Algorithmic skeletons for numerical simulation of coupled problems

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Abstract— This paper presents some theoretical and numerical problems that arise in the analysis of coupled electromagnetic-thermal problems in electromagnetic devices.

The principal objective of the paper is to describe some computational aspects for coupled electromagnetic and thermal fields in the context of the finite element method, with emphasis on the reduction of the computing resources. We present coupled models for magnetic field and thermal field. The mathematical model for magnetic field is based on time-harmonic Maxwell equations in vector magnetic potential formulation for axisymmetric fields. The model for the heat transfer is the heat conduction equation.

We propose simplified numerical models for coupled fields in electromagnetic devices with target examples on the induction heating devices and high-voltage and large power cables. Domain decomposition is presented in the context of the coupled fields. The analysis domain is divided into two overlapping subdomains for the two coupled-fields considering physical significance of the pseudo-boundary of the two subdomains.

Keywords— Coupled fields, Finite-element method.

I. Introduction

The phenomena in the technical devices are not isolated but they were analysed independently because of some justified motivations. Ones of them are:

- **limited computational power** of the conventional computers
- the **complexity** of the **coupled problems**
- the **lack** of a **strong co-operation** between the engineers and mathematicians

There are **two standpoints**, which are not in contradiction, but they are linked. The former is the **mathematician's standpoint** that tries to prove that the problem has a solution and preferably a unique solution. The latter is the **engineer's standpoint** that wants the solution, and in practical cases an approximate solution. An engineer is concerned with large-scale physical achievement. We must not forget that each category is judged by different measures for their activities: a mathematician is judged for his publications in his area, and an engineer is judged by his physical achievements.

It is true that mathematics is with a step before the engineering, that is, sometimes, there are many years or decades between the mathematical researches and the application in the engineering. One of the motivations is limited technology for implementation of the mathematics results in practice. Now the time intervals are reduced. Are we clever? In my opinion, the answer is NO. We have more knowledge, we have a fast access to the information and we cooperate or we must cooperate in different disciplines. We dream more and have the tools to transform the dreams in reality.

Research engineers, that are devoted themselves to scientific research into engineering problems, use mathematics extensively. Mathematics enables the engineer to express his technical knowledge in clear and concise mathematical terms and arrange the components of his knowledge in logical order. Engineering is a science so that an engineer without mathematics is a gardener without his special tools.

A. Motivations for advanced algorithms

It is well known that the nature is complex in its behaviour and the abstract models do not capture accurately the laws of the nature. We work with abstract models. These models describe the phenomena from nature and the technical devices. But it is a great mistake to think that we have perfect models of the natural phenomena. More, many numerical algorithms are not discovered so that, although we limit our discussion to our actual achievements in this area, we must dream and to seek permanently new and modern approaches for the actual problems in science, technics and life.

Analytical solutions for the electrical engineering problems are limited to some simple applications and ignore some physical phenomena. For complex problems the accurate models are necessary and the numerical solutions are efficient approaches for an optimal design and operation.

With the advent of modern digital computers, many numerical models were developed and they become widely used in the scientific computing. We use the old algorithms and transform them for the new architectures but we must invent new algorithms having in our mind the computational power of the new computers.

The efficient design of the electromagnetic devices has resulted in more stringent specifications and a demand for optimal operation, which is very important in high-performance electrical power systems. More exacting specifications have demanded during the design stage the development of accurate methods of predicting the performance characteristics of these devices. Some of the

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performance indicators of concern in the design of the power devices are the electromagnetic forces, iron losses, the eddy-currents effects and the heat transfer between the component parts. Prediction of the flux densities and current densities can be used to compute forces and local heating, both of which are of a serious concern to the designer of the devices of high performance.

B. Motivations for coupled fields

Many areas of electrical engineering require the solution of problem in which the electromagnetic field equations are coupled to other partial differential equations, such as those describing thermal field, fluid flow or stress behaviour. These phenomena are described by equations that are coupled [1]. The coupling between the fields is a natural phenomenon and only in a simplified approach the field analysis can be treated as independent problem.

In several cases, it is possible a decoupling and a cascade solution of the coupled equations. Another attractive and efficient approach of solving coupled differential equations is to consider the set as a single system. In this way a single linear algebraic system for the whole set of differential equations is obtained after discretization, and is solved to a single step. If one or more equations are non-linear, non-linear iterations of the whole system are required.

The equations of the electromagnetic fields and heat dissipation in electrical engineering are coupled because the most of the material properties are temperature dependent and the heat sources represent the effects of the electromagnetic field [3].

The thermal effects of the electromagnetic field are both desirable and undesirable phenomena. Thus, in conducting parts of some electromagnetic devices (coils of the large-power transformers, current bars, cables conductors, conductors of the electric machines etc) the heating is an undesirable phenomenon. The heat is generated by ohmic losses of the driving currents and eddy currents induced in conducting materials. But in induction heating devices for welding the heating is a desirable phenomenon. The thermal effect of the electromagnetic field is the treatment base for many electric materials in industry [5].

II. MODELLING OF THE ELECTROMAGNETIC FIELD

A complete physical description of electromagnetic field is given by Maxwell's equations in terms of five field vectors: the magnetic field **H**, the magnetic flux density **B**, the electric field **E**, the electric field density **D**, and the current density **J**. In low-frequency formulations, the quantities satisfy Maxwell's equations [1]:

$$\nabla \times H = J \qquad (1)$$

$$\nabla \times E = -\frac{\partial B}{\partial t} \qquad (2)$$

$$div\ B = 0 \qquad (3)$$

$$div\ D = \rho_C \qquad (4)$$

with ρ_c the charge density. For simplicity we gave up to the bold notations for vectors

The second set of relationships, called the constitutive relations, is for linear materials:

$$B = \mu H;\ D = \varepsilon E;\ J = \sigma E$$

where σ – the electric conductivity and μ the magnetic permeability.

The B-H relationship is often required to represent non-linear materials. The current density J in Eq. (1) must represent both currents impressed from external sources and the internally generated eddy currents.

The formulation with vector and scalar potentials has the mathematical advantage that boundary conditions are more often easily formed in potentials than in the fields themselves. The magnetic vector potential is a vector A such that the flux density B is derivable from it by the operation *curl* or the operator *(* $\nabla \times$ *)*.

The complexity of the mathematical model for electromagnetic field was one of the main reasons to find and develop new computation methods. All methods can be included in one of the following classes [1]:

- Manipulation of the equations so that some unknowns are eliminated
- Definition of some potential functions from where the field unknowns can be obtained by simple processing
- Finding of some assumptions that simplifies the computation for practical problems

The potential formulations seem attractive because of their computational advantages. One of these consists in the fact the boundary conditions are easily framed in the potentials than in the field themselves.

A. The eddy-current problems

The time-varying magnetic field within a conducting material causes circulating currents to flow within the material. These currents called eddy-currents can be unwanted or desirable phenomena. Thus, the eddy-currents in electrical machines give rise to unwanted power dissipation. On the other hand the induction heating is a wanted phenomenon in industry of the metal treatment.

Industrial equipment in which the eddy currents are essentially can be included in one of the following classes:

- **long structures**, in which the electric field and the current density posses only one component
- **complex structures** in which we use models 3D

In the *long structures*, the currents are generated by an electric field applied at the terminals of the conductor or by a time-varying magnetic field linking the loop formed by the conductors. These structures belong to electric transmission network or the distribution networks (bus bars, large-power cables etc). In these problems the applied voltage of the bar or cable is known and we seek to compute the current density distribution within the conductor in order to determine some electromagnetic quantities of interest (the electrodynamic forces, mutual inductances, local heating etc).

The *complex structures* arise difficulties in simulation and computation of their characteristics although these structures possess construction simplicity. One of these structures is the device for electric heating by electromagnetic induction. In these types of the applications, it is necessary to compute accurately the eddy currents. If the eddy-currents distribution is non-uniform, the resulting high-temperature gradients may crack the workpiece.

The problems are different in the two different types of applications but for any given application the presence of the saturable iron sheets introduce saturation phenomena and the problem becomes non-linear.

For each class we can apply general mathematical methods but it is more efficient to develop a particular algorithm for each kind of classes.

The **effects** of the eddy currents are:

- The time-varying magnetic **flux density** is **nonuniform** within the conductor. The alternating magnetic flux is concentrated toward the outside surface of the material (phenomenon known as the skin effect).
- **Power losses** are increased in the material

Eddy current computation appears in two types of problems:

- **Stationary** problems where the structures are fixed and source currents are time varying
- **Motion** problems where the field source is a coil in moving

Many practical engineering problems involve geometric shape and size invariant in one direction. Let z denote the Cartesian co-ordinate direction in which the structure is invariant in size and shape. This is the case of a **plane-parallel field** or **translational field** problem, where A has one component, namely A_z. It is independent of the z co-ordinate and the Coulomb Gauge is automatically imposed and V is independent of x and y. In such a case both the magnetic vector potential and the source current J_S reduce to a single component oriented entirely in the axial direction and vary only with the co-ordinates x and y.

Consequently, the component A_z (for simplicity we give up the subscript z) satisfies the diffusion equation in fixed [4]:

$$\nabla(\nu\nabla A) - \sigma\frac{\partial A}{\partial t} = -J_S \qquad (5)$$

or in Cartesian co-ordinates:

$$\frac{\partial}{\partial x}(\upsilon\frac{\partial A}{\partial x}) + \frac{\partial}{\partial y}(\upsilon\frac{\partial A}{\partial y}) - \sigma\frac{\partial A}{\partial t} = -J_S \qquad (6)$$

The boundary conditions are set-up for the single component A and can be Dirichlet's and/or Neumann's condition. The interface conditions between two materials with different properties are:

$$A1 = A2; \quad v1\frac{\partial A1}{\partial n} = v2\frac{\partial A2}{\partial n}$$

B. Modelling of time-dependent fields

The time dependent electromagnetic field problems are usually solved using differential models of diffusion type. Many practical problems of great interest in electromagnetics involve time-harmonic fields and this case will be considered in this work.

In general, computer software for time-varying problem can be classified into two classes [7]:

- **time-domain** programs
- **frequency-domain** programs

Time-domain programs generate a solution for a specified time interval at different time moments. Frequency-domain programs solve a problem at one or more fixed frequencies.

The first class has some disadvantages. One of these consists in the large amount of data that must be stored to recover the field behaviour. Although the second class has an essential advantage (a compact and a cheap program in terms of the computer resources), the area of problems that can be solved is limited. It is applicable only to linear problems (all phenomena are sinusoidal).

The usual mathematical model for time dependent electromagnetic field problems is with Maxwell's equations in their normal differential form. For low frequency the displacement current term in Maxwell's equations can be neglected. At a surface of a conducting material the normal component of current density J_n can be assumed to be zero.

In problems with two dimensions, there are two limiting cases:

- A formulation with H field
- A formulation with magnetic vector potential

Both cases are PDEs of the diffusion type. The latter case is of greater practical interest because can be solved by numerical methods.

In general the time dependent problems after a spatial discretization can lead to a lumped-parameter model. For example, Maxwell's equations in differential form for low frequency in 2-D case, after spatial discretization, lead to a system of ordinary differential equations by the form [3]:

$$[S]\left\{\frac{\partial A}{\partial t}\right\} + [R]\left\{A\right\} + \{b\} = 0 \qquad (7)$$

where [R] and [S] are matrices and b is the vector of the free terms.

To simplify the computation, one approach is to separate the spatial domain of the problem in conducting and non-conducting parts, such that A_1 is the solution vector in conducting regions and A_2 is the solution vector in the non-conducting regions. By reordering the matrices, the system of equations is divided in two systems [3]:

$$[R_{11}]\{A_1\} + [R_{12}]\{A_2\} + [S]\left\{\frac{\partial A_1}{\partial t}\right\} + \{b_1\} = 0 \qquad (8)$$

$$[R_{21}]\{A_1\} + [R_{22}]\{A_2\} + \{b_2\} = 0 \qquad (9)$$

The system (9) is formed of algebraic equations; the system (8) is formed of differential equations. These systems are solved by an iterative procedure.

III. Mathematical modelling of the thermal field

The thermal field is described by the heat conduction equation [5]:

$$\frac{\partial}{\partial t}[(c\gamma)(T)\cdot T]+\nabla[-k(T)\cdot\nabla T]=q \qquad (10)$$

where: T (x,t) is the temperature in the spatial point x at the time t; point k is the tensor of thermal conductivity; γ is mass density; c is the specific heat that depends on T; q is the density of the heat sources that depends on T. In the coupled problems we use the formula:

$$q=\rho(T)\cdot J^{2} \qquad (11)$$

with ρ the electrical resistivity of the material. Equation (10) is solved with boundary and initial conditions. The boundary conditions can be of different types: Dirichlet condition for a prescribed temperature on the boundary, convection condition, radiation condition and mixed condition [2].

For many eddy-current problems the magnetic flux penetration into a conductor without internal sources of the magnetic field is confined mainly to surface layer. This is the skin effect. The skin depth δ depends on the material properties μ, ω and σ so that for the small depths all of the effects of the magnetic field are confined to a surface layer.

In steady-state low-frequency eddy current problems in magnetic materials, the mathematical model is the diffusion equation (6).

The skin effect can be exploited in two directions:

- To reduce the space domain in analysis with a fine mesh close to conductor surfaces
- To reduce the material volume since a significant proportion of the conductor is virtually unused

The penetration depth is given by the formula:

$$\delta=\sqrt{\frac{2}{\omega\sigma\mu}} \qquad (12)$$

For example, in a semi-infinite slab of conductor with an externally applied uniform alternating field, parallel to the slab, the amplitude of flux decays exponentially. In other words for problems with the skin depth very small all the effect of the field is confined to a surface layer. In a numerical model based on finite element method (FEM) this effect can be exploited by the use of a special boundary condition, known as the surface impedance condition. In this way we don’t waste run-time of a program based on FEM.

Designer engineers use the formula (12) considering the permeability and the conductivity as numbers. In reality the two physical parameters change during heating. The changes in the value of δ affect the loss in the material and depend on the process (conduction or induction). For example, if the conductivity decreases by x, the depth increases by √x, that is the current penetrates deeper into the metal. If the magnetic material heats, its resistivity (the inverse of the conductivity) rises but its relative permeability remains substantially constant up to the Curie point. In this point it drops suddenly to unit.

Another simplifying assumption for the designer engineers is based on that all heat enters at the surface of the conductor. In reality, this is only true if the frequency of the magnetic field source is very high and the depth of heating is small compared with the geometrical dimensions of the conductor.

For an accurate computation of the penetration depth of the magnetic field we must consider two practical conditions:

- The heat is distributed in the conducting part
- There is an important heat lost by radiation at the conductor surface

Radiation can be regarded as a simple surface loss subtracting from the surface power input. The Stefan-Boltzmann law gives the radiation loss. If the body is radiating to a surface at absolute temperature T_∞ Kelvin, the radiation loss is defined by:

$$P_r=\varepsilon_r c_0(T^4-T_\infty^4)$$

where ε_r is the emissivity coefficient of the surface (dimensionless) and T is the absolute surface temperature in Kelvin (K). The constant c_0 is $5.67.10^{-8}$ W/m^2K^4. For low temperatures the radiation loss is negligible but in the induction-heating device it must be considered.

Consequently, it is convenient to use coupled models and accurate methods for computation of the heat penetration in the conductors, especially in the induction heating.

IV. Iterative algorithms for coupled problem

A complete mathematical model for coupled fields involves Maxwell’s equations and the heat conduction equation. Combining these equations yields a coupled system of non-linear equations. In a discrete form the unknowns are the nodal values of the temperature T and the magnetic vector potential A.

For electromagnetic field we consider the A-formulation, that is we define the magnetic vector potential A by B = *curl* A. More, the domain is the same for temperature and the electromagnetic field although in practice the interest is for different field domains.

The non-linear equations for T and A are straightforwardly obtained by a Galerkin's finite element method. For the 2D steady-state problems we do the approximations at the element level [6]:

$$T(x,y)=\sum_{j=1}^{r}N_j(x,y)T_j$$

$$A(x,y)=\sum_{j=1}^{r}N_j(x,y)A_j$$

where the interpolation functions N_j are basis functions in the mesh over Ω and r is the number of nodes of an element.

The usual procedure for the FEM applications leads to a system of *2p* equations where p is the total number of the unknowns in each field problem. These non-linear equations can be solved by two different basic strategies [1]:

- Solving the equations for T_i and A_i simultaneously
- Solving the equations for the two fields in sequence with an outer iteration, technique known as operator-splitting technique (for example Newton-Raphson procedure)

In the area of the first strategy, Gauss-Seidel and Jacobi methods are well known. We present these methods in brief [1]. For this, let us define the two discrete equations derived from the electromagnetic field model and the thermal field model in the form:

$$f_A(A_1,...,A_p,T_1,...,T_p)=0$$

$$f_T(A_1,...,A_p,T_1,...,T_p)=0$$

where the subscript denotes the original problem (A – for the magnetic field in the magnetic vector potential formulation; T – for the thermal field).

The Gauss-Seidel algorithm for coupled fields has the following pseudo-code [1]:

For m: =1, 2, … until **convergence** *DO*
Solve

$$f_A(A_1^{(m)},...,A_p^{(m)};T_1^{(m-1)},...,T_p^{(m-1)})=0$$

with respect to $A_1^{(m)}$, … $A_p^{(m)}$
Solve

$$f_T(A_1^{(m)},...,A_p^{(m)};T_1^{(m)},...,T_p^{(m)})=0$$

with respect to $T_1^{(m)}$, … $T_p^{(m)}$

In other words, the system is solved firstly with respect to A, using the values of T from the previous iteration. Afterwards, the equation derived from the thermal field model is solved using the computed values of A from the current iteration. The equations f_A=0 or/and f_T=0 are non-linear and must be solved by an iterative procedure (for example Newton-Raphson method).

The algorithm Jacobi-type is similar to Gauss-Seidel method, except that at the iteration number m when we must solve the model for T, the values for A are from the previous iteration, which is $A^{(m-1)}$. The algorithm has the following pseudo-code:

For m: =1, 2, … until **convergence** *DO*
Solve

$$f_A(A_1^{(m)},...,A_p^{(m)};T_1^{(m-1)},...,T_p^{(m-1)})=0$$

with respect to $A_1^{(m)}$, … $A_p^{(m)}$
Solve

$$f_T(A_1^{(m-1)},...,A_p^{(m-1)};T_1^{(m)},...,T_p^{(m)})=0$$

with respect to $T_1^{(m)}$, … $T_p^{(m)}$

The domain decomposition could be determined from mathematical properties of the problem (real boundaries or interfaces between subdomains), or from the geometry of the problem (pseudo-boundaries). For elliptic partial differential equations, there exists a mathematical approach based on the ideas given earlier in 1890 by Schwarz [8]. In Schwarz procedure there is an inherent parallelism with a data communication time for the passage of pseudo-boundary data between processors.

There is no general rule for the domain or/and operator decomposition. It is defined in a somewhat random fashion. The problems and questions that appear in the decomposition technique are:

- do *domain decomposition* or the *operator* decomposition
- Which approach is the best: *disjoint* or *overlapping* sub-domains?
- What *kinds of boundary conditions* are set up on the pseudo-boundaries of the sub-domains
- What kind of domain decomposition is useful for a particular problem: *static* or *dynamic* decomposition?

A. Decomposition techniques

The desire of the scientific community for faster processing on lager amounts of data has driven the computing field to a number of new approaches in this area [8]. The main trend in the last decades has been toward advanced computers that can execute operations simultaneously, called parallel computers. For these new architectures, new algorithms must be developed and the domain decomposition techniques are powerful iterative methods that are promising for parallel computation. Ideal numerical models are those that can be divided into independent tasks, each of which can be executed independently on a processor. Obviously, it is impossible to define totally independent tasks because the tasks are so inter-coupled that it is not known how to break them apart. However, algorithmic skeletons were developed in this direction that enables the problem to be decomposed among different processors. The mathematical relationship between the computed sub-domain solutions and the global solution is difficult to be defined in a general approach.

In the area of the coupled fields we define two levels of decomposition, that is, we define a hierarchy of the decompositions:

- One at the level of the problem
- The other at the level of the field

In other words, we decompose the coupled problem in two sub-problems: an electromagnetic problem and a thermal problem, each of them with disjoint or overlapping spatial domains. This is the first level of decomposition. At the next level, we decompose each field domain in two or more subdomains. The decomposition is guided both by the different physical properties of the materials, and the difference of the mathematical models. At this level of decomposition the Steklov-Poincaré operator can be associated with field problem. This operator reduces the solution of the coupled subdomains to the solution of an equation involving only the interface values. One efficient and practical solution of elliptical partial differential equations is the dual Schur complement method [8].

V. SOME INDUSTRIAL APPLICATIONS

In any electromagnetic device there are power losses that are transformed in heating so that the modelling of device involves coupled mathematical models. In electrical engineering the coupled electromagnetic and thermal fields represent both desirable phenomena and undesirable phenomena. Two examples illustrate this assertion: induction heating and the high-voltage (HV) electrical cables.

Induction heating describes the thermal conductivity problem in which the heat is generated by eddy currents induced in conducting materials, by a varying magnetic field. Induction heating is an efficient procedure for bulk-heating metals to a set temperature. The heating is generated by the eddy-currents induced from a separate source of alternating current.

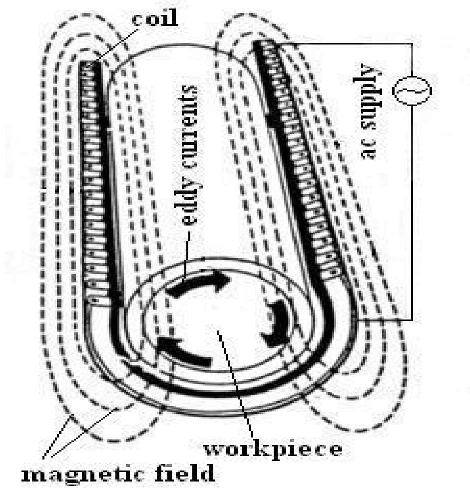

Fig. 1 – Axial section

Figure 1 shows a long cylindrical workpiece excited by a close-coupled axial coil. The device has a cylindrical symmetry so that the problem can be reduced to a 2D-problem in the plane Orz. An axial section is presented in the figure 2 with 1- the workpiece, 2 – the air and 3 – the coil. The coil is assimilated with a massive conductor. In this case we cannot ignore the eddy currents in the coil.

In Fig.2 an axial section is presented. The coil is assimilated with a massive conductor. In this case we cannot ignore the eddy currents in the coil. We consider a low-frequency current in the coil so that the penetration depth is large. In this case we can decompose the whole domain of the field problem into *overlapped subdomains* for the two coupled-fields.

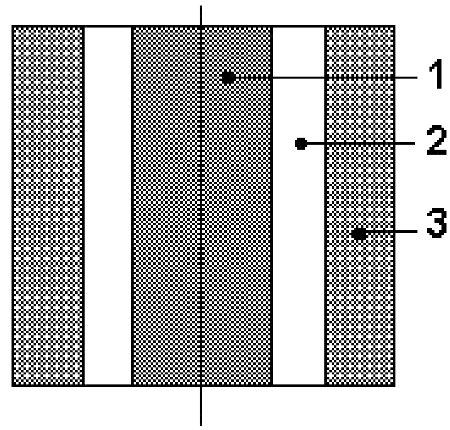

Fig. 2 –Analysis domain

The domain for the magnetic field can be reduced to a quarter of the device bounded by a boundary at a finite distance from the device. For the thermal field we consider the workpiece as the analysis domain. The penetration depth of the magnetic field in the workpiece imposes the overlapping domains for the two fields [8]. The numerical model is considered in a cylindrical co-ordinates with the vertical axis Or and the horizontal axis Oz.

The mathematical model for the electromagnetic field using A-formulation is a 2D-scalar model in (r-z) plane:

$$\frac{\sigma}{r}\frac{\partial(rA)}{\partial t} - \nabla\left[\frac{\upsilon}{r}\nabla(rA)\right] = J_S \qquad (13)$$

For the harmonic-time case, mathematical model is:

$$\frac{\partial}{\partial r}\left[\frac{\upsilon}{r}\frac{\partial(rA)}{\partial r}\right] + \frac{\partial}{\partial z}\left[\frac{\upsilon}{r}\frac{\partial(rA)}{\partial z}\right] - j\sigma\omega\frac{\upsilon}{r}(rA) = -J_S \qquad (14)$$

Another example that we present is a high-voltage cable with three phase conductors and a neutral conductor [9]. The HV cables are important components of the energetic system for distribution of the electric energy. Fig. 3 shows the cross-section of the system. This high-voltage tetra-core cable has three triangle sectors with phase conductors and round neutral conductor in the lesser area of the cross-section above. All the conductors are made of copper. Each conductor is insulated and the cable as a whole has a three-layered insulation. The cable insulation consists of inner and outer insulators and a protective braiding (steel tape). The sharp corners of the phase conductors are chamfered to reduce the field crown. The corners of the conductors are rounded. Empty space between conductors is filled with some insulator (air, oil etc.)

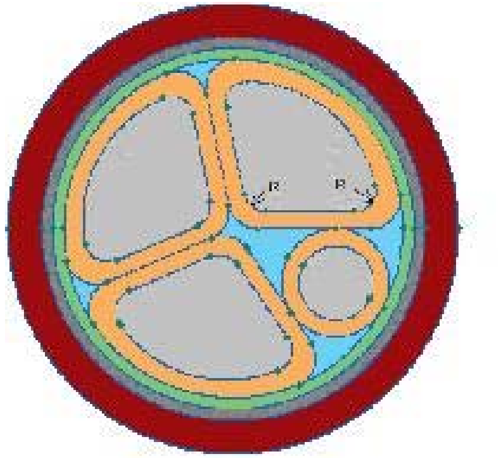

Fig. 3– A cross-section of the cable

VI. Numerical results

We shall present the results of the numerical simulation for the cable. This system can be analysed for different operating regimes. When the cables are in load, the conductor currents can generate local heating that destroys the insulation and finally, the whole system. Consequently, the temperature distribution is of great importance for the designer. Fig. 4 shows the temperature map of the system.

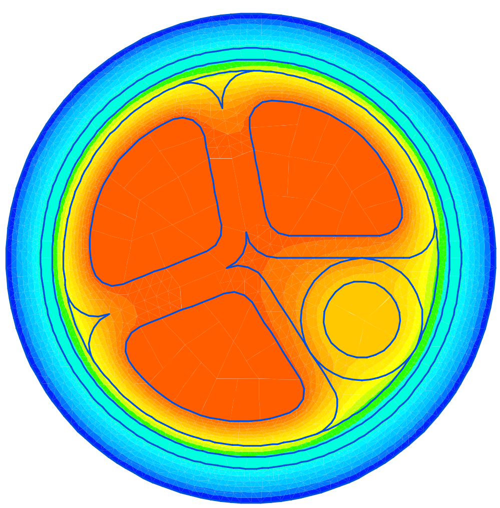

Fig. 4 – The map of the temperature

Each cable-core has its own insulation but there are two layers of insulation: inner cable insulation and outer cable insulation more thick than the internal insulation. Also, there is a protective steel braiding.

The load of the conductors are currents of amplitude equal to 250 A at the frequency of 50 Hz.

In post-processing stage of the FEM program, a lot of physical quantities can be obtained [8]. They are of great importance for the electrical engineers in the evaluation of the device performance. These derived quantities are presented in user's manual of any software CAD [9]. The voltage amplitude is 7000 V.

The non-uniformity of the temperature is due to the non-uniformity of the current density in system. In figure 5 the map of the total current density is shown. In computation of the total current in the cable, the skin effect and proximity effect of the cable cores were considered.

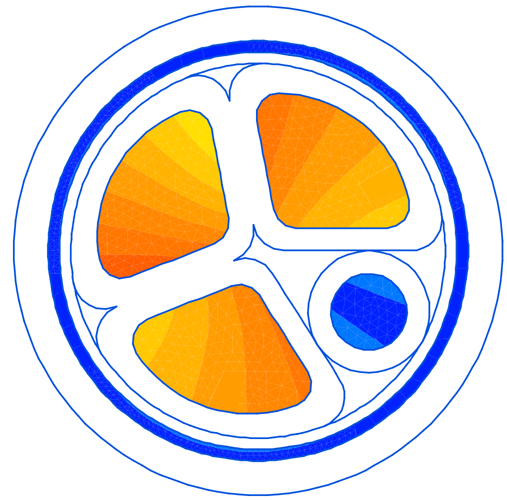

Fig. 5 – The map of the current density

Another important field in the operating regime of an electromagnetic device is the mechanical field. Stress analysis problem is the utmost one that imports the temperature field from the heat transfer problem and the magnetic forces from the magnetic problem. The conducting medium is subjected to both temperature change and Lorenz force. Due to this magnetic and thermal loading the cable shape can be deformed. The electrodynamic force is a vector normal to the magnetic induction ***B*** and the electrical current ***I*** according to the formula **F = *I*** X ***B***.

In a stress analysis problem the displacement, strain and stress are of great importance. The physical quantities for stress analysis are:

- Displacement vector δ
- Strain vector ε and its principal values
- Stress vector σ and its principal values
- Some relevant criteria (Tresca criterion, Drucker-Prager criterion, Mohr-Coulomb criterion, Von Mises stress)

The mathematical models for stress analysis the elasticity theory is used. The equilibrium equations for axisymmetric problems are:

$$\frac{1}{r}\frac{\partial(r\sigma_r)}{\partial r}+\frac{\partial\tau_{rz}}{\partial z}=-f_r$$
$$\frac{1}{r}\frac{\partial(r\tau_{rz})}{\partial r}+\frac{\partial\sigma_z}{\partial z}=-f_z$$

where σ_r , σ_z τ_{rz} are the stress components, and f_r, f_z are components of the volume force vector [9].

Temperature strain is determined by the coefficients of thermal expansion and temperature difference between strained and strainless states. Components of the thermal strain for axisymmetric problem and orthotropic material are defined by the following equation [9]:

$$\varepsilon_0 = \begin{Bmatrix} \alpha_z \\ \alpha_r \\ \alpha_\theta \\ 0 \end{Bmatrix} \cdot \Delta T$$

where α_z , α_r , α_θ are the coefficients of thermal expansion along the corresponding axes for orthotropic material, and ΔT is the temperature difference between strained and strainless states.

For linear elasticity, the stresses are related to the strains by the constitutive law (Hooke's law):

$$\{\sigma\} = [D](\{\varepsilon\} - \{\varepsilon_0\})$$

where [D] is a matrix of elastic constants (Young's modulus, Poisson's ratio, shear modulus), and $\{\varepsilon_0\}$ is the column vector for the initial thermal strain.

The interactions of the three fields increase the problem complexity and finally, the algorithm complexity. The algorithm complexity can be reduced considering some physical properties of the materials. It can be used a predefined temperature profile of a material for updating the magnetic field at specified temperatures. For example, at Curie temperature the material properties change dramatically [4]. After this critical point the magnetic field equation must be updated.

VII. CONCLUSIONS

The problem of coupled fields in electrical engineering is a complex problem in terms of computing resources. In practice the coupled fields are treated independently in some simplified assumptions. The accuracy of the numerical computation is poor. With the new architectures, a multidisciplinary research is possible. Some iterative procedures were presented with emphasis on the coupled problems.

Domain decomposition offers an efficient approach for large-scale problems or complex geometrical configurations. This method in the context of the finite element programs leads to a substantial reduction of the computing resources as the time of the processor. In our target examples, especially in induction heating devices, we can do a dynamic domain decomposition. It is obviously that the penetration depth of the magnetic field in the workpiece, δ, depends on the frequency and electrical conductivity. Practically the depth depends on temperature so that the depth differes from a time step to other.

In coupled problems a hierarchy of decomposition can be defined with a substantial reduction of the computation complexity.

The finite element method was used for the numerical result. The program Quickfield [9] was used in our target examples.

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