The methods of multi attribute analysis in application to assess optimal factor combination in one experiment

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Abstract - Experiments are used by scientists to affirm their hypothesis, these experiments are called tests in research, or to choose the best from available possibilities, these experiments are called valuations in research in which group belongs also optimal factor combination choice in one multifactor and often multivariate experiments. For decreasing influence ever present uncontrolled factors i.e. experimental error researchers make different plans. Mathematical instruments of most effective plans for experiment organization are possible to search on the basis of total random distribution, random block distribution and some special organized block distribution while they can most effectively represent complex multifactor and multivariate experiments. Statistic analysis for any experiment plan is very complex in the standard way with analysis of variance and multiple linear regression and especially in the case of the optimal factor combination choice. From other side multiple criteria analysis like modern science discipline enables an easier way to make analysis of results of one experiment just in the case of optimal factor combination choice of one multifactor and multivariate experiment. Therefore authors propose multiple criteria analysis application in analysis of experiment results and in this paper authors consider application of one subgroup of these methods, so called multi attribute decision methods, to which belong and ELECTRA method. One example of multiple attribute analysis application in analysis of results of one experiment is given in the end of this paper.

Keywords - multifactor experiment, multiple linear regression, multi attribute analysis

I. INTRODUCTION

Considering the influence different treatments and their combination on the unit of experimental examination is the basic task in one experiment. One plan in experiment organization has as primary aim to make smaller always present experimental error because of uncontrolled factors effect and in this way enable to establish a real differences between applied treatments (see[1], [4]-[6]).

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The plans of experiments have developed first in the form of total random distribution then like better and more precise random block distribution and on the end in the form special block distribution (see[1]-[6]).

In the multifactor and multivariate experiments, which are usually object of considering ones experiment therefore they give possibility for greater precision and also considering of interaction and the plans for them are the same like for one factorial experiment. For all of these experiments it is very complicated to make statistical analysis usual like analysis of variance and especially it is very complex to solve a problem of the optimal factor combination choice (see [5]-[6] and [11]-[12]).

Considering the optimal factor combination choice in one multifactor and multivariate experiment towards the aim of this experiment is invention of the minimum or maximum answer dependent variables in this experiment. In the case of univariate experiments we may write for dependent variable y_i which is called the response surface

$$y_i=F(x_{1i},x_{2i},x_{3i},\dots,x_{pi})+e_i \ , \ \text{where}$$

$i=1,2,\dots,n$ represents the n observation in the multi factorial experiment and x_{pi} represents the level of p-th factor in the i-th observation and residual e_i measures the experimental error of the i-th observation. When the mathematical form of function F isn't known, this function can be approximated satisfactorily, for example by a polynomial, different degree, in the independent variables x_{pi}.

Since the fitting of a polynomial can be treated as a particular case of multiple linear regression, we shall use the calculations required to fit a multiple linear regression of y_i on the k variables x_{pi} where $i=1,2,\dots,n$ and $p=1,2,\dots,k$ in the form

$$y_i=\beta_0+\beta_1x_{1i}+\beta_2x_{2i}+\beta_3x_{3i}+\dots+\beta_kx_{ki}+e_i \, .$$

For the plans for multivariate experiment organization it is very difficult to make analysis of results using known apparatus of classical statistics. Especially it is very difficult to solve a problem of the optimal factor combination choice ussualy like canonical analysis and this theory is not subject of considering in this paper (see [4]-[6] and [13]-[16]).

In other way theory of multi criteria analysis gives possibility that we can make in easier way analysis of experiments results. This possibility follows if we use the apparatus of operational research and have already presented general definition of French mathematicians

Descartes in XVII century for Scientific approach and process of decision (see Figure 1).
An application of multi criteria analysis in the optimal factor combination choice on the basis of one experiment results is possible because of that in this experiments exist:

1. More criteria – functions of aim for decision which are defined with defined explicit attributes
2. More and that finite number of discreet alternatives
3. One finite solution

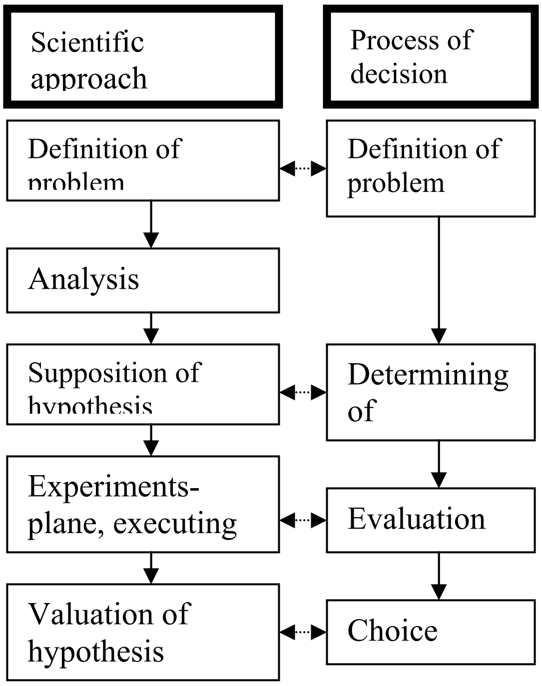

Figure 1. Definition the process of decision

II. MATHEMATICAL APPARATUS FOR OPTIMAL FACTOR COMBINATION CHOICE

Mathematical apparatus for results analysis of univariate experiments towards the aim of optimal factor combination choice can be:

- statistical analysis of multiple linear regression and
- multi attribute decision methods.

In the case of multivariate experiments results analysis towards the aim of optimal factor combination choice we have in basis also two possibilities:

- statistical analysis of (more universal is canonic correlation analysis), but this case is not subject of considering in this paper(see Kovačić Z. (1994)).
- Already described multi attribute decision methods.

A. Analysis of multiple linear regression

Method for examining the influence of more different independent variables (see [4]-[6] and [8]-[10]).for example $x_{1i},x_{2i},x_{3i},\ldots,x_{pi}$ on one dependent variable for example y is called multiple regression and can be given in the form

$y=a+b_1x_1+b_2x_2+b_3x_3+\ldots+b_px_p$.

where b_i $i=1,2,\ldots,p$ are partial coefficients of regression. In the case of fixed values independent variables x when we have and experimental error in each from fully n observation we can present multiple regression in the form

$y_i=\beta_0+\beta_1x_{1i}+\beta_2x_{2i}+\beta_3x_{3i}+\ldots+\beta_kx_{ki}+e_i$.

The calculation of parameters $a,b1,b2,b3,\ldots,b_p$ we can make with the method of smallest quadrates with minimization of expression

$$\sum_{i=1}^{n}(y_i-a-b_1x_{1i}-b_2x_{2i}-\cdots-b_px_{pi})^2.$$

Practically, algebraic algorithm for solving arising system of equation is rarely in use than known Gaussian method of multiplication all the more so this method is already used in calculation for regression valuation and therefore we consider this method.

With differentiation in relation on $a,b1,b2,b3,\ldots,b_p$ and with exchange in notation $b_0=a$ we obtain next normal equation which must be solved to receive parameters:

$b_0(00)+b_1(01)+\ldots+b_p(0p)=(0y)$
$b_0(10)+b_1(11)+\ldots+b_p(1p)=(1y)$
.
.
.
$b_0(p0)+b_1(p1)+\ldots+b_p(pp)=(py)$

where

$(jk)=(kj)=\sum_{i=1}^{n}x_{ji}x_{ki}$

is the sum of products of j-th and k-th variables x_j and y_k,

$(jj)=\sum_{i=1}^{n}(x_{ji})^2$

is the sum of squares j-th column of variable x_i,

$(jy)=\sum_{i=1}^{n}x_{ji}yi$

is the sum of products j-th column of variable x_j and of variable y.

The matrix of independent variables x and vector y are the initial basis for calculation sum of squaresand products of variables and can be given like:

x				y
x_{01}	x_{11}	∘∘∘	x_{p1}	y_1
x_{02}	x_{12}	∘∘∘	x_{p2}	y_2
x_{03}	x_{13}	∘∘∘	x_{p3}	y_3
∘	∘	∘∘∘	∘	∘
x_{0n}	x_{1n}	∘∘∘	x_{pn}	y_n

From this matrix and vector we form sums of square and product of variables x and products of x and y which form system of normal equation:

jk=x'x				jy=x'y
00	01	∘∘∘	op	0y
10	11	∘∘∘	1p	1y
20	21	∘∘∘	2p	2y
∘	∘	∘∘∘	∘	∘
p0	p1	∘∘∘	Pp	py

Partial coefficients of regression are:

$$b_i=\sum_{i=1}^{p}(C_{jk})(jy)$$

i.e. the sum of products of k-th column C_{ij} with the column (jy). When the independent variables are mutually orthogonal normal equations are particularly easy to solve therefore in this case all sums of products (jk) vanish (j≠k) and the normal equations for b_i reduces to $(jj)b_j=(jy)$

Also and the multiplier in inverse matrix becomes values $C_{jj}/(jj)$ and $C_{jk}=0$.

B. Multi attribute decision methods

Multi criteria decision methods are grouped in two basis groups:

- multi target methods
- multi attribute methods

and in each of these two basis groups we have a few methods(see [2] and [13]-[16]).

The subject of interest in this paper is multi attribute methods. In this group we have two different subgroup of methods and that:

- subgroup without heaviness coefficients which typical represent is data envelopment analysis (DEA) method and
- the methods with heaviness coefficients for considered units which well known represent of this group are Elimination et choice translating reality (ELECTRE)method and preference ranking organization method for enrichment evaluations (PROMETHEE) method in the subgroup of standard heaviness coefficients determining and Analytical hierarchical process(AHP)method in the subgroup for objective heaviness coefficients determining.

As we have noticed the multiple factor experiments, which are usually object of considering ones experiment therefore they give possibility for greater precision and also considering of interaction and where practically each treatment consists of one combination of values of each factor the application of multi attribute methods and that one concrete from enumerated method is possible so that it is easy to make the table of criteria which are in columns of this table and alternatives which are rows in this table with values from executed experiments take the values of factor combinations. With the application of method of mathematical programming, which is in the basis of multi attribute methods, today we can produce also information support in the form of suitable software. Multi attribute methods can be given with next mathematical model:

Max $\{f_1(x), f_2(x),\dots, f_n(x), n\geq 2\}$ by restriction $x\in A=[a_1, a_2,\dots, a_m]$, where is:

n-number of criteria(attributes) j=1,2,…,n

m-number of alternatives(actions) i=1,2,…,m

f_j – criteria(attributes) j=1,2,…,n

a_i –alternatives(actions) i=1,2,…,m

A – set of all alternatives(actions).

Also are known values f_{ij} of each considered criteria f_j which are received with each from possible alternatives a_i :

$f_{ij} = f_j\ (a_i)\ \ \forall\ (i,j);\ i=1,2,\dots,m;\ j=1,2,\dots,n.$

Usually the model of some multi criteria method is given with suitable matrix of attributes values for individual alternative :

	max f_1	*max f_2*	∘∘∘	*max f_n*
a_1	f_{11}	F_{12}	∘∘∘	f_{1n}
a_2	f_{21}	F_{22}	∘∘∘	f_{2n}
∘∘∘	∘∘∘	∘∘∘	∘∘∘	∘∘∘
a_m	f_{m1}	f_{m2}	∘∘∘	f_{mn}

Criteria type of minimization can be translated in criteria type of maximization for example with multiplication of their values with -1. For example, method ELECTRE is based on the fact:

When is alternative a better then alternative b for majority criteria and in addition don't exist criteria for which is alternative a strict worse then alternative b we can say ,without risc, alternative a is better then b i.e. alternative a surpassed alternative b.

The base of algorithm of decision for ELECTRE method form two conditions:

- condition of agreement defined trough desired level of agreement P and real index of agreement c(a,b)
- condition of disagreement defined trough desired level of disagreement Q and real index of disagreement d(a,b)

Indexes of agreement and disagreement express quantitative indexes of agreement or disagreement that the alternative a can be ranged before alternative b in the sense of all criteria simultaneously.

Index of agreement is the relation of the sum of relative importance of each criteria which give that the alternative a is better or equals in relation with alternative b and total sum of relative importance w_j criteria K_j in the sense which we make range

$$c(a,b)=\frac{\sum_{j\in J_1}W_j}{\sum_{j=1}^{n}w_j}\cdot 100(\%)$$

where J_1 is the set of all criteria trough which is

alternative a better then alternative b or equals. Indexes of agreement (they are n(n-1)) take values from 0 to 1 end we notice they in matrix of agreement C_{nxn}.
Index of disagreement is defined like maximum normalized interval of disagreement i.e. relation of the maximum of intervals for criteria where is alternative a worse then b and maximum interval of valuation for each criteria

$$d(a,b) = \begin{cases} 0, for I_2 = \emptyset \\ \dfrac{\max_j r(a,b)}{\max_j R_j}, contrary. \end{cases}$$

, where is:

r(|a,b)-difference of valuations criteria values for alternatives a and alternatives b for individual criteria,
R_j – maximum span of valuations for each criteria (max a_j -min a_j)
I_2 – set of each criteria for which is alternative a worse then alternative b.
With the choice the biggest range of agreement(p=1) and the least range of disagreement(q=0) we separate only alternatives which are better for each criteria simultaneously.
The range is determined on the basis of relation index agreement and disagreement for even comparison i.e.

- a is better then alternative b if c(a,b)≥p and d(a,b)≤q

b is better then alternative a if c(b,a)≥p and

- d(b,a)≤q
- in other cases alternatives a and b are incomparable

III. EXPERIMENTS RESULTS

The authors of this paper propose an application of multi attribute methods and that concrete ELECTRA(at any rate and PROMETHEY and AHP can be used) method in the way which is present in next several lines and also in two examples in this section because of that solving a problem of the optimal factor configuration choice with apparatus of multiple regression analysis is very complex. Examples are applied in multifactor experiment in example 1 and multivariate experiment in example 2 and both multifactor and multivariate experiments are with repetition.
As a result of one application of one multifactor experiment with repetition we have results organized in one table with rows which are factor combinations and columns which are repeated results of these factor combinations.
In the ELECTRA method we make the beginning matrix which is given as a table of criteria which are in columns of this table and alternatives which are rows in this table with values from results of executed experiments and this values take the middle value of values of one factor combination. In the last row we have values of heaviness coefficients of this criteria. Sum of values of this heaviness coefficients is normalized on value 1. It is known that there exist a methods for exact determining the heaviness coefficients of applied criteria, which are unfortunately complex. Therefore, without generalization, we understand that the heaviness coefficients for applied criteria are equal for a group of output and a group of input criteria and between them in each group.
In this way with application of multi attribute decision method we obtain the new procedure which evident enables an easier and efficacious way for considering a results of one experiment.
Example 1. Compare effect of three factor experiment in cow feeding organized in four groups each with five cows. First factor is grouped in two sort of fodder - noodles of sugar beet and cornstalks which are quantified with values respectively 1 and 0.5, second factor is grouped in two races of cows - Frisian and domestic variegated which are quantified with values respectively 1 and 0.5 and the third factor is the period of time - first like 28 days and second next 28 days which are quantified with values respectively 1 and 0.5. Gain of quantity of milk for 28 days is given in liter.
Results are given in the table 1.

Table 1. Results of experiment gave in example 1.

Sort of fodder S	Race of cows D	Period of time P	Repetition (Gain of of milk) G					Sum
			1	2	3	4	5	
1	1	1	496.7	438.5	586.6	453.1	518.9	2493.8
		0.5	392.3	284.2	678.9	309.4	576.2	2241.0
	0.5	1	444.9	434.2	485.9	555.5	298.9	2219.4
		0.5	496.9	409.2	411.3	307.9	438.6	2063.9
0.5	1	1	411.0	348.6	781.3	356.1	523.7	2420.7
		0.5	311.2	368.2	514.9	362.6	452.8	2009.7
	0.5	1	553.1	366.2	456.6	323.2	468.6	2167.7
		0.5	286.2	365.3	279.1	382.1	204.0	1516.7
Total			3392.3	3014.4	4194.6	3049.9	3481.7	17132.9

With statistical analysis of multiple linear regression, using Excel Data analysis option, we obtain results which are given in Table 2.

Tablee 2. Results of application multiple linear regression,using Excel Data analysis, for example 1.

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.933755
R Square	0.871898
Adjusted R Square	0.775821
Standard Error	28.46464
Observations	8

ANOVA

	df	*SS*	*MS*	*F*	*SignificancF*
Regression	3	22058.7	7352.899	9.075013	0.029422
Residual	4	3240.943	810.2357		
Total	7	25299.64			

	Coef- ficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	160.49	53.25246	3.013757	0.039404	12.63746	308.3425	12.63746	308.3425
X Variable1	90.33	40.25508	2.243941	0.088236	-21.436	202.096	-21.436	202.096
X Variable2	119.75	40.25508	2.97478	0.04095	7.983986	231.516	7.983986	231.516
X Variable3	147.03	40.25508	3.652458	0.021724	35.26399	258.796	35.26399	258.796

Output multiple linear regression gives us relation between output parameter and factors in example 1
$G = 160,49 + 90,33S + 119,75D + 147,03P$
and we can calculate optimal factor combination like first combination and that first factor like noodles of sugar beet , second factor like Frisian race and third factor first period time of 28 days.

Let us solve example 1 with procedure proposed in this paper with ELECTRE method of multi attribute decision and with heaviness coefficients for criteria i.e factors which are equal between each factor in group and between groups of input and output factors.

Table 3. Beginning matrix for ELECTRA method for example 1.

	x1(S)	**x2(D)**	**X3(P)**	**y(G)**
A1	1	1	1	498.76
A2	1	1	0.5	448.2
A3	1	0.5	1	443.88
A4	1	0.5	0.5	412.78
A5	0.5	1	1	484.14
A6	0.5	1	0.5	401.94
A7	0.5	0.5	1	433.54
A8	0.5	0.5	0.5	303.34
Heavin. Coef.	0.167	0.167	0.167	

Table 4. Result of application ELECTRA method for example 1.

a1 dominant over: a2 a3 a4 a5 a6 a7 a8
a2 dominant over: a4 a6 a8
a3 dominant over: a4 a7 a8
a4 dominant over: a8
a5 dominant over: a6 a7 a8
a6 dominant over: a8
a7 dominant over: a8
a8 non dominant

The obtained results for data in Table 3. showed in Table 4. demonstrate that the a_1 i.e. same alternative like with using multiple linear regression method is dominant.
It is necessary to notice that the application of proposed procedure in the case of multivariate experiments is still efficacious. From this reason, to practically show this fact, we add in experiment given in example 1 second dependent variable (factor, output criteria) and that quality of milk which can have four different values – 1.5 for extra quality, 1 for best quality, o.5 for middle quality, 0 for bad quality and so we get example 2 which dates are given in Table 5. In example 2 we have one multivariate experiment on which also we apply procedure proposed from authors in this paper and results are given in Tables 6 and 7

Table 5. Results of experiment gave in example 2

Sort of fodder S	Race of cows D	Period of time P	Repetition (Gain of of milk) G/ Q (Quality)					Sum
			1 Gain/ quality	2 Gain/ Quality	3 Gain/ quality	4 Gain/ quality	5 Gain/ Quality	
1	1	1	496.7/0.5	438.5/0.5	586.6/0.5	453.1/0.5	518.9/0.5	2493.8/2.5
		0.5	392.3/0.5	284.2/1.0	678.9/0.0	309.4/1.0	576.2/0.0	2241.0/2.5
	0.5	1	444.9/0.5	434.2/0.5	485.9/0.5	555.5/0.0	298.9/1.0	2219.4/2.5
		0.5	496.9/0.0	409.2/0.5	411.3/0.5	307.9/1.0	438.6/0.5	2063.9/2.5
0.5	1	1	411.0/0.5	348.6/1.0	781.3/0.0	356.1/1.0	523.7/0.0	2420.7/2.5
		0.5	311.2/0.5	368.2/1.0	514.9/0.0	362.6/1.0	452.8/0.0	2009.7/2.5
	0.5	1	553.1/0.0	366.2/0.5	456.6/0.5	323.2/1.0	468.6/0.5	2167.7/2.5
		0.5	286.2/0.5	365.3/0.0	279.1/1.0	382.1/0.0	204.0/1.0	1516.7/2.5
Total			3392.3/ 25	3014.4/ 50	4194.6/ 3.0	3049.9/5. 5	3481.7/ 3.5	17132.9/ 19.5

Let us solve example 2 with procedure proposed in this paper with ELECTRE method of multi attribute decision and with heaviness coefficients for criteria i.e. factors which sum is equal l for all input and output criteria i.e. factors and also with heaviness coefficients equal between each factor in group of input i.e. output and between groups of input and output factors.
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Table 6.Beginning matrix for ELECTRA method for example 2.

	x1(S)	X2(D)	x3(P)	y(G)	y1(G1)
a1	1	1	1	498.76	0.5
a2	1	1	0.5	448.2	0.5
a3	1	0.5	1	443.88	0.5
a4	1	0.5	0.5	412.78	0.5
a5	0.5	1	1	484.14	0.5
a6	0.5	1	0.5	401.94	0.5
a7	0.5	0.5	1	433.54	0.5
a8	0.5	0.5	0.5	303.34	0.5
Heavin. Coef.	0.167	0.167	0.167	0.25	0.25
Heavin. Coef.	Input var. $\sum = 0.5$			Output var.$\sum = 0.5$	

Table 7. Result of application ELECTRA method for example

a1 dominant over: a2 a3 a4 a5 a6 a7 a8
a2 dominant over: a4 a6 a8
a3 dominant over: a4 a7 a8
a4 dominant over: a8
a5 dominant over: a6 a7 a8
a6 dominant over: a8
a7 dominant over: a8
a8 non dominant

The obtained results which are given in Table 7. 6. show that the a_1 i.e. same alternative like in example 1 is dominant.

Example 2. Compare effect in three factor experiment for corn . First factor is number of plants in hectare and that 70000, 105800 and 128600 , second factor is density of nitrogen fertilizers in kg/ha and that 50,100 and 150 and the third factor is the time of harvest and that in two ripeness milky and wax which are quantified with respectively with values 0.75 and 1. This 3x3 factorial experiment is performed so that the factors are applied in total random distribution plane of experiment plane with 4 repetition. Gain of dried matter is given in kg/7m^2.

Results are given in the table 8.

Table 8. Results of experiment gave in example 2

Number of plants N	Density of fert. D	Harvest H	Repetition (Gain of dried matter) G 1	2	3	4	Sum
70000	50	0.75	5.28	6.66	7.78	5.78	25.50
		1	8.49	8.20	8.39	8.38	33.46
	100	0.75	9.34	8.28	8.55	8.43	34.60
		1	10.34	8.86	9.81	8.96	37.97
	150	0.75	9.60	10.35	9.08	9.07	38.10
		1	10.10	10.46	11.51	13.80	45.87
105800	50	0.75	7.10	6.33	6.76	7.34	27.53
		1	8.86	9.07	9.11	9.23	36.27
	100	0.75	8.19	7.52	8.66	9.45	33.82
		1	10.17	9.73	10.97	10.27	41.14
	150	0.75	9.94	9.78	9.49	8.81	38.02
		1	11.52	9.94	12.14	12.08	45.68
128600	50	0.75	7.08	6.98	6.67	6.71	27.44
		1	6.77	7.06	8.01	8.12	29.96
	100	0.75	8.17	7.80	8.99	7.94	32.90
		1	9.70	8.37	11.26	10.14	39.47
	150	0.75	11.89	9.03	10.85	8.94	40.71
		1	11.54	11.00	11.93	11.84	46.31
Total			164.08	155.42	169.96	165.29	654.75

Table 9. Analysis of variance of experiment results given in example 2.

Regression Statistics

Multiple R	0.973683
R Square	0.948059
Adjusted R Square	0.936928
Standard Er.	0.039782
Observatios	18

ANOVA

	df	*SS*	*MS*	*F*	*Significane F*
Regression	3	0.404402	0.134801	85.17809	3.13E-09
Residual	14	0.022156	0.001583		
Total	17	0.426558			

	Coefficients	*Standard Er.*	*t Stat*	*P-value*	*Lower 95%*
Intercept	0.023922	0.080496	0.297185	0.770689	-0.14872
X Variable 1	0.015558	0.038877	0.400185	0.69506	-0.06782
X Variable 2	0.310542	0.022968	13.52067	1.99E-09	0.26128
X Variable 3	0.639	0.075013	8.518536	6.54E-07	0.478113

Table 10. Beginning matrix for ELECTRA method for example 2. and for given heaviness coefficients in 1)

	x1(N)	x2(D)	x3(H)	y(G)
a1	0.7	0.5	0.75	0.6375
a2	0.7	0.5	1	0.8365
a3	0.7	1	0.75	0.865
a4	0.7	1	1	0.94925
a5	0.7	1.5	0.75	0.9525
a6	0.7	1.5	1	1.14675
a7	1.058	0.5	0.75	0.68825
a8	1.058	0.5	1	0.90675
a9	1.058	1	0.75	0.8455
a10	1.058	1	1	1.0285
a11	1.058	1.5	0.75	0.9505
a12	1.058	1.5	1	1.142
a13	1.286	0.5	0.75	0.686
a14	1.286	0.5	1	0.749
a15	1.286	1	0.75	0.8225
a16	1.286	1	1	0.98675
a17	1.286	1.5	0.75	1.01775
a18	1.286	1.5	1	1.15775
	0.16665	0.1665	0.1667	0.5

From the relation of values for F distribution we see that only the variance of density of fertilizer and harvest are significant.

Statistical analysis of multiple linear regression, using Excel Data analysis option, we obtain results which are given in also in Table 9.

Output multiple linear regression give us relation between output parameter and factors of experiment in example 2 in the form

$$G=0{,}023922+0{,}015558N+0{,}310542D+0{,}639H$$

from which we can calculate optimal factor combination like sixth combination N=105800 , D= 150 and H=wax ripeness in for example a_6 notation..

Let us to solve example 2 with procedure proposed in this paper with ELECTRE method of multi attribute decision and :

1.) with heaviness coefficients for criteria i.e factors given in Table 10. which are equal between input factors i.e. with values for number of plants N, density of fertilizer D and harvest H equal 0,1666 or equal 0,5 for sum all three input criteria i.e. factors and like authors suppose value 0,5 for gain of dried matter like only one output criteria i.e. factor.

Table 10. Beginning matrix for ELECTRA method for example 2. for given heaviness coefficients in 1)

Obtained results with ELECTRE method and such values for criteria are given in Table 11.

Table 11. Application of ELECTRA method for example 2.

a1 non dominant
a2 dominant over: a1 a7 a13 a14 a15
a3 dominant over: a1 a7 a9 a13 a15
a4 dominant over: a1 a7 a13 a14 a15
a5 dominant over: a11
a6 dominant over: a1 a3 a4 a5 a7 a9 a11 a12 a13 a14 a15 a17
a7 dominant over: a13
a8 dominant over: a1 a3 a7 a9 a13 a14 a15
a9 dominant over: a1 a15
a10 dominant over: a2 a3 a5 a7 a9 a11 a13 a14 a15 a16 a17
a11 non dominant
a12 dominant over: a4 a5 a7 a9 a11 a13 a14 a15 a17
a13 non dominant
a14 non dominant
a15 non dominant
a16 dominant over: a9 a11 a13 a14 a15
a17 dominant over: a1 a15
a18 dominant over: a11 a13 a14 a15 a17

The obtained results shows that same alternative notated with a6 is dominant.

2.) with heaviness coefficients for criteria i.e factors given in Table 12 which are for input factors i.e. criteria with values for number of plants N, density of fertilizer D and harvest H proportional to values corresponding F parameters respectively 0,00274, 0,27737 and 0.21989 and equal 0,5 for sum all three input criteria i.e. factors and like authors suppose value 0,5 for gain of dried matter like only one output criteria i.e. factor.

Table 12. Beginning matrix for ELECTRA method for example 2. for given heaviness coefficients in 2)

	x1(N)	x2(D)	x3(H)	y(G)
a1	0.7	0.5	0.75	0.6375
a2	0.7	0.5	1	0.8365
a3	0.7	1	0.75	0.865
a4	0.7	1	1	0.94925
a5	0.7	1.5	0.75	0.9525
a6	0.7	1.5	1	1.14675
a7	1.058	0.5	0.75	0.68825
a8	1.058	0.5	1	0.90675
a9	1.058	1	0.75	0.8455
a10	1.058	1	1	1.0285
a11	1.058	1.5	0.75	0.9505
a12	1.058	1.5	1	1.142
a13	1.286	0.5	0.75	0.686
a14	1.286	0.5	1	0.749
a15	1.286	1	0.75	0.8225
a16	1.286	1	1	0.98675
a17	1.286	1.5	0.75	1.01775
a18	1.286	1.5	1	1.15775
	0.00274	0.27737	0.21989	0.5

Obtained results with ELECTRE method for such values for criteria are given in Table 13

Table 13. Obtained results with ELECTRE method for values for criteria are given in Table 12.

a1 non dominant
a2 dominant over: a1 a7 a13 a14 a15
a3 dominant over: a9 a15
a4 non dominant
a5 dominant over: a11
a6 dominant over: a5 a11 a12
a7 dominant over: a1 a13
a8 dominant over: a1 a2 a3 a7 a9 a13 a14 a15
a9 dominant over: a15
a10 dominant over: a3 a4 a5 a9 a11 a15 a16 a17
a11 non dominant
a12 dominant over: a5 a11
a13 dominant over: a1
a14 non dominant
a15 non dominant
a16 dominant over: a4 a5 a11 a15
a17 dominant over: a5 a11
a18 dominant over: a5 a6 a11 a12

The obtained results shows that alternatives notated with a_8 and $a_{10}$are dominant.
On the end of this main section of this paper is given analysis of variance for considered example.

Table 14. Analysis of variance of experiment results

Variation source	Range of right	Sum of square	Middle of square	F
Blocks	3	6,1343	2.0448	
Number of plants N	2	1,1424	0,5712	0,9771
Density of fertilizer D	2	115,8935	57,9467	99,122
Harvest H	1	45,9361	45,9361	78,577
Interact. NxD	4	3,0108	0,7527	1,2875
Interact. NxH	2	1,6991	0,8495	1,4531
Interact. DxH	2	0,2963	0,1481	0,2533
Interact. NxDxH	4	2,6451	0,6613	1,1312
Error	51	29,8155	0,5846	
Total	71	206,5731		

From the relation of values for F distribution in the table 14. we see that only the variance of density of fertilizer and harvest are significant.

IV CONCLUSION

The application of the classical statistic mathematical apparatus for result analysis of different multifactor anyhow multivariate experiment organization is difficult and especially in solving a problem of the optimal factor configuration choice. Therefore the authors have proposed in this paper one application mathematical apparatus so called multi attribute analysis for analysis of experiment results. Evidently this procedure process these results of one multifactor, at any rate and multivariate experiment in one easier, efficacious and universal way.

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References

[1] Beth T. et. al., *Design Theory*, Cambridge University Press, 1993.
[2] Nikolić I., Borović S.,*Višekriterijumska optimizacija*,Centar vojnih škola VJ,1996.
[3] Randjelović D. et. al., Multi criteria analysis application in the analysis of experiment results, Journal WSEAS transaction on mathematics, Issue 7, Volume 6, 2007.
[4] Hadzivuković S., *Statistički metodi,* Univerzitet u Novom Sadu, 1991
[5] Kempthorne O, *The design and Analysis of Experiments,* John Wiley&sons Inc,New York 1952.
[6] Cohran W. and Cox G., *Experimental Designs,* John Wiley&sons Inc,New York 1957.
[7] Z. Kovačić, *Multivarijaciona analiza*, Univerzitet Beograd, 1994.
[8] D.M. Randjelović, Objective functions for systolic arrays, WSEAS transaction on computers, Volume 2, 2003., pp. 1015-1020.
[9] D.M. Randjelović, Space objective functions for systolic arrays, WSEAS transaction on mathematics, Volume 3, 2004., pp. 7-13.
[10] D. Randjelović, Objective functions for flowing systolic arrays, WSEAS transaction on computers, Issue 7, Volume 6, 2007., pp. 973-978.
[11]Randjelović D. et. al.,Existence one class of Steiner block - schemas and their application in the agricultural experiment organization, Proceedings of

MASSEE2003., Borovets, Bulgaria, 2003.
[12] Randjelović D. et. al.,Classes of block designs and theirs application in the analysis of agricultural experiment organization, Proceedings of WSEAS VI International Conference on Applied computer science(ACOS'07),Hagzhou, China,2007.,pp 596-601
[13]Aigner M.,*Combinatorial Theory*,Springer, Berlin, 1997.
[14]Beth T. et. al.,*Design Theory*, University Press Cambridge, 1993.
[15]Milovanović I, et. al.,*Diskretna matematika*,Univerzitet Niš ,Niš, 2000.
[16]Charnes A. et al., *Data envelopment analysis:Theory*, *Methodology and Applications*, Kluwer Academic Publisher, Boston, 1993.

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