Double-Spool Single Jet Engine for Aircraft as Controlled Object

Romulus Lungu, Alexandru Nicolae Tudosie, Liviu Dinca

Abstract— The paper deals with an aircraft double-spool single-jet engine identified as possible controlled object. The authors have identified the main parameters and the engine's non-linear equation system (the motion equation and the gas-dynamic characteristics), then they have established the linear non-dimensional model, useful for further studies, and the block diagram with transfer functions. Engine's stability domains were also established, and some simulation were performed, concerning the engine's time behavior (step response), for two different cases of control schemes.

Keywords— air flow rate, exhaust nozzle, fuel, jet engine, rotation speed, spool, temperature, throttle.

I. Introduction

Jet engines for aircraft are covering a large range of performances and types. The continue need for the engine's thrust increasing, as well as for a lower fuel consumption, has as consequence a continue increasing of the engines' combustors temperatures T_3^* and/or of their compressors pressure ratios π_c^*. Meanwhile, the frontal sections (air inlet's section) are becoming larger, especially for the combat aircraft engines, which could also have afterburner components mounted before their exhaust nozzles, as a method of thrust gain.

High pressure ratio for the engine's compressor involves a less gas-dynamic stability, which means a dangerous closing to the stall limit, especially for the extreme operating regimes; such a consequence makes the pressure ratio's growing for a multi-stage compressor an unacceptable alternative, which means that a ratio $\pi_c^* \geq 15$ could not be used.

In order to protect the very high pressure-ratio compressors from stall operating, the manufacturers have divided the compression evolution, so they have "split" the compressor in two or more groups of stages, which are only gas-dynamic bounded and which have their own rotation speed. This constructive solution is the "multi-spool" engine (or the multi-shaft engine); the most used are the double spool engines, a large range of types being nowadays in service (single- or twin-jet double spool engines, as well as the turbo-shafts or turbofans) for civil and combat aircraft and helicopters.

Fig. 1 presents an aircraft double-spool single-jet turbo-engine, showing its main parts and both the main operating sections. The most important parts are the two turbo-compressors (spools): the low pressure spool (LPS) and the high pressure spool (HPS), gas-dynamic bounded, each one having its own speed (n_1 for the LPS and n_2 for the HPS) and using the other's fluid.

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II. Problem Formulation

A model for an engine as controlled object is useful for further studies, concerning some options for its automatic control and for studies about its stability and quality, as well as its behavior and optimization.

One must start with the identifying of the control parameters, as well as the controlled parameters; a mathematical model is the next step and - eventually - the performing of some studies and simulations.

A. Engine's parameters

Amongst the multitude of engine's parameters, one can identify as possible controlled parameters (outputs) the parameters which are easiest to be measured, such as: a) the engine combustor temperature T_3^*; b) the exhaust nozzle gas temperature T_4^*; c) the low pressure spool speed n_1 and d) the high pressure spool speed n_2.

The most important are the spool speeds, because of their interesting properties, as they result from theoretical and experimental studies:

- the speed level offers information about the engine's thrust, about the fuel consumption, as well as about the mechanical charge of the engine's mobile parts;
- the dependence of the thrust and of the fuel consumption on the engine's rotation speed is univoc and can be linear expressed for some large range of operating regimes;
- speed's level measurement uses simple and reliable equipment and methods, both for steady state regimes and for dynamic regimes;

As control parameters (inputs) for a single-jet double spool

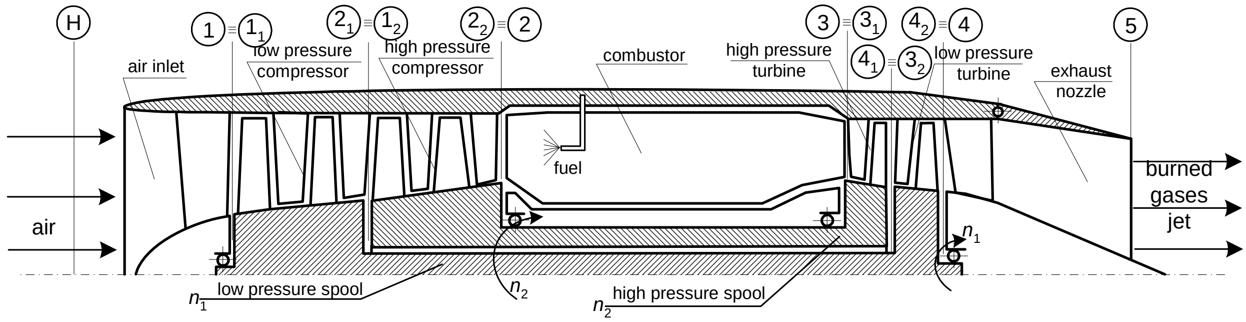

Fig. 1 Double spool single jet engine

engine, only two inputs can be identified: the fuel flow rate Q_c (which is the most important control parameter) and the exhaust nozzle opening A_5.

As consequence, only two control laws (two independent equations or formulas), which can bound the inputs to the outputs, could be chosen:

a) $Q_c \to n_1$ and $A_5 \to n_2$;

or

b) $Q_c \to n_2$ and $A_5 \to n_1$.

The experimental studies mentioned in [8], [13] and [14] have shown that the second control law is more appropriate, closer to the real engine's behavior. The experiments are also proving a complex dependence, because of the gas-dynamic mutual influence of the spools.

Some control possibilities for a double spool engine are shown in the diagrams in fig.2, as given by [14] and [17].

B. Engine's equations

The equation set which describes the engine's behavior ([13], [14] and [17]) consists of:

- spools' (turbo-compressor rotors') motion equations:

$$\frac{\pi J_i}{30}\frac{\mathrm{d}n}{\mathrm{d}t} = M_{Ti} - \left(M_{Ci} + M_{fi} + M_{bi}\right), \qquad (1)$$

where indices $i = \overline{1,2}$, 1-for the low pressure spool (LPS), 2-for the high pressure spool (HPS), $J =$ spool inertial momentum; $M_T =$ turbine's (active) torque, $M_C =$ compressor's (resistant) torque, $M_f =$ equipment (fuel and hydraulic pumps, generators) necessary torque (considered as constant), $M_b =$ bearings friction torque (considered as constant);

- turbo-compressor's universal characteristics

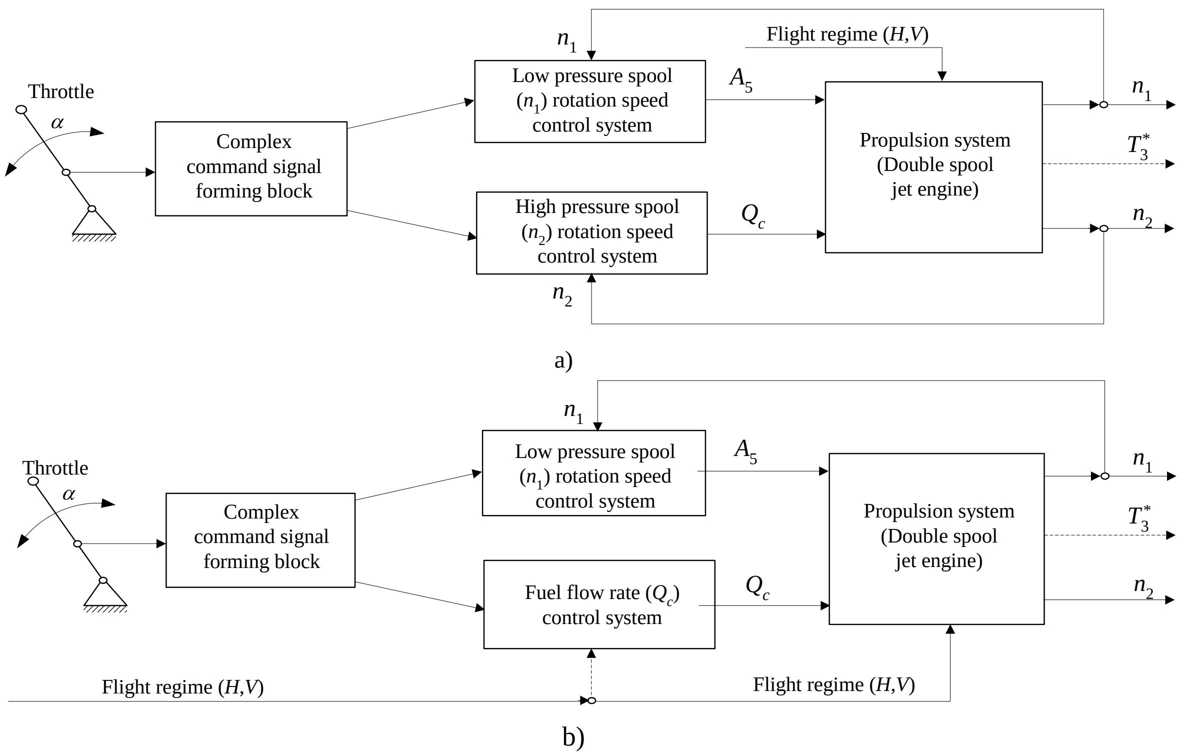

Fig. 2 Double spool jet engine's control possibilities

$$\frac{h_{2i}^*}{h_{1i}^*} = f(\pi_{ci}^*, \eta_{ci}, Q_{ai}), \tag{2}$$

$$\frac{h_{3i}^*}{h_{4i}^*} = f(\delta_{Ti}^*, \eta_{Ti}^*, Q_{gi}), \tag{3}$$

where $h_x^* =$ total specific enthalpy in current engine's section x (1-for the compressor's inlet, 2-for the compressor's exhaust, 3-for the turbine's inlet, 4-for the turbine's exhaust), $\pi_c^* =$ compressors' pressure ratio, $\delta_T^* =$ turbines' pressure ratio, $\eta_c =$ compressor efficiency (randament), $\eta_T^* =$ turbine efficiency (randament) – considering the exhaust speed, Q_a, $Q_g =$ air and exhaust (burned) gases flow rate. Usually, the turbo-compressors' characteristics are presented graphic (see fig. 3) or graph-analytical, the most important element in this characteristic being the "engine's work line" (operating regimes line or curve);

- combustor's equation

$$Q_a h_2^* + \xi_{CA} Q_c P_c = Q_g h_3^*, \tag{4}$$

where $Q_c =$ fuel flow rate, $\xi_{CA} =$ combustor's perfection co-efficient; $P_c =$ fuel caloric characteristic;

- engine's fluid flow rate equation (conservation)

$$Q_g = Q_a;\; Q_g = Q_g(A_5, p_4^*). \tag{5}$$

These are the double spool engine's equations, which will be used for engine's modeling.

III. System's Mathematical Model

The above presented equation system is a non-linear one and it is very difficult to use them as shown for engine's modeling as controlled object, so one need to bring them, if possible, to a linear form.

The arguments in these equations are mostly all of them non-linear, depending on more than one parameter, e.g.

$$M_T = M_T\left(T_3^*, Q_g, n, \delta_T^*\right) = \frac{30}{\pi}\frac{\chi_g R_g \eta_T^*}{\chi_g - 1}\frac{Q_g(n)}{n}T_3^*\left[1-\left(\frac{1}{\delta_T^*}\right)^{\frac{\chi_g-1}{\chi_g}}\right], \tag{6}$$

$$M_C = M_C\left(T_1^*, Q_a, n, \pi_c^*\right) = \frac{30}{\pi}\frac{\chi\ R\ \eta_C}{\chi\ -1}\frac{Q_a(n)}{n}T_1^*\left[\left(\pi_c^*\right)^{\frac{\chi_g-1}{\chi_g}} - 1\right], \tag{7}$$

$$T_3^* = T_3^*(Q_c, Q_a) = T_3^*(Q_c, n_1, n_2) \text{ etc}, \tag{8}$$

where $k, k_g =$ air, respectively burned gases adiabatic exponents, $R, R_g =$ air, respectively burned gases thermodynamic constants.

The quantities in the right side of (6) and (7) are the specific adiabatic works of the turbine and of the compressor, expressed as:

$$l_T = \frac{\chi_g \eta_T^*}{\chi_g - 1}T_3^*\left[1-\left(\frac{1}{\delta_T^*}\right)^{\frac{\chi_g-1}{\chi_g}}\right] \tag{6'}$$

and

$$l_C = \frac{\chi\ \eta_C}{\chi\ -1}T_1^*\left[\left(\pi_c^*\right)^{\frac{\chi_g-1}{\chi_g}} - 1\right]. \tag{7'}$$

Their dependence on the rotation speed's non-dimensional

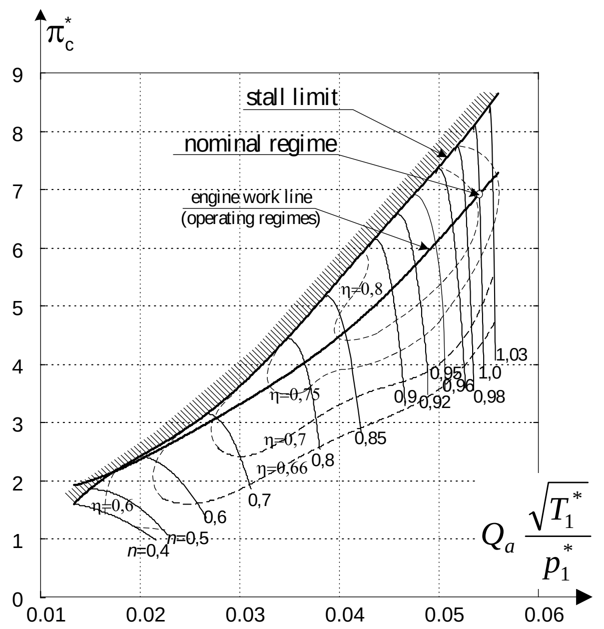

Fig. 3 Turbo-compressor universal characteristics [17]

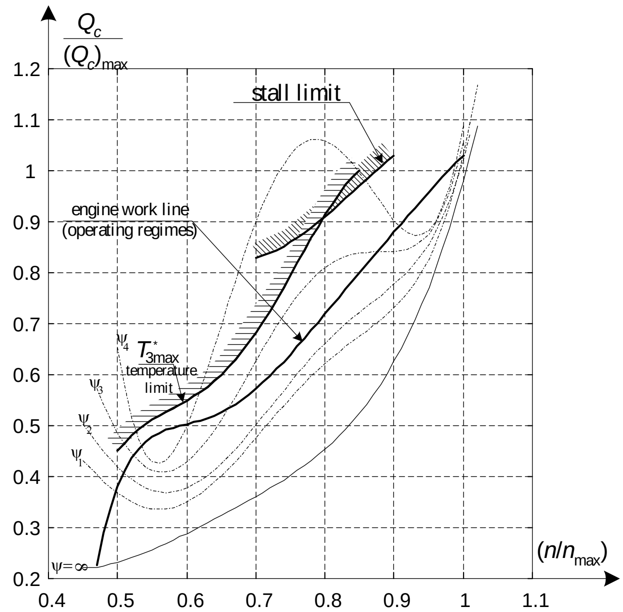

Fig. 4 Engine's flow-rate/speed characteristics [17]

parameter $\overline{n} = \dfrac{n}{n_{\max}}$ is presented in fig. 5; the turbine's work becomes constant when the critical regime is reached (see the horizontal segment in fig. 5).

Concerning the fluid's flow rate along the engine, the variations are negligible (as fig. 6. shows) so, the assumption of the fluid flow rate's constant value (for a constant engine's operating regime) is correct.

In order to make the study easier, the equation system must be linearised, assuming the small-disturbances hypothesis.

A method of description for each argument X can be used (where X can be $n_1, n_2, Q_a, Q_c, Q_g, ...$ etc):

$$X = X_0 + \frac{\Delta X}{1!} + \frac{(\Delta X)^2}{2!} + ... + \frac{(\Delta X)^n}{n!}, \qquad (9)$$

(where X_0 is the steady state regime's X-value and ΔX - deviation or static error) and neglecting the terms which contains $(\Delta X)^r, r \geq 2$, one obtains a new form of the equation system, particularly in the neighborhood of a steady state operating regime.

A. *Linear mathematical model*

One can assume, after the above observations, that (1) can be expressed as:

$$\frac{\pi J_1}{30}\frac{\mathrm{d}n_1}{\mathrm{d}t} = M_{T1} - M_{C1} = M_{T1}\left(Q_g, n_1, \delta^*_{T1}, T^*_{41}\right) - ,$$
$$- M_{C1}\left(Q_a, n_1, \pi^*_{c1}, T^*_1\right), \qquad (10)$$

$$\frac{\pi J_2}{30}\frac{\mathrm{d}n_2}{\mathrm{d}t} = M_{T2} - M_{C2} = M_{T2}\left(Q_g, n_2, \delta^*_{T2}, T^*_3\right) - ,$$
$$- M_{C2}\left(Q_a, n_2, \pi^*_{c2}, T^*_1\right). \qquad (11)$$

Considering (2) to (8), as well as the forms

$$\delta^*_{Ti} = \delta^*_{Ti}(n_i, Q_g),\ T^*_{41} = T^*_{41}\left(\delta^*_{T2}, Q_g, T^*_3\right),$$
$$Q_g = Q_g\left(\delta^*_{T1}, A_5\right), Q_a = Q_a\left(n_1, \pi^*_{c1}\right)$$

one can conclude that

$$M_{T2} = M_{T2}(Q_c, n_2, n_1), M_{C2} = M_{C2}(n_1, n_2), \qquad (12)$$

$$M_{T1} = M_{T1}(Q_c, n_2, n_1, A_5), M_{C2} = M_{C2}(n_1, n_2), \qquad (13)$$

$$T^*_3 = T^*_3(Q_c, n_2, n_1). \qquad (14)$$

So, equations (10) and (11), based on the assumption (9), are becoming:

$$\frac{\pi J_1}{30}\frac{\mathrm{d}}{\mathrm{d}t}\Delta n_1 = \left(\frac{\partial M_{T1}}{\partial Q_c}\right)_0 \Delta Q_c + \left[\left(\frac{\partial M_{T1}}{\partial n_1}\right)_0 - \left(\frac{\partial M_{C1}}{\partial n_1}\right)_0\right] \cdot$$
$$\Delta n_1 + \left[\left(\frac{\partial M_{T1}}{\partial n_2}\right)_0 - \left(\frac{\partial M_{C1}}{\partial n_2}\right)_0\right]\Delta n_2 + \left(\frac{\partial M_{T1}}{\partial A_5}\right)_0 \Delta A_5, \qquad (15)$$

$$\frac{\pi J_2}{30}\frac{\mathrm{d}}{\mathrm{d}t}\Delta n_1 = \left(\frac{\partial M_{T2}}{\partial Q_c}\right)_0 \Delta Q_c + \left[\left(\frac{\partial M_{T2}}{\partial n_1}\right)_0 - \left(\frac{\partial M_{C2}}{\partial n_1}\right)_0\right] \cdot$$
$$\Delta n_1 + \left[\left(\frac{\partial M_{T2}}{\partial n_2}\right)_0 - \left(\frac{\partial M_{C2}}{\partial n_2}\right)_0\right]\Delta n_2. \qquad (16)$$

The equation which gives the combustor's gases temperature (before the turbine) becomes also:

$$\Delta T^*_3 = \left(\frac{\partial T^*_3}{\partial Q_c}\right)_0 \Delta Q_c + \left(\frac{\partial T^*_3}{\partial n_1}\right)_0 \Delta n_1 + \left(\frac{\partial T^*_3}{\partial n_2}\right)_0 \Delta n_2. \qquad (17)$$

The above used partial derivates can be calculated using the formulas (6) and (7), or (6') and (7'), as well as the graphics in fig. 3, fig. 4, fig. 5 and fig.6.

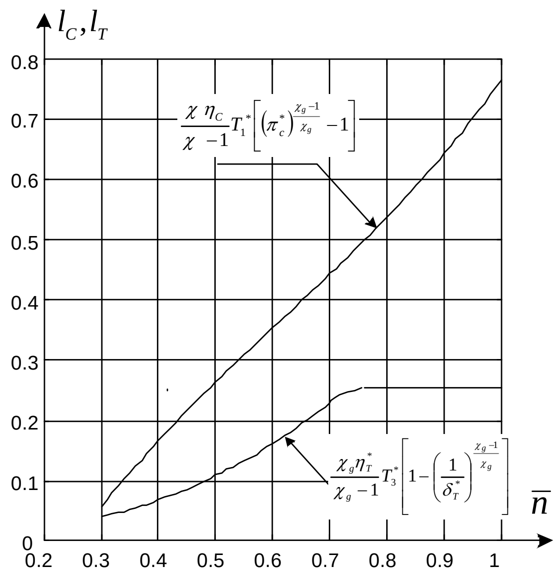

Fig. 5 Specific adiabatic work for compressor and turbine

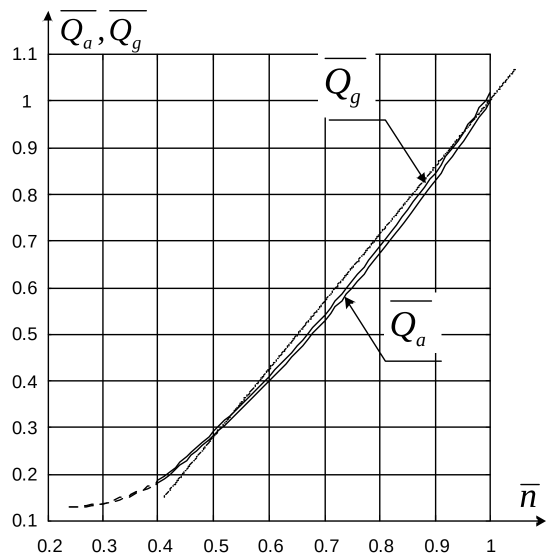

Fig. 6 Fluid's flow rate along the engine with respect to the engine's non-dimensional speed parameter

B. Non-dimensional mathematical model

One must group in (15) the terms which contains Δn_1, respectively in (16) the terms which contains Δn_2, to the left side, then divide the equations by their left side's co-efficient. Using, also, the generic annotation $\overline{X} = \frac{\Delta X}{X_0}$ and applying the Laplace transformation for both equations, it results, eventually:

$$(\tau_{r1}s+1)\overline{n_1} = k_{c1}\overline{Q_c} + k_{1n2}\overline{n_2} + k_{1A}\overline{A_5}\,, \tag{18}$$

$$(\tau_{r2}s+1)\overline{n_2} = k_{c2}\overline{Q_c} + k_{2n1}\overline{n_1}\,, \tag{19}$$

where the used annotations are:

$$K_{Ri} = \frac{1}{\left(\frac{\partial M_{Ci}}{\partial n_i}\right)_0 - \left(\frac{\partial M_{Ti}}{\partial n_i}\right)_0},\ \tau_{ri} = \frac{\pi J_i}{30}K_{Ri},\ i=\overline{1,2}\,,$$

$$k_{ci} = K_{Ri}\frac{Q_{c0}}{n_{i0}}\left(\frac{\partial M_{Ti}}{\partial Q_c}\right)_0,\ k_{1A} = K_{R1}\frac{Q_{c0}}{n_{10}}\left(\frac{\partial M_{T1}}{\partial Q_c}\right)_0\,,$$

$$k_{1n2} = K_{R1}\frac{n_{20}}{n_{10}}\left[\left(\frac{\partial M_{T1}}{\partial n_2}\right)_0 - \left(\frac{\partial M_{C1}}{\partial n_2}\right)_0\right],$$

$$k_{2n1} = K_{R2}\frac{n_{10}}{n_{20}}\left[\left(\frac{\partial M_{T2}}{\partial n_1}\right)_0 - \left(\frac{\partial M_{C2}}{\partial n_1}\right)_0\right]. \tag{20}$$

Both speeds have similar expressions, but some important particularities are occurred.

Each spool can be assimilated to an independent spool of a single-jet single-spool engine, but operating as a couple, being gas-dynamic bounded. Consequently, analyzing the (18) and (19) forms, an observation can be made, concerning the existence of a mutual influence between the spool's speeds, accomplished by the co-efficient k_{1n2} and k_{2n1} which appear in (18) and (19). These co-efficient are the mutual co-efficient and have a lot of influence in the engine's stability (as controlled object).

These mutual co-efficient k_{1n2} and k_{2n1} are not constant, but they depends on the flight regime (altitude H and speed V), as fig, 7 shows.

Another observation is that the exhaust nozzle's opening A_5 influences only the LPS speed, as (18) shows, which is the consequence of the burned gas flow rate's dependence on the above mentioned parameter (A_5).

For (17) one can obtain a similar form:

$$\overline{T_3^*} = k_{3c}\overline{Q_c} + k_{3n1}\overline{n_1} + k_{3n2}\overline{n_2}\,, \tag{21}$$

where

$$k_{3c} = \left(\frac{Q_{c0}}{T_{30}^*}\right)\left(\frac{\partial T_3^*}{\partial Q_c}\right)_0,$$

$$k_{3n1} = \left(\frac{n_{10}}{T_{30}^*}\right)\left(\frac{\partial T_3^*}{\partial n_1}\right)_0,$$

$$k_{3n2} = \left(\frac{n_{20}}{T_{30}^*}\right)\left(\frac{\partial T_3^*}{\partial n_2}\right)_0. \tag{22}$$

The equations (18), (19) and (21), with the annotations (20) and (22), represent the linear non-dimensional mathematical model.

If the disturbance(s) (represented by the flight regime) must be taken into account, it shall affect the LPS equation, because it is given by the terms containing the pressure in the front of the compressor (p_1^*). The flight regime has a direct influence above the low pressure compressor's inlet, as well as above

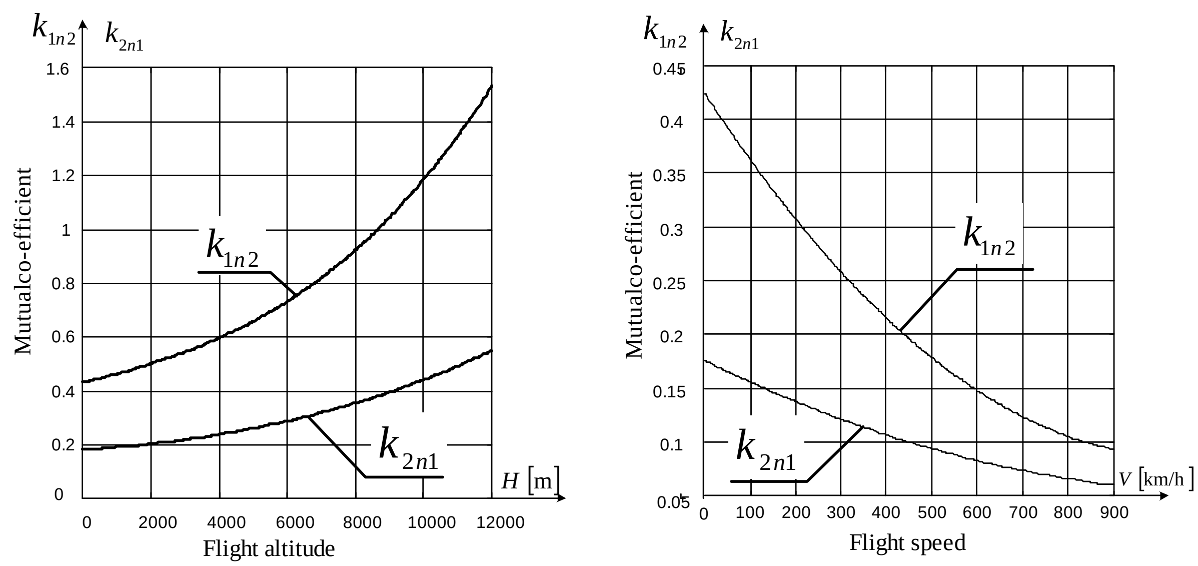

Fig. 7. Mutual co-efficient k_{1n2} and k_{2n1} dependence on the flight regime

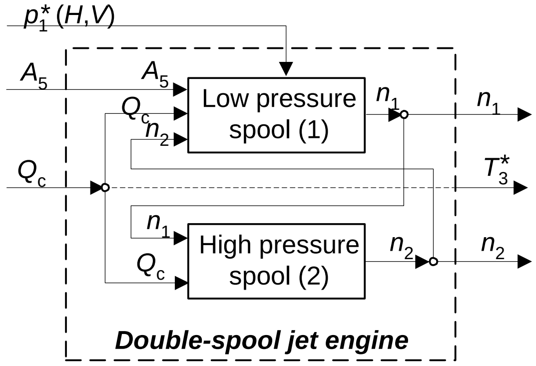

Fig. 8. Engine's functional diagram

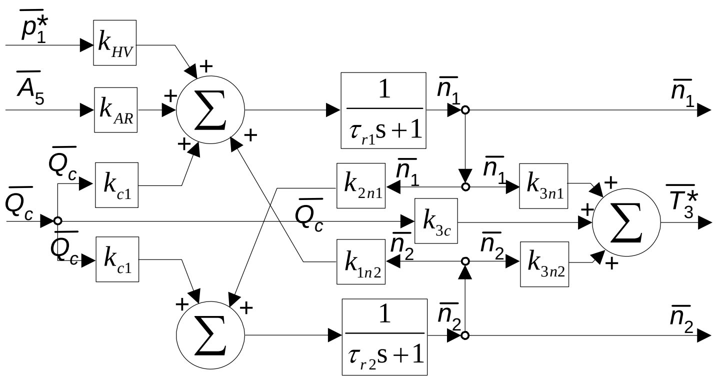

Fig. 9. Engine's block diagram with transfer functions

the low pressure turbine's exhaust, which explains why the flight regime affects only the LPS. Consequently, (18) gets a new term, involving the mentioned pressure $\left(p_1^*\right)$ and becomes

$$\left(\tau_{r1}s+1\right)\overline{n_1}=k_{c1}\overline{Q_c}+k_{1n2}\overline{n_2}+k_{1A}\overline{A_5}+k_{HV}\overline{p_1^*}, \quad (23)$$

where $k_{HV}=k_{c1}\left(\dfrac{p_{10}^*}{Q_{c0}}\right)\left(\dfrac{Q_c}{p_1^*}\right)_{H=0,V=0}$.

An experimental (empirical) formula for k_{HV} is given in [17], with respect to its value at sea level k_{00} and to the total pressure in the front of the compressor p_1^*:

$$k_{HV}=k_{00}\left(\frac{p_1^*}{p_0}\right)^{\frac{2}{3}}. \quad (24)$$

Double-spool engine's functional diagram is represented in fig. 8, and its block diagram with transfer functions in fig. 9. This block diagram was built using the equation system (18), (19) and (21).

Same observation can be made:

a) the inputs are the combustor's fuel flow rate and the exhaust nozzle's opening area;
b) the outputs are both engine's rotation speeds.

C. Engine's transfer functions

One can observe that only two equations are independent (equations (18) and (19)) and the third is a linear combination of these ones (21). So, using (18) and (19), the expressions for the engine's spools rotation speed non-dimensional parameters are:

$$\overline{n_1}=\frac{k_{c1}\tau_{r2}s+\left(k_{c1}+k_{c2}k_{1n2}\right)\overline{Q_c}+k_{1A}\left(\tau_{r2}s+1\right)\overline{A_5}}{\tau_{r1}\tau_{r2}s^2+\left(\tau_{r1}+\tau_{r2}\right)s+\left(1-k_{1n2}k_{2n1}\right)}, \quad (25)$$

$$\overline{n_2}=\frac{k_{c2}\tau_{r1}s+\left(k_{c2}+k_{c1}k_{2n1}\right)\overline{Q_c}+k_{1A}k_{2n1}\overline{A_5}}{\tau_{r1}\tau_{r2}s^2+\left(\tau_{r1}+\tau_{r2}\right)s+\left(1-k_{1n2}k_{2n1}\right)}; \quad (26)$$

the system's characteristic equation is:

$$\tau_{r1}\tau_{r2}s^2+\left(\tau_{r1}+\tau_{r2}\right)s+\left(1-k_{1n2}k_{2n1}\right)=0. \quad (27)$$

Introducing the expressions for $\overline{n_1}$ and $\overline{n_2}$, given by (25) and (26), in the combustor's temperature non-dimensional parameter $\left(\overline{T_3^*}\right)$ (expression (21), see above), it results a similar form:

$$\overline{T_3^*}=\frac{\left(a_2s^2+a_1s+a_0\right)\overline{Q_c}+\left(b_1s+b_0\right)\overline{A_5}}{\tau_{r1}\tau_{r2}s^2+\left(\tau_{r1}+\tau_{r2}\right)s+\left(1-k_{1n2}k_{2n1}\right)}, \quad (28)$$

where

$$a_2=k_{3c}\tau_{r1}\tau_{r2},$$
$$a_1=k_{3c}\left(\tau_{r1}+\tau_{r2}\right)+k_{3n1}k_{c1}\tau_{r2}+k_{3n2}k_{c2}\tau_{r1},$$
$$a_0=k_{3c}+k_{3n1}\left(k_{c1}+k_{c2}k_{1n2}\right)+k_{3n2}\left(k_{c2}+k_{c1}k_{2n1}\right). \quad (29)$$

IV. About system's stability

Because of the characteristic equation's form and degree, one can apply the algebraic Routh-Hurwitz criteria in order to determine the system's stability, which leads to the next conditions:

$$\tau_{r1}\tau_{r2}>0, \quad (30)$$

$$\tau_{r1}+\tau_{r2}>0, \quad (31)$$

$$1-k_{1n2}k_{2n1}>0. \quad (32)$$

Because the time constants τ_{r1},τ_{r2} are strictly positive quantities, the condition (30) and (31) are always identical satisfied; so, for the stability, only the third condition (32) remains to be studied; its equivalent form gives a hyperbolic relation between the mutual co-efficient k_{1n2}, k_{2n1} (meaning that $k_{2n1}<\dfrac{1}{k_{1n2}}$), as fig. 10 shows.

The stability domain is the one below the hyperbola $k_{2n1}=\dfrac{1}{k_{1n2}}$, so in order to be a stable object, the double-spool jet engine must have the product of its mutual co-efficient little than 1.

Inside the stability domain, one can also insulate the non-periodic stability domain, studying the condition for real (and

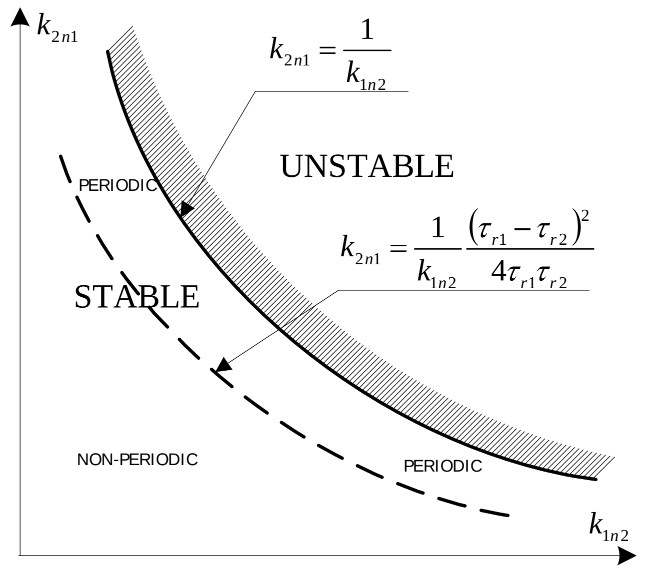

Fig. 10 Engine's stability domains

negative) roots of the characteristic polynom:

$$(\tau_{r1} + \tau_{r2})^2 - 4\tau_{r1}\tau_{r2}(1 - k_{1n2}k_{2n1}) > 0\,, \tag{33}$$

which gives another semi-plan, limited by the hyperbola of equation

$$k_{2n1} = \frac{1}{k_{1n2}} \frac{(\tau_{r1} - \tau_{r2})^2}{4\tau_{r1}\tau_{r2}}\,. \tag{34}$$

For the periodic stability, the engine's stability degree (or reserve) δ is defined as the distance (measured on the horizontal - real axis) between the complex plan's imaginary axis and the nearest root. So, the maximum value of the stability reserve is obtained for the double root, when

$$(\tau_{r1} + \tau_{r2})^2 - 4\tau_{r1}\tau_{r2}(1 - k_{1n2}k_{2n1}) = 0\,, \tag{35}$$

is given by

$$\delta_{\max} = -\frac{\tau_{r1} + \tau_{r2}}{2\tau_{r1}\tau_{r2}} = -\frac{1}{2}\left(\frac{1}{\tau_{r1}} + \frac{1}{\tau_{r2}}\right). \tag{36}$$

For a stable engine $\delta < 0$ and the bigger is the absolute $\delta -$value, the higher is the stability.

The stability degree can be expressed as

$$\delta = \delta_{\max}(1 - \Delta\delta), \tag{37}$$

where

$$\Delta\delta = \sqrt{1 - \frac{4\tau_{r1}\tau_{r2}(1 - k_{1n2}k_{2n1})}{(\tau_{r1} + \tau_{r2})^2}}\,. \tag{38}$$

If the transfer function's characteristic polynom is expressed as

$$P(\mathrm{s}) = \mathrm{s}^2 + 2\xi\omega\mathrm{s} + \omega^2\,, \tag{39}$$

the pulsation of the self-oscillations is given by

$$\omega = \sqrt{\frac{1 - k_{1n2}k_{2n1}}{\tau_{r1}\tau_{r2}}} \tag{40}$$

and the self-oscillations damping co-efficient becomes

$$\xi = \frac{\tau_{r1} + \tau_{r2}}{2\sqrt{\tau_{r1}\tau_{r2}(1 - k_{1n2}k_{2n1})}}\,. \tag{41}$$

If the engine is an unstable object, it is compulsory that one or more controllers assist it. These controllers could assist the spools (controlling the speed, feed-back type), or could assist the input parameters (feed-before type).

V. System's quality. Engine time behavior

One has identified the engine as controlled object with two inputs and two outputs. A simulation can be performed, for an existing engine (R11-F300); for this type of engine one could estimate the amount of the co-efficient, as follows:

$\tau_{r1} = 0.3102$ s; $\tau_{r2} = 0.4498$ s; $k_{c1} = 0.3511$; $k_{c2} = 0.4091$; $k_{1n2} = 0.2981$; $k_{1A} = 0.5917$; $k_{2n1} = 0.1911$; $a_2 = 0.0899$; $a_1 = 0.4194$; $a_0 = 0.4217$; $b_1 = 0.1039$; $b_0 = 0.2407$.

The simulation can be performed for two different cases:

A) independent input signals;

B) correlated input signals.

A) Considering that the input signals, for the non-dimensional parameters $\overline{Q_c}$ and $\overline{A_5}$, are independent, so each one of them is realized (hypothetically) by an outer system, independent of the engine's operation, one has obtained the system step response, imposing as input a step signal, both for $\overline{Q_c}$ and $\overline{A_5}$; the results are shown in figure 11.

Some observations can be made:

1) the engine is a stable system and its stability is non-periodic;

2) for both output parameters $\overline{n_1}$ and $\overline{n_2}$, the stabilizations is realized with static error (error which is bigger for the high pressure spool speed), as fig. 11 shows;

3) the combustor's temperature has an initial step growing, a small override and furthermore it shows the same non-periodic stabilization;

4) the combustor's temperature is not a directly controlled parameter, but it depends on the fuel flow rate injection (as (21) shows) , as well as on the air flow rate (indirectly represented in (21) by the speeds). If a temperature limitation is necessary, the most efficient input parameter for control is the fuel flow rate, which will affect both the temperature and the speeds $\overline{n_1}$ and $\overline{n_2}$.

B) If one considers the real situation, for an existing engine, where the input parameters are given by some other automatic systems, the results are changing.

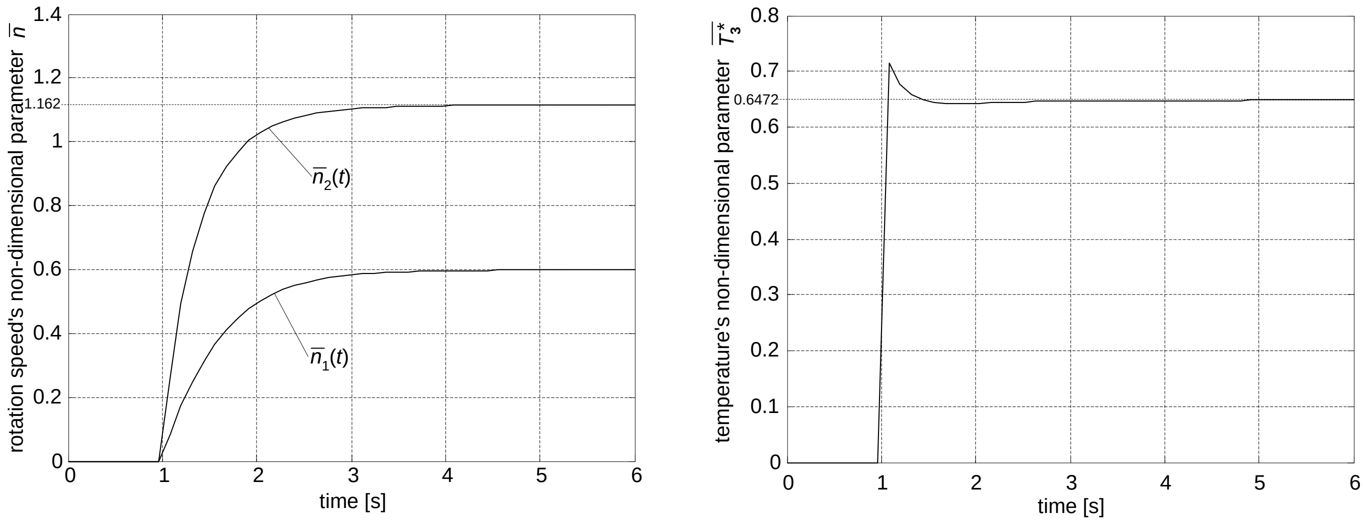

Fig. 11 Engine's step response (for independent input parameters)

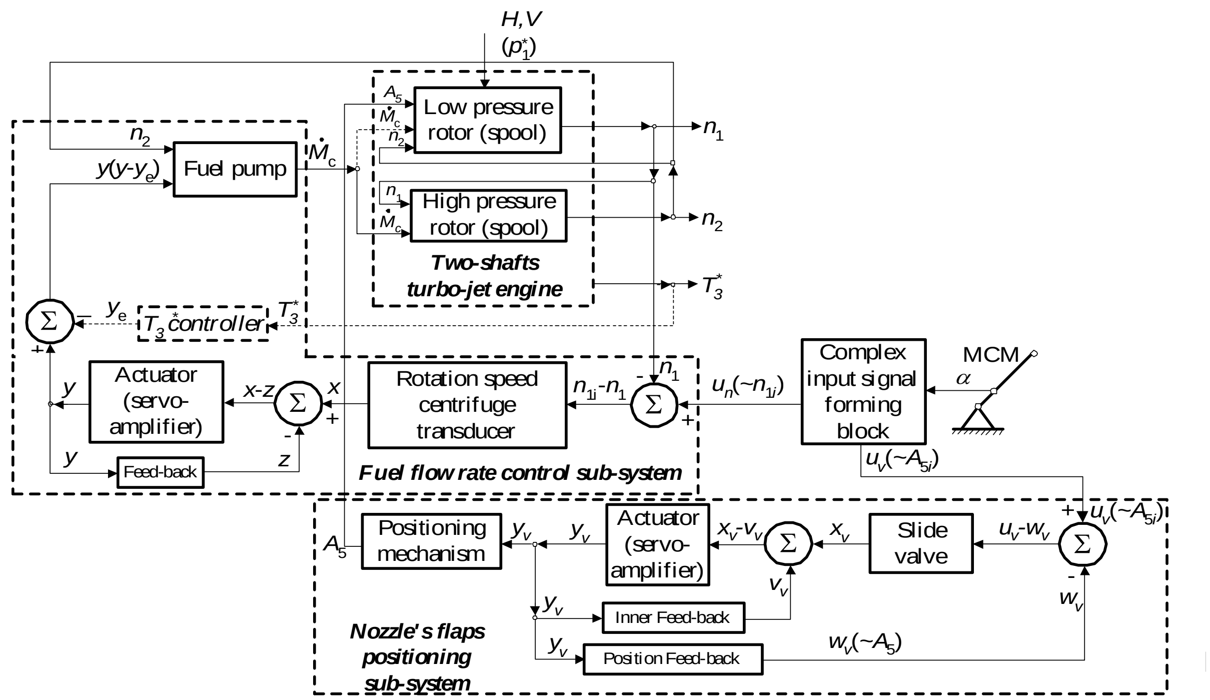

Fig. 12 Double spool jet-engine's constructive block diagram (for correlated input parameters)

Most of the nowadays operating engines have an unique input, which is the throttle's positioning α .

More specific, for an engine R11-F300 type, the throttle realizes the input for the two auxiliary systems S1 and S2, as the block diagram in fig. 12 shows:

S1) the fuel's flow rate controller (fuel pump's regulator) [18]. The pump is coupled to the high pressure spool, and the regulator's speed transducer is coupled to the low pressure spool;

S2) the exhaust nozzle's flaps positioning system (a follower system) [2].

The throttle realizes an unique input, which is split into two command signals by the CISFB (complex input signal forming block), which are the inputs for the sub-systems S1 and S2.

Fig. 13 shows the block diagram with transfer functions for such a complex system; the engine has, also, an automatic system for the combustor's temperature limitation (a so-called

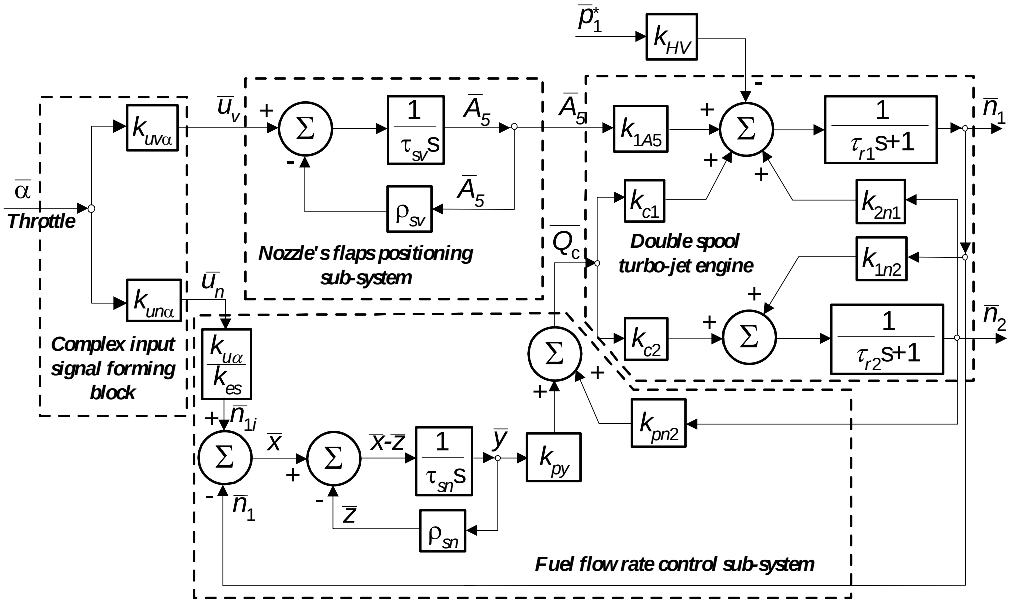

Fig. 13 Block diagram with transfer functions for a double spool jet engine with throttle operated input signal forming block, fuel flow rate controller and exhaust nozzle's opening controller

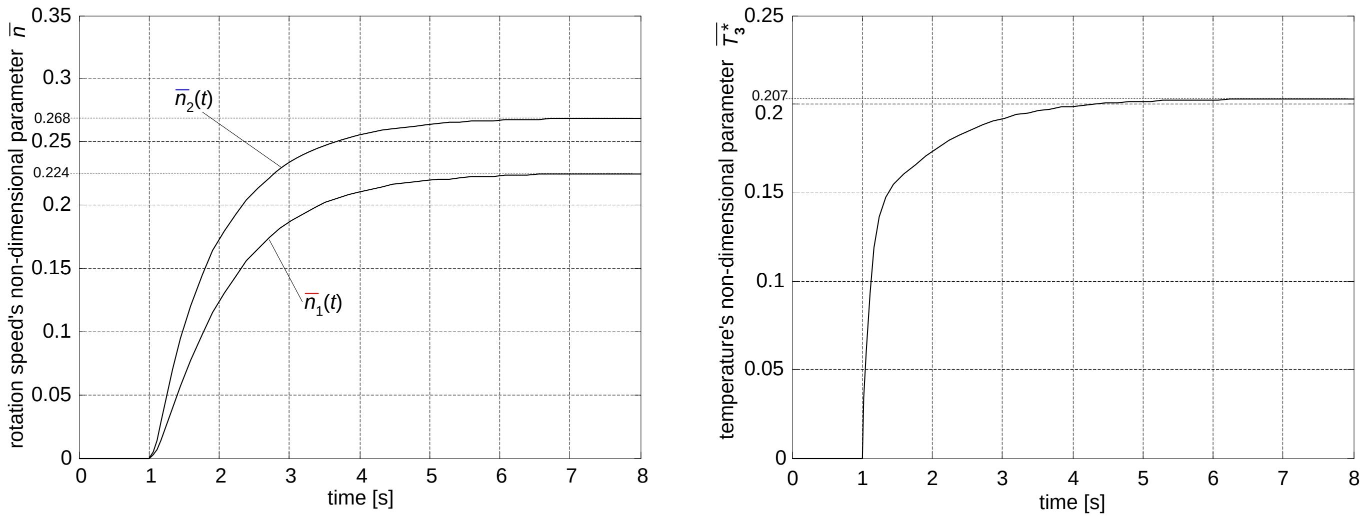

Fig. 14 Engine's step response (engine with input signal forming block)

T_3^*-controller), which is a supplementary controller for the fuel flow rate, acting on the fuel pump's actuator ([17] and [14]), by discharging one of its active chamber. Obviously, the controller's activation, when the maximum value for T_3^* is overlapped, reduces the fuel flow rate and, consequently, the speeds n_1 and n_2 will be reduced too.

The diagrams in fig. 14 show the step response for this kind of engine, considering as input the step growing of the throttle's non-dimensional displacement $\overline{\alpha}$.

The main observation, concerning the engine's time behavior and stability, remains the same. Supplementary, one can see that both of speeds have smaller static errors than the errors in fig. 11.

Meanwhile, the temperature's behavior is improved, the initial override has disappeared and its stabilization is non-periodic.

As a final conclusion, this kind of model for a double-spool jet engine, based on a linear equation system and as well as on the mechanical and gas-dynamic characteristics and co-efficient, can offer an image about the engine as controlled object and about its stability and quality.

For other types of double-spool jet engines, such as the turbofan, the turbo-shaft or the twin-jet engine (see [8], [14], [17] and [20]), a similar approach can be used, leading to similar conclusions.

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