$H\infty$ Approach Control for Regulation of Active Car Suspension

Jamal Ezzine, Francesco Tedesco

***Abstract*—There are many types of car suspensions control. $H\infty$ control of vehicle suspension is studied in the literature for a brave time ago. We can define active suspensions as control systems incorporating a parallel spring and an electronically controlled damper. The contribution of this paper relies on $H\infty$ control design to improve comfort and road holding of the car, and on control validation through simulation on an quarter Car and Half Car model with seat-passengers of the suspensions system. In this paper an $H\infty$ controller is designed for a actuated active suspension system of a quarter-modelled and half-modelled with seat-passengers vehicle in a cascade feedback structure. In this paper we will make a comparison between application of quarter car and half car model. In the framework of Linear Matrix Inequality (LMI) optimization, constrained $H\infty$ active suspensions are designed on half-car models.**

***Index Terms*—Active Control, Suspensions Control, Structure Dynamic.**

I. Introduction

There are three type of vehicle suspension: passive, semi-active and active. In this paper we will focus only on active and passive suspensions. Differences depend on the operation mode to improve vehicle ride comfort, vehicle safety, road damage minimization and the overall vehicle performance. Normally, conventional passive suspensions are effective only in a certain frequency range and no on-line feedback action is used. Thus, optimal design performance cannot be achieved when the system and its operating conditions are changed. On the contrary, active suspensions can improve the performance of the suspension systems over a wide range of frequency and can adapt to the system variations based on on-line changes of the actuating force.

A. Intelligent suspension systems

An intelligent system's response depends not only on the physical quantities which affect the response directly, but also on physical quantities which do not affect the response directly. A physical quantity that affects the response of the suspension system directly is, for example, the damper velocity, while the vehicle body roll speed can be used as an example of a physical quantity that has no direct effect on the function of the suspension. The idea of a passive and intelligent suspension system can be more easily appreciable with the figures below

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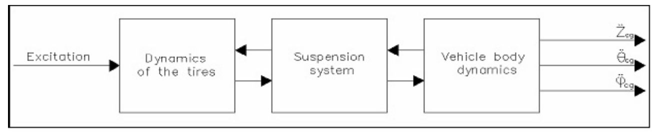

Fig. 1. A passive suspension system

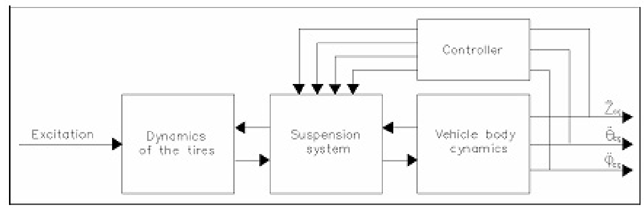

Fig. 2. An intelligent suspension system

B. The Goal of Active suspensions

Demands for ride comfort and controllability of road vehicle are pursued by many automotive industries by using active suspension. These electronically controlled suspension system can improve the ride comfort as well as the road handling of the vehicle. The suspension system can be categorized into three groups: passive suspension systems including conventional springs and dampers. Systems contain no electronic sensor and control. Semi-active suspension systems provide controlled real-time dissipation of energy. Active suspension systems use a hydraulic or pneumatic actuator in parallel with a passive spring and shock absorber, and, the measurement of body vibration is used to decide instantaneously the amount of force needed by the actuator. Different characteristics can be considered in a suspension system design namely, ride comfort, body movement, road holding and suspension travel. No suspension system can simultaneously optimize all four mentioned parameters. But a better trade-off among these parameters can be achieved in active suspension system [6]. Many researches in recent years have concentrated on active vehicle suspension system. In these researches a variety of models including quarter, half, and full-car model with passenger have considered. In this paper two $H\infty$ controllers are designed for the quarter-car and half-car case structure, considering robust performance in both cases. The solution of mixed sensitivity problem is significantly reducing the vehicle vibration in the human sensitivity frequency range. Statistical analysis of the simulation results using random input as road roughness illustrates that the proposed strategy can provide a suitable trade-off between ride comfort and road holding.

II. Derivation of models

Two different models will be used in this paper, one is based on a Quarter Car model and the other on the Half Car model, but in our case we take in consideration a model with the presence of the passengers inside the vehicle.

A. Half Car Suspension Model

Fig. 3 shows an half-car model system, which has six degrees of freedom. The model consists of the chassis, two axles, and two passengers [4]. It is assumed that the chassis has both bounce and pitch, the axles have independent bounce, and the passengers have only vertical oscillations. The suspension, tyre, and passenger seats are modelled using linear springs in parallel with viscous dampers [3]. Two actuators are used to provide controllable forces, and are located parallel to the suspension springs and shock absorbers. To derive the equations of motion for the half-car. The six degrees of freedom are: x (vertical displacement of the chassis), θ (rotatational displacement of the chassis), x_{p1} (vertical displacement of passenger 1), x_{p2} (vertical displacement of passenger 2), x_{t1} (vertical displacement of tyre 1), and x_{t1} (vertical displacement of tyre 2), y_1 and y_2 are the displacements of the road surface at the front wheel and rear wheel respectively,I_p is the moment of inertia at the vehicle body's center of gravity.
Summing all the forces on all 5 masses and summing the moments about the centre of gravity of the chassis leads to the following equations:

car.pdf

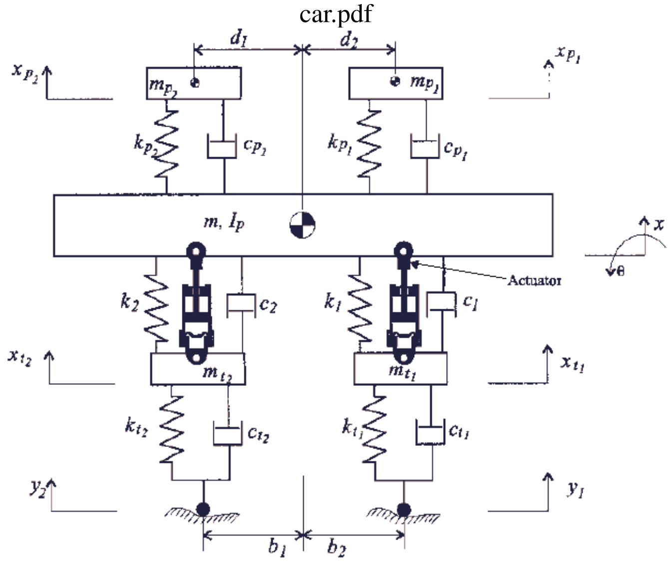

Fig. 3. Half-car model with passengers

$$\begin{aligned}\ddot{x} = & -\frac{1}{m}(k_1 + k_2 + k_{p1} + k_{p2})x \\ & -\frac{1}{m}(k_1 b_1 + k_2 b_2 + k_{p1} d_1 + k_{p1} d_2)\theta \\ & +\frac{k_{p1}}{m}x_{p1} + \frac{k_{p2}}{m}x_{p2} + \frac{k_1}{m}x_{t1} + \frac{k_2}{m}x_{t2} \\ & +\frac{c_{p1}}{m}\dot{x}_{p1} + \frac{c_{p2}}{m}\dot{x}_{p2} + \frac{c_1}{m}\dot{x}_{t1} + \frac{c_2}{m}\dot{x}_{t1} \\ & -\frac{1}{m}(c_1 + c_2 + c_{p1} + c_{p2})\dot{x} \\ & -\frac{1}{m}(c_1 b_1 - c_2 b_2 + c_{p1} d_1 - c_{p1} d_2)\dot{\theta} \\ & +\frac{F_1 + F_2}{m}\end{aligned} \tag{1}$$

$$\begin{aligned}\ddot{\theta} = & -\frac{1}{I_p}(k_1 b_1 - k_2 b_2 + k_{p1} d_1 - k_{p2} d_2)x \\ & -\frac{1}{I_p}(k_1 b_1^2 + k_2 b_2^2 + k_{p1} d_1^2 - k_{p1} d_2^2)\theta \\ & +\frac{k_{p1} d_1}{I_p}x_{p1} + \frac{k_{p2} d_2}{I_p}x_{p2} - \frac{k_1 b_1}{I_p}x_{t1} + \frac{k_2 b_2}{I_p}x_{t2} \\ & -\frac{c_{p1} d_1}{I_p}\dot{x}_{p1} + \frac{c_{p2} d_2}{I_p}\dot{x}_{p2} - \frac{c_1 b1}{I_p}\dot{x}_{t1} + \frac{c_2 b_2}{I_p}\dot{x}_{t2} \\ & -\frac{1}{I_p}(c_1 b_1 - c_2 b_2 + c_{p1} d_1 - c_{p2} d_2)\dot{x} \\ & -\frac{1}{I_p}(c_1 b_1^2 + c_2 b_2^2 + c_{p1} d_1^2 + c_{p2} d_2^2)\dot{\theta} \\ & +\frac{1}{I_p}(F_1 b_1 + F_2 b_2)\end{aligned} \tag{2}$$

$$\begin{aligned}\ddot{x}_{p1} = & \frac{1}{m_{p1}}(k_{p1}x + k_{p1} d_1 \theta - k_{p1} x_{p1}) \\ & +\frac{1}{m_{p1}}(c_{p1}\dot{x} + c_{p1} d_1 \dot{\theta} - c_{p1}\dot{x}_{p1})\end{aligned} \tag{3}$$

$$\begin{aligned}\ddot{x}_{p2} = & \frac{1}{m_{p2}}(k_{p2}x - k_{p2} d_2 \theta - k_{p2} x_{p2}) \\ & +\frac{1}{m_{p2}}(c_{p2}\dot{x} + c_{p2} d_2 \dot{\theta} - c_{p2}\dot{x}_{p2})\end{aligned} \tag{4}$$

$$\begin{aligned}\ddot{x}_{t1} = & \frac{1}{m_{t1}}(k_1 x + k_1 b_1 \theta - (k_1 + k_{t1})x_{t1} + k_{t1} y_1 - F_1) \\ & +\frac{1}{m_{t1}}(c_1 \dot{x} + c_1 b_1 \dot{\theta} - (c_1 + c_{t1})\dot{x}_{t1} + c_{t1}\dot{y}_1)\end{aligned} \tag{5}$$

$$\begin{aligned}\ddot{x}_{t2} = & \frac{1}{m_{t2}}(k_2 x + k_2 b_2 \theta - (k_2 + k_{t2})x_{t2} + k_{t2} y_2 - F_1) \\ & +\frac{1}{m_{t1}}(c_2 \dot{x} + c_2 b_2 \dot{\theta} - (c_2 + c_{t2})\dot{x}_{t2} + c_{t2}\dot{y}_2)\end{aligned} \tag{6}$$

Notation	Description	Units	Values
m	Chassis mass	kg	1794
I_p	Chassis intertia	kg m^2	3443.05
m_{p1}	Driver mass	kg	75
m_{p2}	Passenger mass	kg	75
m_{t1}	Front axle mass	kg	87.15
m_{t2}	Rear axle mass	kg	140.04
k_1	Front main stiffness	N/m	66824.2
k_2	Rear main stiffness	N/m	18615.0
k_{p1}	Front seat stiffness	N/m	14000.0
k_{p2}	Rear seat stiffness	N/m	14000.0
b_1	Distance	m	1.271
b_2	Distance	m	1.713
d_1	Distance	m	0.481
d_2	Distance	m	1.313
C_1	Front main damping	N s/m	1190
C_2	Rear main damping	N s/m	1000
c_{p1}	Front seat damping	N s/m	1190
c_{p2}	Rear seat damping	N s/m	1000
c_{t1}	Front tyre damping	N s/m	1190
c_{t2}	Rear tyre damping	N s/m	1000

Fig. 4. Suspension parameters

The equations of motion of the suspension system can be written in compact form as:

$$\dot{z} = Az + Bu + Gw \tag{7}$$

where the state vector z, is given by:

$$z = [x\ \theta\ x_{p1}\ x_{p2}\ x_{t1}\ x_{t2}\ \dot{x}\ \dot{\theta}\ \dot{x_{p1}}\ \dot{x_{p2}}\ \dot{x_{t1}}\ \dot{x_{t2}}]^T \tag{8}$$

The vector w represents the road disturbance:

$$w = [y_1\ y_2\ \dot{y_1}\ \dot{y_2}]^T \tag{9}$$

and the vector u is:

$$u = [F_1\ F_2]^T \tag{10}$$

represents the damper forces. The matrices $A \in R^{12\times 12}$ $B \in R^{12\times 2}, andG \in R^{12\times 4}$

B. Quarter Car Suspension Model

Quarter-car model with passenger consist of the wheel, unsprung mass, sprung mass, seat with passenger and suspension components Fig. 5, Wheel is represented by the tire, which has the spring character. The equations of motion of the suspensions system are:

$$\begin{aligned}\ddot{x} = &-\frac{1}{m_q}(k+k_p)x + \frac{k_p}{m_q}x_p + \frac{k}{m_q}x_t + \frac{c_p}{m_q}\dot{x}_p \\ &+ \frac{c}{m_q}\dot{x}_t - \frac{1}{m_q}(c+c_p)\dot{x} + \frac{F}{m_q}\end{aligned} \tag{11}$$

$$\ddot{x}_p = -\frac{1}{m_p}(k_p x - kp x_p) + \frac{1}{m_p}(c_p \dot{x}_p) \tag{12}$$

$$\begin{aligned}\ddot{x}_t = &\frac{1}{m_t}(k_x - (k+k_t)x_t + k_t y - F) \\ &+ \frac{1}{m_t}(c\dot{x} - (c+c_t)\dot{x}_t + c_t\dot{y})\end{aligned} \tag{13}$$

The equations of motion of the suspension system can be written in compact form as:

$$\dot{z} = Az + Bu + Gw \tag{14}$$

where the state vector z, is given by:

$$z = [x\ x_p\ x_t \dot{x}\ \dot{x}_p\ \dot{x}_t]^T \tag{15}$$

The vector w represents the road disturbance:

$$w = [y\ \dot{y}]^T \tag{16}$$

and the vector u represents the damper forces:

$$u = [F]^T \tag{17}$$

The matrices $A \in R^{6\times 6}, B \in R^{6\times 1}, andG \in R^{6\times 2}$

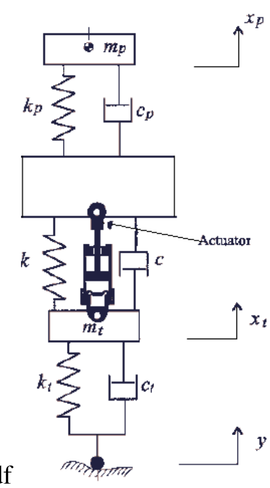

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Fig. 5. Quarter-car model with passengers

III. $H\infty$ Control

There is a wide range of optimal controllers derived for a quarter car model and half Car Model. The approach they use is usually based on $H\infty$. $H\infty$ is a method in control theory for the design of optimal controllers. Essentially it is an optimization method, that takes into consideration a strong mathematical definition of the restrictions on the expected behavior of the closed loop and strict stability of it.

Any advanced control methodology consist in finding a dynamic controller $K(s)$ for a plant model $G(s)$ such that the closed-loop system has good performances and robustness properties. The main advantage of the $H\infty$ theory based control is that both the level of plant uncertainty and the signal gain from disturbance inputs to controlled outputs can be specified in the frequency domain. The $H\infty$ theory is appropriate for $MIMO$ systems.

Let us first recall the $H\infty$ control problem and the way of solving it; Next the mixed sensibility problem is presented as a particular case of our application on suspensions car with the $H\infty$ control problem.

A. Statement of $H\infty$ Control

Any control problem can be reformulated as in fig. 6 where $P(s)$ is a linear system defined as follows:

$$\begin{pmatrix} \mathbf{e} \\ \mathbf{y} \end{pmatrix} = \begin{pmatrix} P_{11}(s) & P_{12}(s) \\ P_{21}(s) & P_{22}(s) \end{pmatrix} \begin{pmatrix} w \\ u \end{pmatrix}$$

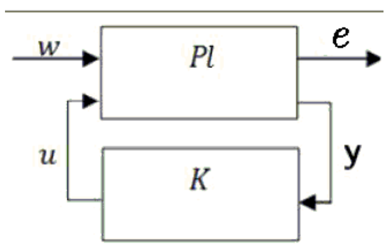

Fig. 6. General control configuration

where w is the disturbance vector, u is the control vector, e is the controlled output vector, y is the measurements vector.

Given γ, a prespecified attenuation level, a $H\infty$ suboptimal control problem is to design a stabilizing controller that insures:

$$\|T_{ew}(s)\|_\infty = max_\omega \bar{\sigma}(T_{ew}(jw)) \leq \gamma \tag{18}$$

where

$$T_{ew} := F_1(P,k) := P_{11} + P_{22}k(I - P_{22}k)^{-1}P_{21} \tag{19}$$

T_{ew} is the closed-loop transfer matrix from the disturbances w to the controlled outputs e and $\bar{\sigma}(T_{ew}(jw)) \leq \gamma$ is the maximal singular value of $T_{ew}(jw)$.

B. The Mixed Sensitivity Problem

In this part, the mixed sensitivity problem, which is a particular application of the $H\infty$ control problem, is presented. Fig. 7 gives the structure of a process $G(s)$ and its controller $K(s)$. u_p is the controlled input, y the output, r the reference, n a noise input due to the measurement and d_y the controlled output disturbance. Let us defined the usual sensitivity functions:

$$\begin{cases} S_y = (I+GK)^{-1} \\ T_y = (I+GK)^{-1}GK \end{cases} \quad \begin{cases} S_u = (I+KG)^{-1} \\ T_u = (I+KG)^{-1}KG \end{cases}$$

With this notation we have:

$$\begin{aligned} y &= S_y d_y - T_y n \\ u_p &= -KS_y d_y - KS_y n \end{aligned} \tag{20}$$

The problem given in fig. 7 can be rewritten in an $H\infty$ problem. Indeed, considering the general control configuration $P(s)$ of the plant model $G(s)$, the $H\infty$ control problem [18] and [19] consists in finding $K(s)$ such that the closed-loop system is stable and :

$$\left\| \begin{matrix} W_y S_y & W_y T_y \\ W_u K S_y & W_u K S_y \end{matrix} \right\|_\infty \leq \gamma \tag{21}$$

sensitivity problem.pdf

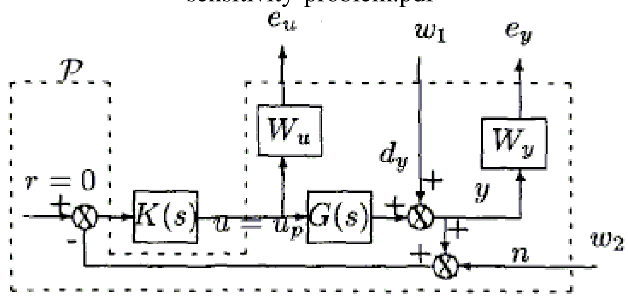

Fig. 7. Mixed sensitivity problem.

This problem is a mixed sensitivity problem. It allows to tackle the performances objectives by a frequency analysis of the sensitivity functions. So the disturbance rejection d_y impels the sensitivity functions S_y and KS_y to be small. And for a good rejection of the measurement noise n the functions T_y and KS_y must be small. And if the aim is for the output y to follow the reference T,the function T_y must be unitary. So these goals have to be achieved for different frequency ranges.

C. Problem LMI based Hinf Problem

Before introducing the main result of this paper, which uses well known $H\infty$, let us first recall some basic facts on LMI based $H\infty$ problem resolution for suspensions systems, we can see that a $H\infty$ controller is defined by:

$$S : \begin{bmatrix} \dot{x}_c \\ u \end{bmatrix} = \begin{bmatrix} A_k & B_k \\ C_k & D_k \end{bmatrix} \begin{bmatrix} x_k \\ y \end{bmatrix}$$

where x_k, y and u are the state, the control input and output, respectively, of the controller.

$A_k \in R^{n\times n}$, $B_k \in R^{n\times n_y}$, $C_k \in R^{n_u \times n}$, $D_k \in R^{n_u \times n_y}$

To make the LMI problem, we must do the following positions. Let k be the size of the controller:

$$P = \begin{bmatrix} X & N \\ N^T & F \end{bmatrix} \in R^{n+k\times n+k}$$

$$P^{-1} = \begin{bmatrix} Y & M \\ N^T & Z \end{bmatrix} \in R^{n+k\times n+k}$$

con: $X = X' \in R^{(n)\times(n)}, Y = Y' \in R^{(n)\times(n)}, N \in R^{(n)\times(k)}, M \in R^{n\times k}$

Of course $PP^{-1} = I$, but if this is true then surely we can write:

$$P = \begin{bmatrix} X \\ M^T \end{bmatrix} = \begin{bmatrix} I_n \\ 0 \end{bmatrix}$$

Now if we introduce the following matrices:

$$\pi_y = \begin{bmatrix} Y & I_n \\ M^T & 0 \end{bmatrix}$$

$$\pi_x = \begin{bmatrix} I_n & X \\ 0 & N^T \end{bmatrix}$$

We can certainly write that:

$P\pi_y = \pi_x$ Introduce these two matricesπ_y and π_x we need because they help us to derive the terms of LMI.

Going to apply them, there are known the following results:

$$R_1 = \pi_y^T P A_{cl} \pi_y = \pi_x^T A_{cl} \pi_y = = \begin{bmatrix} AY + B_1 \hat{C}_K & A + B_1 \hat{D}_k C \\ \hat{A}_k & XA + \hat{B}_k C \end{bmatrix}$$

$$R_2 = \pi_y^T P \pi_y = \pi_x^T \pi_y = \begin{bmatrix} Y & I \\ I & X \end{bmatrix}$$

$$R_3 = \pi_y^T P B_{cl} = \pi_x^T B_{cl} = = \begin{bmatrix} B_2 + B\hat{D}_k D \\ XB_2 + \hat{B}\hat{B}_k D \end{bmatrix}$$

$$R_{4,i} = \overline{C}_i \pi_y = = \begin{bmatrix} C_i Y + D_{i,2} \hat{C}_k \\ C_i + D_{i2} \hat{D}_k C \end{bmatrix}^T$$

Note that each of these is a combination of affine matrices where the auxiliary variables are:

$$\begin{cases} \widetilde{A} = NA_k M^T + NB_k CY + XBC_k M^T + \\ \quad X(A + BD_k C)Y \\ \widetilde{B} = NB_k + XBD_k \\ \widetilde{C} = C_k M^T + D_k CY \\ \widetilde{D} = D_k \end{cases}$$

The Hinf problem can be formulated as:

$$\begin{bmatrix} \pi_y A_{cl}^T P\infty \pi_y + \pi_y^T P\infty A_{cl} \pi_y & \pi_y P\infty B_{cl} & \pi_y^T \overline{C}_1^T \\ * & -\gamma\infty I & \overline{D}_1^T \\ * & * & -\gamma\infty I \end{bmatrix} < 0$$

from that:

$$\begin{bmatrix} R_1^T + R_1 & R_3 & R_{4,1} \\ * & -\gamma\infty I & \overline{D}_1^T \\ * & * & -\gamma\infty I \end{bmatrix} < 0$$

This inequality is a bilinear matrix inequality (BMI) so far, hence a non-convex problem has to be solved. Via a change of basis expressed in [11]. Solving follow relations leads to the $H\infty$ optimal solution. Then, choosing $M$and N such that $MN^T = I_n - XY$, the controller is obtained like:

If M and N are square matrices we can have M and N non-singular and we obtain:

$$\begin{cases} D_k = \hat{D}_k \\ C_k = (\hat{C}_k - D_k CY)(M^T)^{-1} \\ B_k = N^{-1}(\hat{B}_k - XBD_k) \\ A_k = N^{-1}(\hat{A}_k - NB_k CY - XBC_k M^T - \\ \quad X(A + BD_k C)Y)(M^T)^{-1} \end{cases}$$

IV. $H\infty$ Control Suspensions Car

Our aim is here to design one controller for each system and then to give the global controller. In those applications not all the controlled outputs are available, see figure [8]

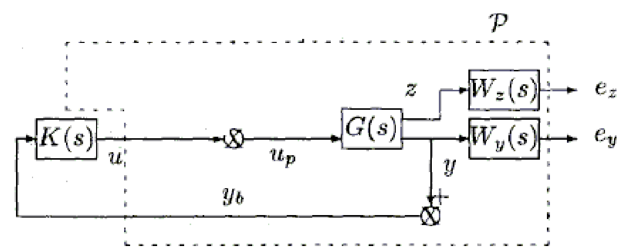

Fig. 8. General control configuration for system with some no measured controlled output.

Let us dispatch the G system transfer in:

$$\begin{pmatrix} \mathbf{z} \\ \mathbf{y} \end{pmatrix} = \begin{pmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{pmatrix} \begin{pmatrix} d_p \\ u_p \end{pmatrix}$$

Where z is the unmeasured outputs and y the measured one. To set this problem in a general control configuration the disturbances inputs w and the output e, which must be as small as possible, are defined by:

$$e = [e_z \, e_y]^T w = [0]^T \tag{22}$$

The equivalent $H\infty$ control problem is then to find a controller $K(s)$ that insure the stability of the closed loop system.

In order to make a difference between the vertical subsystem and the roll-lateral one, the weighting functions are renamed in relation with the name of the weighted signal, as:

$$W_z = \begin{bmatrix} W_{\ddot{z}s} & 0 \\ 0 & W_{z_u - r_z} \end{bmatrix}$$

$W_y = W_{z_s - z_u} Wu = W_{f_{az}}$

In particular, as the human body is more sensitive to vibrations in a frequency range of 3 to 8Hz, a low pass filter with a cutoff frequency of 10Hz is used for:

W_{r_z} W_y W_{f_z} $W_{f_{az}}$ are high pass filter in order to limit the control input for the high frequencies.

V. Simulation Results

The performance of the $H\infty$ control approach has been demonstrated through two suspension model, first on the Half car model and then on the Quarter-Car model. In the sequel the *active* and *passive* terms represent respectively the closed loop $(--)$ case and the open loop $(-.)$ case.

A. Results of Half-car model case

1) Frequency-Domain Analysis: It is well known that ride comfort is frequency sensitive. From the *ISO2361*, the human body is much sensitive to vibrations of $4-8$ Hz in the vertical direction and of $1-2$ Hz in the horizontal direction [15]. Hence, we need to evaluate the $H\infty$ active suspension with the state feedback the controller K in the frequency domain.

In this case the measure entering in controller are the acceleration of passengers $\ddot{x}_{p1}$ and $\ddot{x}_{p2}$ while the unmeasured weighted controlled output are the position chassis x, the chassis acceleration $\ddot{x}$ and the pitch acceleration $\ddot{\theta}$.

On Fig. [9] and Fig. [10] we can see the responses of the transfer functions from vertical road profiles y_1 and y_2 to vertical chassis mass position x in passive and active case are compared.

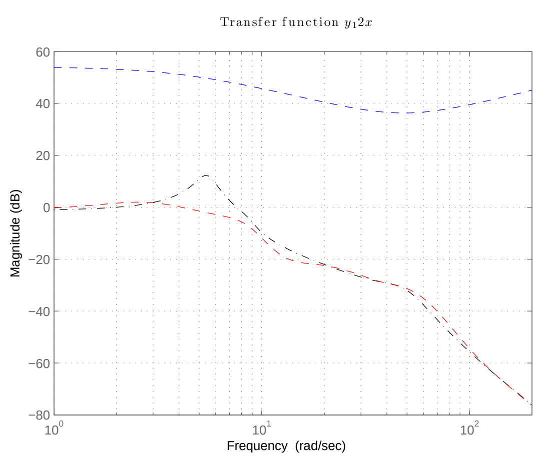

Fig. 9. Half-car Transfer Functions between road profiles y_1 and chassis position x, $(--)$ Active suspension, $(-.)$ Passive suspension

On Fig. [11] and Fig. [12] we can note the responses of the transfer functions from vertical road profiles y_1 and y_2 to vertical chassis acceleration $\ddot{x}$ in passive and active case are compared.

On Fig. [13] and Fig. [14] we can note comparison between passive and active case involves the response of transfer functions from vertical road profiles y_1 and y_2 to pitch acceleration $\ddot{\theta}$.

2) Time-Domain Analysis: On Fig. [15] In the time response simulates half-vehicle running at speed of 70 km/h on road hump of 1 cm high and 10 cm long. By reducing the vertical chassis mass in the frequency range of human sensivity, the passengers comfort is improved as expected.

Fig. 10. Half-car Transfer Functions between road profiles y_2 and chassis position x, $(--)$ Active suspension, $(-.)$ Passive suspension

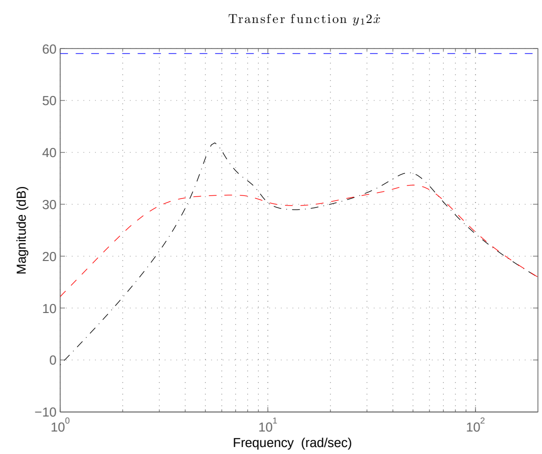

Fig. 11. Half-car Transfer Functions between road profiles y_1 and chassis acceleration $\ddot{x}$, $(--)$ Active suspension, $(-.)$ Passive suspension

B. Results of Quarter-car model

1) Frequency-Domain Analysis: In this case the measure entering in controller are the acceleration of passengers $\ddot{x}_p$ while the unmeasured weighted controlled output are the position chassis x andthe chassis acceleration $\ddot{x}$.

On Fig. 16 and Fig. 17the responses of the transfer functions from vertical road profiles y to vertical sprung mass position x and acceleration $\ddot{x}$ respectively in passive and active case are compared

2) Time-Domain Analysis: In the time simulation for the quarter-vehicle is used the same road profile of the half-car case. The passenger comfort is more improved in this case, like in Fig. 18.

VI. Conclusion

In this paper, $H\infty$ control for a half car and quarter car active suspension system with the simulation of the passengers has been investigated. Two $H\infty$ controllers have been designed for this system. Both controllers give favourable performance compare to passive suspension system . The

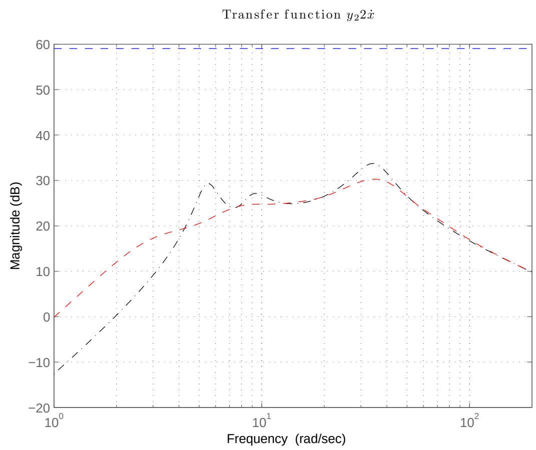

Fig. 12. Half-car Transfer Functions between road profiles y_2 and chassis acceleration $\ddot{x}$, (−−) Active suspension, (−.) Passive suspension

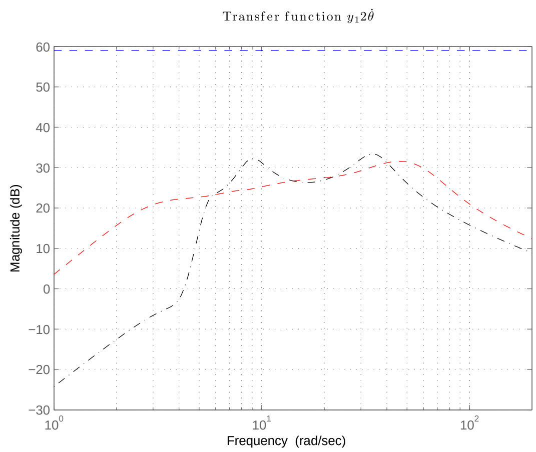

Fig. 13. Half-car Transfer Functions between road profiles y_1 and pitch acceleration $\ddot{\theta}$, (−−) Active suspension, (−.) Passive suspension

controller used in quarter car simulation significantly has better performance than half-car case, this is due to a lower number of constraints used in resolution of $H\infty$ problem.

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REFERENCES

[1] Yu, Z., 1999, *Automotive Engineering* 2nd ed., China Machine Press, Beijing.
[2] D.E. Ivers and L.R. Miller, *Experimental comparison of passive, semi-active on/off, and semi-active continuous suspensions*.SAE Technical Paper Series No. 892484 (1989).
[3] H.D. Taghirad and E. Esmailzadeh, *Automobile passenger comfort assured through LQG/LQR active suspension*, Journal of Vibration and Control, 4, pp. 603-618, 1998.
[4] Mansour A. Karkoub a; Mohamed Zribi, *Active/semi-active suspension control using magnetorheological actuators* International Journal of Systems Science, 37:1,35 44, 2006.

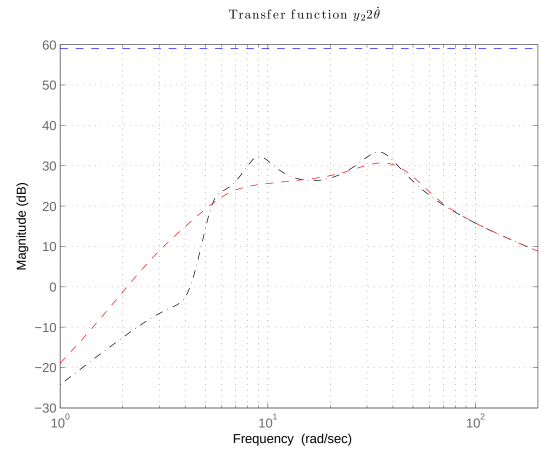

Fig. 14. Half-car Transfer Functions between road profiles y_2 and pitch acceleration $\ddot{\theta}$, (−−) Active suspension, (−.) Passive suspension

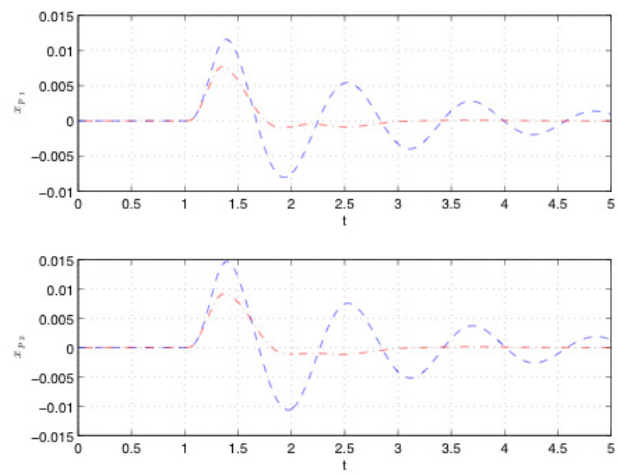

Fig. 15. Half-car, Responses of passenger positions, (−−) Active suspension, (−.) Passive suspension in the time response

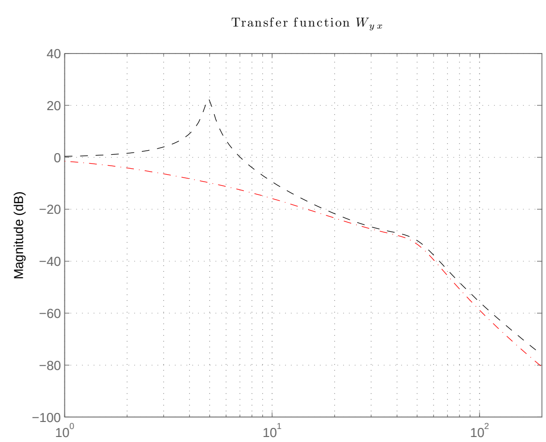

Fig. 16. Quarter-car Transfer Functions between road profiles y and chassis position x, (−−) Active suspension, (−.) Passive suspension

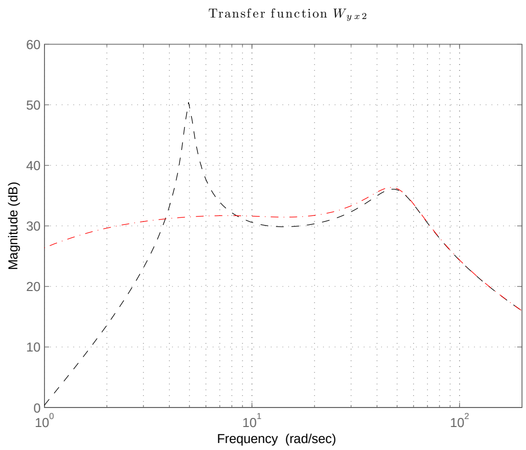

Fig. 17. Quarter-car Transfer Functions between road profiles y and chassis position $\ddot{x}$, $(--)$ Active suspension, $(-.)$ Passive suspension

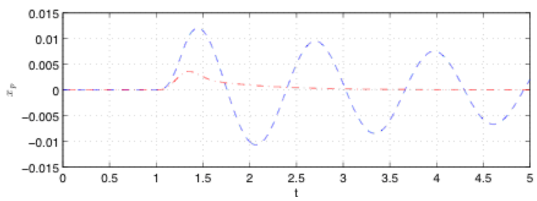

Fig. 18. Quarter-car, Responses of passenger positions, $(--)$ Active suspension, $(-.)$ Passive suspension in the time response

[5] J.D. Carlson and B.F. Spencer Jr. *Magneto-rheological fluid dampers for semi-active seismic control*, Proceedings of the 3rd International Conference on Motion and Vibration Control, Vol. 3, Japan: Chiba, 1996, pp. 3540.

[6] K. Zhou, J. Doyle, and K. Glover. *Robust Optimal Control*. Prentice Hall, 1996.

[7] D. Sammier, O. Sename, L. Dugard, *$H\infty$ control of active vehicle suspensions* Proceedings of the 2000 IEEE International Conference on Control Applications Anchorage, Alaska, USA September 2527,2000.

[8] Boyd, S., El-Ghaoui, L., Feron, E., Balakrishnan, V. *Linear matrix inequalities in system and control theory. SIAM studies in applied mathematics.* (1994).

[9] Apkarian, P., Gahinet, P. *A convex characterization of gain scheduled $H\infty$ controllers.* IEEE Transaction on Automatic Control, 40(5), 853864. (1995).

[10] Chilali, M., Gahinet, P., Apkarian, P. *Robust pole placement in LMI regions. IEEE Transaction on Automatic Control*, 44(12), 22572270. (1999).

[11] Scherer, C., Gahinet, P., Chilali, M. *Multiobjective output-feedback control via LMI optimization. IEEE Transaction on Automatic Control.* 42(7), 896911. (1997).

[12] A.R. TAVAKOLPOUR, I.Z. MAT DARUS, M. MAILAH *Numerical Simulation of a Flexible Plate System for Vibration Control* WSEAS Transactions on Systems and Control, Issue 3, Volume 4, March 2009.

[13] DAmbrosio S., Guarnaccia C., Guida1 D., Lenza T.L.L., Quartieri J. *System Parameters Identification in a General Class of Non-linear Mechanical Systems* International Juornal of Mechanics,Issue 4, Volume 1, 2007.

[14] JAO-HONG CHENG, CHIH-HUEI TANG *An Application of Fuzzy Delphi and Fuzzy AHP for Multi-criteria Evaluation on Bicycle Industry Supply Chains* WSEAS Transactions on Systems and Control, Issue 1, Volume 4, January 2009.

[15] Hrovat, D., 1993, *Applications of Optimal Control to Advanced Automotive Suspension Design* ASME J. Dyn. Syst., Meas., Control, 115, pp. 328342.

[16] Hrovat, D., 1997, *Survey of Advanced Suspension Developments and Related Optimal Control Applications* Automatica, 33(10), pp. 17811817.

[17] H. Chen, Z. -Y. Liu, P. -Y. Sun, *Application of Constrained $H\infty$ Control to Active Suspension Systems on Half-Car Models* Journal of Dynamic Systems, Measurement, and Control, SEPTEMBER 2005, Vol. 127 345

[18] Gordon, T. J., Marsh, C., and Milsted, M. G., 1991, *A Comparison of Adaptive LQG and Nonlinear Controllers for Vehicle Suspension Systems* Veh.Syst. Dyn., 20, pp. 321340.

[19] Fialho, I., and Balas, G. J., 2002, *Road Adaptive Active Suspension Design Using Linear Parameter-Varying Gain-Scheduling* IEEE Trans. Control Syst.Technol., 10(1), pp. 4354.