Efficient Mixing in Microchannel by using Magnetic Nanoparticles

T.N Le, Y.K. Suh and S. Kang

***Abstract*—**Rapid and efficient mixing in microchannel using magnetic nanoparticles has been numerically investigated. The magnetic nanoparticles are introduced into the microchannel and are exerted by the external magnetic force to cause the vortex motion of the fluid for mixing. The velocity field of the flow and trajectories of the particles are solved implicitly by using the Finite Volume Method (FVM). The obtained results illustrate the significant effects of the magnetic actuation force, the switching frequency, number of magnetic nanoparticles on the mixing efficiency. The mixing properties of the flow predicted by numerical simulation are studied under the concentration field, mixing index and Poincaré section.

***Keywords*—**Microchannel, Magnetic Force, Magnetic Nanoparticle, Mixing Index, Poincaré section.

$C, \overline{C}$	: Concentration
D	: Diffusion coefficient
f	: Switching frequency
F_d	: Drag force
F_m	: Magnetic force
H	: Half height of microchannel
H_m	: Magnetic field intensity
I	: Mixing index
L	: Length of microchannel
m_p	: Mass of magnetic particle
M_p	: Mangetization of particle
N	: Total number of grid point.
N_p	: Number of magnetic nanoparticle
p	: Pressure
Pe	: Peclet number
Re	: Reynolds number
R_p	: Radius of magnetic particle
t	: Time
V_p	: Volume of magnetic particle
u_i	: Velocity components
u_p	: Velocity of magnetic particle
U_0	: Referent velocity
Δt	: Transient timescale
α, γ, ρ	: Runge-Kutta coefficients
ρ_f	: Fluid density
ρ_{Np}	: Density of magnetic particle per fluid volume
ρ_p	: Magnetic particle density
μ_0	: Permeability of free space
υ	: Fluid kinematic viscosity

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I. Introduction

RECENTLY, the biological and chemical analysis in micro-fluidic systems have been widely researched and developed for many applications. Chemical reactions, bio-analytical techniques etc need the rapid, efficient mixing processing. However, the mixing in micro-scale with low Reynolds number, Re (less than 100), is a big challenge for many researchers due to dominated molecular diffusion. If we want to get full mixing which relies on pure diffusion, the microchannel has to be extended to extremely long.

To achieve a faster mixing with shorter channel length, there are two kinds of mixers used: passive and active mixers [5-9]. A number of research works have been reported regarding both active and passive mixing, both of them have some advantage and disadvantage. So far, active mixers are more preferred to investigate because they can produce excellent mixing on conditions of short channel length and limit time performance.

In this paper, we have considered an active mixer with magnetic nanoparticles inserted in the microchannel flow. By using a time dependent magnetic field, we show the possibility to induce a particle-fluid interaction that causes a vortex motion of the fluid in microchannel. This is the primary principle in order to create a good mixing.

The present paper is organized as follows. In section 2, we describe mathematical model, numerical methods used to solve the problem. Then the evaluations of the influence of the obtained results by quantities like external magnetic force, switching frequency etc are explained in section 3. Finally, conclusion is given in section 4.

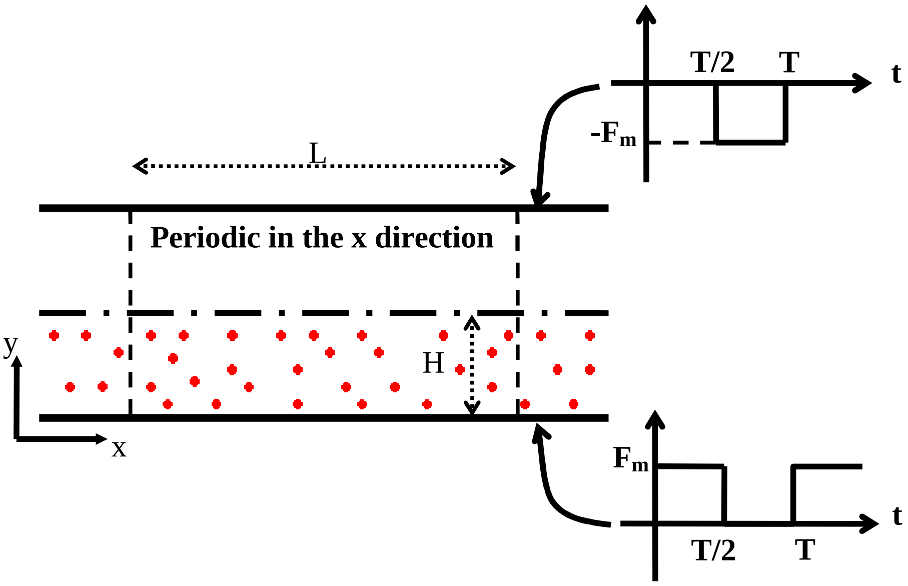


Fig. 1. Two dimensional microchannel (L=3H) with the external magnetic force changing in time.

II. Mathematical Model

We use a simple microchannel shown in figure 1 to illustrate the motion of an incompressible fluid in the channel. The subsections will mention the method of solving the above physical problem which is based fundamentally on Wang et all with some assumed dimensional parameters in table 1[5].

A. Governing equations for magnetic nanoparticles

Magnetic nanoparticles introduced into the microchannel under external magnetic field are exerted by several forces: magnetic force, fluidic drag, particle-particle interactions, inertia, gravity, thermal kinetics. To facilitate in simulation, here, we are interested in the behavior of the magnetic force and drag force in mixing effect. Therefore, we use the classical Newtonian equation as follow:

$$m_p \frac{du_{p,i}}{dt} = -F_{d,i} + F_{m,i} \tag{1}$$

Where m_p, u_p are the mass and velocity of the particle, F_m, F_d are the magnetic force and drag force, respectively.

Drag force

Particles flowing in the microchannel have an interaction with the fluid, expressed by Stoke's law

$$F_{d,i} = 6\pi\rho_f \upsilon R_p \left[u_{p,i} - u_i \delta\left(r, r_p\right)\right] \tag{2}$$

With ρ_f, υ are the density and viscosity of the fluid, respectively. And R_p is the radius of the magnetic nanoparticle.

To obtain drag force, we need the velocity of the particle u_p and velocity of the fluid at the particle position u_i. From neighboring fluid velocity, we can evaluate the fluid velocity u_i at the particle position by using bilinear interpolation function.

Magnetic force

Under the external magnetic field with the applied magnetic intensity H_m at the center of particle, magnetic force on particles is given by

$$F_{m,i} = \mu_0 V_p \left(M_p \cdot \nabla\right) H_m \tag{3}$$

Where V_p, M_p are the volume and magnetization of the particle. The permeability of free space is $\mu_0 = 4\pi \times 10^{-7}\ H/m$.

B. Governing equations for the flow

The incompressible flow in microchannel can be illustrated by using the following dimensional equations written in Cartesian coordinates x, y, z

Momentum equation

$$\frac{\partial u_i}{\partial t} + \frac{\partial\left(u_i u_j\right)}{\partial x_j} = -\frac{1}{\rho_f}\frac{\partial p}{\partial x_i} - \frac{1}{\rho_f}\frac{dP}{dx_1}\delta_{i1} + \upsilon\frac{\partial^2 u_i}{\partial x_j \partial x_j} + f_{d,i}\delta\left(r, r_p\right) \tag{4}$$

This equation will be solved together with the continuity equation,

$$\frac{\partial u_j}{\partial x_j} = 0 \tag{5}$$

where the second term in the right hand side of eq (4) is the constant pressure gradient in x direction $\frac{dP}{dx_1} = \frac{3\rho_f \upsilon U_0}{H^2}$.

The fluidic drag between particle and fluid per unit volume $f_{d,i}$ is evaluated as

$$\oint_{\partial V} \rho_f f_{d,i} dV \approx \rho_f f_{d,i} \delta V = 6\pi \rho_f \upsilon R_p \left(u_{p,i} - u_i \right) \tag{6}$$

C. Numerical method

The above equations (1)-(6) are non-dimensionalised and are integrated in time below

Momentum equation

$$\begin{aligned} \frac{\widehat{u_i} - u_i^{k-1}}{\Delta t} &= \alpha_k \left(\widehat{L_i} + L_i^{k-1} \right) - 2\alpha_k \frac{\partial p^{k-1}}{\partial x_i} \\ &+ 2\alpha_k \frac{3}{\mathrm{Re}} \delta_{i1} - \left(\gamma_k N_i^{k-1} + \rho_k N_i^{k-2} \right) \\ &+ \alpha_k f_{d,i} \delta\left(r, r_p\right) + \alpha_k f_{d,i}^{k-1} \delta^{k-1}\left(r, r_p\right) \end{aligned} \tag{7}$$

With $L_i = \frac{1}{\mathrm{Re}} \frac{\partial^2 u_i}{\partial x_j \partial x_j}$ is the diffusive term, $N_i = \frac{\partial \left(u_i u_j \right)}{\partial x_j}$ is the convective term and $f_{d,i} = \frac{6\pi R_p}{\mathrm{Re}\,\delta V} \left(u_{p,i} - u_i \right)$ is the fluid drag per volume.

Equation of motion of particle

$$\begin{aligned} m_p \frac{u_{p,i}^k - u_{p,i}^{k-1}}{\Delta t} &= -\alpha_k A \left(u_{p,i}^k + u_{p,i}^{k-1} \right) \\ &+ A\left(\gamma_k u_i^{k-1} \delta^{k-1} + \rho_k u_i^{k-2} \delta^{k-2} \right) + 2\alpha_k F_m \delta_{i2} \end{aligned} \tag{8}$$

With the constant magnetic force in vertical direction y $F_m \delta_{i2}$ and the constant $A = \frac{6\pi R_p}{\mathrm{Re}}$.

Here, we use three kinds of complementary simulation to examine the mixing processing: concentration field, mixing index, Poincaré section. These methods are used to make the mixing mechanism in the microchannel to be clearly understood.

The concentration distribution is obtained by solving the diffusion-convection equation

$$\frac{\partial C}{\partial t} + \frac{\partial \left(C u_j \right)}{\partial x_j} = \frac{1}{Pe} \frac{\partial^2 C}{\partial x_j \partial x_j} \tag{9}$$

Where C is the concentration, Pe is the Peclet number which can be defined as follows,

$$Pe = \frac{U_0 H}{D} \tag{10}$$

Where U_0, H, D are average velocity of flow, characteristic length (here half of the channel), the molecular diffusion coefficient, respectively.

With the concentration field and Poincaré section (will be mentioned in the next section), we can determine the local microstructure of mixing patterns, but to evaluate exactly the mixing rates, we should use another parameter, the so called mixing index I, defined as,

$$I = \frac{1}{\overline{C}} \sqrt{\frac{1}{N} \sum_{i}^{N} \left(C_i - \overline{C} \right)^2} \tag{11}$$

Where $\overline{C}$ is the average concentration of the calculation domain, defined as $\overline{C} = \sum_i^N C_i / N$, C_i is the concentration at grid points, N is the total number of grid points in the region.

The governing equations (7)-(9) are discretized by using spatial second-order central difference scheme on a staggered mesh with the number of grid points 90 x 60. The semi-implicit Fractional Step Method: a third-order Runge-Kutta method for body force term and a second-order Crank Nicolson method for the diffusion term, are used to solve the Navier-Stokes equation and the time integration of the particle movement equation. In addition, the Power law scheme [4] is applied in the convective-diffusive equation because the second-order central different scheme can produce a negative concentration. The numerical procedure is schematized described in figure 2.

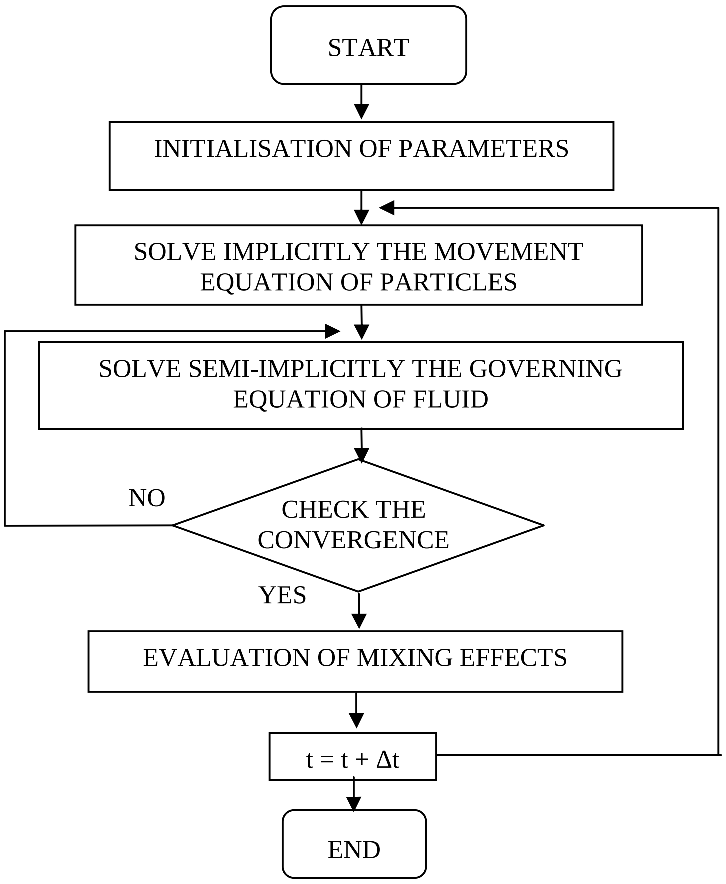
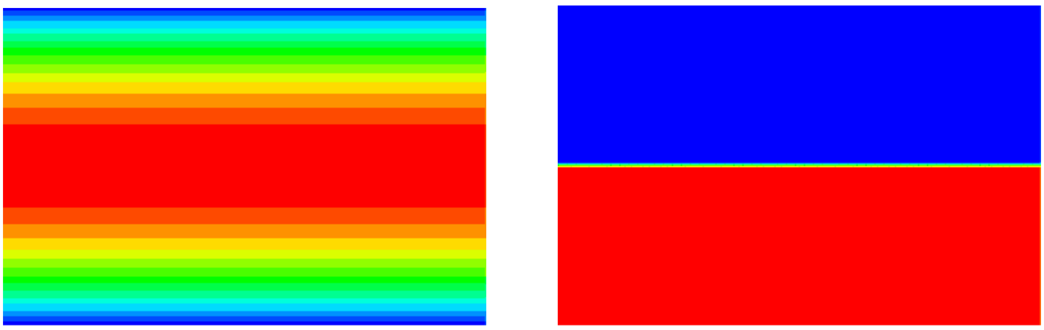


Fig. 2. Numerical procedure of the flow mixing in microchannel.



(a) (b)

Fig. 3. Initial condition of (a) the flow and (b) the concentration distribution in microchannel.

The fully developed flow is set as the initial condition for the flow field in figure 3(a) and for the concentration in figure 3(b). No slip boundary conditions are used at the top and bottom channel wall. In addition, we have considered only the calculation domain: height and length 2x3 with the periodic fluid flow. The random distribution shown in figure 5(a) in the bottom half of channel is the initial condition of the magnetic nanoparticles at which velocity of particles equals velocity of the fluid. With the above boundary conditions and suitable dimensionless parameters from table 1, the mixing in microchannel using magnetic nanoparticles can be solved.

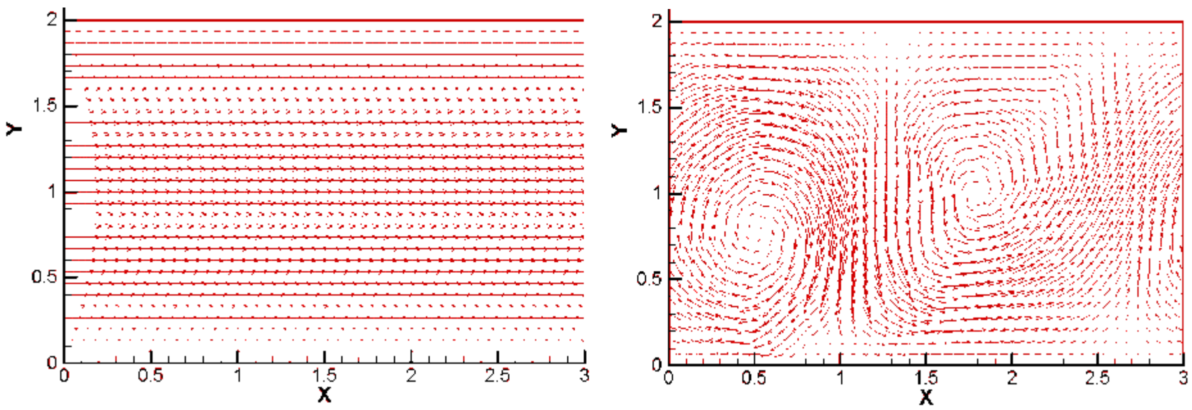
Table 1. Parameters used

Dimension		Dimensionless	
ρ_p	1580 kg/m^3	ρ_p/ρ_f	1.58
R_p	0.5x10^{-6} m	R_p	0.01
ρ_f	1000 kg/m^3	Re	0.05
υ	1x10^{-6} m^2 /s	Pe	1000, 10000
U_0	10^{-3} m/s	U_0	1
H	50x10^{-6} m	H	1
ρ_{Np}	0.3308 kg/m^3	N_p	300
F_m	7.5 pN	F_m	5
t	0.05s	t	1

III. Results And Discussion

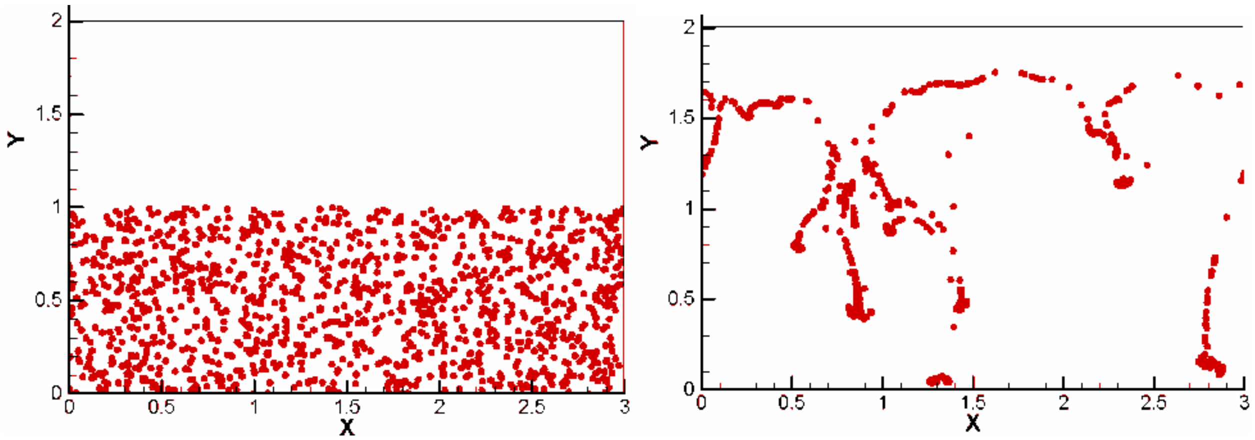
In this section, the numerical results will be illustrated in detail. If magnetic nanoparticles are put in microchannel and are not exerted by any external magnetic field, they will follow the fluidic flow without mixing occurrence. But when we apply the actuation magnetic force corresponds in time, we achieve the streamline of the flow shown in figure 4(b). Magnetic particles forced by magnetic field will accumulate and create some chains of a number of magnetic particles shown in figure 5(b). This mechanism makes the flow in microchannel like the flow pattern shown in figure 4(b) passes over the chains of magnetic nanoparticles. Hence we can see the variation of flow streamline patterns in the microchannel with and without the applied magnetic force.

Since Peclet number depends on diffusion coefficient, the larger Peclet number will achieve low diffusion which is more difficult for mixing. In figure 6, we can see that the mixing index is slightly dependent on Peclet number. This is an indication that this kind of mixing mechanism is not affected considerably by Peclet number and it also means that this mixer can be used for various types of liquid instead of water mentioned in this paper.



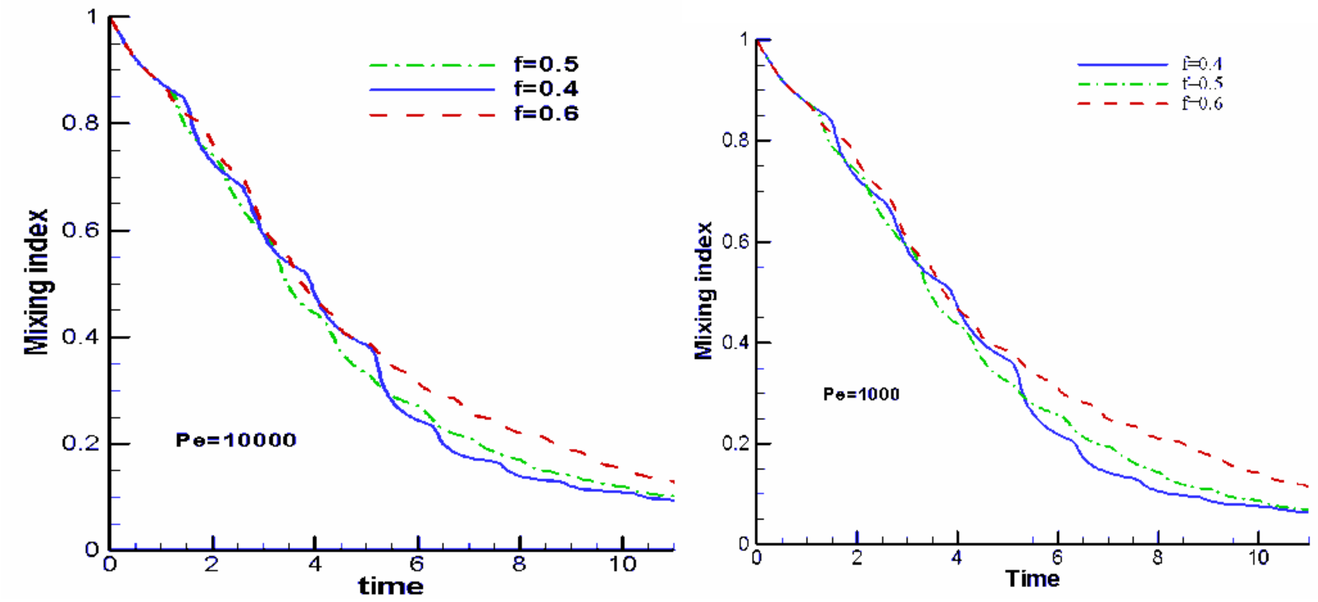
(a) (b)

Fig. 4. Streamline of the flow in microchannel (a) without the applied magnetic force (b) with the applied magnetic force F_m=5, switching frequency f=0.4 and number of magnetic particles N_p=1200.



(a) (b)

Fig. 5. Magnetic particle distribution (a) at initial time and (b) at time t=4 with the applied magnetic force F_m=5, switching frequency f=0.4 and number of magnetic particles N_p=1200.

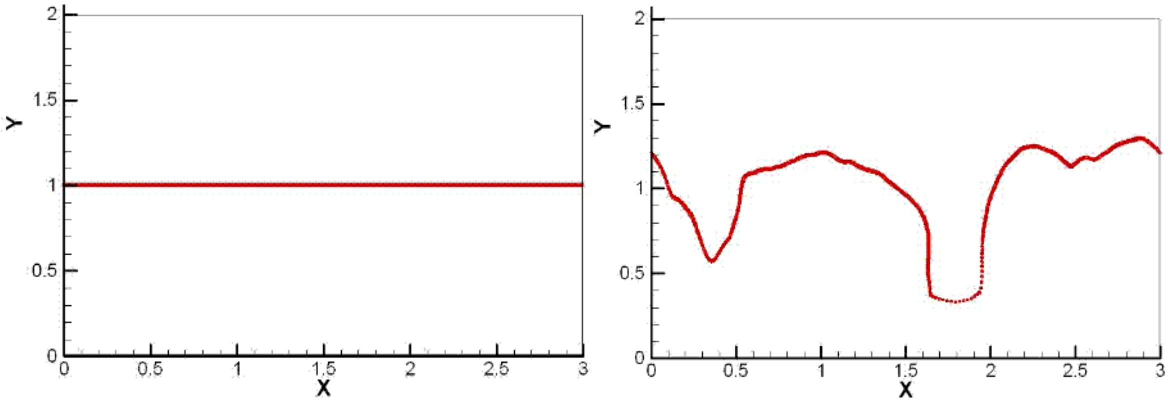


(a) (b)

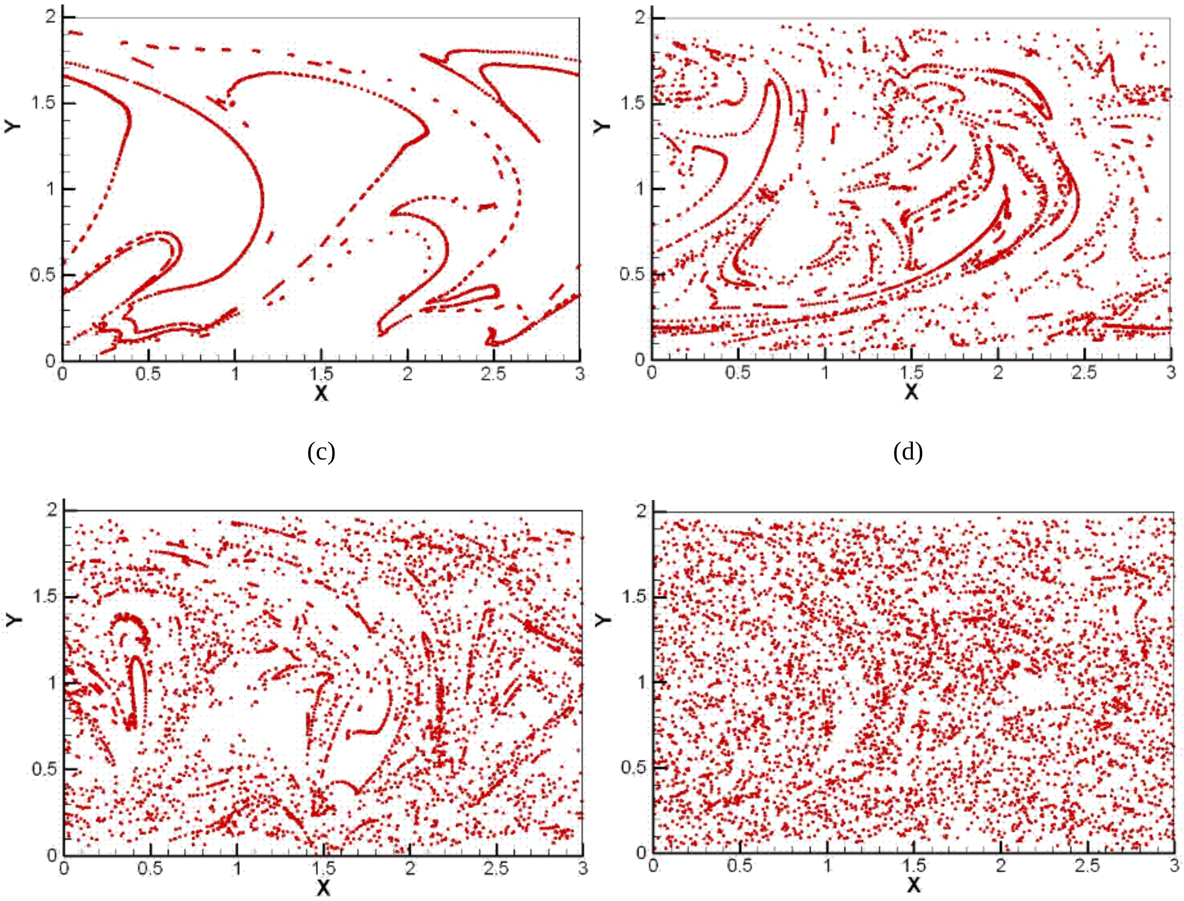
Fig. 6. Mixing index with different Peclet number (a) 1000 and (b) 10000 with F_m=5, f=0.4, N_p=1200.

Our primary objective is to quantify differences in mixing efficiency with the altering of switching frequencies, magnetic force magnitudes and number of particles, so we first evaluate mixing behavior in microchannel by computing Poincaré sections. In order to calculate a Poincaré section in the microchannel, we put 5000 passive fluid particles in horizontal lines at the center of channel. Particle trajectories are obtained by a fourth-order Runge-Kutta method.

From the observation of the particle lines deformation, the mechanism to create folding and stretching, is illustrated clearly in figure 7. When the magnetic particles accumulate together enough to create some bigger spheres, the fluid flow between those spheres will be accelerated backward to the sphere velocity direction that induces the big folding like figure 7(b). Then the folding will be stretched by the center flow with height velocity and narrowed again by slower velocity flow near the walls. This processing manipulates repeatedly to manifest chaotic mixing. And a good mixing pattern is eventually created as shown in figure 7(f).



(a) (b)



(c) (d)

(e) (f)

Fig. 7. Evolution of the Poincaré section in respect of times (a) 0, (b) 2, (c) 5, (d) 9, (e) 12, (f) 20 with F_m=5, f=0.3, N_p=1200.

A. *Effect of switching frequency*

By making the agitation of 1200 magnetic particles with an external magnetic force F_m=5, we can investigate the variation of mixing efficiency which corresponds to each operating frequency.

When the magnetic force is fixed, particle trajectory depends only on the switching frequency. The shorter distance magnetic particle travels with higher frequency. However, the mixing process is affected strongly by the particle trajectories. The magnetic particles oscillate within longer distance we will get a good mixing in shorter time of manipulation. In case of the frequency f=0.7, the mixing index is approximately 0.125 at time t=11 because of the small amount of magnetic nanoparticles that move to the upper half of channel and that is difficult to collect more particles for mixing. From observation of the variation of mixing index in the different switching frequencies, we can see that the mixing process is achieved faster and more efficient when the frequency is reduced, but the switching frequency reach the optimum frequency. If we reduce sequentially the frequency f=0.2, the mixing will become inefficient because the magnetic nanoparticles are maintained their location at wall region, which prevents the mixing process. From figure 8, we can predict the optimum frequency for the magnetic force F_m=5 as f=0.3.

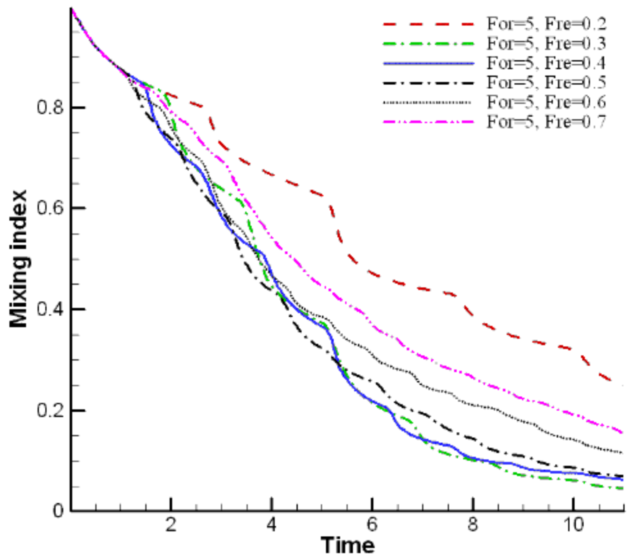


Fig. 8. Mixing index in the case of magnetic force F_m=5, N_p=1200 with different switching frequencies.

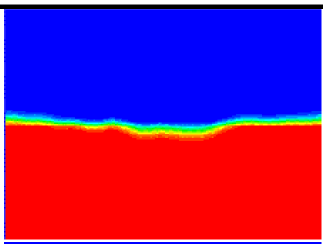
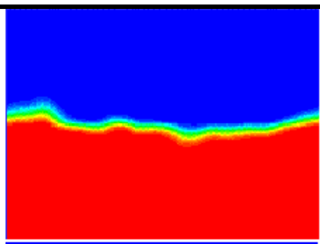
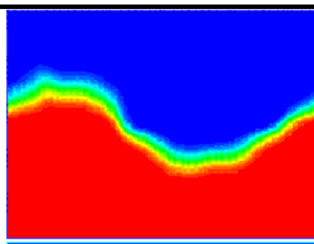
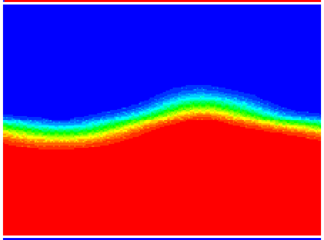
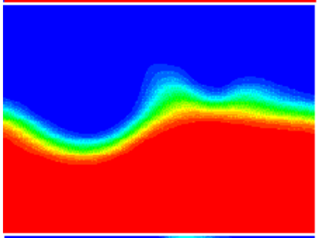
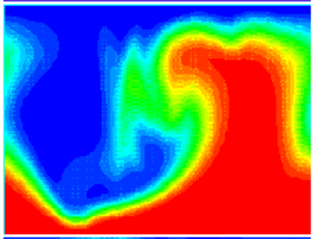
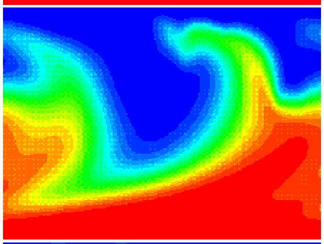
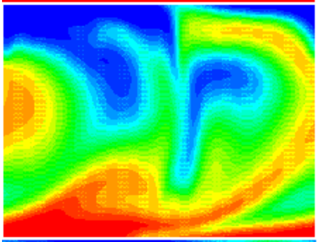
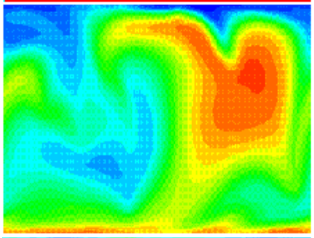
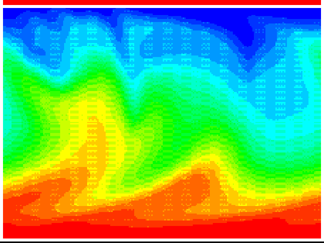
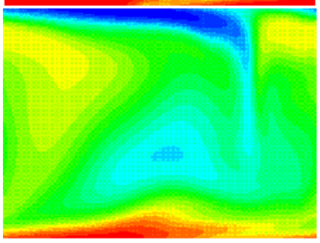
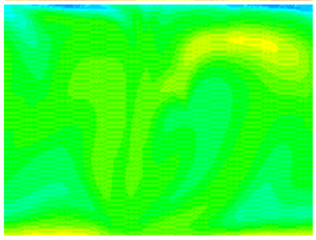
Time	300 particles	600 particles	1200 particles
0.3			
1.6			
4			
6.5			

Fig. 9. The distribution of the concentration with different number of particles: 300, 600, 1200 at times with F_m=5, f=0.3.

B. Effect of number of magnetic particles

The mixing efficiency also depends on the number of magnetic nanoparticles. The more particles are introduced into microchannel, the more mixing we get.

Here, to evaluate the influence of the number of magnetic particles, we applied a magnetic force F_m=5 with the optimum frequency defined as 0.3 in the previous subsection. Although we used those conditions for good mixing, the mixing process also depends on number of magnetic particles. As mentioned, the magnetic particles oscillate and accumulate together, but with the small amount of particles, the obtained particle collections will be a few, that is the main reason for long time duration for good mixing.

Figure 9 reveal the concentration field with respect to time for three cases of particle number using: 300, 900, 1200 particles. From figure 10, we can observe that the mixing rate in case of number of magnetic particle=1200 is fastest within the time t=11. For other cases, to get the same mixing quantity,

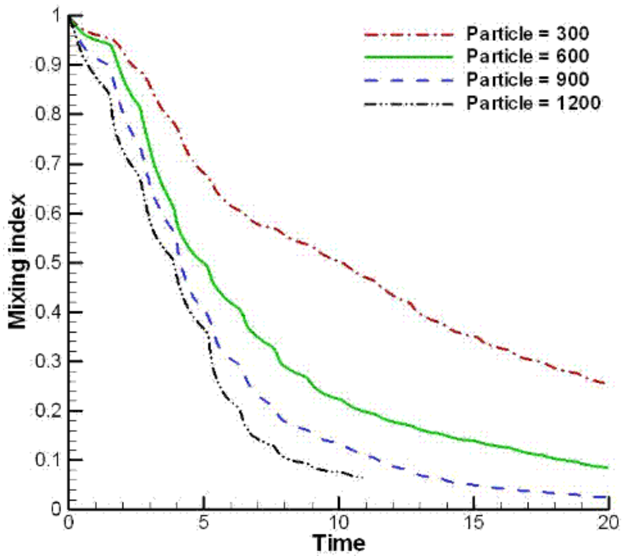


Fig. 10 Mixing index with different number of magnetic nanoparticle, F_m=5, f=0.3.

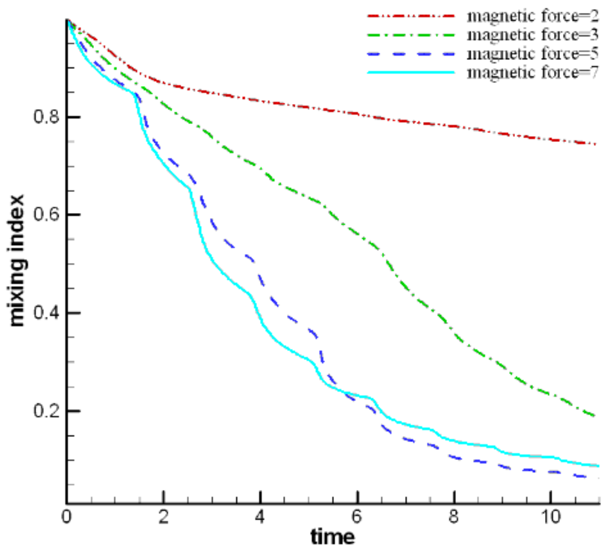


Fig. 11. Mixing index with different magnetic force magnitude, f=0.4, N_p=1200.

it takes longer time. The mixing index is approximately 0.03 at t=20 for 900 particles using and more than the time t=20 for the case of less than 900 particles. The number of magnetic particles using is not only dependent on the mixing possibility but also on the efficient economy. The more particle number is used, the better mixing in the shorter time we get, but the price for that is also more expensive and the amount of mixing fluid is of course limited.

C. Effect of magnitude of magnetic force

The magnetic force magnitude and switching frequency is the two main parameters that influence the particle trajectories. When we fix the operating frequency f=0.4 and change the magnetic force, we can obtain the mixing index comparison shown in figure 11. With the small actuation magnetic force, the particles just only travel within the short distance, so the enhancement of the mixing is limited. To improve the mixing efficiency, we should apply the lower operating frequency. To get good mixing for each magnetic force used, it is essential to find the optimum switching frequency. The smaller magnetic force is used, the lower frequency should be applied to obtain the expected mixing.

IV. Conclusion

By developing a Fortran code with Finite Volume Method, we investigated the rapid and efficient mixing in microchannel using mangetic nanoparticles. The mechanism of the efficient mixing is to the creation of the chaotic flow which is obtained by the chains of magnetic particles moving rapidly in microchannel. Although Peclet number is an important parameter affecting the mixing, with this kind of mixing mechanism, the mixer is not affected by different Peclet number. To show clearly the mixing behavior with the folding and stretching, we also computed a Poincaré section with 5000 passive fluid particles placed in microchannel.

We addressed the effect of the magnetic force magnitude and the switching frequency on the mixing efficiency. Both parameters are correlated. With the fixed magnetic force magnitude F_m=5, we obtained the optimum frequency f=0.3. With each applied magnetic force magnitude, we can find one optimum frequency to determine the maximum mixing.

Finally, the mixing is studied for different number of magnetic nanoparticles which is also the important parameter influencing the efficient mixing. Different cases with number of particles: 300, 600, 900, 1200 are investigated. The more number of magnetic nanoparticles are used, we obtain the better mixing in the shorter time. The mixing index is approximately 0.03 at time t=11 for 1200 particles, at time t=20 for 900 particles and more than t=20 for less than 900 particles.

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