Analysis of Dynamic Characteristics of a Minimal-Time Circuit Optimization Process

A. M. Zemliak

Abstract—The design process for analog network design is formulated as a dynamic controllable system. A special control vector is defined to redistribute the compute expensive between a network analysis and a parametric optimization. This redistribution permits the minimization of a computer time. The problem of the minimal-time network design can be formulated in this case as a classical problem of the optimal control for some functional minimization. This approach generalizes the design problem and generates an infinite number of the different design strategies inside the same optimization procedure. By this methodology the aim of the system design process with minimal computer time is presented as a transition process of a dynamic system that has the minimal transition time. The conception of the Lyapunov function of dynamic controllable system is used to analyze the principal characteristics of the design process. The different forms of the Lyapunov function were proposed to analyze the behavior of a design process. The special function that is a combination of the Lyapunov function and its time derivative was proposed to compare the different design strategies and to predict the strategy with the minimal computer design time.

Keywords—Minimal-time system design, circuit optimization, control theory application, Lyapunov function.

I. INTRODUCTION

THE computer time reduction of a large system design is one of the sources of the total quality design improvement. This problem has a great significance because it has a lot of applications, for example on VLSI electronic circuit design. Any traditional system design strategy includes two main parts: the mathematical model of the physical system that can be defined by the algebraic equations or differential-integral equations and optimization procedure that achieves the optimum point of the design objective function. In limits of this conception it is possible to change optimization strategy and use the different models and different methods of analysis but in each step of the circuit optimization process there are a fixed number of the equations of the mathematical model and a fixed number of the independent parameters of the optimization procedure.

Besides the traditionally used ideas of sparse matrix techniques and decomposition techniques [1]-[9] some another ways were proposed to reduce the total computer design time [10]-[13].

The generalized approach for the analog system design on the basis of the control theory formulation was elaborated in some previous works, for example [14]. This approach serves for the minimal-time design algorithm definition. On the other hand this approach gives the possibility to analyze with a great clearness the design process while moving along the trajectory curve into the design space.

The main conception of this theory is the introduction of the special control functions, which, on the one hand generalize the design process and, on the other hand, they give the possibility to control the design process to achieve the optimum of the design cost function for the minimal computer time. This possibility appears because practically an infinite number of the different design strategies that exist within the bounds of the theory. The different design strategies have the different operation number and executed computer time. On the bounds of this conception, the traditional design strategy is only a one representative of the enormous set of different design strategies. As shown in [14] the potential computer time gain that can be obtained by the new design problem formulation increases when the size and complexity of the system increase. However it is can be realized when the algorithm for the optimal design strategy is constructed in practice.

The main properties and the special conditions for the optimal design strategy construction are the first problems that need to be solved for the optimal algorithm searching.

II. PROBLEM FORMULATION

The design process for any analog system design can be defined in discrete form [14] as the problem of the generalized cost function $F(X,U)$ minimization by means of the system (1) with the constraints (2):

$$x_i^{s+1} = x_i^s + t_s \cdot f_i(X,U) \tag{1}$$

$$i = 1,2,...,N$$

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$$(1 - u_j) g_j(X) = 0 \qquad (2)$$

$$j = 1, 2, \ldots, M$$

where $X \in R^N$, $X = (X', X'')$, $X' \in R^K$ is the vector of the independent variables and the vector $X'' \in R^M$ is the vector of dependent variables $(N = K + M)$, $g_j(X)$ for all j presents the network model, s is the iterations number, t_s is the iteration parameter, $t_s \in R^1$, $H \equiv H(X, U)$ is the direction of the generalized cost function $F(X, U)$ decreasing, U is the vector of the special control functions $U = (u_1, u_2, \ldots, u_m)$, where $u_j \in \Omega$; $\Omega = \{0; 1\}$. The functions $f_i(X, U)$ for example for the gradient method are defined as:

$$f_i(X, U) = -\frac{\delta}{\delta x_i} F(X, U) \qquad (3)$$

$$i = 1, 2, \ldots, K,$$

$$f_i(X, U) = -u_{i-K} \frac{\delta}{\delta x_i} F(X, U) + \frac{(1 - u_{i-K})}{t_s} \{ -x_i^s + \eta_i(X) \} \qquad (3')$$

$$i = K + 1, K + 2, \ldots, N,$$

where the operator $\dfrac{\delta}{\delta x_i}$ hear and below means

$\dfrac{\delta}{\delta x_i} \varphi(X) = \dfrac{\partial \varphi(X)}{\partial x_i} + \displaystyle\sum_{p=K+1}^{K+M} \dfrac{\partial \varphi(X)}{\partial x_p} \dfrac{\partial x_p}{\partial x_i}$, x_i^s is equal to $x_i(t - dt)$; $\eta_i(X)$ is the implicit function $(x_i = \eta_i(X))$ that is determined by the system (2). The generalized cost function $F(X, U)$ can be defined for example as:

$$F(X, U) = C(X) + \psi(X, U) \qquad (4)$$

where $C(X)$ is the non negative cost function of the design process, and $\psi(X, U)$ is the additional penalty function:

$$\psi(X, U) = \frac{1}{\varepsilon} \sum_{j=1}^{M} u_j \cdot g_j^2(X) \qquad (5)$$

This formulation of the design process permits the redistribution of the computer time expense between the solution of problem (2) and the optimization procedure (1) for the function $F(X, U)$. The control vector U is the main tool for the redistribution process in this case. Practically an infinite number of the different design strategies are produced because the vector U depends on the optimization procedure current step. The problem of the optimal design strategy search is formulated now as the typical problem for the functional minimization of the control theory. The functional that needs to minimize is the total CPU time T of the design process. This functional depends directly on the operations number and on the design strategy that has been realized. The main difficulty of this definition is unknown optimal dependencies of all control functions u_j. It is necessary to find the optimal behavior of the control functions u_j during the design process to minimize the total computer design time. This problem can be solved by some approximate methods of the control theory [15]-[16].

The idea of the system design problem formulation as the functional minimization problem of the control theory is not depend on the optimization method and can be embedded into any optimization procedures. The gradient method is used as a principal method in this paper. Nevertheless other optimization methods can be used successfully too. For example the right hand parts of the system (1) $f_i(X, U)$ for the Newton method or Davidon-Fletcher-Powell (DFP) are defined by the next formulas:

$$f_i(X, U) = -\sum_{k=1}^{N} b_{ik} \frac{\delta}{\delta x_k} F(X, U) \qquad (6)$$

$$i = 1, 2, \ldots, K,$$

$$f_i(X, U) = -u_{i-K} \sum_{k=1}^{N} b_{ik} \frac{\delta}{\delta x_k} F(X, U) + \frac{(1 - u_{i-K})}{t_s} \{ -x_i^s + \eta_i(X) \} \qquad (6')$$

$$i = K + 1, K + 2, \ldots, N,$$

where b_{ik} is the element of the matrix $\{ F''(X, U) \}^{-1}$ for the Newton's method and the element of the matrix $B(X, U)$ for the DFP method. In the last case this element is defined by the following expression:

$$B_{s+1} = B_s + \frac{R^s (R^s)^T}{(R^s)^T Q^s} - \frac{(B_s Q^s)(B_s Q^s)^T}{(Q^s)^T B_s Q^s} \qquad (7)$$

where B_0 is the unit matrix, $s = 0, 1, \ldots$ and

$$R^s = X^{s+1} - X^s, Q^s = F(X^{s+1}, U^{s+1}) - F(X^s, U^s). \qquad (8)$$

The gradient method is used below for all examples as the principle optimization method.

Now the process for analog network design is formulated as

a dynamic controllable system. The minimal-time design process can be defined as the dynamic system with the minimal transition time in this case. So, we need to find the special conditions to minimize the transition time for this dynamic system. These conditions can be defined as the basis for the optimal design algorithm construction.

III. LYAPUNOV FUNCTION OF DESIGN PROCESS

On the basis of the analysis in previous section we can conclude that the minimal-time algorithm has one or some switch points in control vector where the switching is realize among different design strategies. As shown in [17] it is necessary to switch the control vector from the like modified traditional design strategy (MTDS) when all the components of the vector U are equal to 1, to the like traditional design strategy (TDS) when all the components of the vector U are equal to 0, with some adjusting. Some principal features of the time-optimal algorithm were determined previously. These are: 1) an additional acceleration effect that appeared under special circumstances [18]; 2) the start point special selection outside the separate hyper-surface to guarantee the acceleration effect [19]; at least one negative component of the start value of the vector X is needed in this case; 3) the most perspective strategies of the structural basis that serve for an optimal design process construction. The two first problems were discussed in [18]-[19]. The third problem is discussed in the present paper.

The main problem of the time-optimal algorithm construction is unknown optimal sequence of the control vector switch points during the design process. We need to define a special criterion that permits to realize the optimal or quasi-optimal algorithm by means of the optimal switch points searching. A Lyapunov function of dynamic system serves as a very informative object to any system analysis in limits of the control theory. We propose to use a Lyapunov function of the design process to detect the optimal algorithm, particularly for the comparison of the different design strategies on computer time. The Lyapunov function properties can help us to solve this problem.

There is a freedom of the Lyapunov function definition because of a non-unique form of this function. Let us define the Lyapunov function of the design process (1)-(5) by the following expression:

$$V(X) = \sum_i (x_i - a_i)^2 \qquad (9)$$

where a_i is the stationary value of the coordinate x_i, in other words the set of all the coefficients a_i is the main objective of the design process. The function (9) satisfies all of the conditions of the standard Lyapunov function definition for the variables $y_i = x_i - a_i$. In fact the function

$$V(Y) = \sum_i y_i^2 \qquad (10)$$

is the piecewise continue. Besides there are three characteristics of this function: i) $V(Y)>0$, ii) $V(0)=0$, and iii) $V(Y) \to \infty$ when $\|Y\| \to \infty$.

Inconvenience of the formulas (9) or (10) is an unknown point $a=(a_1, a_2, ..., a_N)$, because this point can be reached at the end of the design process only. We can use this form of the Lyapunov function if we already found the design solution someway. On the other hand, it is very important to control the stability of the design process during the optimization procedure. In this case we need to construct other form of the Lyapunov function that doesn't depend on the unknown stationary point. Let us define two new forms of the Lyapunov function by the next formulas:

$$V(X, U) = [F(X, U)]^r \qquad (11)$$

$$V(X, U) = \sum_i \left(\frac{\partial F(X, U)}{\partial x_i} \right)^2 \qquad (12)$$

where $F(X,U)$ is the generalized cost function of the design process. The formula (11) can be used when the general cost function is non-negative and has zero value at the stationary point a. Other formula can be used always because all derivatives $\partial F / \partial x_i$ are equal to zero in the stationary point a. So, the function V for both formulas has properties: $V(a,U)=0$, $V(X,U)>0$ for all X and at last, this function increases in a sufficient large neighborhood of the stationary point. Besides, the function V is the function of the vector U too, because all coordinates x_i are the functions of the control vector U.

We can define now the design process as a transition process for controllable dynamic system that can provide the stationary point (optimal point of the design procedure) during some time. The problem of the time-optimal design algorithm construction can be formulated now as the problem of the transition process searching with the minimal transition time. There is a well-known idea [20]-[21] to minimize the time of the transition process by means of the special choice of the right hand part of the principal system of equations; in our case these are the functions $f_i(X,U)$. It is necessary to change the functions $f_i(X,U)$ by means of the control vector U selection to obtain the maximum speed of the Lyapunov function decreasing (the maximum absolute value of the Lyapunov function time derivative $\dot{V} = dV/dt$).

IV. STABILITY ANALYSIS

Normally the time derivative of the Lyapunov function is non-positive for the stable processes. The design process is stable if the Lyapunov function time derivative is negative. On the other hand, the direct method of Lyapunov gives the sufficient stability conditions but not necessary [20], so the process loses the stability (or not loses) if this derivative becomes positive. The stability of the different design strategies for a nonlinear network of transistor amplifier in Fig. 1 was analyzed by the Lyapunov direct method.
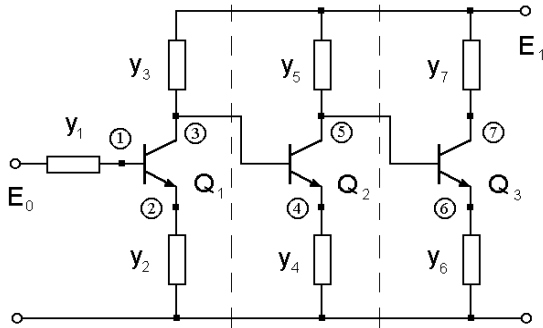

Fig. 1 active nonlinear network

This circuit was analyzed for one, two and three stages separately. The Ebers-Moll static model of the transistor has been used for all examples. The Lyapunov function time derivative dV/dt is a negative for all trajectories on the initial part of the design process; i.e. all admissible strategies are stable at the beginning. It is supposed that the integration step is sufficiently small. However, when the current point of the trajectory gets to the ε-neighborhood of the stationary point a, some strategies can lose the stability because the time derivative of the Lyapunov function becomes positive. It means that all trajectories of this group do not guarantee the convergence from the ε-neighborhood. In fact, each of the trajectories of this group has own critical ε-neighborhood, which defines the maximum achievable precision. Another consideration is important too: the design process convergence slow down strongly before the ε-neighborhood reaching for all strategies of this group. It means that the derivative dV/dt is the negative but very small on the absolute value. The critical values of the ε-neighborhood of some design trajectories for the two-stage transistor amplifier are shown in Table I.

The termination of the design process has been defined by means of the special criterion: the time derivative $\dot{V}$ has positive value in the set of positive measure. The optimization process was provided on the basis of the formula (1) with variable step. As a result the significant variations of the derivative value $\dot{V}$ have been observed. We average the derivative values for 30 integration steps to smooth these variations.

TABLE I
CRITICAL ε VALUE FOR TWO-STAGE AMPLIFIER

N	Control vector U(u1,u2,u3,u4,u5)	Iterations number	Computer time (sec)	Critical value of ε-neighborhood
1	(0 0 0 0 0)	3177	7.25	2.78E-08
2	(0 0 0 0 1)	3074	8.02	3.36E-07
3	(0 0 0 1 1)	11438	26.36	8.18E-07
4	(0 0 1 0 1)	799	1.16	9.38E-09
5	(0 0 1 1 0)	1798	2.61	1.61E-08
6	(0 1 0 1 1)	43431	76.89	3.16E-05
7	(0 1 1 0 0)	1378	2.25	1.67E-08
8	(0 1 1 0 1)	571	0.72	6.83E-09
9	(0 1 1 1 0)	1542	2.03	2.05E-08
10	(1 0 0 1 1)	11839	21.37	1.68E-05
11	(1 0 1 0 0)	2097	3.57	5.47E-07
12	(1 0 1 1 0)	6026	8.31	4.94E-07
13	(1 1 1 0 0)	6602	8.84	7.41E-07
14	(1 1 1 0 1)	935	0.71	1.33E-08
15	(1 1 1 1 0)	2340	2.31	1.62E-07
16	(1 1 1 1 1)	1502	0.38	1.09E-08

The analysis of the results in Table 1 gives the next regularities: there is a correlation between processor design time and a critical ε-neighborhood. The processor design time is lesser for the strategies that have lesser critical ε-neighborhood. We can regulate all of the strategies of Table 1 in order of the computer time or in order of the critical ε-neighborhood. The results of this regulation are presented in Table II.

TABLE II
STRATEGY REGULATION FOR TWO-STAGE AMPLIFIER

Number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Number of strategies regulated by the computer tim	16	14	8	4	9	7	15	5	11	1	2	12	13	10	3	6
Number of strategies regulated by the ε-neighborhood	8	4	16	14	5	15	7	9	1	2	12	11	13	3	10	6

The places of each strategy, which defined by two different methods of regulation, are differed very small. This place is the same for two strategies (13 and 6). The one place difference is appeared seven times, the two places difference is appeared four times and the three places difference is appeared three times. The average value of this difference is 1.5. We can consider that the correlation between a computer time and a critical ε-neighborhood is acceptable. From the other hand the critical ε-neighborhood values were obtained by the analysis of the Lyapunov function and its derivative. So, we can state that there is a close relation between the computer time and the properties of the Lyapunov function of the design process.

The analysis of the three-stage amplifier of Fig. 1 shows very similar results. The critical value of ε-neighborhood of some design trajectories for the three-stage transistor amplifier is shown in Table III. As for the first example we

can formulate the rule – the less value of the critical ε -neighborhood for some strategy provoke the less value of the computer time for this strategy. The regulations of all the strategies of Table III in order of the computer time and in order of the critical ε -neighborhood are presented in Table IV. The places of each strategy, which defined by two different methods of regulation, are differed small. The average value of this difference is 2.6. We can conclude that the Lyapunov function can serves as the informative source for the searching of the perspective strategies that have the minimal computer time. The additional useful information is

in the Lyapunov function time derivative $\overset{\bullet}{V} = dV/dt$ behavior.

TABLE III
CRITICAL ε VALUE FOR THREE-STAGE AMPLIFIER

N	Control vector U(u1,u2,u3,u4,u5,u6,u7)	Iterations number	Computer time (sec)	Critical value of ε -neighborhood
1	(0000000)	9106	45.75	7.27E-07
2	(0010110)	1844	8.63	1.91E-07
3	(0011100)	3067	15.65	6.77E-07
4	(0011111)	647	1.87	4.73E-09
5	(0110101)	627	1.42	4.69E-09
6	(0110110)	1513	4.01	5.62E-07
7	(0110111)	643	1.21	9.07E-09
8	(0111110)	3229	7.31	7.54E-07
9	(1010101)	2069	4.06	1.43E-07
10	(1011101)	1657	2.53	2.13E-07
11	(1110101)	1477	2.09	1.33E-07
12	(1110110)	3931	6.48	9.21E-07
13	(1111100)	3626	7.85	6.25E-07
14	(1111101)	1793	2.03	2.12E-07
15	(1111110)	2345	3.51	2.08E-07
16	(1111111)	2149	0.61	7.59E-08

TABLE IV
STRATEGY REGULATION FOR THREE-STAGE AMPLIFIER

Number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Number of strategies regulted by the computer time	16	7	5	4	14	11	10	15	6	9	12	8	13	2	3	1
Number of strategies regulated by the ε-neighborhood	5	4	7	16	11	9	2	15	14	10	6	13	3	1	8	12

V. STRUCTURAL BASIS ANALYSIS

There is a possibility to define more informative function as a time derivative of the Lyapunov function relatively the Lyapunov function: $W = \overset{\bullet}{V}/V$. In this case we can compare the different design strategies by means of the function $W(t)$ behavior and we can search of the control vector optimal structure.

The results of the analysis of some nonlinear networks are presented below. Functions $V(t)$ and $W(t)$ were the main objects of the analysis and its behavior has been analyzed for some strategies that compose the structural basis of the general design methodology.

The behavior of the functions $V(t)$ and $W(t)$ for the network of Fig. 2 is shown in Fig. 3a, and Fig. 3b.
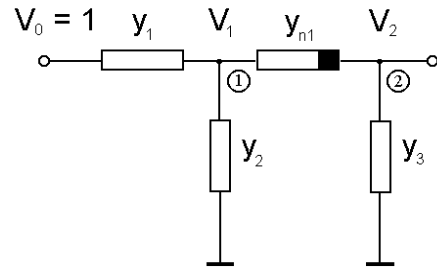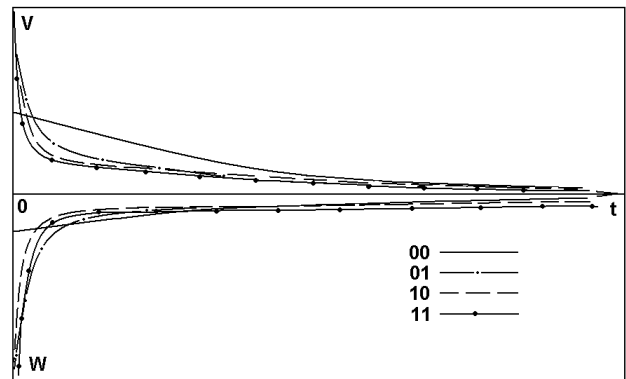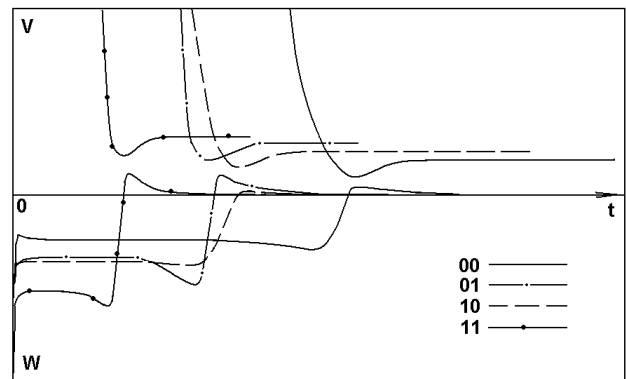

Fig. 2 two-node nonlinear passive network

(a)

(b)

Fig. 3 behavior of the functions $V(t)$ and $W(t)$ for four design strategies during the design process for network in Fig.2; (a) – initial part of the design process, (b) – design process the whole with the final part in detail

The nonlinear element has the following dependency $y_{n1} = y_0 + b(V_1 - V_2)^2$. The vector X includes five components: $x_1^2 = y_1$, $x_2^2 = y_2$, $x_3^2 = y_3$, $x_4 = V_1$, $x_5 = V_2$.

The model of this network (2) includes two equations (M=2) and the optimization procedure (1) includes five equations. The objective function $C(X)$ has been determined as the sum of the squared differences between beforehand-defined values and current values of the nodal voltages for two nodes with additional inequalities for some circuit elements.

The network in Fig. 2 is characterized by two dependent parameters (two nodes) and the control vector includes two control functions: $U = (u_1, u_2)$.

The structural basis of the design strategies includes four design strategies; 00, 01, 10, 11. The Lyapunov function was calculated by formula (11) for r=0.5. As we can see from Fig. 3 the functions $V(t)$ and $W(t)$ can give an exhaustive explanation for the design process characteristics. Fig. 3a shows these functions behavior for the initial part of the design process (2% of the total design time). First of all we can conclude that the speed of decreasing of the Lyapunov function is inversely proportional to the design time. The minimal value of the Lyapunov function that corresponds to the maximum precision is approximately equal for all strategies and exactly is equal to 8.7_{10}-6, 1.7_{10}-5, 1.3_{10}-5, 2.0_{10}-5 for the strategies 00, 01, 10, 11 accordingly. We can see from Fig. 3b that after the minimal value decision the Lyapunov function increases a little. This small increasing corresponds to the small positive value of the Lyapunov function time derivative. Later on this derivative aspire to zero and the Lyapunov function has a permanent value.

The relative design time for four design strategies is equal to 1, 0.44, 0.78 and 0.3 for the strategies 00, 01, 10, 11 accordingly. This time was defined for the time point with the minimal value of the function V. As we can see from Fig. 3b a large absolute value of the function $W(t)$ corresponds to a more rapid decreasing of the function $V(t)$ and a smaller computer design time.

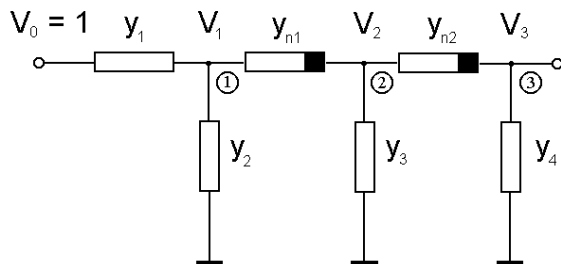Another passive nonlinear network with three nodes (Fig. 4) was analyzed below.

The nonlinear elements have been defined by following dependencies: $y_{nl} = a_{nl} + b_{nl} \cdot (V_1 - V_2)^2$, $y_{n2} = a_{n2} + b_{n2} \cdot (V_2 - V_3)^2$. The vector X includes seven components: $x_1^2 = y_1$, $x_2^2 = y_2$, $x_3^2 = y_3$, $x_4^2 = y_4$, $x_5 = V_1$, $x_6 = V_2$, $x_7 = V_3$. The model of this network (2) includes three equations (M=3) and the optimization procedure (1) includes seven equations. This network is characterized by three dependent parameters and the control vector includes three control functions: $U = (u_1, u_2, u_3)$. The behavior of the functions $V(t)$ and $W(t)$ for this network is shown in Fig. 5.

The structural basis of design strategies includes eight design strategies: 000, 001, 010, 011, 100, 101, 110 and 111. Fig. 5a shows the behavior of $V(t)$ and $W(t)$ functions for the initial part of the design process. We can conclude that the speed of decreasing of the Lyapunov function is inversely proportional to the design time for the network in Fig.4. The minimal value of the Lyapunov function that corresponds to the maximum precision is in the limits from 1.2_{10}-5 for strategy 000 to 5.9_{10}-5 for strategy 111.
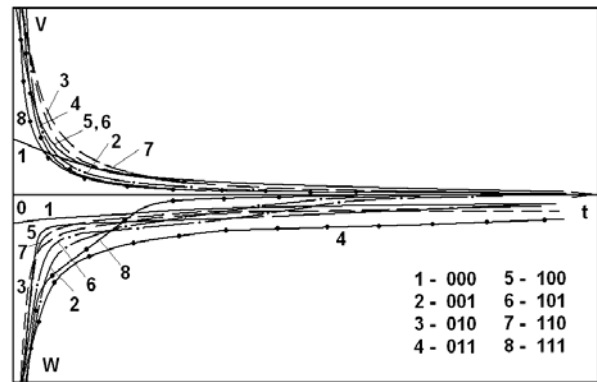
(a)
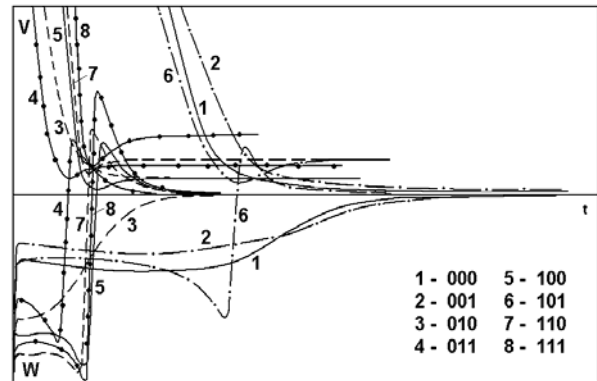
(b)

Fig. 5 behavior of the functions $V(t)$ and $W(t)$ for eight design strategies during the design process for network in Fig.4;
(a) – initial part of the design process,
(b) – design process the whole with the final part in detail

Fig. 4 three-node nonlinear passive network

We can see from Fig. 5b that the Lyapunov function increases a little for some strategies after the minimal value decision. The relative design time for all design strategies is equal to 1, 0.886, 0.569, 0.091, 0.129, 0.25, 0.131 and 0.105 for the strategies 000, 001, 010, 011, 100, 101, 110 and 111 accordingly. This time was defined for the time point with the minimal value of the function V. Anew we can see from Fig. 5b that a large absolute value of the function $W(t)$ corresponds to a more rapid decreasing of the function $V(t)$ and a smaller computer design time. The strategies 011, 100, 110 and 111 have a large value of the function $W(t)$ during all the design process till a small value of the function $V(t)$. That is why these strategies have a relative little computer time.

The four-node nonlinear circuit is analyzed below (Fig. 6) on basis of the proposed methodology.
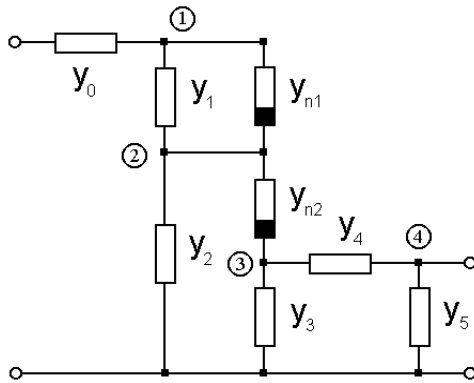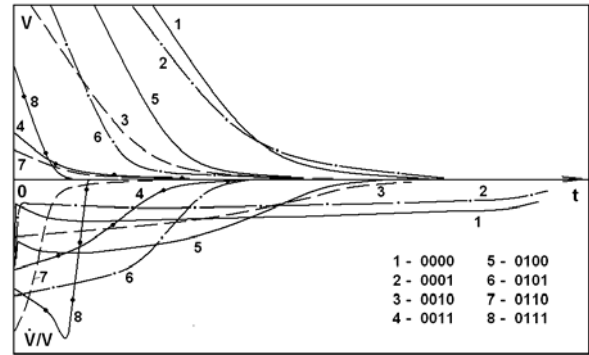

Fig. 6 four-node nonlinear passive network

The problem includes five independent parameters $(x_1, x_2, x_3, x_4, x_5)$, where $x_1^2 = y_1$, $x_2^2 = y_2$, $x_3^2 = y_3$, $x_4^2 = y_4$, $x_5^2 = y_5$, and four originally dependent parameters (x_6, x_7, x_8, x_9), where $x_6 = V_1$, $x_7 = V_2$, $x_8 = V_3$, $x_9 = V_4$. The model of this network (2) includes four equations ($M=4$) and the optimization procedure (1) includes nine equations. The control vector includes four control functions: $U = (u_1, u_2, u_3, u_4)$. The structural basis of design strategies includes 16 design strategies. The iterations number and computer time for all strategies of the structural basis are shown in Table V. The behavior of the functions $V(t)$ and $W(t)$ for this network is shown in Fig. 7. We can see that three design strategies number 8, 12, and 16 from the Table V have the minimal value of computer time. At the same time these strategies have a very specific behavior of the function $W(t)$ (curves 8 from Fig. 7a and 4 and 8 from Fig. 7b). The function W has a large value and increases during the initial part of the design process for these strategies. It means that the Lyapunov function decrease very fast for these strategies. As a result we have very small value of the design process computer time for these design strategies. On the other hand the strategies 1, 2, 3, 5, and 9 from Table V have
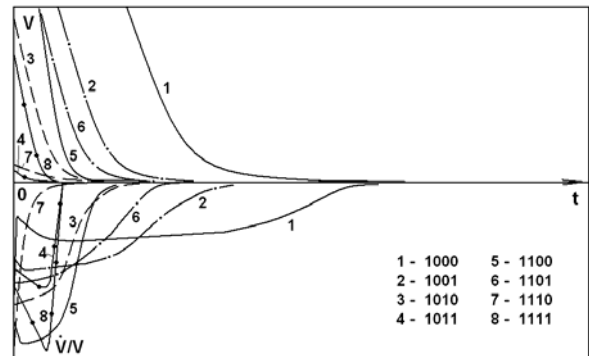
the large computer time and at the same time its have the minimal value of the function W during the design process (curves 1, 2, 3, 5 from Fig. 7a, and curve 1 from Fig. 7b).

TABLE V
NUMBER OF ITERATIONS AND COMPUTER TIME
FOR COMPLETE STRUCTURAL BASIS

N	Control vector	Iterations number	Total design time (sec)
1	(0 0 0 0)	39875	8.62
2	(0 0 0 1)	78570	10.27
3	(0 0 1 0)	302865	13.51
4	(0 0 1 1)	120983	5.38
5	(0 1 0 0)	150885	21.92
6	(0 1 0 1)	44339	6.43
7	(0 1 1 0)	165593	7.19
8	(0 1 1 1)	52044	1.82
9	(1 0 0 0)	144446	17.14
10	(1 0 0 1)	105750	4.72
11	(1 0 1 0)	176329	8.02
12	(1 0 1 1)	26353	0.93
13	(1 1 0 0)	155509	6.71
14	(1 1 0 1)	312015	10.98
15	(1 1 1 0)	170710	4.39
16	(1 1 1 1)	52664	1.05

1 - 0000	5 - 0100
2 - 0001	6 - 0101
3 - 0010	7 - 0110
4 - 0011	8 - 0111

(a)

1 - 1000	5 - 1100
2 - 1001	6 - 1101
3 - 1010	7 - 1110
4 - 1011	8 - 1111

(b)

Fig. 7 behavior of the functions $V(t)$ and $W(t)$ for full structural basis of the design strategies for network in Fig.6; (a) – first eight design strategies, (b) – next design strategies

In Fig. 8 there is a circuit that has 6 independent variables as admittance $y_1, y_2, y_3, y_4, y_5, y_6$ (K=6) and 5 dependent variables as nodal voltages V_1, V_2, V_3, V_4, V_5 (M=5) at the nodes 1, 2, 3, 4, 5.
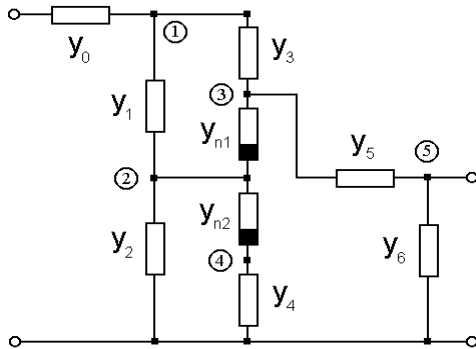

Fig. 8 five-node circuit topology.

The nonlinear elements have next dependency: $y_{n1} = a_{n1} + b_{n1} \cdot (V_3 - V_2)^2$, $y_{n2} = a_{n2} + b_{n2} \cdot (V_4 - V_2)^2$. The vector X includes eleven components. The first six components are defined by means of the admittance as: $x_1^2 = y_1$, $x_2^2 = y_2$, $x_3^2 = y_3$, $x_4^2 = y_4$, $x_5^2 = y_5$, $x_6^2 = y_6$. The others components are defined as nodal voltages: $x_7 = V_1$, $x_8 = V_2$, $x_9 = V_3$, $x_{10} = V_4$, $x_{11} = V_5$. The control vector U includes five components $U = (u_1, u_2, u_3, u_4, u_5)$ and the structural basis consists of 32 different design strategies. The iterations number and computer time for some strategies of the structural basis are shown in Table VI. The behavior of the functions $V(t)$ and $W(t)$ for this network is shown in Fig. 9.

TABLE VI
NUMBER OF ITERATIONS AND COMPUTER TIME
FOR SOME STRATEGIES OF STRUCTURAL BASIS

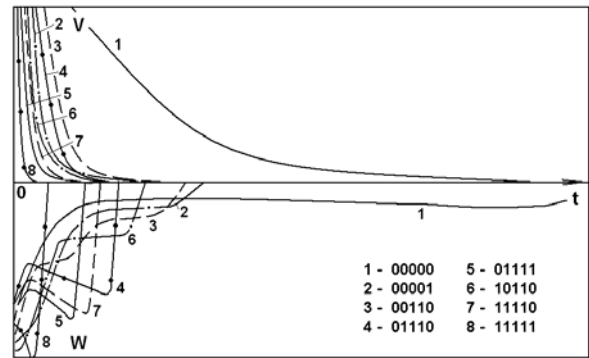| N	Control vector	Iterations number	Total design time (sec)
1	(0 0 0 0 0)	33456	14.12
2	(0 0 0 0 1)	10837	5.63
3	(0 0 1 1 0)	15490	5.16
4	(0 1 1 1 0)	35567	3.91
5	(0 1 1 1 1)	28360	2.41
6	(1 0 1 1 0)	20756	4.18
7	(1 1 1 1 0)	36049	3.46
8	(1 1 1 1 1)	29002	1.21

Fig. 9 behavior of the functions $V(t)$ and $W(t)$ for some strategies of the structural basis for network in Fig.8

It is clear that the design strategies number 4, 5, 7, and 8 from the Table VI have the minimal value of computer time. The same strategies have a larger value of the function $W(t)$ during the optimization process as we can see from Fig. 9. We can decide that due to this specific behavior of the function W the computer time for these strategies is lesser than for others. On the other hand the strategy 1 has the minimal value of the function W during the design process and that is why this strategy has a larger computer time value.

The next example corresponds to the active nonlinear circuit, one-stage transistor amplifier in Fig. 10.
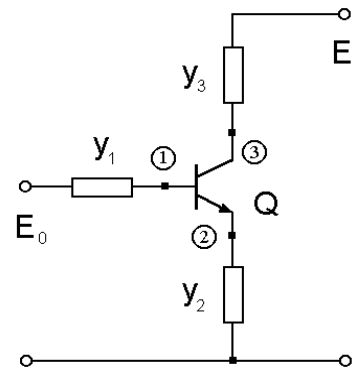

Fig. 10 one-stage transistor amplifier

In this case the vector X includes six components: $x_1^2 = y_1$, $x_2^2 = y_2$, $x_3^2 = y_3$, $x_4 = V_1$, $x_5 = V_2$, $x_6 = V_6$. The model of this network (2) includes three equations (M=3) and the optimization procedure (1) includes six equations. The total structural basis contains eight different strategies. The control vector has three components in this case $U = (u_1, u_2, u_3)$ and the structural basis consists of eight design strategies.

The results of the analysis of complete structural basis of the design strategies are shown in Table VII. The behavior of

TABLE VII

NUMBER OF ITERATIONS AND COMPUTER TIME
FOR COMPLETE STRUCTURAL BASIS

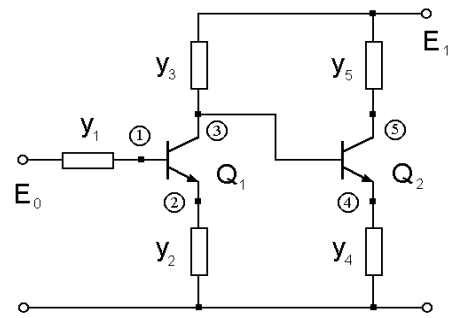| N	Control vector	Iterations number	Total design time (sec)
1	(0 0 0)	7683758	518.22
2	(0 0 1)	45900	2.42
3	(0 1 0)	1151505	60.14
4	(0 1 1)	47464	2.53
5	(1 0 0)	109784	5.87
6	(1 0 1)	4753	0.25
7	(1 1 0)	303579	14.83
8	(1 1 1)	4940	0.08

Fig. 12 two-stage transistor amplifier

This network is characterized by five dependent parameters and the control vector includes five control functions: $U=(u_1, u_2, u_3, u_4, u_5)$. The structural basis consists of 32 design strategies. The results of the analysis of some design strategies from the structural basis for the iterations number and computer time are shown in Table VIII. The behavior of the functions $V(t)$ and $W(t)$ that correspond to the data of the Table VIII is shown in Fig. 13.

the functions $V(t)$ and $W(t)$ for all possible strategies of the structural basis is shown in Fig. 11. These graphs correspond to a time interval when all of the design strategies are finished.

The strategies with control vector 101 and 111 have extremely large value of the relative derivative $W(t)$ from the beginning of the design process and that is why the Lyapunov function is decreases very rapidly. The design time is very small for two these strategies and it is equal to 0.25 sec and 0.08 sec accordingly. The strategies with the control vector 001, 011 and 100 have the sufficient level of the function W during the analyzed interval and the relative design time is equal to 2.42 sec, 2.53 sec, and 5.87 sec accordingly. Nevertheless three other design strategies with the control vector 000, 010 and 110 have a large value of the computer time, especially the traditional design strategy with control vector 000. It occurs because the function W for these strategies decreases rapidly while the Lyapunov function had a relatively large value. After this the Lyapunov function decreases very slowly and the design time is very large for these strategies.

TABLE VIII
NUMBER OF ITERATIONS AND COMPUTER TIME
FOR SOME STRATEGIES OF STRUCTURAL BASIS

N	Control vector	Iterations number	Total design time (sec)
1	(0 0 0 0 0)	165962	299.56
2	(0 0 0 0 1)	337487	737.55
3	(0 0 1 0 0)	44118	68.87
4	(0 0 1 0 1)	14941	19.06
5	(0 0 1 1 1)	21971	22.03
6	(0 1 1 0 1)	4544	4.56
7	(1 0 1 0 1)	2485	1.65
8	(1 0 1 1 1)	7106	3.57
9	(1 1 1 0 1)	2668	1.32
10	(1 1 1 1 1)	79330	10.11

1 - 000   5 - 100
2 - 001   6 - 101
3 - 010   7 - 110
4 - 011   8 - 111
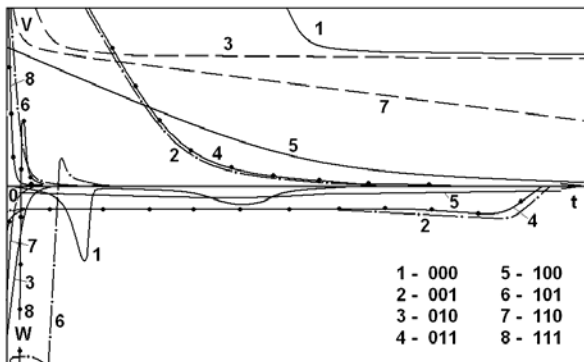
Fig. 11 behavior of the functions $V(t)$ and $W(t)$ for all strategies of the structural basis for network in Fig.10

Other example corresponds to the two-stage transistor amplifier in Fig.12.

1 - 00000   6 - 01101
2 - 00001   7 - 10101
3 - 00100   8 - 10111
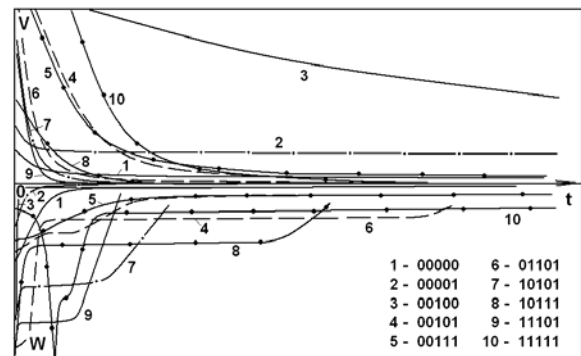4 - 00101   9 - 11101
5 - 00111   10 - 11111

Fig. 13 behavior of the functions $V(t)$ and $W(t)$ for some strategies during the design process for network in Fig.12

These graphs correspond to a time interval when the majority of the design strategies are finished. The strategies 6, 7, 8 and 9 have a large value of the relative derivative W at the beginning of the design process. This property provides extremely fast decreasing of the Lyapunov function. We can see that just these strategies 6, 7, 8 and 9 have the design time lesser than other strategies. The strategies 4, 5 and 10 have an average value of the function W in the initial part of the design process and these strategies have an average value of the design time. At last, the strategies 1, 2, and 3 have a large design time and just these strategies have a very fast decreasing of the function W during initial part of the design process when the Lyapunov function had a relatively large value. After this the Lyapunov function decreases very slowly and the design time for these strategies is large.

We can state that the large absolute values of the function $W(t)$ on the initial part of the design process provoke the less computer time.

So, the main feature of the analyzed examples can be formulated by the next manner: the behavior of the Lyapunov function V and the relative time derivative W with confidence determine the design time. It means that it is possible be guided by means of these functions to predict the computer design time for any design strategy. We could analyzed the functions $V(t)$ and $W(t)$ behavior for the initial time interval only for the different strategies and on the basis of this analysis we can predict the strategies that have a minimal computer design time.

VI. CONCLUSION

The problem of the minimal-time design algorithm construction can be solved adequately on the basis of the control theory. The design process in this case is formulated as the controllable dynamic system. The Lyapunov function of the design process and its time derivative include the sufficient information to select more perspective design strategies from the infinite set of the different design strategies that exist in the generalized design methodology. The special function $W(t)$ was proposed to predict the structure of the time optimal design strategy. This function serves as a principal criterion to compare the different design strategies. The best strategy that has the minimal computer time at the same time has the largest value of the function $W(t)$ at the beginning of the circuit optimization process.

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