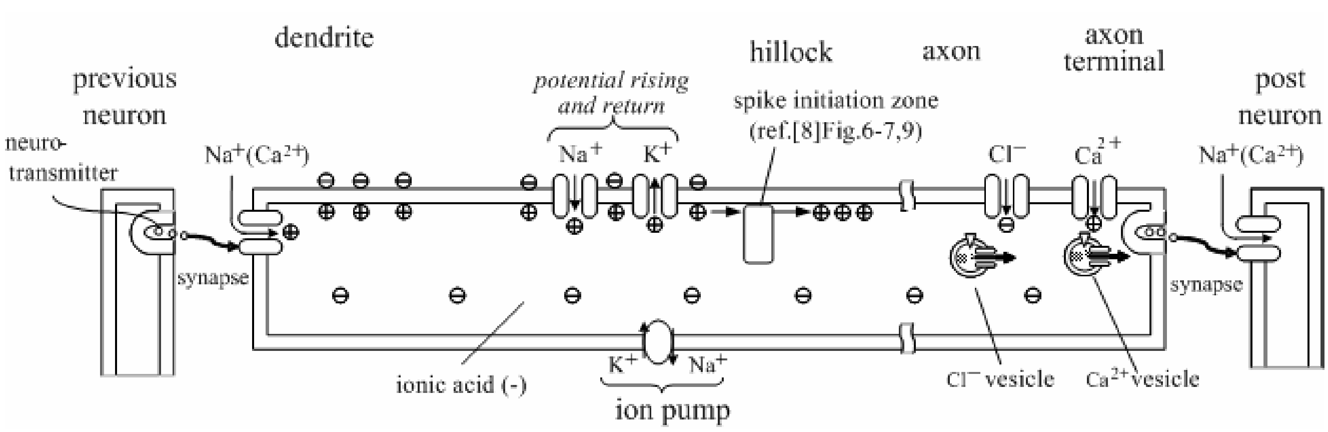


Fig. 6 The membrane model of a neuron in activation.

ACKNOWLEDGMENT

The authors express sincere gratitude for Professor Tomoyuki Higuchi, director-general, and Professor Hiroe Tsubaki, vice director-general, and Professor Noboru Sonehara, project leader of System of Human and Society for their leaderships and kind supports for this research.

The grater part of this study is supported by the project of Function and Induction with the Research Organization of Information and Systems.

The authors also express sincere gratitude for Professor Toshiharu Horie, Chiba University, Professor Kazuhiko Natori,, Toho University, and Professor Yoshikazu Miyanaga, Hokkaido University for their

REFERENCES

[1] A. Fukasawa, Y. Takizawa, "Activity of a Neuron and Formulation of a Neural Group for Synchronization and Signal Processing," The recipient of Best Paper Award on NEUROLOGY'12 from WSEAS/NAUN, Proc. of the Int. Conf. on Neurology, pp.242-247, Kos, Greece, July 2012.

[2] A. Fukasawa, Y. Takizawa, "Activity of a Neuron and Formulation of a Synchronous Neural System," Proceedings of the International Conference on Mathematical Methods, Computational Techniques and Intelligent Systems (MAMECTIS '13), pp.66-73, Cyprus, Mar. 2013.

[3] Takizawa Y., Fukasawa A., "Formulation of a Neural System and Application to Topographical Mapping," Proceedings of the 3rd International Conference on Neurology (NEUROLOGY '12), pp. 248-253, Kos Island, Greece, July 14-17, 2012.

[4] Takizawa Y., Fukasawa A., "Formulation of Topographical Mapping in Brain with a Synchronous Neural System," Proceedings of the International Conference on Mathematical Methods, Computational Techniques and Intelligent Systems (MAMECTIS '13), pp.60-65, Cyprus, Mar. 2013.

[5] Y. Takizawa, A. Fukasawa, "Signal Processing by a Neural System and its Application to Location of Multiple Events," Int. Jour. of Applied Mathematics and Informatics, Vol. 6, Issue 3, pp. 126-133, 2012.

[6] A. L. Hodgkin, A. F. Huxley, "A quantitative description of membrane current and its application to conduction and excitation in nerve," Bulletin of Mathematical Biology, Vol. 52, No. 1/2, pp.25-71, 1990.

[7] B. Hille, "Ion Channels of Excitable Membranes, 3rd ed." Sinauer Associates, Ma., USA, 2001.

[8] F. Delcomyn, "Foundations of neurobiology," Freeman and co., New York, 1998.

[9] W. Shockley, "Electrons and holes in semiconductors," D. Van, Nostrand, New York, 1950.

[10] A. Fukasawa, "Active electric circuit and its application to antenna system, - Low noise amplifier with Esaki Diode," Master Thesis of Waseda Univ., Apr.. 1966.

[11] Y. Takizawa, G. Rose, M. Kawasaki, "Resolving Competing Theories for Control of the Jamming Avoidance Response: The Role of Amplitude Modulations in Electric Organ Discharge Decelerations," Journal of Exp. Biol. 202, pp. 1377-1386, 1999.

[12] K. Yanagisawa, "Transistor and its electric circuit," (Japanese) Kyoritsu ,Tokyo, Dec. 1955.

Yumi Takizawa received the B.S. degree in Physics from Shinshu University, Japan, in 1984, and the Ph.D. degree from the University of Tokyo in 1994. She was a research leader of signal processing for communication system Lab., OKI Elecric Industry Co. Ltd., Tokyo since 1984. She joined the Institute of Statistical Mathematics, Japan as an associate professor in 1995.

She has been engaged nonstationary signal processing methods and its application to wide areas of industrial systems. She also has been engaged in digital mobile communication system of Wideband CDMA. Then she has been engaged in neuroethology of weakly electric fish as a visiting researcher at University of Virginia, 1997-1998 and 1999-2000. She continued research of biological signal processing including behavior of neural systems.

She received the Prize on Telecommunication System Technology from the Foundation of Telecommunication Association, Japan in 2004. She is an associate Professor.

Atsushi Fukasawa received the B.S. degree in Electrical Engineering from Chiba Unversity in 1962. He received the Master of Arts degree in Electrical communication from Waseda University, Tokyo, in 1967 and the Ph.D. degree from Waseda University in 1983.

He joined Graduate University of Science and Technology, Chiba University as a professor in 1997. He joined the department of Electrical and Electronics Engineering, faculty of Engineering, Chiba University in 1998, and the department of Urban Environmental and System Engineering, Chiba University in 1999. He joined Transdisciplinary Research Integration Center, Research Organization of Information and Systems as a project professor in 2004.

He received the Award of the Ministry if Science and Technology, Japan in 1967, and received the Ohm Science and Technology Prize from Ohmsha in 1994 respectively. He received also the Prize on Telecommunication System Technology from the Foundation of Telecommunication Association, Japan in 2004.

He is a senior member of the IEEE.