Computer-aided simulation on the reversing operation of the two-phase induction machine

Alecsandru Simion, Leonard Livadaru, and Dorin Lucache

***Abstract*—**The paper presents a new mathematical model of the two-phase induction machine, called "in total fluxes", which is very appropriate for the study of the reversing regime. The equations of the model use as main quantities the rotation angle and the total fluxes of the windings and exclude the rotation speed, which now become a secondary quantity that can be calculated from rotation angle expression. On the basis of this model, the computer simulation looks into the behavior of a two-phase induction servomotor with low inertia when the supply voltages of the two separate windings have the same magnitude but a different frequency, under load and no load operation. The reversing regime is also simulated under unbalanced supply conditions. The results offer the perspective to design electromechanical systems with speed or rotation angle expressed as harmonic, quasi-harmonic or even random variation laws.

Keywords—Induction machine, mathematical model with fluxes, reversing operation, simulation.

I. Introduction

THE two-phase induction machine (denoted with the acronym T-PIM in this paper) is the machine with minimal number of windings that operates on the basis of rotating magnetic field. It has two immobile windings on stator, 90 electrical degrees shifted in space. The other two equivalent windings, placed on rotor, are obviously in rotation. The position angle, θ_R, between the homologous phase axes, reference *as* from stator and *ar* from rotor, is time dependent under a linear rule if the machine operates in a stationary regime [1,2,3]. This machine is used as motor (two-phase induction servomotor) in low power electric drives or as position or speed transducer (under generating duty) for the movement control of some rotating mechanical elements of positioning systems [4-16]. The two-phase induction motor with hollow-rotor made of non-magnetic metals (aluminum or alloy) and with high rotor resistance is frequently used for positioning. Due to its low inertia, this machine is proper for reversing rotation (bidirectional servomotor). This operation regime can be obtained by supplying the control phase winding with a voltage that has a different frequency from the excitation phase winding. The T-PIM is also frequently used as mathematical model (αβ-αβ, ab-ab, ab-dq, DQ-dq) for the study of the multiphase induction machines, mainly the three-phase ones, which operate as part of variable electric drives.

The study of this machine, operating under different regimes, uses 4 differential equations given by the II Kirchhoff theorem (voltage balance) and the torque equilibrium equation where both angular speed ω_R and its derivative $d\omega_R/dt$ are present. The second order movement equation can be replaced by two differential equations of the first order. Thus, the final system contains 6 differential equations of the first order and non-linear. The scientific literature generally uses as variable quantities the winding currents and the electromagnetic torque [2,3]. If the total fluxes produced by the windings are expressed in terms of currents and inductances, then the model becomes a hybrid one, denoted as *in total fluxes and currents*. A coordinate transformation can bring a linearization of the system. But these transformations bring new variable quantities which usually have a different frequency related to real ones. A new model is obtained, with two rotor windings and collinear stator and rotor axes, called *dq-dq (DQ-dq)* [1,2,3]. The models mostly used in the scientific literature contain inside the circuit equations the rotor speed ω_R. The resolution of the system takes into consideration a maintaining constant of the speed or acceptance of a “slow variation”. This is an important error source mainly when the speed varies rapidly (start-up or reversal of the two-phase induction servomotor, for example) [4].

This paper presents a new model, called *in total fluxes*, which has simpler equations, the variables represent real quantities (no transformation is required) and the rotation speed, ω_R, is not present in an explicit form. The electric circuits have as independent variable the rotation angle, $\theta_R=var$ (its variation law is not necessary to be prior imposed). This model gives more accurate results for the study of variable speed regimes, the calculus time could be shorter, the precision of the unbalanced duty analysis is higher, and the implementation together with a position transducer and eventually with a rotor flux transducer is more effective.

Manuscript received January 31, 2009.

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II. THE TWO-PHASE INDUCTION MACHINE MODEL WITH VARIABLE QUANTITIES IN TOTAL FLUXES

The T-PIM is presented in Fig. 1a), the 4 windings with their characteristics in Fig. 1b) and the reduced rotor quantities [5] in Fig. 1c). The II Kirchhoff theorem gives 4 non-linear equations of the first order. The total fluxes depends both on time and rotation angle, θ_R, which is itself variable with time as well.

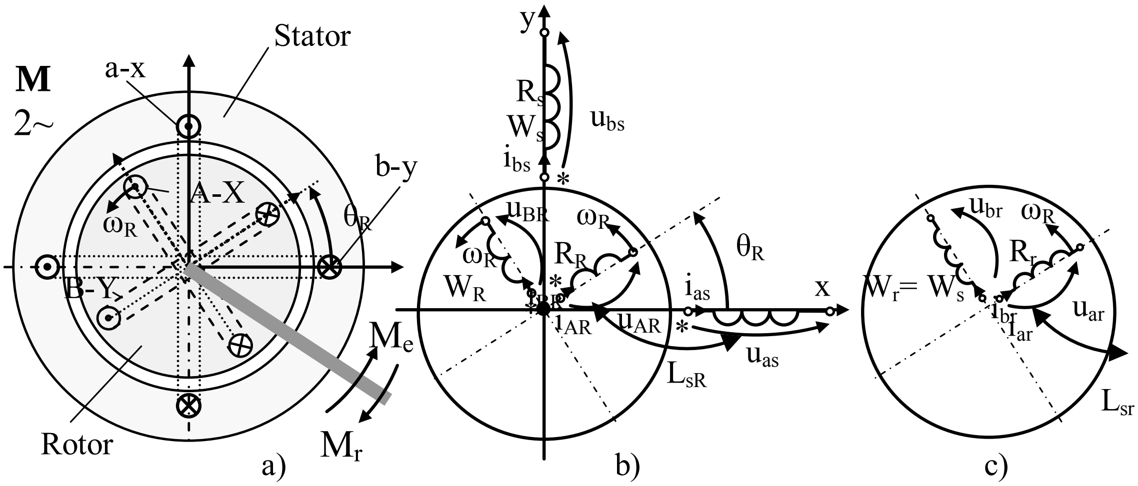


Fig.1 Two-phase induction machine: a) Physical model, b) Simplified depiction, c) Reduced rotor

If the total fluxes are expressed in terms of currents and inductances then a matrix equation can be obtained [1]:

$$
\begin{cases}
[u_{absr}] = [R_{s,r}][i_{absr}] + \dfrac{d[\psi_{absr}]}{dt}, \\
where \quad [\psi_{absr}] = [L_{abssrr}][i_{absr}], \\
[L_{abssrr}] = L_{hs}\begin{bmatrix} 1+l_{\sigma s} & 0 & \cos\theta_R & -\sin\theta_R \\ 0 & 1+l_{\sigma s} & \sin\theta_R & \cos\theta_R \\ \cos\theta_R & \sin\theta_R & 1+l_{\sigma r} & 0 \\ -\sin\theta_R & \cos\theta_R & 0 & 1+l_{\sigma r} \end{bmatrix}, \\
with: \dfrac{L_{\sigma s}}{L_{hs}} = l_{\sigma s}, \dfrac{L_{\sigma r}}{L_{hs}} = l_{\sigma r}.
\end{cases}
\tag{1}
$$

To obtain the currents, the left term is amplified with the reciprocal matrix:

$$
\begin{aligned}
&[L_{abssrr}]^{-1}[\psi_{absr}] = [L_{abssrr}]^{-1}[L_{abssrr}][i_{absr}], \\
&or \quad [i_{absr}] = [L_{abssrr}]^{-1}[\psi_{absr}]
\end{aligned}
\tag{2}
$$

The main issue is now to identify the *reciprocal matrix* [17]. This matrix is supposed to be similar to the direct matrix and by identification term by term and using the notations:

$$
\begin{aligned}
&\Lambda_h = \frac{1}{L_{hs}}; \quad k = \frac{1}{l_{\sigma s} + l_{\sigma r} + l_{\sigma s} l_{\sigma r}}; \\
&k_s = (1 + l_{\sigma s})k; \quad k_r = (1 + l_{\sigma r})k
\end{aligned}
\tag{3}
$$

one obtains:

$$
[L_{abssrr}]^{-1} = \\
= \begin{bmatrix} k_r\Lambda_h & 0 & -k\Lambda_h\cos\theta_R & k\Lambda_h\sin\theta_R \\ 0 & k_r\Lambda_h & -k\Lambda_h\sin\theta_R & -k\Lambda_h\cos\theta_R \\ -k\Lambda_h\cos\theta_R & -k\Lambda_h\sin\theta_R & k_s\Lambda_h & 0 \\ k\Lambda_h\sin\theta_R & -k\Lambda_h\cos\theta_R & 0 & k_s\Lambda_h \end{bmatrix}
\tag{4}
$$

Then follows the product of the matrix $[R_{s,r}][L_{abssrr}]^{-1}[\psi_{absr}]$, and the voltage equation (1) using reduced quantities becomes after convenient grouping:

$$\frac{d\psi_{as}}{dt} + k_r\Lambda_h R_s \psi_{as} = u_{as} + k\Lambda_h R_s(\psi_{ar}\cos\theta_R - \psi_{br}\sin\theta_R) \tag{5-a}$$

$$\frac{d\psi_{bs}}{dt} + k_r\Lambda_h R_s \psi_{bs} = u_{bs} + k\Lambda_h R_s(\psi_{ar}\sin\theta_R + \psi_{br}\cos\theta_R) \tag{5-b}$$

$$\frac{d\psi_{ar}}{dt} + k_s\Lambda_h R_r \psi_{ar} = u_{ar} + k\Lambda_h R_r(\psi_{as}\cos\theta_R + \psi_{bs}\sin\theta_R) \tag{5-c}$$

$$\frac{d\psi_{br}}{dt} + k_s\Lambda_h R_r \psi_{br} = u_{br} + k\Lambda_h R_r(-\psi_{as}\sin\theta_R + \psi_{bs}\cos\theta_R) \tag{5-d}$$

The next step is the quantification of the electromagnetic torque, which is possible by means of the stored energy principle or through the magnetic energy stored in the machine circuits [2,8,14] according to current values. The expression of the torque [3,4] together with (2) becomes:

$$
\begin{aligned}
M_e &= \frac{p}{2}\left\{[i_{absr}]_t \cdot \frac{d[L_{abssrr}]}{d\theta_R}[i_{absr}]\right\} = \\
&= \frac{p}{2}\left\{[\psi_{absr}]_t \cdot [L_{abssrr}]_t^{-1}\frac{d[L_{abssrr}]}{d\theta_R}[L_{abssrr}]^{-1} \cdot [\psi_{absr}]\right\}
\end{aligned}
\tag{6}
$$

The following equality is valid:

$$\left[L_{absssrr}\right]_t^{-1}\frac{d\left[L_{absssrr}\right]}{d\theta_R}\left[L_{absssrr}\right]^{-1}=-\frac{d\left[L_{absssrr}\right]^{-1}}{d\theta_R}\qquad(7)$$

For validation, (7) is amplified to the left with $\left[L_{absssrr}\right]=\left[L_{absssrr}\right]_t$ and results:

$$\left[L_{absssrr}\right]\cdot\left[L_{absssrr}\right]^{-1}\frac{d\left[L_{absssrr}\right]}{d\theta_R}\left[L_{absssrr}\right]^{-1}=-\left[L_{absssrr}\right]\frac{d\left[L_{absssrr}\right]^{-1}}{d\theta_R}$$

The first product is the unit matrix $[I]$, so: $\frac{d\left[L_{absssrr}\right]}{d\theta_R}\left[L_{absssrr}\right]^{-1}=-\left[L_{absssrr}\right]\frac{d\left[L_{absssrr}\right]^{-1}}{d\theta_R}$, an obvious expression as a consequence of derivation rule of a constant product. Consequently, the active electromagnetic torque, given by (6), is:

$$M_e=-\frac{p}{2}\left[\psi_{absr}\right]_t\frac{d\left[L_{absssrr}\right]^{-1}}{d\theta_R}\left[\psi_{absr}\right]\qquad(8)$$

a similar expression to the well known equation in currents. For the nonce, the torque depends with *total fluxes* alone. The calculus gives forwards:

$$\begin{aligned}M_e=pk\Lambda_h\big[&-\left(\psi_{as}\psi_{ar}+\psi_{bs}\psi_{br}\right)\sin\theta_R+\\&+\left(\psi_{bs}\psi_{ar}-\psi_{as}\psi_{br}\right)\cos\theta_R\big]\end{aligned}\qquad(8')$$

which proves that M_e depends on *total fluxes* and *rotor position angle*. The torque equation can be written as follows:

$$\begin{aligned}&\frac{d\dot{\theta}_R}{dt}+\frac{k_z}{J}\dot{\theta}_R=\frac{p}{J}\left\{pk\Lambda_h\left[-\left(\psi_{as}\psi_{ar}+\psi_{bs}\psi_{br}\right)\sin\theta_R+\right.\right.\\&\left.\left.+\left(\psi_{bs}\psi_{ar}+\psi_{as}\psi_{br}\right)\cos\theta_R\right]-M_{st}\right\}\\&d\theta_R/dt=\dot{\theta}_R\end{aligned}\qquad(9)$$

Now, the 6 equations system of the machine can be written by getting together the equation group (5-a to 5-d which has as variables the total fluxes and θ_R, but no speed) with the other 2 equations (9) which contain as variables only the total fluxes of the 4 windings, the speed and the position angle of the rotor.

III. Simulation Study with the Model "In Total Fluxes" for a Reversing T-PIM

In order to simplify the locution of the equation system (5-a to 5-d), new notations for the total fluxes from the right member (those which depends on θ_R) will be introduced. They are determined as:

- sums of projections corresponding to rotor fluxes from the axes *ar* and *br* on the two equivalent stator axes, *as* and *bs* (denoted with *d* and *q*, that is ψ_{dr}, ψ_{qr}) when apply to the fluxes from the right member of the first two equations;
- sums of projections corresponding to stator fluxes from the axes *as* and *bs* on the two equivalent rotor axes, *as* and *bs* (denoted with *D* and *Q*, that is ψ_{DS}, ψ_{QS}) when apply to the fluxes from the last two equations.

The expressions become:

$$\begin{aligned}&\psi_{dr}=\psi_{ar}\cos\theta_R-\psi_{br}\sin\theta_R,\ \psi_{qr}=\psi_{ar}\sin\theta_R+\psi_{br}\cos\theta_R\\&\psi_{DS}=\psi_{as}\cos\theta_R+\psi_{bs}\sin\theta_R,\psi_{QS}=-\psi_{as}\sin\theta_R+\psi_{bs}\cos\theta_R\end{aligned}\qquad(10)$$

The equations (5-a to 5-d) can be expressed in a symbolic form as follows:

$$\overline{\psi}_{as}(\overline{s}+k_r\Lambda_hR_s)=\overline{u}_{as}+k_h\Lambda_hR_s\overline{\psi}_{dr},\qquad(11\text{-a})$$

$$\overline{\psi}_{bs}(\overline{s}+k_r\Lambda_hR_s)=\overline{u}_{bs}+k_h\Lambda_hR_s\overline{\psi}_{qr},\qquad(11\text{-b})$$

$$\overline{\psi}_{ar}(\overline{s}+k_s\Lambda_hR_r)=\overline{u}_{ar}+k_h\Lambda_hR_r\overline{\psi}_{DS},\qquad(11\text{-c})$$

$$\overline{\psi}_{br}(\overline{s}+k_s\Lambda_hR_r)=\overline{u}_{br}+k_h\Lambda_hR_r\overline{\psi}_{QS},\qquad(11\text{-d})$$

The following notations (12) will be used as well. The coefficients c_s and c_r correspond to stator and rotor respectively and the total leakage inductance, $L_{\sigma\sigma}$, is the sum of the phase leakage inductances:

$$\begin{aligned}&k_h\Lambda_h=\frac{L_{hs}}{L_{\sigma s}+L_{\sigma r}+L_{\sigma s}L_{\sigma r}/L_{hs}}\cdot\frac{1}{L_{hs}}=\frac{1}{L_{\sigma\sigma}}\cong\frac{1}{L_{\sigma s}+L_{\sigma r}},\\&k_s\Lambda_h=\frac{L_{ss}}{L_{\sigma\sigma}L_{hs}}=\frac{1+l_{\sigma s}}{L_{\sigma\sigma}}=\frac{c_s}{L_{\sigma\sigma}};\ k_r\Lambda_h=\frac{L_{rr}}{L_{\sigma\sigma}L_{hs}}=\frac{1+l_{\sigma r}}{L_{\sigma\sigma}}=\frac{c_r}{L_{\sigma\sigma}}\end{aligned}\qquad(12)$$

For a more appropriate description of the machine is necessary, in our opinion, to introduce *the total leakage time constants* of the machine windings. For example, the *stator constant* is defined as the ratio of *the total leakage inductance* (which represents the sum of the total leakage inductance of a stator phase and the total leakage inductance of a rotor phase) and a *stator phase resistance*. Similarly, for the rotor:

$$\begin{aligned}&T_{\sigma s}=\frac{L_{\sigma\sigma}}{R_s},\ T_{\sigma r}=\frac{L_{\sigma\sigma}}{R_r},\\&k_r\Lambda_hR_s=\frac{c_r}{T_{\sigma s}},\ k_s\Lambda_hR_r=\frac{c_s}{T_{\sigma r}},\ k_h\Lambda_hR_s=\frac{1}{T_{\sigma s}}\\&k_h\Lambda_hR_r=\frac{1}{T_{\sigma r}},\\&where: L_{\sigma\sigma}=L_{\sigma s}+L_{\sigma r}+(L_{\sigma s}L_{\sigma r}/L_{hs})\cong L_{\sigma s}+L_{\sigma r}.\end{aligned}\qquad(13)$$

With the new notations, the 4 equations (5) can be written as follows:

$$\frac{d\psi_{as}}{dt}+c_r\frac{\psi_{as}}{T_{\sigma s}}=u_{as}+\frac{\psi_{dr}}{T_{\sigma s}};\qquad(14\text{-a})$$

$$\frac{d\psi_{bs}}{dt}+c_r\frac{\psi_{bs}}{T_{\sigma s}}=u_{bs}+\frac{\psi_{qr}}{T_{\sigma s}};\qquad(14\text{-b})$$

$$\frac{d\psi_{ar}}{dt}+c_s\frac{\psi_{ar}}{T_{\sigma r}}=u_{ar}+\frac{\psi_{DS}}{T_{\sigma r}};\qquad(14\text{-c})$$

$$\frac{d\psi_{br}}{dt}+c_s\frac{\psi_{br}}{T_{\sigma r}}=u_{br}+\frac{\psi_{QS}}{T_{\sigma r}};\qquad(14\text{-d})$$

These expressions can be effortless expressed in a symbolic form. The next challenge is to obtain the electromagnetic torque expression dependent on *total fluxes* and *rotor position angle* alone. For this purpose, (8) can be transformed in (15') or (15") by using (12) and (13):

$$M_e = \frac{p}{L_{\sigma\sigma}}\left[\psi_{bs}\left(\psi_{ar}\cos\theta_R - \psi_{br}\sin\theta_R\right) - \right.$$
$$\left. - \psi_{as}\left(\psi_{ar}\sin\theta_R + \psi_{br}\cos\theta_R\right)\right] = \frac{p}{L_{\sigma\sigma}}(\psi_{bs}\psi_{dr} - \psi_{as}\psi_{qr}) \quad (15')$$

$$M_e = \frac{p}{L_{\sigma\sigma}}\left[\psi_{ar}\left(-\psi_{as}\sin\theta_R + \psi_{bs}\cos\theta_R\right) - \right.$$
$$\left. - \psi_{br}\left(\psi_{as}\cos\theta_R + \psi_{bs}\sin\theta_R\right)\right] = \frac{p}{L_{\sigma\sigma}}(\psi_{ar}\psi_{QS} - \psi_{br}\psi_{DS}). \quad (15'')$$

These expressions show "the total symmetry" of the two-phase machine versus the stator and the rotor.

The balance equation of the torques, (9), can be modified by using (15'):

$$\frac{d\theta_R}{dt} = \dot{\theta}_R, \quad \frac{d\dot{\theta}_R}{dt} + \frac{k_z}{J}\dot{\theta}_R = \frac{p}{J}\left[\frac{p}{L_{\sigma\sigma}}(\psi_{bs}\psi_{dr} - \psi_{as}\psi_{qr}) - M_{st}\right] \quad (16)$$

and subsequently can be written in a *symbolic* form. The result is a 6 equations system that includes only the total fluxes of the 4 windings and the position angle between the equivalent windings placed on *as* and *ar* axes. Finally, the *symbolic* form of the system, which takes into consideration (14-a to 14-d) and (16), is:

$$\overline{\psi}_{as}(\overline{s} + \frac{k_r}{T_{\sigma s}}) = \overline{u}_{as} + \frac{\overline{\psi}_{dr}}{T_{\sigma s}}, \quad \text{(17-a)}$$

$$\overline{\psi}_{bs}(\overline{s} + \frac{k_r}{T_{\sigma s}}) = \overline{u}_{bs} + \frac{\overline{\psi}_{qr}}{T_{\sigma s}}, \quad \text{(17-b)}$$

$$\overline{\psi}_{ar}(\overline{s} + \frac{k_s}{T_{\sigma r}}) = \overline{u}_{ar} + \frac{\overline{\psi}_{DS}}{T_{\sigma r}}, \quad \text{(17-c)}$$

$$\overline{\psi}_{br}(\overline{s} + \frac{k_s}{T_{\sigma r}}) = \overline{u}_{br} + \frac{\overline{\psi}_{QS}}{T_{\sigma r}}, \quad \text{(17-d)}$$

$$\overline{\dot{\theta}_R}\left(\overline{s} + \frac{k_z}{J}\right) = \left(\frac{p}{J}\right)\left[\frac{p}{L_{\sigma\sigma}}(\overline{\psi}_{bs}\overline{\psi}_{dr} - \overline{\psi}_{as}\overline{\psi}_{qr}) - M_{st}\right], \quad \text{(17-e)}$$

$$\overline{s}\theta_R = \dot{\theta}_R. \quad \text{(17-f)}$$

where the expressions of the transformed fluxes, (10), has to be added. On the basis of these equations, the block-diagram from Fig. 2 has been conceived.

The next step is a simulated analysis by using the proposed mathematical model for the study of a two-phase induction servomotor with low inertia rotor. On the basis of the diagram-block (Fig. 2), the following three situations are taken into consideration: a) supply voltages with coequal magnitudes and different frequencies – *no-load operation*; b) supply voltages with coequal magnitudes and different frequencies – *under load operation* with a constant load torque of gravitational nature; c) unbalanced supply system (different magnitudes and frequencies on the two phases) - *under load operation* with a constant load torque of gravitational nature. The input electrical parameters are: $L_{ss}=L_{rr}=0.1H$; $L_{hs}=0.9H$; $L_{\sigma s}=0.01H=L_{\sigma r}$; $R_s=2\Omega$; $R_r=20\Omega$; $J=0.01$; $k_z=0.004$; $p=1$; $\omega_{as}=314$; $\omega_{bs}=307.72$.

A. Symmetric reversing operation, no-load

The two supply voltages have coequal magnitudes - $U_{asmax}=U_{bsmax}=600\ V$, but their frequencies are different - $f_{as}=50\ Hz$, $f_{bs}=49\ Hz$. The load torque is equal to zero, $M_{st}=0$, and moreover the opponent torque determined by the friction is neglected. The simulation time is rather high (2 seconds) in order to put in view the complete behavior of the machine. Fig. 3 and Fig. 4 present the variation of the applied voltages on the two phases (the representation take into consideration only the last tithe of the simulation period for a better recognition of the continuous alteration of phase of the two voltages). The variation of the speed and rotation angle with time put in view the following remarks:

- If we consider as origin the moment when the difference of phase between the two voltages is π/2 rad – positive, then the servomotor start running in the "positive" direction, denoted with 1, up to an angular velocity close to synchronism (~ 310 rad/s) in 0.3 seconds (Fig.5). The difference of phase increases then up to π rad and the electromagnetic torque becomes null. Next, the electromagnetic torque becomes negative, the rotor decelerates and then starts turning in the opposite ("negative") direction, denoted with 2, up to ~ -310 rad/s. This operation cycle recommence when the difference of phase goes beyond 2π rad. Practically, the rotor performs an "oscillatory" rotation, each direction change corresponding to a time period equivalent to a π rad of the difference of phase. The period of oscillation can be calculated by evaluating the electric angles when the difference of phase is equal to π rad, that is: $(\omega_{as} - \omega_{bs})T_{mec}/2 = \pi$, or $f_{mec} = (\omega_{as} - \omega_{as})/(2\pi) = f_{as} - f_{bs}$. Therefore, the rotor oscillation frequency is equal to the difference between the two supply frequencies. In our case, this oscillation frequency is of 1 Hz. Interesting to be mentioned is the fact that variation rate changes according to a *quasi-sinusoidal law* (excluding the initial transient phenomenon corresponding to the supply moment).
- The rotation angle of the rotor varies according to a harmonic law as a consequence of the harmonic variation of the angular speed. After start, $\theta_R=0$, during a second-period, the rotor perform a rotation of 106 rad (~17 rotations), then it comes to the initial position (Fig. 6) resulting a *come-and-go* movement. We can talk here about a movement called *smooth-running reversing piston*.

The time variation of the total rotor flux corresponding to *ar* phase is presented in Fig. 7 and for the stator *as* phase in Fig. 8. It is a fact that the total rotor flux has a frequency close to zero value when the rotor is close to synchronism and rises to 50 Hz for blocked rotor. The amplitude of this flux is about 2 Wb (except initial transients corresponding to the supply source connection). The total stator flux has a 50 Hz frequency and amplitude of 2 Wb with some variations.

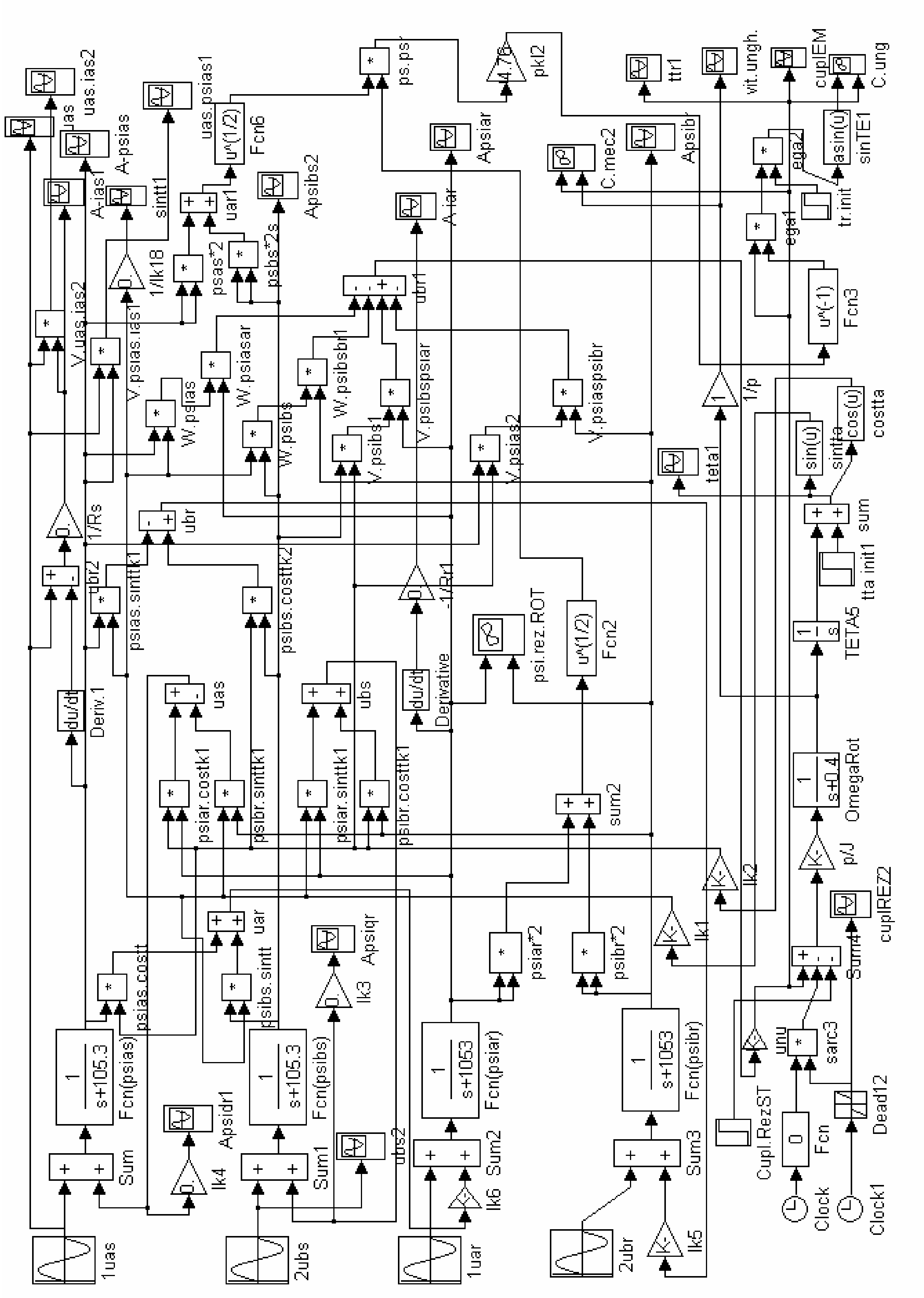
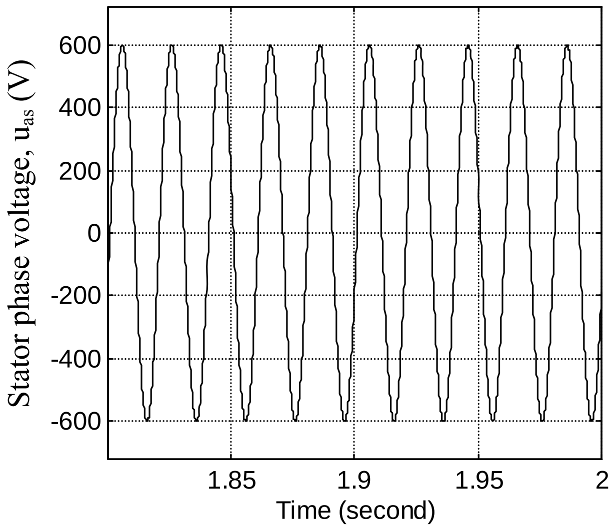


Fig.2 Diagram-block of the two-phase induction oscillator servomotor



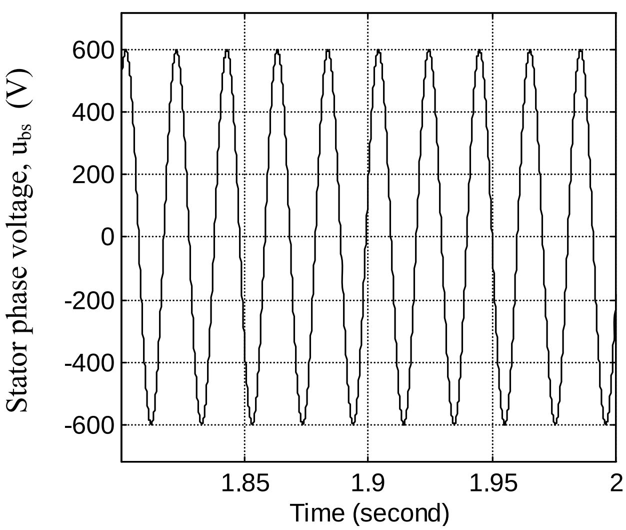
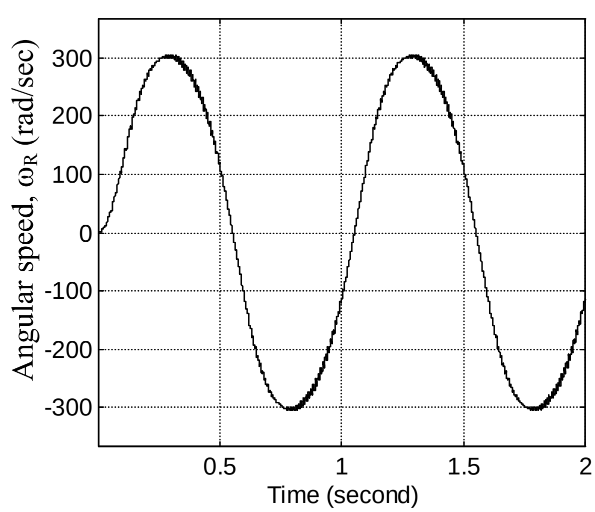


Fig.3 Stator phase voltage, $u_{as}=f(t)$



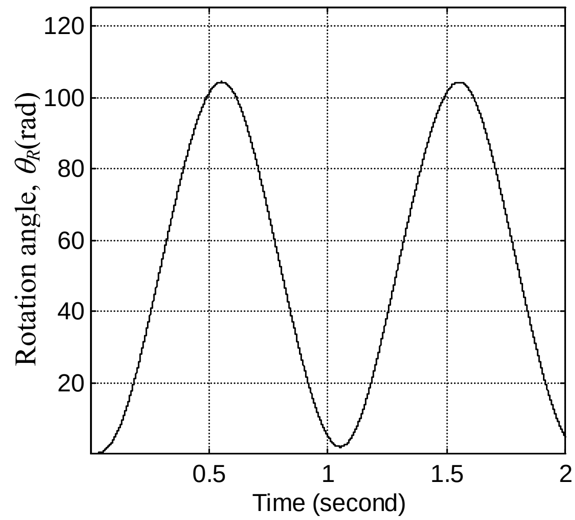


Fig.4 Stator phase voltage, $u_{bs}=f(t)$

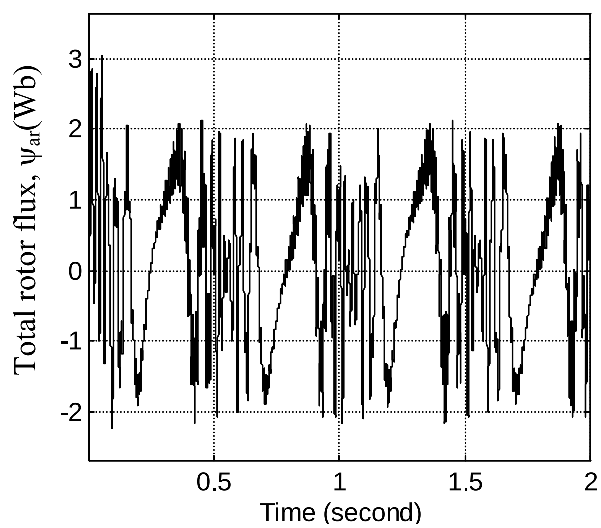
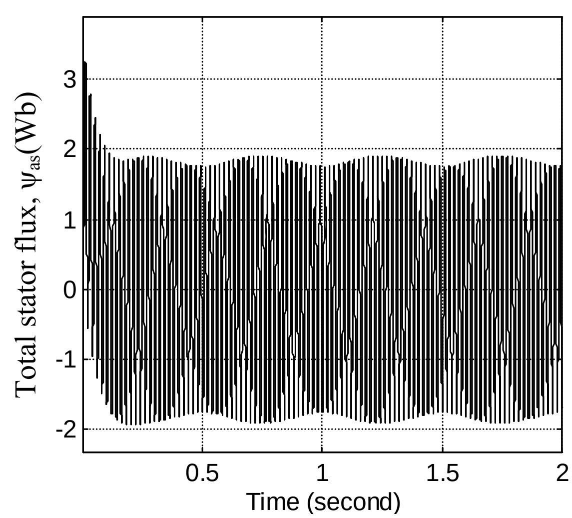


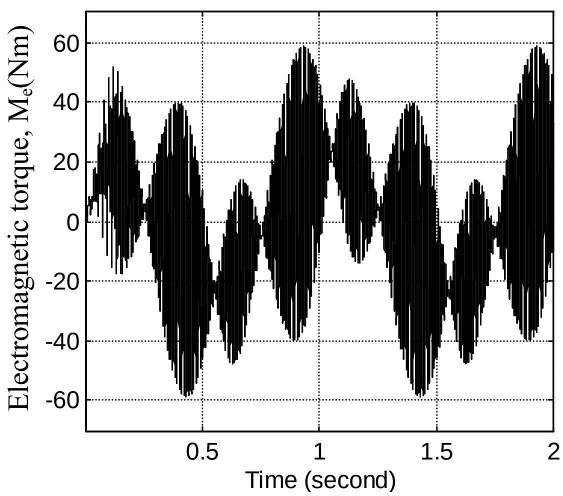
Fig.5 Angular speed, $\omega_R=f(t)$



During the simulation, the active instantaneous torque oscillates around zero value, Fig. 9 (sometimes the torque is positive then negative). This repetitive sequence determines the reversing rotation of the rotor.

As regards the current variation of the *as* stator phase, a „beat" phenomenon is noticeable. Close to synchronism (quasi no-load operation), the current has a reduced amplitude, around 10 A, but for null speed (short-circuit operation) the amplitude rises up to 30 A, Fig. 10.

Useful information about the reversing operation of the two-phase induction servomotor are given by the mechanical ($\omega_R=f(M_e)$ – Fig. 11) and angular ($M_e=f(\delta)$ – Fig. 12) characteristics. One can see a variation of the speed in the range +310 rad/s to -310 rad/s and a variation of the torque between +60 Nm and -60 Nm with a symmetrical behavior for the two rotation directions.



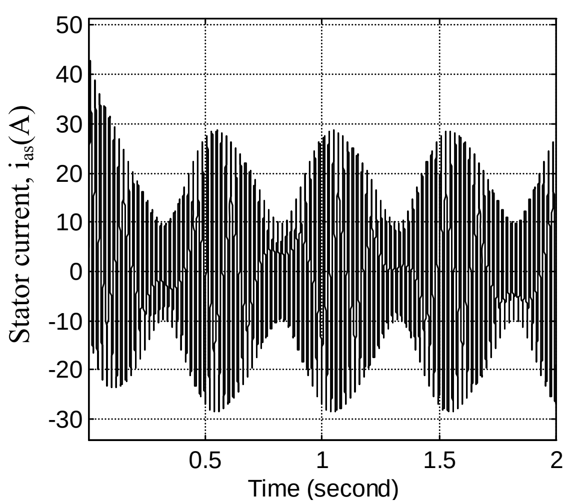
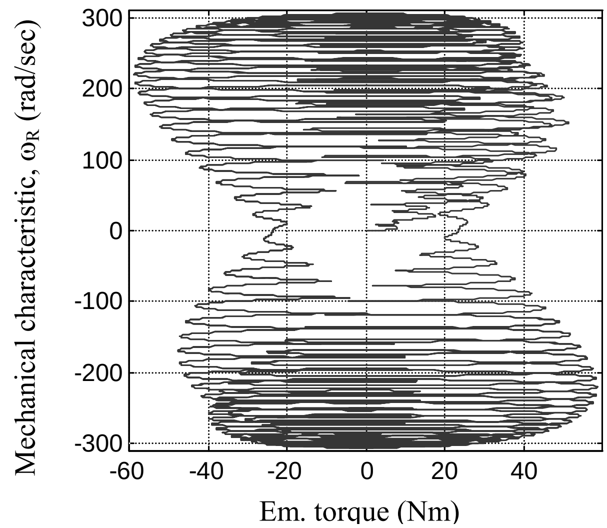


Fig.9 Electromagnetic torque, $M_e=f(t)$

Fig.10 Stator current, $i_{as}=f(t)$

Fig. 12 presents the angular characteristics where δ is the angle between the stator and rotor flux representative phase vectors. When the difference of phase between the two supply voltages is close to π/2 rad. and the rotor speed is close to synchronism then the electromagnetic torque is high, about 60 Nm and the internal angle is small since the load torque is small (close to zero). When the phase difference is zero then the torque is small and the internal angle is high, close to 1.57 rad (π/2). Again symmetry behavior as reversing servomotor is noticeable.



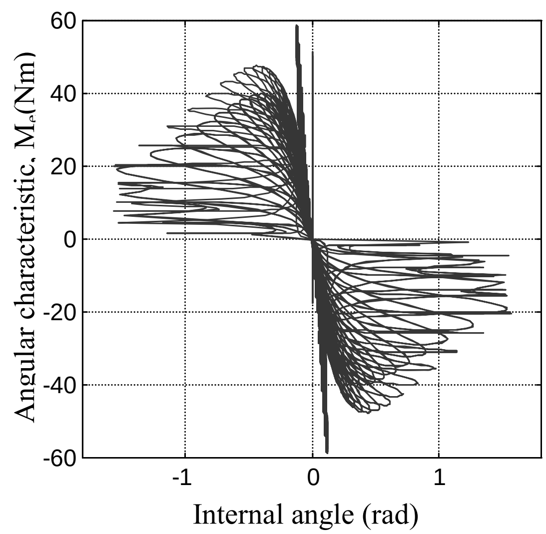


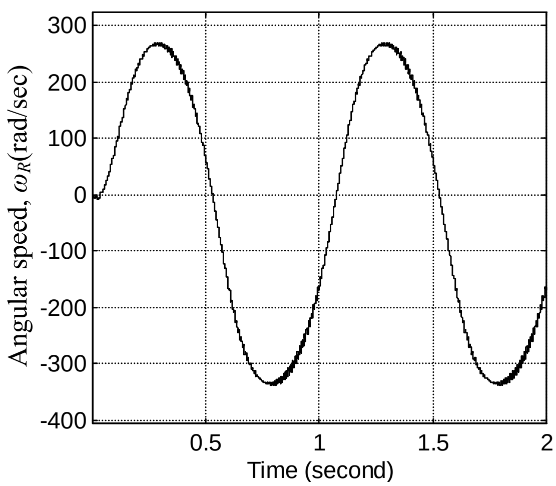
Fig.11 Mechanical characteristic, $\omega_R=f(M_e)$

Fig.12 Angular characteristic, $M_e=f(\delta)$

B. Symmetric reversing operation, under load

The two supply voltages have coequal magnitudes, $U_{asmax}=U_{bsmax}=600$ V, but their frequencies are different, $f_{as}=50$ *Hz*, $f_{bs}=49$ *Hz*. The load torque is constant in value and direction, $M_{st}=10Nm$, no matter the rotation direction. Such a situation is equivalent, for example, to the electric drive that controls a winch barrel. The following remarks have to be pointed out regarding the variation of the angular speed and rotation angle with time:

- If we consider as origin the moment when the difference of phase is π/2 rad – positive, then the servomotor start running in the "positive" direction, denoted with 1, up to an angular velocity *much lower* then synchronism (~ 270 rad/s) in 0.3 seconds (Fig.13). The magnitude of the load torque has a capital influence upon this value. The difference of phase increases then up to π rad and the electromagnetic torque turns negative, the rotor decelerates and then starts turning in the opposite ("negative") direction, denoted with 2, up to ~ -330 rad/s (higher then synchronism). The machine operates for a short time as generator. Then the rotor repeats the cycle with a forward-backward rotation, but the time-periods corresponding to the two directions are different (greater in the "negative" direction). Therefore, the rotor oscillation frequency stands equal to the difference between the two supply frequencies (1 Hz in our example). The variation rate changes according to a *quasi-harmonic law*, which includes a constant term (of approx. -30 rad/s) and a harmonic function



with amplitude of 300 rad/s (excluding the initial transient regime corresponding to the supply moment).

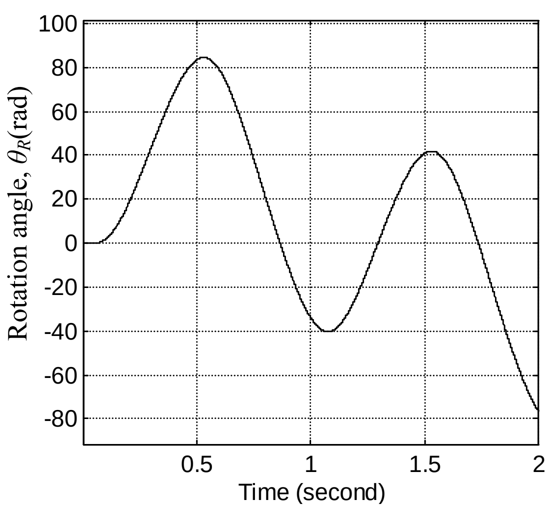
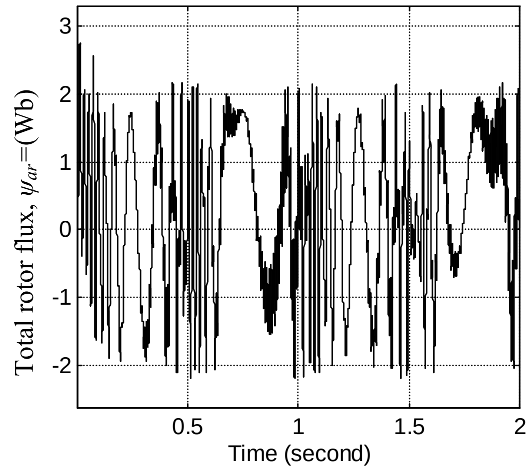


Fig.13 Angular speed, $\omega_R=f(t)$

Fig.14 Rotation angle, $\theta_R=f(t)$



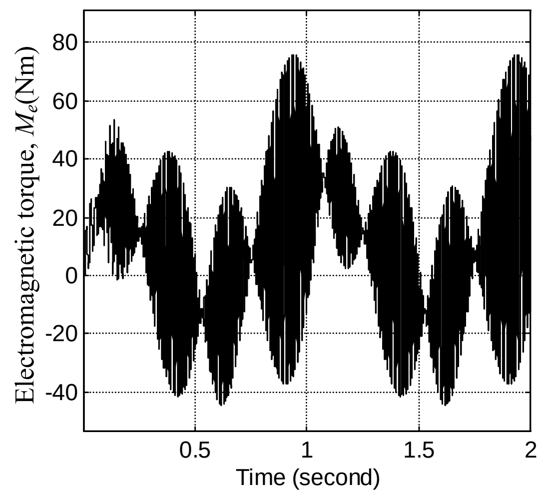


Fig. 15 Total rotor flux, $\psi_{ar}=f(t)$

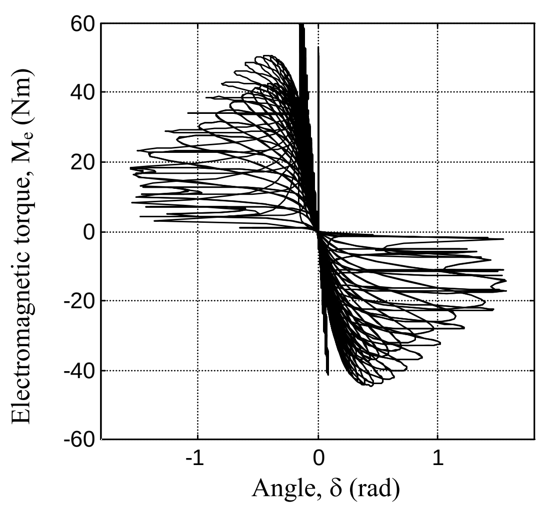


Fig.16 Electromagnetic torque, $M_e=f(t)$

Fig.17 Angular characteristic, $M_e=f(\delta)$

- The rotation angle of the rotor varies according to a harmonic law superimposed by a straight line of negative slope (this is a consequence of the *quasi-harmonic* variation of the angular speed). After start, $\theta_R=0$, during a second-period, the rotor perform a rotation of 87 rad (~14 rotations), then it comes back to a different position than initial one, that is $\theta_R=-40$ *rad*, which means a backward movement of ~6 rotations (Fig. 14). It is also a *smooth-running reversing piston* but with negative feed. It is an unbalanced behavior of the machine versus the two rotation directions.

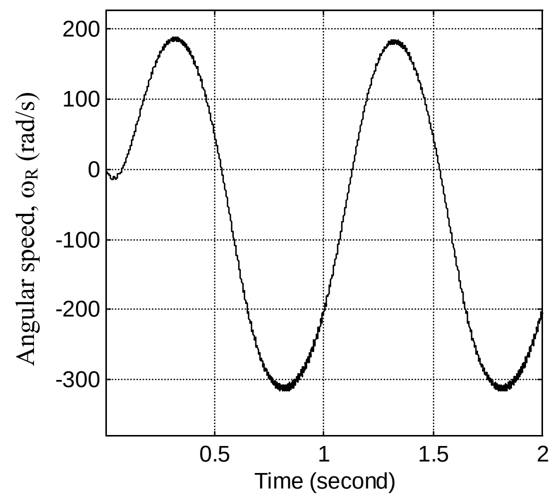
This fact is also attested by the rotor flux variation (Fig. 15) where a significant difference appears during the positive and negative alternances. The electromagnetic torque has an unbalanced time-variation as well (Fig. 16). The mean value corresponding to positive alternances is higher in comparison with negative alternances. This is an understandable fact since the electromagnetic torque stands against a load torque with constant direction. The same unbalanced operation is proved by the angular characteristics (Fig. 17) where the electromagnetic torque has values over 60 Nm during the positive rotation and -45 Nm for negative rotation.

C. Reversing operation, unbalanced supply system - under load

The two supply voltages have different magnitudes, U_{asmax}= 600 V, U_{bsmax}=300 V, and their frequencies are also different, $f_{as}=50$ *Hz*, $f_{bs}=49$ *Hz*. The load torque is constant in value and direction, $M_{st}=10Nm$, no matter the rotation direction. The following remarks have to be pointed out regarding the variation of the angular speed and rotation angle with time:

- As previously, the servomotor starts running in the "positive" direction, denoted with 1, up to an angular velocity *far lower* then synchronism (~ 180 rad/s) in 0.3 seconds (Fig.18). Since the supply system is unbalanced, the resultant electromagnetic torque is weaker. There is a backward rotating field which practically creates an electromagnetic torque that acts as a supplementary load torque. The maximum angular velocity corresponding to the "positive" direction is also lower while during the negative direction movement the velocity stands close to synchronism (-310 rad/s). The rotor oscillation frequency stands equal to the difference between the two supply frequencies (1 Hz in our example), and the variation rate changes according to a *quasi-harmonic law*, which includes a constant term (of approx. -65 rad/s) and a harmonic function with amplitude of 245 rad/s.
- The rotation angle of the rotor varies according to a harmonic law superimposed by a straight line of more negative slope (this is a consequence of the *quasi-harmonic* variation of the angular speed). After start, $\theta_R=0$, during a second-period, the rotor perform a rotation of 51 rad (~8 rotations), then it comes back to a different position than initial one, that is $\theta_R=-70$ *rad*, which means a backward movement of ~11 rotations (Fig. 19). It is also a *smooth-running reversing piston* but with significant negative feed. It is an unbalanced behavior of the machine versus the two rotation directions, more significant than the previous case.

This fact is also attested by the rotor flux variation (Fig. 20) and the time-variation of the electromagnetic torque (Fig. 21) where a significant difference appears during the positive and negative alternances. The electromagnetic torque has an unbalanced time-variation shown in Fig. 22 as well, and the mean values corresponding to positive alternances are higher in comparison with negative alternances.



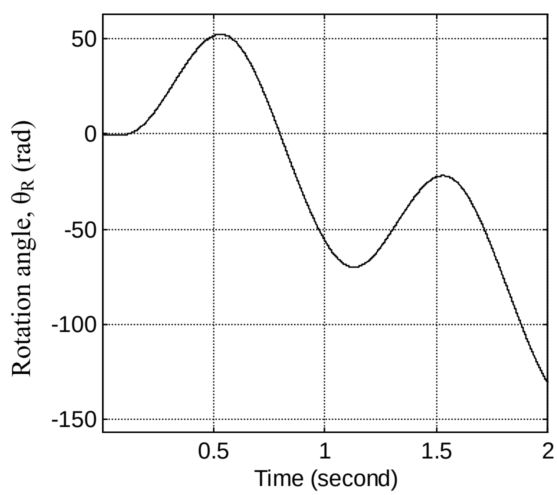
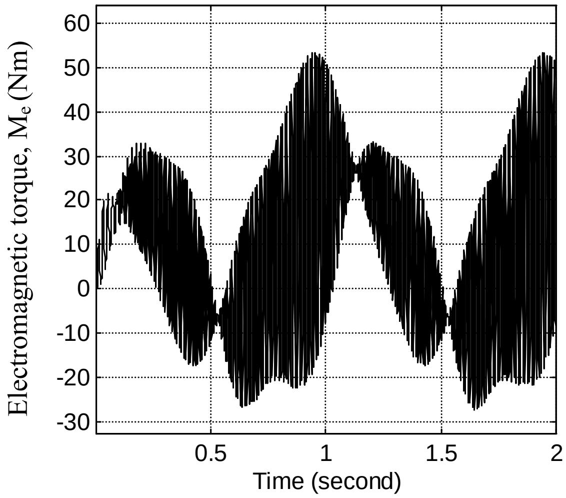


Fig.18 Angular speed, $\omega_R=f(t)$

Fig.19 Rotation angle, $\theta_R=f(t)$



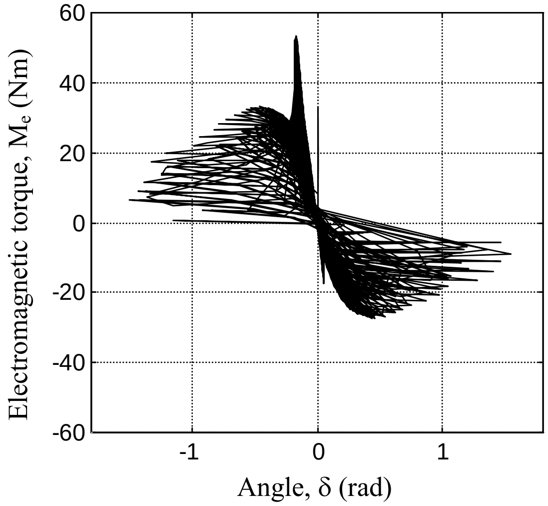


Fig.20 Total rotor flux, $\psi_{ar}=f(t)$

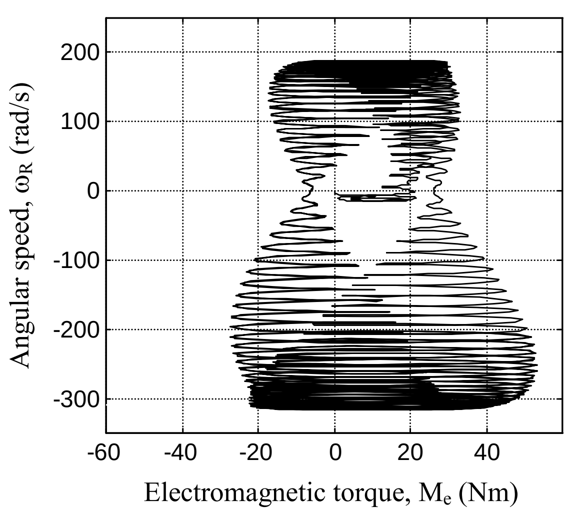


Fig.21 Electromagnetic torque, $M_e=f(t)$

Fig.22 Angular characteristic, $M_e=f(\delta)$

Fig.23 Mechanical characteristic, $\omega_R=f(M_e)$

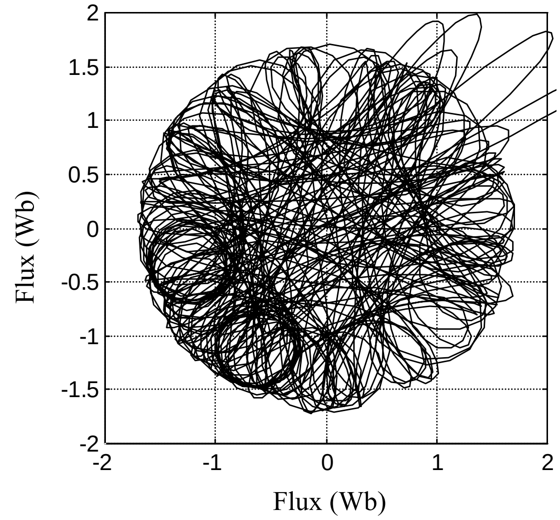


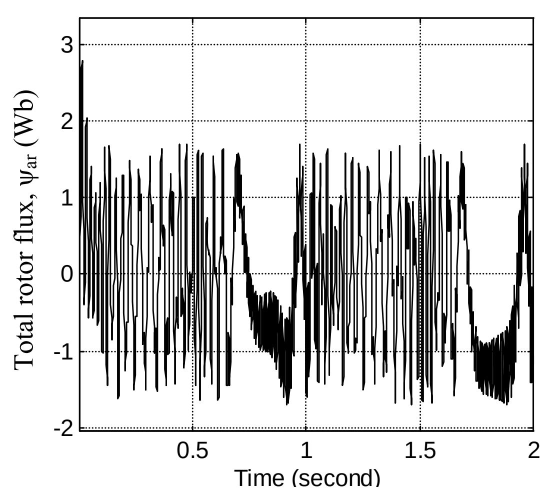
Fig.24 Hodograph of the resultant rotor flux

The unbalanced operation of the machine is also proved by the mechanical characteristic (Fig. 23) where one more interesting fact has to be pointed out: the operation cycle determine the machine to operate as motor, generator and also brake with very different limit values both for electromagnetic torque and angular velocity. Fig. 24 presents the hodograph of the resultant rotor flux which proves that during rotor reversing, the apex of this phase vector is situated inside the circle with 1.7 Wb radius value (it is a known fact that in the stationary and symmetric regime the locus of the apex of the flux phase vector is a circle with 1.7 Wb radius value).

IV. Conclusions

The model *in total fluxes* has simpler equations and diagram-block consequently. It allows a proper study of the reversing two-phase servomotor.

The variables (stator and rotor fluxes) represent real quantities and the rotation speed, ω_R, which varies within large limits in this operation regime, is no longer present in the equations in an explicit way. This fact determines a more precisely analysis on the basis of this model.



In comparison with the classical models, the proposed model uses as variables nothing but *total fluxes* and *rotation angle*. As consequence, there is more comfort in using it for the study of the unbalanced duties such as reversing rotation with frequencies between a few Hz to 10 Hz.

The presented study, based on a new mathematical model, proves the possibility of using variation laws expressed as sums of harmonic functions (Fourier expansions for example) for both angular velocity and rotation angle by means of a physical model of electrical machine – the two phase induction servomotor with low rotor inertia.

The movement parameters can be easy modified through frequency value applied to one of the windings, or voltage magnitude, or initial difference of phase or their combinations.

The reversing operation by using a supply system with two different frequencies is no doubt more advantageous in comparison with the "classical" solution of reversing the current from one of the two windings.

The model has a significant didactical importance and clear up that supply of the two-phase induction servomotor with different frequencies on the two windings determine a "continuous" phase control of the machine accompanied by the successive reversal of the electromagnetic torque sense. From analytically viewpoint, this is a validation of the experimental tests [8].

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