Mathematical Modeling of Multiple Intelligence Theory with Bayesian Theorem

Bahadtin Ruzgar and Nursel Selver Ruzgar

Abstract— In this work, the multiple intelligence theory proposed by Gardner, a professor of education at Harvard University, is modeled by Bayesian Theorem under two hypotheses. Howard Gardner initially formulated a list of seven intelligences, and then added two more. As a different approach, if set theory for multiple intelligences is used, the structure of multiple intelligences to set theory under four properties of intelligence algebra can be generalized. Assuming that the number of intelligences increases, Boolean algebra in set theory can be applicable. Bayesian theorem with application of conditional probability generates a good structure for multiple intelligences. Bayesian Theorem was applied to two hypotheses; mutual intersections of n intelligences are empty and non-empty sets, and using conditional probability, it can be shown that multiple intelligences and Bayesian Theorem are in good harmony or multiple intelligences can be explained by Bayesian theorem.

Keywords-Bayesian Theorem, Modeling, Multiple intelligences

I. INTRODUCTION

Tp until twenty years ago or so, when teaching was still held in high regard by European society, you had an equal mix of good and bad teachers for a variety of reasons but as the position has declined in held regard combined with falling pay and stricter academic-based selection, the distribution of good and bad has become much more bunched together but with the important exception that there is less creativity in the average teacher nowadays because academia doesn't give much weight to creativity. Creative people tend to be good at putting themselves in good positions, actual ability is of less importance) [1]. Every child is a genius. That doesn't mean that every child can paint like Picasso, compose like Mozart, or score 150 on an I.Q. test. But every child is a genius according to the original meanings of the word "genius," which are: "to give birth" (related to the word genesis) and "to be zestful or joyous," (related to the word genial). Essentially, the real meaning of genius is to "give birth to the joy" that is within each child. Every child is born with that capacity. Each child comes into life with wonder, curiosity, awe, spontaneity, vitality, flexibility, and

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many other characteristics of a joyous being. An infant has twice as many brain connections as an adult [2]. In the heyday of the psychometric and behaviorist eras, it was generally believed that intelligence was a single entity that was inherited; and that human beings - initially a blank slate could be trained to learn anything, provided that it was presented in an appropriate way. Nowadays an increasing number of researchers believe precisely the opposite; that there exists a multitude of intelligences, quite independent of each other; that each intelligence has its own strengths and constraints; that the mind is far from unencumbered at birth; and that it is unexpectedly difficult to teach things that go against early 'naive' theories of that challenge the natural lines of force within an intelligence and its matching domains [3]. The theory of the multiple intelligences" (Thinking style: multiple intelligence theorem) suggested by Harward Gardner in 1983 eliminated the influence of intelligence on societies and education that lasted for years; that is the traditional intelligence test and intelligence definition that only considers the language and mathematic intelligence [4]. Howard Gardner, who is a psychologist and Professor at Harvard University's Graduate School of Education, claims that all human beings have multiple intelligences. Howard Gardner defined the first seven intelligences in "Frames of Mind" (1983). He added the last two in "Intelligence Reframed" (1999). These multiple intelligences can be nurtured and strengthened, or ignored and weakened. He believes each individual intelligences: Verbal-Linguistic has nine Intelligence (well-developed verbal skills and sensitivity to the sounds, meanings and rhythms of words), Mathematical-Logical Intelligence (ability to think conceptually and abstractly, and capacity to discern logical or numerical patterns), Musical Intelligence (ability to produce and appreciate rhythm, pitch and timber), Visual-Spatial Intelligence (capacity to think in images and pictures, to visualize accurately and abstractly), Bodily-Kinesthetic Intelligence (ability to control one's body movements and to handle objects skillfully), Interpersonal Intelligence (capacity to detect and respond appropriately to the moods, motivations and desires of others), Intrapersonal Intelligence (capacity to be self-aware and in tune with inner feelings, values, beliefs and thinking processes), Naturalist Intelligence (ability to recognize and categorize plants, animals and other objects in nature) and Existential Intelligence (sensitivity and capacity to tackle deep questions about human existence, such as the meaning of life, why do we die, and how did we get here) [5]. According to Gardner,

• All human beings possess all nine intelligences in varying amounts.

• Each person has a different intellectual composition.

• We can improve education by addressing the multiple intelligences of our students.

• These intelligences are located in different areas of the brain and can either work independently or together.

• These intelligences may define the human species [5].

The theory of multiple intelligences proposes a major transformation in the way our schools are run. It suggests that teachers be trained to present their lessons in a wide variety of ways using music, cooperative learning, art activities, role play, multimedia, field trips, inner reflection, and much more [6].

Intelligence is the integration of abilities and skills peculiar to each person in order to be able to adapt to life and changes in the changing world. According to Binet, intelligence is the capacity of good reasoning, good judging and self-criticizing [7]. According to Wechsler, intelligence is the capacity used by an individual to behave purposeful, think intelligently and cope with his environment effectively [8]. According to Woolfolk, intelligence is the ability of an individual to use his inherited or learned mental functions to acquire knowledge, remember, recall, solve problems or adapt to world [9]. According to Piaget, intelligence is a definite harmonized behavior model that means organization of action with thought and its reorganization [10]. According to Gardner, intelligence is the integration of abilities and skills peculiar to each person in order to be able to live in the changing world and adapt to changes [11]. Thorndike grouped intelligence in three kinds; abstract, mechanical and social [2]. Guilford determined 120 intelligence factors. According to the Group Factor Theory of Thurnstone, intelligence can be divided into primary abilities in some definite number. "The primary ability" term contains seven factors; numeric problem solving, oral recognition, memorizing, general reasoning, oral smoothness (word producing etc), recognizing shape relations, sensitive speed. According to Stenberg, intelligence consists of various combinations. These are; experiencing and learning ability from life, abstract thinking or reasoning ability, adaptation ability to caprices of changing and unclear world, and fast performing ability of the works that must be performed and motivation ability [12]. Psychologists define intelligence as a capacity whereas educators define it as ability [13].

II. BAYESIAN THEOREM

Bayesian theorem is named after the Reverend Thomas Bayes (1702–1761), who studied how to compute a distribution for the parameter of a binomial distribution (to use modern terminology). His friend, Richard Price, edited and presented the work in 1763, after Bayes' death, as *An Essay towards solving a Problem in the Doctrine of Chances*. Pierre-Simon Laplace replicated and extended these results in an essay of 1774, apparently unaware of Bayes' work.

One of Bayes' results gives a simple description of conditional probability, and shows that it can be expressed independently

of the order in which things occur: If there be two subsequent events, the probability of the second b/N and the probability of both together P/N, and it being first discovered that the second event has also happened, from hence I guess that the first event has also happened, the probability I am right [i.e., the conditional probability of the first event being true given that the second has also happened] is P/b [14-15]

Bayes's Theorem is central to these enterprises both because it simplifies the calculation of conditional probabilities and because it clarifies significant features of subjectivist position. Indeed, the Theorem's central insight that a hypothesis is confirmed by any body data that its truth renders probable — is the cornerstone of all subjectivist methodology.

Let A and B_i be sets. Conditional probability requires that

$$P(A \cap B_{j}) = P(A)P(B_{j} / A), \qquad (1)$$

where I denotes intersection ("and"), and also that

$$P(A \cap B_j) = P(B_j \cap A) = P(B_j)P(A / B_j)$$
(2)

Therefore,

$$P(B_{j} / A) = \frac{P(B_{j})P(A / B_{j})}{P(A)}$$
(3)

Now, let
$$S \equiv \bigcup_{i=1}^{N} A_i$$
 (4)

So A_i is an event in S and $A_i \cap A_i = \{ \}$ for, $i \neq j$ then

$$A = A \cap S = A \cap \left(\bigcup_{i=1}^{N} A_i \right) = \bigcup_{i=1}^{N} (A \cap A_i)$$
(5)

$$P(A) = P\left(\bigcup_{i=1}^{N} (A \cap A_i)\right) = \sum_{i=1}^{N} P(A \cap A_i)$$
(6)

But this can be written

$$P(A) = \sum_{i=1}^{N} P(A_i) P(A / A_i)$$
(7)

$$P(A_{i} / A) = \frac{P(A_{i})P(A / A_{i})}{\sum_{j=1}^{N} P(A_{j})P(A / A_{j})}$$
⁽⁸⁾

Bayesian statistics still provide the core to reasoning in many uncertain reasoning systems with suitable enhancement to overcome the many problems. Although there are many applications of Bayesian Theorem in different fields in literature, such as business, engineering, marketing and so on, multiple intelligence theory has not been modeled mathematically yet, even Bayesian Theorem.

III. MODELING OF MULTIPLE INTELLIGENCES

Intelligence is the capacity to solve problems in different cases experienced throughout of life and to create new products. For this reason, intelligence must be defined with clear dimensions as in mathematical operation. A mathematical operation requires mathematics to establish a system and can be defined as a system in that matter as it ensures representation of the system features; so intelligence can also be defined with its clear dimensions. Gardner stated that each of the abilities can not be considered intelligence on their own alone; and in order that abilities can be defined as intelligence, 4 features must exist. These are:

- to have a series symbols
- to include a value in cultural structure (or to have value in society)
- to be able produce a service or goods through its intervention (to make production through the ability used)
- to be able to solve a problem inside it.

Based on the above, 8 kinds of intelligence defined by Gardner with such features can be evaluated separately from each other. They are defined as follows:

 $B_1 = Verbal/Linguistic Intelligence$

- B₂= Mathematical / Logical Intelligence
- B₃= Visual/Spatial Intelligence
- B₄= Musical/ Rhythmical Intelligence
- B₅= Bodily/Kinesthetic Intelligence
- B_6 = Social/Interpersonal Intelligence
- B_7 = Intrapersonal Intelligence

 B_8 = Naturalist Intelligence [17-21]. However, if we consider intelligence as theoretical sets, we must accept that such sets may be extended on condition that they will have the above said features.

In that case, B_9 , B_{10} , ..., B_n (n= finite) can be accepted as separate intelligences. Gardner explained in an article in 2003 that there were new intelligences and he worked on them [14]. According to the multiple intelligences theorem, an individual owns all the intelligences and uses them in some definite rates. Then, the total intelligence of an individual will be combination of B_1 , B_2 , ..., B_n intelligences.

$$U=B_1 \cup B_2 \cup B_3 \cup \dots \cup B_n \tag{9}$$

Besides, each of intelligences is a separate ability having 4 features; situation of intelligences in respect to each other can be established on two hypotheses. First of them depends on the hypothesis that mutual intersections are empty set.

$$\mathbf{B}_{i} \cap \mathbf{B}_{j} = \left\{ \right\} \text{for } \forall \ i \neq j \tag{10a}$$

Second one depends on the hypothesis that mutual intersections are not empty set.

$$B_i \cap B_i \neq \{\} \text{ for } \forall i \neq j \tag{10b}$$

A. Hypothesis 1: For $B_1, B_2, ..., B_n$ intelligences of each individual, $B_i \cap B_j = \{ \}$ for $\forall i \neq j$.

The basic thought in hypothesis 1 is depended on the view of Gardner. Neuro psychology and development expert Garner who examined damaged brains after an accident or disease observed separate abilities that worked independently from each other in a manner when one of them was damaged others remained healthy. It was concluded that human brain consisted of different sections each having special functions. As a result of the research findings on mental disorders resulting from brain damages it was concluded that when some section of human brain was damaged, people could show performance in some definite fields and continued their life with the remaining sections of their brain. Losing of one of the abilities because of damage on brain or its isolation is independent from other abilities [3]. Depending on this thought, we can say that intelligences are separate from each other. As the intelligence levels in B₁, B₂, ..., B_n intelligences of different individuals will be different, for instance intelligence levels for X individual in age of k can be shown as $P(B_{k1})$, $P(B_{k2})$, $P(B_{k3})$,..., $P(B_{kn})$. Total intelligence of X individual in age of k is;

$$\mathbf{U}_{\mathbf{k}} = \mathbf{B}_{\mathbf{k}1} \cup \mathbf{B}_{\mathbf{k}2} \cup \mathbf{B}_{\mathbf{k}3} \cup \dots \cup \mathbf{B}_{\mathbf{k}n} \tag{11}$$

and total intelligence level of individual X of age k is;

$$P(U_k) = P(B_{k1}) + P(B_{k2}) + P(B_{k3}) + \dots + P(B_{kn})$$
(12)

If we try to explain the total intelligence level of Individual X of age k through Bayesian Theorem, then S indicates the total intelligence of individual X of age k and P(S) indicates his intelligence level. His intelligence used in each intelligence level will be $S \cap B_i$ and his intelligence level will be $P(S \cap B_i)$. Then, the total intelligence used by individual X of age k is;

$$S = (S \cap B_1) \cup (S \cap B_2) \cup (S \cap B_3) \cup \dots \cup (S \cap B_n)$$
(13)

and his intelligence level is

$$P(S) = P(S \cap B_1) + P(S \cap B_2) + P(S \cap B_3) + \dots + P(S \cap B_n)$$
(14)

The individuals who are criticized with some definite intelligence level in the social evaluation (no numeric intelligence) are in fact considered with their intelligence levels they got from abilities and not with the potential intelligences they own. This case is the part of intelligence level owned by Individual X of age k and the conditional probability is:

$$P(S / B_i) = \frac{P(S \cap B_i)}{P(B_i)}$$
(15)

This rate expresses which percentage is explained or shown from B_i intelligence in total S intelligence used by Individual X of age k. Then, this demonstrated intelligence level is not the intelligence of the individual but it is his B_i intelligence share in his S total. This event is shown with Venn diagram in Fig. 1. Let's try to explain the above statement for B_3 intelligence by using Fig. 1. The rate of B_3 intelligence of individual in S total intelligence can be defined as the ratio of A(EFGH) to A(ABCD).

$$P(S/B_3) = \frac{A(EFGH)}{A(ABCD)}$$
(16)

If we reconsider (15) for the explanation of intelligence with conditional probability then, the rate will be;

$$P(S/B_i) = \frac{P(S \cap B_i)}{P(B_i)}$$
(17)

If we draw $P(S \cap B_i)$ from that equation, it will be;

$$P(S \cap B_i) = P(B_i)P(S / B_i)$$
(18)

S indicates the total intelligence of Individual X of age k and P(S) indicates the measure of his intelligence. Then;

$$P(S) = \sum_{i=1}^{n} P(S \cap B_i)$$
(19)

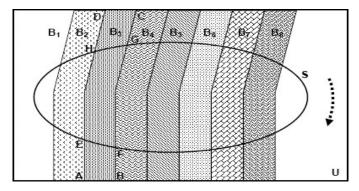


Fig. 1 Indication of intelligences separately from each other

If (18) is written in place of (19), then;

$$P(S) = \sum_{i=1}^{n} P(S \cap B_i) = \sum_{i=1}^{n} P(B_i) P(S / B_i)$$
(20)

Now, by considering the intelligences acquired from abilities, let's try to find the intelligence of individual belongs to only one of the abilities from the intelligence used by individual X of age k. Total of intelligences owned by Individual X of age k from all of his abilities was S. If S is known, its share in B_i can be found with conditional probability in the following equation;

$$P(B_i / S) = \frac{P(S \cap B_i)}{P(S)}$$
(21)

If $P(S \cap B_i)$ is drawn from (21), then;

$$P(S \cap B_i) = P(S)P(B_i / S)$$
(22)

If (18) and (22) are equalized;

$$P(S \cap B_i) = P(B_i)P(S/B_i) \text{ and } P(S \cap B_i) = P(S)P(B_i/S),$$

$$P(S)P(B_i/S) = P(B_i)P(S/B_i)$$
(23)

If $P(B_i / S)$ is drawn from (22), then;

$$P(B_i / S) = \frac{P(B_i)P(S / B_i)}{P(S)}$$
(24)

This statement shows that Bayesian Theorem is compatible with the multiple intelligences or multiple intelligences can be explained with Bayesian Theorem. As the intelligence level used by Individual X of age k consists of his abilities that are different from his actual intelligence level, it follows that the intelligence suggested by Gardner can be changed and developed. When P(S) intelligence level can be extended, $P(B_i)$ intelligence level will also be extended.

Example: To be able to be a teacher, 25% B_1 intelligence, 30% B_2 intelligence, 10% B_3 intelligence and 35% B_4 intelligence are required. Mr. Baha is known as a teacher. Mr. Baha uses 2% of B_1 intelligence, 7% of B_2 intelligence, 5% of B_3 intelligence and 12% of B_4 intelligence. Mr. Baha exhibits behavior like a teacher. What is the probability of such behavior that may result from B_3 intelligence?

Let's consider the behavior exhibited by Mr. Baha is S.

$$P(B_{3} / S) = \frac{P(B_{3})P(S / B_{3})}{P(S)}$$

$$= \frac{P(B_{3})P(S / B_{3})}{P(B_{1})P(S / B_{1}) + P(B_{2})P(S / B_{2}) + P(B_{3})P(S / B_{3}) + P(B_{4})P(S / B_{4})}$$

$$P(B_{1}) = 0.25, P(B_{2}) = 0.30, P(B_{3}) = 0.10, P(B_{4}) = 0.35,$$

$$P(S / B_{1}) = 0.02, P(S / B_{2}) = 0.07,$$

$$P(S / B_{3}) = 0.05, P(S / B_{4}) = 0.12,$$

$$P(B_{3})P(S / B_{3})$$

$$P(B_3/S) = \frac{(3)(3)}{P(B_1)P(S/B_1)+P(B_2)P(S/B_2)+P(B_3)P(S/B_3)+P(B_4)P(S/B_4)}$$
$$\frac{(0.10)(0.05)}{(0.25)(0.02)+(0.30)(0.07)+(0.10)(0.05)+(0.35)(0.12)}$$
$$= \frac{5}{73} = 0.0685$$

The probability of behavior exhibited by Mr. Baha that may result from B_3 intelligence is 6.85%.

Example: Acquisition or establishment of ability depends on 50% B_1 intelligence, 35% B_2 intelligence and 15% B_3 intelligence. In establishment of that behavior, Ahmet uses 6% of B_1 intelligence, 6% of B_2 intelligence and 15% of B_3 intelligence. Ahmet exhibits that behavior. What is the probability of that behavior that may result from B_1 intelligence?

Let's consider the behavior exhibited by Ahmet is S.

$$P(B_{1} / S) = \frac{P(B_{1})P(S / B_{1})}{P(S)}$$

$$P(B_{1} / S) = \frac{P(B_{1})P(S / B_{1})}{P(B_{1})P(S / B_{1}) + P(B_{2})P(S / B_{2}) + P(B_{3})P(S / B_{3})}$$

$$P(B_{1}) = 0.50 \quad P(B_{2}) = 0.35 \quad P(B_{3}) = 0.15$$

 $P(S/B_1) = 0.06$ $P(S/B_2) = 0.03$ $P(S/B_3) = 0.15$

$$P(B_1 / S) = \frac{P(B_1)P(S/B_1)}{P(B_1)P(S/B_1) + P(B_2)P(S/B_2) + P(B_3)P(S/B_3)}$$
$$P(B_1 / S) = \frac{0.50 * 0.06}{(0.50 * 0.06) + (0.35 * 0.03) + (0.15 * 0.15)}$$

$$P(B_1 / S) = \frac{30}{63} = 0.476$$

The probability of that behavior that may result from B_1 intelligence is 47.6%.

B. Hypothesis 2: For $B_1, B_2, ..., B_n$ intelligences of each individual, $\forall i \neq j$ için $B_j \cap B_j \neq \{\}$.

The following statements can be made regarding the principles of multiple intelligence theory;

- People have many different intelligence kinds.
- Each people have a special intelligence mixture consists of actively used intelligences. Each people have a peculiar intelligence profile.
- Each of intelligences has a different development process.
- All the intelligences are dynamic.
- Intelligences can be defined and developed.
- Each people have the chance to develop and recognize his own intelligence.
- Development of each intelligence can be evaluated within itself.
- Each of intelligence has a different system with respect to memory, attention, cognition and problem solving.
- While using one of intelligences other intelligences can also be used.
- Personal substructure, culture, heritage and beliefs have an effect on development of intelligence.
- All intelligences are different and special sources in way of realization of himself of each individual.
- All the scientific theories that evaluate human development support the intelligence theory.
- There may be intelligences different from the existing intelligence kinds.

One of the statements above, the statement "While using one of intelligences other intelligences can also be used "means there are common values between two intelligences [22]. Intelligences generally work together by establishing confusing structures because of none of intelligences in real life works alone except the idiots and brain damaged patients. Intelligences are always in interaction. The eight intelligence kinds work together peculiar to each person [6]. From such statements we can say that intelligences can not be separated from each other.

As the intelligence levels of different individuals in their $B_1, B_2, ..., B_n$ intelligences will be different, the intelligence levels of Individual X of age k can be shown as $P(B_{k1})$, $P(B_{k2}), P(B_{k3}), ..., P(B_{kn})$. Total intelligence of Individual X of

age k is;

$$U_k = B_{k1} \cup B_{k2} \cup B_{k3} \cup \dots \cup B_{kn}$$

$$(25)$$

Total intelligence level of Individual X of age k is

$$P(U_k) < P(B_{k1}) + P(B_{k2}) + P(B_{k3}) + \dots + P(B_{kn})$$
(26)

If we try to explain the total intelligence level of Individual X of age k through Bayesian Theorem, then S indicates the total intelligence of Individual X of age k and P(S) indicates his intelligence level. His intelligence used in each intelligence level will be $S \cap B_i$ and his intelligence level will be $P(S \cap B_i)$. Then, the total intelligence used by Individual X of age k is;

$$\mathbf{S} = (\mathbf{S} \cap \mathbf{B}_1) \cup (\mathbf{S} \cap \mathbf{B}_2) \cup (\mathbf{S} \cap \mathbf{B}_3) \cup \dots \cup (\mathbf{S} \cap \mathbf{B}_n)$$
(27)

and his intelligence level is,

$$P(S) < P(S \cap B_1) + P(S \cap B_2) + P(S \cap B_3) + \ldots + P(S \cap B_n)$$

$$(28)$$

If we consider Hypothesis 2 for two intelligences like B_1 and B_2 to explain the statement in simpler way, Fig. 2 can be used for graphical representation.

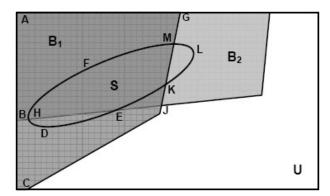


Fig. 2 Indication of B₁ and B₂ intelligences

The rate of B_1 intelligence of an individual in his S total intelligence;

$$P(S/B_1) = \frac{P(S \cap B_1)}{P(B_1)} \text{ and } P(S \cap B_1) = P(B_1)P(S/B_1)$$
(29)

If S is known, its share in B_1 will be found from the equation below.

$$P(B_1 / S) = \frac{P(S \cap B_1)}{P(S)} = \frac{A(DEKMFH)}{A(S)}$$
(30)

If $P(S \cap B_1)$ is drawn from (30), then:

$$P(S \cap B_1) = P(S)P(B_1 / S)$$
(31)

Similarly, if S intelligence of individual is known, it shares in B_2 intelligence will be found as follows.

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$$P(B_2 / S) = \frac{P(S \cap B_2)}{P(S)} = \frac{A(HEKLMF)}{A(S)}$$
(32)

If we draw $P(S \cap B_2)$ from (32), then;

$$P(S \cap B_2) = P(S)P(B_2 / S)$$
(33)

As intersection of B_1 and B_2 intelligences is not empty, the intelligence level of intersection will be

$$P(B_1 \cap B_2) = A(BJGA) \text{ and } P(S) = P[(S \cap B_1) \cup (S \cap B_2)]$$
$$P(S) = P(S \cap B_1) + P(S \cap B_2) - P(S \cap B_1 \cap B_2)$$

If we show such intelligence levels as an area;

$$P(S \cap B_1) \cup (S \cap B_2) = A(DEKMF) + A(HEKLM) - A(HEKMF)$$

If we equalize (29) and (31), then;

$$P(S \cap B_1) = P(B_1)P(S / B_1)$$
 and $P(S \cap B_1) = P(S)P(B_1 / S)$, then

$$P(S)P(B_1/S) = P(B_1)P(S/B_1)$$
(34)

If $P(B_1 / S)$ is drawn from here, then;

$$P(B_1/S) = \frac{P(B_1)P(S/B_1)}{P(S)} = \frac{P(B_1 \cap S)}{P(S \cap B_1) + P(S \cap B_2) - P(S \cap B_1 \cap B_2)}$$
(35)

Now, let's try to explain in which percentage the intelligence of Individual X of age k consists of his existing abilities that uses his total intelligence. The Venn diagram of intersections of $B_1, B_2, ..., B_7, B_8,$ intelligences that are not empty is given in Fig. 3. If we write (30) for any B_i , the following equation is found.

$$P(B_i / S) = \frac{P(S \cap B_i)}{P(S)}$$
(36)

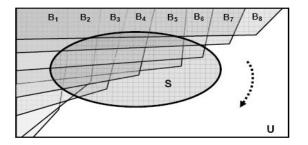


Fig. 3 Indication of intelligences of which intersections are not empty

The difficulty here, as it can be seen in (30), is that there is no P(S). As the intelligences to be operated increase, the term number in calculation of P(S) will also increase. For example, for B_1 and B_2 intelligences, the term number in calculation of

P(S) is
$$\binom{2}{1} + \binom{2}{2} = 3$$
 and for B₁, B₂, B₃ and B₄ intelligences, the

term number in calculation of P(S) is $\binom{4}{1} + \binom{4}{2} + \binom{4}{3} + \binom{4}{4} = 15$.

Similarly, the term number necessary in calculation of P(S) for 8 intelligences is a total of 245. In other words, the expression of

$$P(S) = P[(S \cap B_1) \cup (S \cap B_2) \cup (S \cap B_3) \cup \dots (S \cap B_7) \cup (S \cap B_8)]$$

must be calculated. This means calculation will be very difficult.

Example: To be able to be an architect, 65% B₁ and 50% B₂ intelligences are required. Mr. Altan is known as an architect and can use B₁ and B₂ intelligences together in rate of 15%. Architect Mr. Altan exhibits a behavior for which he does not use 10% of B₁ intelligence and 5% of B₂ intelligence. What is the probability of that behavior that may result from B₁ intelligence?

Let's consider that behavior exhibited by Mr. Altan is S event.

$$P(B_{1}/S) = \frac{P(B_{1})P(S/B_{1})}{P(S)} = \frac{P(B_{1} \cap S)}{P(S \cap B_{1}) + P(S \cap B_{2}) - P(S \cap B_{1} \cap B_{2})}$$
$$P(B_{1}/S) = \cdot \frac{0.55}{0.55 + 0.45 - 0.15} = \frac{55}{85} = 0.647$$

The probability of that behavior that may result from B_1 intelligence is 64.7%.

IV. CONCLUSION

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Although there are many applications of Bayesian Theorem in different fields of academic literature, such as business, engineering, marketing and so on, multiple intelligence theory has not been modeled mathematically yet, including the Bayesian Theorem. In this study, it is shown that multiple intelligence theory can be modeled mathematically with Bayesian theorem. It is also shown that the number of multiple intelligences is generalized to n. Based on the hypothesis of Gardner that intelligences must be considered separate from each other, a modeling was made for separated intelligences. Based on the hypothesis of Armstrong that intelligences always interacted from each other and eight different intelligences work together peculiar to each individual, a modeling for none-separated intelligences was made. While the modeling process for each of intelligences that is separate from each other was realized very clearly, modeling when intersections of intelligences are not empty and requires great number of operations. The modeling is suitable for the intelligences yet to be established and also supports the statement given by Gardner in the 2003 article that he continued his studies on multiple intelligences. Following discussions with other academicians, there was no clear opinion as to which hypothesis as definitely valid.

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Therefore, both hypotheses were included in the study.

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