Multi-market bidding strategy of power suppliers in China

Xingping Zhang, Runlian Wu, Ling Chen

Abstract**—**The 5 power generation groups of China have a feature of monopoly, so we apply a non-basic auction theory to make bidding rules in order to reduce the market power of power suppliers. Using non-basic auction theory, order statistics, and Monte Carlo simulation, we calculate the bidding probabilities in different cases. Taking into account bidding probabilities and the revenue, we put forward the bidding strategy models of the spot market and the spinning reserve market, which can be applied to calculate the power volume and bid in the multi-market so as to maximize profit and minimize the bidding risk. Enough spinning reserve volume is important to the security and reliability of the electrical system, the bidding strategy of spinning reserve market prompts power suppliers to bid, because they can get a rational profit and reduce the bidding risk from the spinning reserve market. An example supports the validity of the multi-market bidding model.

Keywords**—**Bidding strategy, Electrical market, Non-basic auction theory, Order statistics

I. INTRODUCTION

IN recent years many scholars have studied the application of Game Theory to the bidding strategy of power suppliers. Based on a game theoretic matrix model, Ferrero denotes the candidate's bidding strategy as a discrete quantum. When the bidding strategy features a few discrete points, a matrix of power suppliers' revenue can be structured when they adopt different mixed strategies, and then a balanced mixed bidding strategy can be found which corresponds to the optimum bidding strategy [1]. Kwang-Ho establishes a revenue matrix reflecting many power suppliers' participation in bidding and having a mixed strategy equilibrium solution [2]. Ferrero describes competition in a power pool as a problem of non-cooperative game theory. Each participant has only incomplete information and knows only its own operating cost and nothing of its opponents'. The problem is solved through a Nash equilibrium and translated into an imperfect-information game, and the optimum price is at the Nash equilibrium point [3]. Considering transmission constraints in a power pool market, Correia establishes a Bayesian equilibrium model under incomplete and non-symmetric information and finds the optimum bidding strategy solution through a Monte Carlo model [4]. Hou Jingzhou puts forward a stochastic model of bidding strategy and establishes the revenue matrix by considering the incompleteness of market information and estimating the opponents' lowest bid. By a min/max decision rule, the optimum bidding strategy is directly chosen by the model for a single period from the revenue matrix [5]. Torre proposes a three-step method to get a Nash equilibrium solution of power suppliers' bidding [6]. Wu Zhiyong puts forward a math model of optimum bidding, and gives a solution based on game theory and probability theory, which solves many tough multi-person game and incomplete information problems by applying the virtual opponent and the method of parameter estimation [7]. Chen Qi-an establishes a game theoretic bidding strategy model of power suppliers under relatively wide assumption. The research shows that when electricity demand volume in the market is less than or equals to the amount of electricity offered into the grid, the optimum bidding by power suppliers will slightly fluctuate between the unit generation cost and the price cap in the power market [8]. Hao has supposed that each power supplier offers the same amount of electricity, and a bidding game model of power suppliers is obtained by using auction theory. Actually, the bidding volume is far larger than one unit and the power generator has incomplete information on the costs of its opponents in the bidding game, the optimum bidding strategy is even more complicated [9]. Adopting first-price sealed-bid auction theory, Ren Yu-long studies incomplete information bidding strategy model of power suppliers when their bidding volume varies and the contract market is considered. The model takes into account unit cost, the probability of successful bidding, contract volume, contract price and expected market-clearing price [10]. Liu Ya-an *et al.* analyze the correlation between unit profit, electricity clearing volume and bidding strategy coefficient, and gets the optimum bidding strategy of a single unit and the optimum readjustment method when bid goes beyond that point, and also proposes a model of a joint bidding strategy between different units of a single power supplier [11]. A similar research is shown by Gao xin *et al.*[12]. In the process of bidding, the bidding risks are measured by different methods, such as risk utility [13], financial risk management [14]-[15].

There are 5 power generation groups in China: China Power Investment Corporation, China Guodian Corporation, China Datang Corporation, China Huadian Corporation and China

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X. Z. is with the Business and Administration School, North China Electric Power University, Beijing City, 102206, China. (phone: 86-10-80795308; e-mail: zhangxingping302@ 163.com).

R. W. is with the department of Science and Technology, North China Electric Power University, Beijing City, 102206, China. (e-mail: zxp@ncepu.edu.cn).

L. C. is with the Electric Power Corp. of Quanzhou, Fujian Province, 362800 China (e-mail: phdxpz@sohu.com).

Huaneng Corporation. So the electric market is a monopolistic market in China. In that case the winning price is bid upward by them to obtain monopoly profit. In the mechanism of a non-basic auction, the market supervisor defines the benchmark bidding interval prior to bid in order to reduce the market power of electricity suppliers. So the theory of non-basic auction is applied to analyze bidding strategies in this paper.

Combining order statistics, non-basic auction [16], and Monte Carlo simulation method, we analyze the probabilities of successful bidding in different cases. Taking into account probabilities of successful bidding and the maximum expected profit, we put forward an optimum bidding strategy model. Considering the generation volume and price of both contract market and the spot market, we obtain a bidding strategy in the spinning reserve market by a Cournot game model.

II. Bidding Probability in a Non-Basic Auction Theory

Suppose there are *N* power suppliers, b_i is the bid of the i^{th} power supplier, and $b_i \sim N(u_i, \sigma_i^2)$. u_i and σ_i can be estimated by the prior distribution. According to the non-basic auction theory, the valid bid *b* must meet the condition of $(1-s)\bar{b} \le b \le (1+s)\bar{b}$, where $\bar{b} = \sum_{i=1}^{n} b_i / N$, and *s* is a coefficient determined by the supervisor. So all the bids are restrict in an interval by the supervisor, and the market power that the power suppliers exert on the market price is reduced.

In this paper, a order statistics is applied to calculate the probability of b_i , So b_i is sorted by size, that is $b_i < b_j (i < j)$. There are possible cases in calculating the bidding probability.

(1) If bid *b* of a power supplier is $b = b_1 < b_i \ (i = 2,3,...,N)$.

According to the characteristics of order statistics, the probability of $b = b_1 < b_i \ (i = 2,3,...,N)$ is calculated by:

$$g_1(b) = [1 - F(b)]^{N-1} \tag{1}$$

where *F(b)* is the distribution function of bid *b*.

In this case the probability of successful bidding is obtained by (2).

$$\begin{gathered} h_1(b) = P((1-s)\bar{b} \le b \le (1+s)\bar{b} \,|\, b < b_i) \\ i = 2,3,...,N \end{gathered} \tag{2}$$

(2) If bid *b* is at the price level *k*, that is, $b = b_k > b_i$, $(i = 1,2,...,k-1)$,and $b = b_k < b_j$, $j = k+1, k+2, ..., N$.

According to the characteristics of order statistics, the probability of $b = b_k$ is calculated by (3).

$$g_k(b) = C_{N-1}^{k-1} \cdot [P(b)]^{k-1} [1 - P(b)]^{N-k} \tag{3}$$

According to non-basic auction theory, if bid b_k is successful, it is required that all the other bids which are less than b_k are invalid, that is, $b_i < (1-s)\bar{b}, i = 1,2,...,k-1$. So the probability of successful bidding in this case is obtained by (4).

$$\begin{gathered} h_k(b) = P((1-s)\bar{b} \le b \le (1+s)\bar{b}, \\ b_i < (1-s)\bar{b} \,|\, b > b_i, b < b_j) \\ i = 1,2,...,k-1,\ j = k+1, k+2, ..., N \end{gathered} \tag{4}$$

(3) When bid *b* is at the price cap, that is, $b = b_N > b_i \ (i = 1,2,...,N-1)$, the successful probability in this case is $h_N(b_N) = 0$.

As the above *N* cases are independent, the probability of successful bidding can be obtained by (5).

$$P(b) = \sum_{k=1}^{N} h_k(b) g_k(b) \tag{5}$$

III. Bidding Probability with Incomplete Information

A. *Probability of Unsuccessful Bidding*

If power supplier *i* fails bid, while *m* suppliers are successful, then the bids of *m* power suppliers are lower than b_i . As the transaction volume of each power supplier and the electricity demand in the corresponding period are known, *m* can be calculated.

Suppose q_i is the bidding power generation of supplier *i*, q_{ig} is its power generation to the grid, and let $\eta_j = q_{ig} / q_i$.

Let $L_i(b_i)$ be the probability that a bidder fails in the auction, and $L_i(b_i)$ is calculated by (6).

$$\begin{cases} L_i(b_i) = \sum_{j=1}^{m} h_j(b_i) g_j(b_i) \\ \eta_m q_m + \sum_{k=1}^{m-1} q_k = Q \\ s.t. \quad (1) \sim (5) \end{cases} \tag{6}$$

where *Q* is the electric power demand.

B. *Probability of Successful Bidding*

Suppose a single market clearing price is *w*. There are two possible situations in successful bidding.

(1) If $b_i = w$

In this case, q_{ig} may not equal to q_i , and q_{ig} may has several possible values. If x_j power suppliers bid successfully, then there are $x_j - 1$ power suppliers whose bids lower than b_i ; that is, power supplier *i* is at the price level of x_j . If $b_i = w$, suppose $H_i^j(b_i)$ is the probability of η_j , which is calculated by (7).

$$\begin{cases} H_i^j(b_i) = h_{x_j}(b_i) g_{x_j}(b_i) \\ \eta_j q_i + \sum_{k=1}^{x_j-1} q_k = Q \\ s.t. \quad (1) \sim (5) \end{cases} \tag{7}$$

Then the successful bidding probability of $H_i(b_i)$ is obtained by (8).

$$H_i(b_i) = \sum_j H_i^j(b_i) \tag{8}$$

(2) If $b_i < w$

Suppose $R_i(b_i)$ is the probability of successful bidding in the condition of $b_i < w$, which can calculated by (9).

$$R_i(b_i) = 1 - L_i(b_i) - H_i(b_i) \tag{9}$$

IV. THE INTERVAL OF OPTIMIZATION POWER GENERATION

Suppose M_c is the marginal cost, the power generation of q_{i0} is supplied by the i^{th} power supplier in its minimum of M_c, and the power generation of q_{i2} is supplied by the i^{th} power supplier in the condition of $w = M_c$.

Suppose *r* is the expected rate of return, then the opportunity cost is $C_i(1+r)$, and the return is wq_i. So the minimum of opportunity losses is calculated by (10).

$$\min L = C_i(1+r) - wq_i \tag{10}$$

The optimization root of (10) must meet the following conditions:

$$\begin{cases} dL/dq_i = (1+r)M_C - w = 0 \\ d^2L/dq_i^{\,2} = (dM_C/dq_i)r > 0 \end{cases} \tag{11}$$

According to (11), the internal of the optimization power generation is defined by:

$$q_{i0} < q_{opt} < q_{i2}$$

where q_{opt} is the optimization value of power generation.

Let $q_{i\min}$ and $q_{i\max}$ denote the minimum and maximum power generation of power supplier *i* respectively, the rational interval of q_{opt} is defined by:

$$\max(q_{i\min}, q_{i0}) < q_{opt} < \min(q_{i\max}, q_{i2}) \tag{12}$$

V. MULTI-MARKET BIDDING STRATEGY MODEL

If c_i is the unit cost of power supplier *i*, then c_i is the lowest unit price for electricity that power supplier *i* is ready to offer. We assume that all power suppliers are rational, and other power suppliers don't know the value of b_i, but know only that b_i is a random variable drawn from a distribution over [u , v], whose density function is $f(b)$.

A. Profit of power suppliers

Suppose the cost function of power suppliers is

$$c_i(q_i) = a_i + d_i q_i^2/2 \tag{13}$$

where a_i and d_i represent the cost coefficients of generating sets.

There are 3 cases in calculating profit.

(1) If the bid of b_i is successful and $b_i < w$.

In this case, the payoff of the bidder is calculated by:

$$\pi = q_i(w - c_i) \tag{14}$$

(2) If the bid of b_i is successful and $b_i = w$.

In this case, there are many possible values of q_{ig}, and $q_{ig} = \eta q_i$, where $\eta = \sum_j \frac{H_i^j(b_i)}{H_i(b_i)}$. The payoff of the bidder is calculated by:

$$\pi = q_i[\eta b_i - \eta c_i(\eta b_i)]] \tag{15}$$

According to (13), (15) can be transformed to the following:

$$\pi = q_i[\eta b_i - c_i + d_i q_i(1-\eta^2)/2]$$

(3) If the bid of b_i is unsuccessful, the payoff of the bidder is $\pi = 0$.

Suppose $\pi_i(b_i)$ is the revenue function of power supplier *i*, $\pi_i(b_i)$ of different cases is calculated by

$$\pi_i(b_i) = \begin{cases} q_i[w - c_i] & if\, b_i < w \\ q_i[\eta b_i - c_i + d_i q_i(1-\eta^2)/2] & if\, b_i = w \\ 0 & if\, b_i > w \end{cases} \tag{16}$$

B. Bidding Strategy Model in spot market

The common objective for bidders is to maximize their expected payoffs. Assuming the clear market price is w, the payoff for bidder *i* can be seen from(16). Thus the expected payoff function is the sum of above two terms weighted by their probabilities of occurrence:

$$\begin{aligned} \max \pi_i(b_i) = {} & q_i(w - c_i) \cdot R_i(b_i) + \\ & q_i[\eta b_i - c_i + d_i q_i(1-\eta^2)/2] \cdot H_i(b_i) \end{aligned} \tag{17}$$

To obtain the optimal solution of the bid b_i, the following necessary differential condition should be satisfied:

$$\begin{aligned} \frac{d\pi_i(b_i)}{db_i} = {} & \eta H_i(b_i) + (w - c_i)\frac{dR_i(b_i)}{db_i} + \\ & [\eta b_i - c_i + d_i q_i(1-\eta^2)/2]\frac{dH_i(b_i)}{db_i} = 0 \end{aligned} \tag{18}$$

Denoting ($dH_i(b_i)/db_i$) and ($d\mathrm{R}_i(b_i)/db_i$) as H' and R', we can simplify (18) as follows:

$$\eta H + \eta bH' = c(H' + R') - \frac{dq(1-\eta^2)}{2}H' - wR' \quad (19)$$

Integrating (19) over the range from b to v:

$$\begin{aligned}\eta bH = \int_b^v (H' + R')c\,dc - \frac{dq(1-\eta^2)/2}{2} \\ \times[H(b_2) - H(b)] - w[R(b_2) - R(b)]\end{aligned} \quad (20)$$

The successful probability of b=v is zero, so the boundary condition is obtained:

$$H_i(b_i = v) = R_i(b_i = v) = 0 \quad (21)$$

According to the integration-by-parts and (21), the bid b is obtained from (20):

$$b = \frac{dq(1-\eta^2)}{4\eta} + \frac{wR(b)}{\eta H(b)} - \frac{c[H(b) + R(b)]}{\eta H(b)} \quad (22)$$

C. Bidding Strategy Model in Reserve Market

When the electricity volume of the spot market and contract market are calculated, the available spinning reserve of power supplier i can be calculated. Assuming that the dispatched-reserves share is Ψ, and that the volume and price in the contract market of power supplier i are q_f and p_f respectively, q_g and p_g can be determined beforehand. $q_{i\max}$ and $q_{i\min}$ are the maximum and minimum generation of power supplier i. In this case, suppose the inverse function of the electric power demand in reserve market is a linear function, which can be calculated by:

$$p_R = x - m(\sum_{i=1}^{N} Q_i) \quad (23)$$

where p_R is the price in the reserve market, x and m are the intercept and slope of the inverse demand function, Q_i is the reserve capacity of power supplier i in the spinning reserve market, and N is the number of power suppliers.

Considering the impact of the spot market and the contract market on the spinning reserve market, we can calculate the maximum profit of power supplier i by:

$$\begin{aligned}\max \pi_i = q_g p_g + q_f p_f + Q_i p_R \\ - c_i(q_g + q_f + \Psi Q_i)\end{aligned} \quad (24)$$
$$s.t \quad q_{i\min} \le q_g + q_f \le q_{i\max} - Q_i$$

The profit-maximizing condition is:

$$\frac{\partial \pi_i}{\partial Q_i} = \frac{\partial p_R}{\partial Q_i}Q_i + p_R - \frac{\partial c_i(q_g + q_f + \Psi Q_i)}{\partial Q_i} = 0 \quad (25)$$
$$s.t \quad q_{i\min} \le q_g + q_f \le q_{i\max} - Q_i$$

Using (13), (23), and (25), we get (26) :

$$\begin{aligned}-mQ_i + p_R - \Psi(a_i + d_i(q_g + q_f + \Psi Q_i)) = 0 \\ s.t \quad q_{i\min} \le q_g + q_f \le q_{i\max} - Q_i\end{aligned} \quad (26)$$

Then the price in the spinning reserve market is obtained by rearranging (26):

$$p_R = mQ_i + \Psi(a_i + d_i(q_g + q_f + \Psi Q_i)) \quad (27)$$

VI. EXAMPLE

To facilitate the calculation, all power suppliers are assumed to predict their rivals' bidding price with uniform distribution in the interval of [9\$/(MWh),21\$/(MWh)], and the contract market is not considered. Assuming there are three power suppliers in the market, therefore, that is $n = 3$, $x_j = 2$. The load is 250 MW; and the parameters are shown in Table I.

TABLE I
Parameters of generating sets

supplier	a	d	$q_{\min}$ (MW)	$q_{\max}$ (MW)
1	3.5	0.05	50	200
2	4	0.04	50	150
3	5	0.03	50	150

Suppose $w = 15$ \$/(MWh); the optimal output and the corresponding bid can be calculated by using (22) in the respective bidding markets. The results are shown in Table II.

TABLE II
Parameters in the spot markets

supplier	$R(b_i)$	$H(b_i)$	q_g (MW)	p_g \$/MWh
1	0.2675	0.5001	111.87	13.91
2	0.2998	0.4897	128.96	13.23
3	0 3547	0 4765	141 87	12 49

The bids in the spinning reserve market are calculated by using (27) The results are shown in Table III

Table III
Bidding parameters in the reserve market

suppliers	Ψ	x	m	p_R \$/MWh
1	0.32	25	0.02	18.59
2	0.32	25	0.02	10.22
3	0.32	25	0.02	9.91

VII. CONCLUSION

The bidding rules made by a non-basic auction theory reduce the market power of the monopoly power suppliers. Combined non-basic auction theory, order statistics and Monte Carlo simulation, the bidding probabilities in different cases are calculated.

Taking into account the bid probability and the revenue, we put forward the bidding strategy models of the spot market and the spinning reserve market. The bid and electricity volume bidding in the spot market and reserve market are calculated by the bidding strategy models.

Enough spinning reserve volume is important to the security and reliability of the electrical system, the bidding strategy of spinning reserve market prompts power suppliers to bid in the spinning reserve market, because they can get a rational profit and reduce the bidding risk from the spinning reserve market.

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Xingping Zhang was born in ShanXi province, China. B.Sc. in Management, Shanxi Finance and Economics University, Taiyuan City, 1993. M.Sc. in Management, China University of Mining and Technology, Beijing City, 1997. Dr. in Management, China University of Mining and Technology, Beijing City, 2001. Postdoctoral Fellow, Liaoning Engineering and Technology University, Fuxin City, 2004.

He is Full Professor at School of Business and Administration, North China Electric Power University, Beijing City. Research interests include Energy Economics and Electric Power Market.

Dr. Zhang is member of the Energy Economic Institute of Beijing.

Runlian Wu was born in ShanXi province, China. B.Sc. in Management, Shanxi Finance and Economics University, Taiyuan City, 1993.

She is an assistant of Dr. Zhang.

Ling Cheng was born in Fujian Province, China. B.Sc. in Mathematics, North China Electric Power University, Beijing City, 2003. M.Sc. in Management, North China Electric Power University, Beijing City,2007.

She was graduate student tutored by Dr. Zhang. She is with the Electric Power Corp. of Quanzhou, Fujian Province.