Dynamic of Electrical Drive Systems with Heating Consideration

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Abstract**—** In this paper it is considered an electric drive with static torque with constant component and speed proportional component. Using the classic calculus of variations is determined the extremal control and trajectory and the overheating that ensures maximum exploitation of the system resources represented by the achievement of a maximum variation of speed in the acceleration processes

Keywords**—** analytic and numerical model, extremal trajectory, extremal control, optimal control, overheating

I. Introduction

In the case of electric drives working in continuous duty type, it is necessary to perform the start-up process and in the case of those electric drives working in continuous duty type with periodical change of speed, it is necessary to perform changes of speed.

To estimate the heating process at the drive system acceleration, as performance number can be adopted the maximum exploitation of the system resources. Using the classic calculus of variations can be solved this optimization problem.

II. Problem Formulation

Considered an electric drive with static torque with constant component, speed and square speed proportional component [1], [2], [3]

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$$M_S = M_0 + k_1\omega + k_2\omega^2$$
or (1)
$$F_S = F_0 + k_1 v + k_2 v^2 \ .$$

Neglecting the electromagnetic inertia in respect of the mechanics inertia, supposing a constant inertia moment, the electric drive will be described by the general movement equation

$$M = M_s + J\frac{d\omega}{dt}, \quad \text{or} \quad F = F_s + J\frac{dv}{dt} \tag{2}$$

and by the dependence between speed and acceleration.

$$\omega = \int \dot{\omega} dt \quad \text{or} \quad v = \int \dot{v}\, dt \,. \tag{3}$$

To expand the interpretations and the conclusions, with and for the restraint of the value intervals, will be introduced relative coordinates. In this sense, considering as a reference for time the mechanical constant of time

$$T = \frac{J\omega_N}{M_N} \tag{4}$$

and for electricity, couple and speed, their nominal values will be obtained the relative values

$$\begin{gathered} \tau = \frac{t}{T},\ i = \frac{i}{I_N},\ \mu = \frac{M}{M_N} = \frac{F}{F_N}, \quad \nu = \frac{\omega}{\omega_N} = \frac{v}{v_N}, \\ \mu_s = \frac{M_S}{M_N} = \frac{F_s}{F_N}, \qquad \mu_0 = \frac{M_0}{M_N} = \frac{F_0}{F_N}, \\ k_1 = \frac{k_1\omega_N}{M_N} = \frac{k_1 v_N}{F_N},\ k_2 = \frac{k_2\omega_N^2}{M_N} = \frac{k_2 v_N^2}{F_N} \end{gathered} \tag{5}$$

and for relative acceleration there will be the relation

$$\dot{\nu} = \frac{\dot{\omega}}{\omega_N / T} = \frac{\dot{v}}{v_N / T}\,. \tag{6}$$

In the hypothesis of proportionality between the electromagnetic couple and the burden power, the equations (1), (2) and (3) in the relative coordinates it becomes [18]

$$\begin{gathered} \mu_S = \mu_0 + k_1\nu + k_2\nu^2 \\ \mu_0 + k_1 + k_2 = 0, \qquad \nu = \int \dot{\nu} d\tau \end{gathered} \tag{7}$$

$$i = \mu = \mu_S + \dot{v} = \mu_0 + k_1 v + k_2 v^2 + \dot{v}, \tag{8}$$

with the initial and fixed conditions

$$\tau = \tau_1, \quad v(\tau_1) = v_1, \quad \tau = \tau_2, \quad v(\tau_2) = v_2. \tag{9}$$

The multitude of the conclusions admitted and the multitude of the trajectories that will be admitted will be considered marginal and open multitudes. If from electromechanical point of view, the drive is described by the general movement equation (7) and by the dependence between acceleration and speed given by (8), then from heating point of view, considering that the driving motor is a homogenous object and all the motor points have the same temperature in the same time, based on the infinitesimal heat balance results

$$Q d\theta + q\theta dt = \Delta P\, dt, \tag{10}$$

where θ is the motor heating with respect to the temperature of the environment, Q is the quantity of heat necessary to rise with one degree the drive temperature, q is the quantity of heat yielded to the environment in time unit and at a temperature difference of one degree and ΔP is the motor power loss which is transformed in heat.
The differential equation of the driving motor heating is

$$T_\theta\, d\theta / dt + \theta = \Delta P / q, \tag{11}$$

where by T_θ (heating time constant) it is noted the ratio Q/q. Taking into account only the heat determined by the load current through Joule effect and relating the equation with the nominal heating $\theta / \theta_N = \vartheta$, and the time with the time mechanics constant

$$\frac{\Delta P / q}{\Delta P_N / q} = \frac{R i^2 / q}{R I_N^2 / q} = i^2, \tag{12}$$

the differential equation in relatives coordinates becomes

$$\frac{T_\theta}{T} \frac{d\vartheta}{d\tau} + \vartheta = i^2. \tag{13}$$

It is noted with m the ratio between the time mechanics constant that takes values of seconds or seconds fractions size and the heating constant that takes values of tenth of minutes size and taking into account the equation (7), the heating differential equation becomes

$$\dot{\vartheta} / m + \vartheta = (\mu_0 + k_1 v + k_2 v^2 + \dot{v})^2. \tag{14}$$

The set of accepted controls and trajectories are considered as open and bounded sets. To use the driving motor at its whole capacity, the set of heating trajectories is considered as close and bounded set, that means will exists the heating upper restriction

$$\vartheta \le \vartheta_{max} \le 1. \tag{15}$$

III. The Optimization Criterion

To estimate the drive system working, a maximum exploitation criterion of the system resources can be adopted. This criterion is represented by the achievement of a maximum variation of speed and is expressed by the integral

$$J\left[v(\tau)\right] = \Delta v = v_2 - v_1 = \int_{\tau_1}^{\tau_2} \dot{v}\, d\tau. \tag{16}$$

IV. Formulation Of Optimization Problem

The optimization problem consists in determining the admitted optimal control $i^*(\tau)$ or $\mu^*(\tau)$, that assures the system evolution from the initial conditions ($\tau_1, v(\tau_1), \vartheta(\tau_1)$) to final conditions ($\tau_2, v(\tau_2), \vartheta(\tau_2)$), on an admitted trajectories represented by the speed extremal $v^*(\tau)$ and by the motor overheating extremal $\vartheta^*(\tau)$, so that is obtained the maximizing of the speed variation that is the maximizing of the criterion functional

$$J\left[v(\tau)\right] = \Delta v = v_2 - v_1 = \int_{\tau_1}^{\tau_2} \dot{v}\, d\tau = max. \tag{17}$$

for a given value of the time interval expressed by

$$\tau_2 - \tau_1 = \int_{\tau_1}^{\tau_2} 1 \quad d\tau, \tag{18}$$

satisfying the differential link (14), the initial and final conditions and the temperature restriction (15). To solve the isometric extreme problem it is necessary to reduce it to an unconditional extreme problem, by determining Lagrange auxiliary function with the help of Lagrange multiplier $\lambda(\tau)$

$$L = \dot{v} + \lambda(\tau)\left[\dot{\vartheta}/m + \vartheta - (\mu_0 + k_1 v + k_2 v^2 + \dot{v})^2\right], \tag{19}$$

and determining the unconditional extreme with the following functional [17]

$$J\left[v(\tau), \vartheta(\tau)\right] = \\ = \int_{\tau_1}^{\tau_2} L\left[\vartheta(\tau), \dot{\vartheta}(\tau), \dot{v}(\tau)\right] \quad d\tau = min. \tag{20}$$

V. The Extreme Condition

The low extreme necessary condition is expressed by Euler-Lagrange equation [17], [18]

$$\frac{\partial L}{\partial \vartheta} - \frac{d}{d\tau} \frac{\partial L}{\partial \dot{\vartheta}} = 0 \tag{21}$$

where, having

$$\frac{\partial L}{\partial \vartheta} = \lambda, \quad \frac{\partial L}{\partial \dot{\vartheta}} = \frac{1}{m}\lambda \text{ and } \frac{d}{d\tau}\frac{\partial L}{\partial \dot{\vartheta}} = \frac{1}{m}\dot{\lambda}, \tag{22}$$

results the homogeneous differential equation in λ

$$\dot{\lambda} - m\lambda = 0, \tag{23}$$

and from Euler-Lagrange equation for the function ν [4], [5]

$$\frac{\partial L}{\partial \nu} - \frac{d}{d\tau}\frac{\partial L}{\partial \dot{\nu}} = 0 \cdot \tag{24}$$

Where

$$\frac{\partial L}{\partial \nu} = -2\lambda\left(\mu_0 + k_1\nu + k_2\nu^2 + \dot{\nu}\right)\left(k_1 + 2k_2\nu\right)$$
$$\frac{\partial L}{\partial \dot{\nu}} = 1 - 2\lambda\left(\mu_0 + k_1\nu + k_2\nu^2 + \dot{\nu}\right) \tag{25}$$
$$\frac{d}{d\tau}\frac{\partial L}{\partial \dot{\nu}} = -2\dot{\lambda}\left(\mu_0 + k_1\nu + k_2\nu^2\right) - \\ - 2\lambda\left(k_1\dot{\nu} + 2k_2\nu\dot{\nu} + \dot{\nu}\ddot{\nu}\right)$$

results the linear differential equation of the second order

$$\lambda\left(k_1\dot{\nu} + 2k_2\nu\dot{\nu} + \ddot{\nu}\right) + \dot{\lambda}\left(\mu_0 + k_1\nu + k_2\nu^2 + \dot{\nu}\right) - \\ \lambda\left(\mu_0 + k_1\nu + k_2\nu^2 + \dot{\nu}\right)\left(k_1 + 2k_2\nu\right) = 0. \tag{26}$$

VI. Optimal Solution For Constant Static Torque In The Conditional Extreme Cass, With Heating Emphasis

Considering the transportation equipment loaded, that is with a constant static torque, by particularizing

$$\mu_0 \neq 0, \quad k_1 = 0, \quad k_2 = 0, \quad \mu_S = \mu_0, \tag{27}$$

the extreme condition expressed by Euler-Lagrange equation (24), where having

$$\frac{\partial L}{\partial \nu} = 0, \qquad \frac{d}{d\tau}\frac{\partial L}{\partial \dot{\nu}} = \frac{d}{d\tau}\left[2\lambda\left(\mu_0 + \dot{\nu}\right)\right] = 0 \tag{28}$$

results the differential equation

$$\lambda\left(\mu_0 + \dot{\nu}\right) = C_1 . \tag{29}$$

By integrating the differential equation in λ (23) and taking into account the characteristic attached equation

$$r - m = 0, \qquad r = m. \tag{30}$$

Lagrange multiplier is determined

$$\lambda(\tau) = C_2 e^{m\tau} . \tag{31}$$

Based on the equation (29) and taking into account the movement general equation (8) and Lagrange multiplier expression (31) the time current (torque) evolution is obtained

$$i = \mu = \frac{C_1}{C_2} e^{-m\tau} . \tag{32}$$

Considering the current initial condition, the arbitrary constant is determined

$$\tau_1 = 0, \qquad i(\tau_1) = i(0) = \frac{C_1}{C_2}, \tag{33}$$

and the current (torque) optimal equation becomes

$$i^*(\tau) = \mu^*(\tau) = \mu_0 + \dot{\nu} = i(0)e^{-m\tau} . \tag{34}$$

Expressing the acceleration from the current expression

$$\dot{\nu}(\tau) = i(0)\, e^{-m\tau} - \mu_0 \tag{35}$$

and by integrating, the speed extremal is resulted

$$\nu = -\frac{i(0)}{m} e^{-m\tau} - \mu_0\tau + C_3 . \tag{36}$$

The initial condition for speed allows determining the integrating constant

$$\tau_1 = 0, \quad \nu(0) = \nu_1, \quad -\frac{i(0)}{m} + C_3 = \nu_1 \\ \Rightarrow \quad C_3 = \nu_1 + \frac{i(0)}{m}, \tag{37}$$

the speed extremal having the expression

$$\nu^*(\tau) = \frac{i(0)}{m}\left(1 - e^{-m\tau}\right) - \mu_0\tau + \nu_1 . \tag{38}$$

From the equation:

$$i(0)\, e^{-m\tau} - \mu_0 = 0, \tag{39}$$

can be calculated the critical time

$$m\tau_{cr} = \ln\frac{i(0)}{\mu_0}, \tag{40}$$

for which the acceleration is canceled and the speed has the maximum value.

Considering again the heating differential equation (14) and allowance the optimal current expression (34)

$$\dot{\vartheta} + m\vartheta = m i^2 = m i^2(0)\, e^{-2m\tau} \tag{41}$$

a differential equation is obtained, such as

$$\dot{y} + P(x)\, y = Q(x) \tag{42}$$

with a general solution

$$y = e^{-\int P dx}\left[\int Q e^{\int P dx} dx + C\right]. \tag{43}$$

Having

$$\int P dx = \int m\, d\tau = m\tau \tag{44}$$

and

$$\int Qe^{\int Pdx}dx = \int m i^2(0)\, e^{-2m\tau} e^{m\tau} d\tau = -i^2(0) e^{-m\tau} \qquad (45)$$

the general solution of the temperature differential equation (41) is

$$\vartheta = e^{-m\tau}\left[-i^2(0)\, e^{-m\tau} + C\right] = -i^2(0) e^{-2m\tau} + C\, e^{-m\tau} \qquad (46)$$

and by determining the integrating constant from the initial condition

$$\tau_1 = 0, \quad \vartheta(0) = \vartheta_1, \quad -i^2(0) + C = \vartheta_1, \quad \Rightarrow\ C = i^2(0) + \vartheta_1 \qquad (47)$$

the time evolution of the over optimal temperature becomes (fig.1)

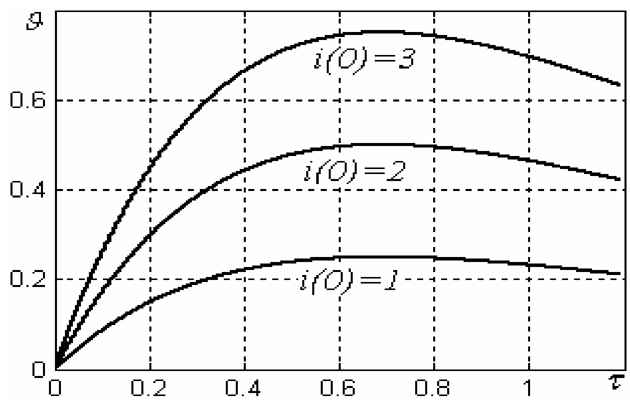


Fig. 1 Time over temperature evolution for different values of the initial current and $\vartheta_1 = 0$

$$\vartheta^*(\tau) = i^2(0)\left(e^{-m\tau} - e^{-2m\tau}\right) + \vartheta_1 e^{-m\tau}. \qquad (48)$$

For an exponential evolution of the current (34), the over temperature expression has a maximum. The maximum condition of the over temperature is given by the equation resulted by canceling the over temperature derivate

$$i^2(0)\left(-1 + 2e^{-m\tau}\right) - \vartheta_1 = 0 \qquad (49)$$

from which can be obtained the critic time value for which results this maximum

$$m\tau_{cr} = \ln\frac{2}{1 + \frac{\vartheta_1}{i^2(0)}} = \ln 2 - \ln\left(1 + \frac{\vartheta_1}{i^2(0)}\right). \qquad (50)$$

For a final time smaller than the critical time $\tau_2 \le \tau_{cr}$ the over temperature $\vartheta(\tau)$ increases, having its maximum value at the end of the interval $[0,\ \tau_2]$, and the maximum speed variation can be obtained on the extremal trajectory $v(\tau)$ and $\vartheta^*(\tau)$ under the optimal control action $i^*(\tau) = \mu^*(\tau)$. The over temperature restriction $\vartheta \le 1$ marked the end of the domain on which the extremal is defined. For using the driving motor at its entire capacity under thermal aspect, the equality sign from the restriction $\vartheta \le 1$ is imposed to be obtained in the point of the final moment $\tau = \tau_2$, that is

$$\vartheta(\tau_1) = i^2(0)\left(e^{-m\tau_2} - e^{-2m\tau_2}\right) + \vartheta_1 e^{-m\tau_2} = 1 \qquad (51)$$

from where can be calculated the initial value of the optimum current (fig.2)

$$i(0) = \sqrt{\frac{1 - \vartheta_1 e^{-m\tau_2}}{e^{-m\tau_2} - e^{-2m\tau_2}}}. \qquad (52)$$

This initial current value is than replaced in all the relations previous obtained.

The existence of the initial over temperature ϑ_1 has as a result the decreasing of the critical time (50) and of the initial value of the current (52).

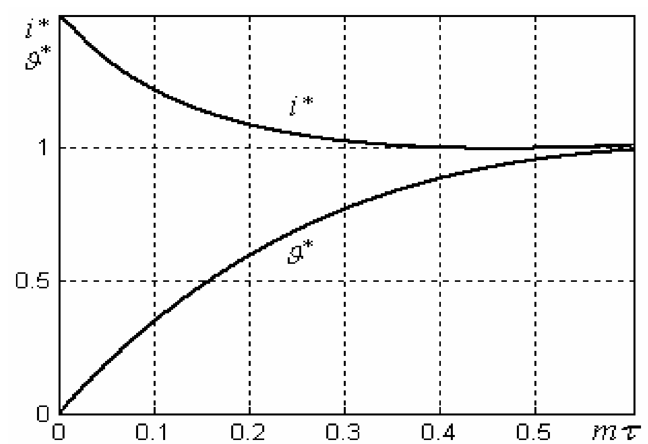


Fig. 2 Optimal evolution of the current and temperature (ϑ_1=0, $\tau_2 > \tau_{cr}$)

For the case in which the final time overtakes the critical time $\tau_2 > \tau_{cr}$, the maximum of the over temperature is touched inside the interval $[0,\ \tau_2]$, and the extreme of the functional, that is the speed maximum variation is obtained on the trajectory made from the extremal $\vartheta(\tau)$ for $\tau \in [0, \tau_{cr}]$ and from the boundary of the domain $\vartheta = 1$ for $\tau \in [\tau_{cr}, \tau_2]$.

VII. Optimal Solution For Constant Static Torque In The Unconditional Extreme

Another modality of solving the optimization problem consists in building such of expression of the optimization criterion so that involves, from the beginning, un unconditional extreme. In this way, following further the maximization of the speed variation, the acceleration is explained from the equation (14)

$$\dot{v} = \sqrt{\frac{1}{m}\dot{\vartheta} + \vartheta} - \mu_0 \tag{53}$$

and it is replaced in the optimization criterion (17), having as a result the expression

$$J = v_2 - v_1 = \int_{\tau_1}^{\tau_2} \dot{v}\, d\tau = \\ = \int_{\tau_1}^{\tau_2} \left[\left(\frac{1}{m}\dot{\vartheta} + \vartheta \right)^{\frac{1}{2}} - \mu_0 \right] d\tau = max\,. \tag{54}$$

having

$$F = \left(\frac{1}{m}\dot{\vartheta} + \vartheta \right)^{\frac{1}{2}} - \mu_0\,. \tag{55}$$

The necessary condition of low relative extreme, expressed by Euler equation

$$\frac{\partial F}{\partial \vartheta} - \frac{d}{d\tau}\frac{\partial F}{\partial \dot{\vartheta}} = 0 \tag{56}$$

in which

$$\frac{\partial F}{\partial \vartheta} = \frac{1}{2}\left(\frac{1}{m}\dot{\vartheta} + \vartheta \right)^{-\frac{1}{2}}, \\ \frac{\partial F}{\partial \dot{\vartheta}} = \frac{1}{2m}\left(\frac{1}{m}\dot{\vartheta} + \vartheta \right)^{-\frac{1}{2}}, \\ \frac{d}{d\tau}\frac{\partial F}{\partial \dot{\vartheta}} = -\frac{1}{4m}\left(\frac{1}{m}\dot{\vartheta} + \vartheta \right)^{-\frac{3}{2}}\left(\frac{1}{m}\ddot{\vartheta} + \vartheta \right) \tag{57}$$

becomes a differential linear equation of second range, with constant coefficients and homogeneous.

$$\frac{1}{2}\frac{1}{\sqrt{\frac{1}{m}\dot{\vartheta} + \vartheta}} + \frac{1}{4m}\frac{\frac{1}{m}\ddot{\vartheta} + \dot{\vartheta}}{\sqrt{\left(\frac{1}{m}\dot{\vartheta} + \vartheta \right)^3}} = 0$$

or

$$\ddot{\vartheta} + 3m\dot{\vartheta} + 2m^2\vartheta = 0\,. \tag{58}$$

Based on the characteristic attached equation

$$r^2 + 3mr + 2m^2 = 0 \tag{59}$$

with the solutions

$$r_1 = -m, \quad r_2 = -2m \tag{60}$$

the general solution, that is the family of trajectories for the over temperature, is as

$$\vartheta = C_1 e^{-m\tau} + C_2 e^{-2m\tau}\,. \tag{61}$$

Using the initial condition for over temperature $\tau_1 = 0$

$$\dot{\vartheta}(0) = \vartheta_1, C_1 + C_2 = \vartheta_1, \Rightarrow C_1 = -C_2 + \vartheta_1 \tag{62}$$

the over temperature can be written as:

$$\vartheta = -C_2\left(e^{-m\tau} - e^{-2m\tau} \right) + \vartheta_1 e^{-m\tau}\,. \tag{63}$$

By derivation the over temperature and by replacing in the equation (14) is obtained the following equation

$$C_2\left(e^{-m\tau} - 2e^{-2m\tau} \right) - \vartheta_1 e^{-m\tau} - C_2\left(e^{-m\tau} - e^{-2m\tau} \right) + \\ + \vartheta_1 e^{-m\tau} = \left(\mu_0 + \dot{v} \right)^2, \tag{64}$$

or, the equation

$$i^2 = \left(\mu_0 + \dot{v} \right)^2 = -C_2 e^{-2m\tau} \tag{65}$$

considering the general movement equation (8).

Determining the integration constant value from the initial condition for current

$$\tau = \tau_1 = 0, \qquad i^2(0) = -C_2 \tag{66}$$

the optimal control value will be

$$i^*(\tau) = \mu^*(\tau) = \mu_0 + \dot{v} = i(0)e^{-m\tau} \tag{67}$$

and from this can be distinguished the acceleration respectively the dynamic torque

$$\dot{v}^*(\tau) = \mu_0^*(\tau) = i(0)e^{-m\tau} - \mu_0\,. \tag{68}$$

By integrating the acceleration is determined the speed

$$v = \int \dot{v}\, d\tau = \\ = \int \left[i(0)e^{-m\tau} - \mu_0 \right] d\tau = \frac{i(0)}{m} e^{-m\tau} - \mu_0\tau + C_3 \tag{69}$$

and based on the initial condition, the integration constant value is obtained

$$\tau = \tau_1 = 0, \qquad v(0) = v_1, \\ -\frac{i(0)}{m} + C_3 = v_1, \quad \Rightarrow \quad C_3 = \frac{i(0)}{m} + v_1 \tag{70}$$

having as a result the following expression for speed:

$$v^*(\tau) = \frac{i(0)}{m}\left(1 - e^{-m\tau} \right) - \mu_0\tau + v_1\,. \tag{71}$$

Being determined the constant value C_2, the extremal over temperature trajectory becomes

$$\vartheta^*(\tau) = i^2(0)\left(e^{-m\tau} - e^{-2m\tau}\right) + \theta_1 e^{-m\tau} . \quad (72)$$

As a conclusion, the results obtained in this way are similarly with the previous obtained results, those from the conditional extreme case.

VIII. The Extreme Nature

To analyzed the extreme nature, Legendre condition is determined

$$\frac{\partial^2 F}{\partial \vartheta^2} = -\frac{1}{4m^2}\left(\frac{1}{m}\dot{\vartheta} + \vartheta\right) < 0, \quad (73)$$

having as a result that the realized extreme is a maximum.

IX. The Optimal Solution For Static Torque Wit Constant Component And Speed Proportional Component

Considering electric drive loaded with static constant component and a component proportional to speed, making customizations

$$\mu_0 \neq 0, \quad k_1 \neq 0, \quad k_2 = 0, \quad \mu_0 + k_1 = 1 \quad (74)$$

static torque becoms

$$\mu_s = \mu_0 + k_1 \nu. \quad (75)$$

The extreme necessary condition expressed by the differential equation (25) becomes

$$\lambda\left(k_1\dot{\nu} + \ddot{\nu}\right) + \dot{\lambda}\left(\mu_0 + k_1\nu + \dot{\nu}\right) - \\ - k_1\lambda\left(\mu_0 + k_1\nu + \dot{\nu}\right) = 0 . \quad (76)$$

The characteristic equation attached to the differential equation (21) [17]

$$r - m = 0 , r = m \quad (77)$$

has the general solution

$$\lambda = C_0 e^{m\tau} . \quad (78)$$

Replacing λ and its derivative in the differential equation (24), it is obtained

$$C_0 e^{m\tau}\left(k_1\dot{\nu} + \ddot{\nu}\right) + m C_0 e^{m\tau}\left(\mu_0 + k_1\nu + \dot{\nu}\right) - \\ k_1 C_0 e^{m\tau}\left(\mu_0 + k_1\nu + \dot{\nu}\right) = 0 \quad (79)$$

and then the differential equation for speed

$$\ddot{\nu} + m\dot{\nu} - k_1(k_1 - m)\nu = \mu_0(k_1 - m). \quad (80)$$

Having the particular solution of the heterogeneous differential equation

$$r^2 + mr - k_1(k_1 - m) = 0 \quad (81)$$

with

$$r_1 = k_1 - m , \quad r_2 = -k_1 \quad (82)$$

with the general solution of the homogeneous differential equation

$$\nu_g = C_1 e^{(k_1 - m)\tau} + C_2 e^{-k_1\tau}$$

will results the general solution of the heterogeneous differential equation, that is the speed trajectories family

$$\nu = -\frac{\mu_0}{k_1} + C_1 e^{(k_1 - m)\tau} + C_2 e^{-k_1\tau} . \quad (83)$$

Using the speed initial condition

$$\tau_1 = 0 , \nu(0) = \nu_1 , \quad -\frac{\mu_0}{k_1} + C_1 + C_2 = \nu \quad . \quad (84)$$

Obtained

$$C_2 = \frac{\mu_0}{k_1} - C_1 + \nu_1 \quad (85)$$

the speed trajectories are

$$\nu = -\frac{\mu_0}{k_1}\left(1 - e^{-k_1}\right) + C_1\left(e^{(k_1 - m)\tau} - e^{-k_1\tau}\right) + \nu_1 e^{-k_1\tau} . \quad (86)$$

The acceleration is determined by speed differentiation

$$\dot{\nu} = -\mu_0 e^{-k_1\tau} + C_1\left[(k_1 - m)e^{(k_1 - m)\tau} + k_1 e^{-k_1\tau}\right] - k_1\nu_1 e^{-k_1\tau} . \quad (87)$$

Corresponding to the movement general equation, the current (the torque) is

$$i = \mu = \mu_0 - \mu_0\left(1 - e^{-k_1\tau}\right) + k_1 C_1\left(e^{(k_1 - m)\tau} - e^{-k_1\tau}\right) \\ + k_1\nu_1 e^{-k_1\tau} + C_1\left[(k_1 - m)e^{(k_1 - m)\tau} + k_1 e^{-k_1\tau}\right] - \\ - \mu_0 e^{-k_1\tau} - k_1\nu_1 e^{-k_1\tau} = C_1(2k_1 - m)e^{(k_1 - m)\tau} . \quad (88)$$

Considering the current initial condition $\tau_1 = 0$,

$$i(0) = C_1(2k - m) . \quad (89)$$

To obtain

$$C_1 = \frac{i(0)}{2k_1 - m} , \quad (90)$$

The optimal current (torque) has the exponential expression (fig.3)

$$i^*(\tau) = \mu^*(\tau) = i(0)\, e^{(k_1 - m)}\tau . \quad (91)$$

The arbitrary constant C_1 being determined, extremals for speed and acceleration will by (fig.3)

$$v^*(\tau)=\frac{i(0)}{2k_1-m}\left(e^{(k_1-m)\tau}-e^{-k_1\tau}\right)- \\ -\frac{\mu_0}{k_1}+\left(\frac{\mu_0}{k_1}+v_1\right)e^{-k_1\tau} \quad (92)$$

$$\dot{v}^*(\tau)=\frac{i(0)}{2k_1-m}\left[(k_1-m)e^{(k_1-m)\tau}+k_1e^{-k_1\tau}\right]-(\mu_0+k_1v_1)e^{-k_1\tau} \quad (93)$$

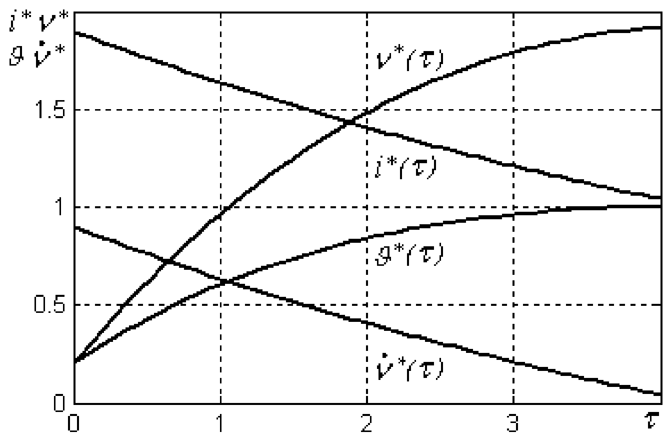


Fig.3 Evolution speed, acceleration, temperature and current extremals

(μ_0=0.9,k_1=0.1, ν_1=0.2, ϑ_1=0.4,m=0.1)

Imposing the condition like the speed has the value ν_2 at the end of the acceleration interval

$$\nu(\tau_2)=\frac{i(0)}{2k_1-m}\left[e^{(k_1-m)\tau_2}-e^{-k_1\tau_2}\right]- \\ -\frac{\mu_0}{k_1}+\left(\frac{\mu_0}{k_1}+\nu_1\right)e^{-k_1\tau_2}=\nu_2 \quad (94)$$

it can be determined the initial value of the acceleration current

$$i(0)=\frac{(2k_1-m)}{k_1}\frac{\left[(\mu_0+k_1\nu_2)-(\mu_0+k_1\nu_1)e^{-k_1\tau}\right]}{e^{(k_1-m)\tau_2-e^{-k_1\tau_2}}} \quad (95)$$

that assures the requested speed variation.

The performance number that is the maximum speed variation has the value

$$J(\nu)=\Delta\nu=\nu_2-\nu_1=\frac{i(0)}{2k_1-m}\left(e^{(2k_1-m)\tau_2}-e^{-k_1\tau}\right) \\ -\left(\frac{\mu_0}{k_1}+\nu_1\right)\left(1-e^{-k_1\tau_2}\right) \quad (96)$$

The heating differential equation (41), for *K2= 0*, taking into account the movement general equation and the optimal current expression (4), can be written as

$$\dot{\vartheta}+m\,\vartheta=m\,i^2(0)e^{2(k_1-m)\tau} \quad (97)$$

being of type

$$\dot{y}+P_0y=Q_0(x) \quad (98)$$

with the general solution

$$\dot{y}=e^{-\int P_0dx}\left(\int Q_0e^{\int P_0dx}+C\right). \quad (99)$$

Having

$$P_0=m,\ \int P_0dx=\int md\tau=m\tau\ , \quad (100)$$

and

$$\int Q_0e^{\int P_0\,dx}dx=\int mi^2(0)e^{2(k_1-m)\tau}e^{m\tau}d\tau= \\ =\frac{m}{2k_1-m}i^2(0)e^{(2k_1-m)\tau} \quad (101)$$

he overheating will be

$$\vartheta=e^{-m\tau}\left(\frac{m}{2k_1-m}i^2(0)e^{(2k_1-m)\tau}+C\right)= \\ \frac{m}{2k_1-m}i^2(0)e^{2(k_1-m)\tau}+Ce^{-m\tau}\ . \quad (102)$$

Using the overheating initial condition, it is determined the arbitrary constant

$$\tau_1=0,\vartheta(0)=\vartheta_1,\vartheta_1=\frac{m}{2k_1-m}i^2(0)+C \\ \Rightarrow C=\vartheta_1-\frac{m}{2k_1-m}i^2(0) \quad (103)$$

and the overheating extremal trajectory

$$\vartheta^*(\tau)=\frac{m}{2k_1-m}i^2(0)\left(e^{2(k_1-m)\tau}-e^{-m\tau}\right)= \\ +\vartheta_1e^{-m\tau}\ . \quad (104)$$

This expression is consistent as time $\theta\le1$. The acceleration current (torque) for $m>k_1$ (excepting the case m=2k_1) is time decreasing (fig. 4 and fig.5), for m=k_1 is constant (fig. 6), and for $m<k_1$ (fig. 7) is time increasing.

Initial values of the acceleration current $i(0)$ and of the acceleration $\dot{\nu}(0)$, that exceed the admissible values because of the big heating time constant, result by calculating the static current (torque) initial value from the final condition of heating $\vartheta(\tau_2)=1$, at reduced values of the initial heating and specially at reduced values of the acceleration time interval. The speed exceeds the necessary values at the end of the acceleration interval. Such situations determine a bigger heating and power loss.

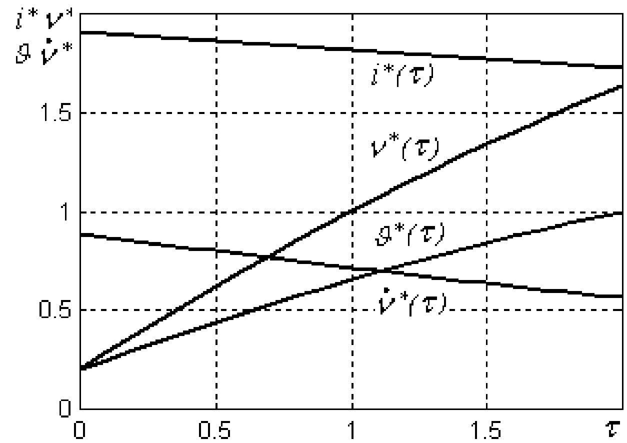


Fig. 4 Speed, acceleration, temperature and current extremals in the case of acceleration *(μ_0=0.9, k_1=0.1,m=0.15, ν_1=0.2, ν_1=0.2, k_1<m<2k_1)*

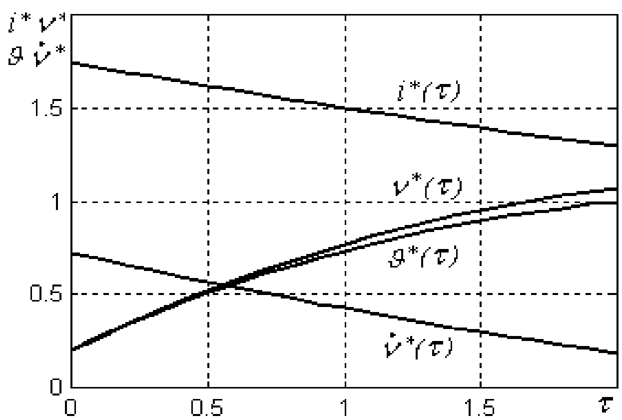


Fig. 5 Speed, acceleration, temperature and current extremals in the case of acceleration *(μ_0=0.9, k_1=0.1,m=0.25, ν_1=0.2, ν_1=0.2, m>2k_1)*

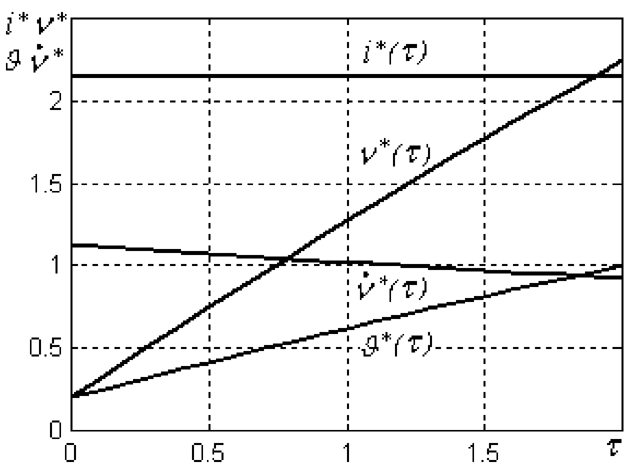


Fig. 6 Speed, acceleration, temperature and current extremals in the case of acceleration *(μ_0=.9, k_1=0.1,m=0.1, ν_1=0.2, ν_1=0.2,m=k_1,)*

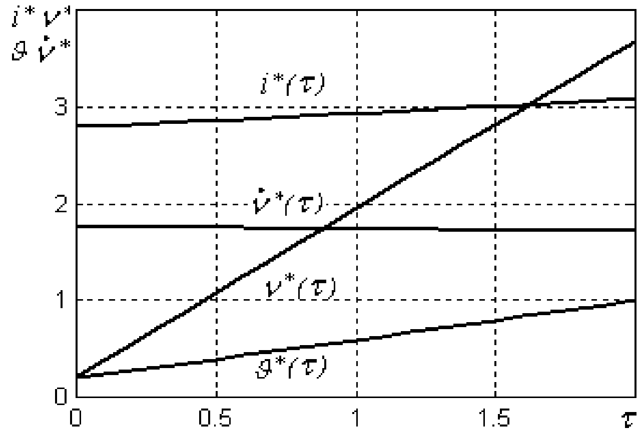


Fig. 7 Eextremals in the case of acceleration *(μ_0=.9,k_1=0.1,m=0.05, ν_1=0.2, ν_2=0.2,0<m<k_1)*

For this reason, it is imposed to calculate the initial value of the acceleration current *i(0)* both from the final speed condition $\nu(\tau_2)=\nu_2$ and from the overheating final condition $\vartheta(\tau_2)=1$ and to use the least value for the other expressions (current, acceleration, speed and overheating).

X. Processes Monitored With Thermography In Ir For Heating Consideration

Thermovision/thermography in infrared is a recent technique in the domain of modern methods of diagnosis in industry, and it offers high precision results which reduce the time to detect faults and which evaluate very precisely the state of equipments during work, without their stop or their removal and transportation to a diagnosis centre.

In some industrial processes there are systems or parts of the process which don't need to be permanently observed and diagnosed, but which regularly need a kind of inspection or analysis based on previous behavior within the process. Usually, when a problem appears at a part of a working system, it overheats. It emits more heat than before, in normal functioning conditions. There are equipments which are more resistant in time, but still have a point where they fail. This failure point can be predicted with a lot of time before the actual failure happens. For this kind of equipments or systems we thought up a system capable of extracting the useful data about it's temperature in infrared, administrate it over a period of time which is not critical in case of fault appearance, and process it at a certain point, established from the beginning. The result would be whether there is a change compared to the previous check-up, and if there is, establish the cause for that, and of course, acting to stop the problem to appear. The logical scheme for the proposed system is in figure 8.

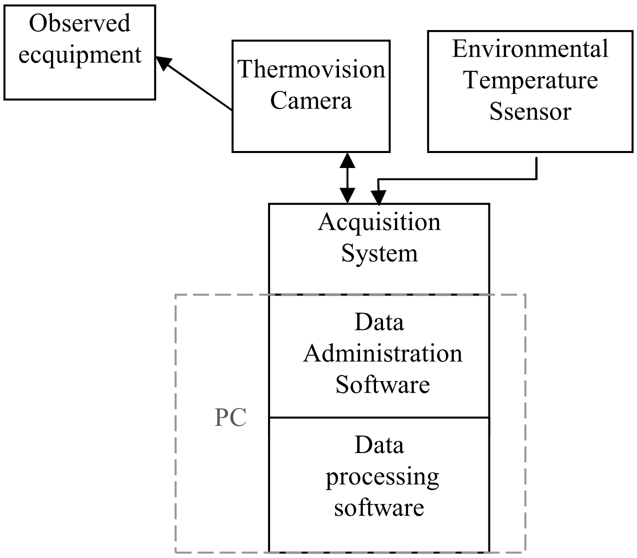


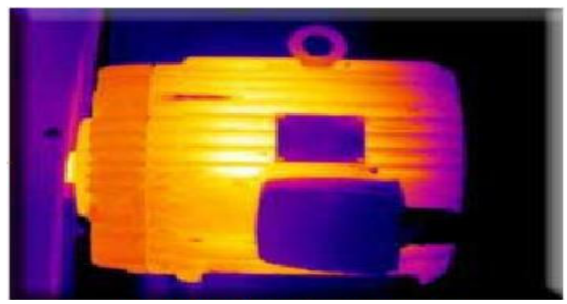
Fig. 8. The logic scheme of the portable system

In the power production, transport, distribution and use of electric energy installations, the unprogrammed stops may lead to significant increase of the exploitation costs.

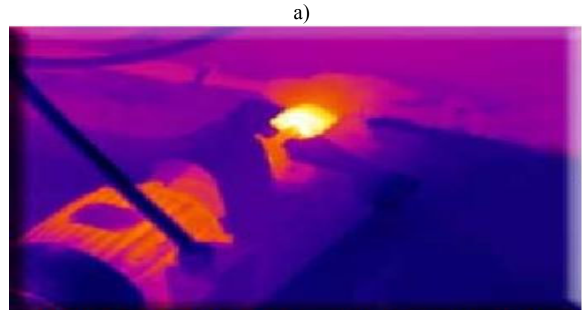
The collected data consists of a number of infrared images of our equipment, captured at a certain period of time (established by the inspector) with a simple thermovision (thermography in infrared) camera, which has no capacity to record the images or perform operations on them. If, for example, the camera is programmed to capture an image each second, there would be 86400 pictures per day. Collected data comes, also, from a simple environmental temperature sensor. Thermovision or thermography refers to a high performance, modern technique which allows real-time visualization and (depending of the type the camera is) generation of thermal maps ("thermal images", thermographs) of the technical (of biological) systems under observation. All objects which have a temperature above 0^{o}K (- $273{,}15^{o}$C) have a molecular movement, thus emit an energy we feel as temperature, which can only be seen in infrared specter. Thermography made it possible to "see" the distribution of temperature at the surface of the objects which are measured, and also measure it. These cameras measure the infrared radiation using specialized sensors which then convert it in order for us to see it as thermal images. Modern thermovision cameras are able to measure temperatures from -40°C and to +1500°C, and can identify temperature differences of up to $0{,}05^{o}$C.In the electric domain, not interfering is very important for measurements, monitoring and diagnosis of the equipments and processes, because it doesn't emit harmful radiations, nor influences the temperature and material of the monitored components, and mainly because it needs no contact (useful for electric equipments under voltage, inaccessible installations, moving objects etc), can be used in hazardous environments, and most of all, there is no need to stop the process or to transport the equipments to a specialized lab. Even though there are so many advantages, quality cameras are expensive, images can be hard to interpret accurately, and the IR expert must always be prepared, and he only obtains a surface temperature, which, of course, is a consequence of an interior temperature.

With the help of thermography in infrared, specialists can identify: problems related to the lubrication of the assemblies from rotative components (bearings, balls, axels, transmissions); problems related to aligning and equilibration of assemblies in movement;

- overheating of the coilings due to overload of isolation problems. In the following pictures there are some examplesfor figure 9 and figure 10.



a)



b)

Fig. 9: a) Overheating at an alternative current motor;b) Overheat at the bearing of the axel of a hydraulic pump;

Fig. 10 Overheating at one of the motors at a pump assembly.

XI. CONCLUSION

The obtained results expressed throw the extremal control and trajectory can be used both in design and in optimal control of electric drive systems with static torque speed dependent working in continuous duty or in continuous duty with periodical change of speed. An increase of the quality and of the efficiency of those electric drive systems is obtained due to these results.

The system thermography in infrared is very efficient when used for non real-time observation. It is, indeed, a little expensive, but the cost of problems it prevents or stops from happening would exceed the cost of acquisition.

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