Probability of Failure on Demand for Systems with Partial Stroke Test

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***Abstract*—**The average Probability of Failure on Demand (PFD) considering the Proof Test interval is one possibility to compare different safety-related systems. In this paper we intend to derive the average PFD for a 1oo1 system taking into account the Proof Test as well as the Partial Stroke Test (PST). Thereby we will specify a unique mathematic function without any help of a probability band. Doing so, we get, on the one hand, additional correlations between the reduction of PFD and the diagnostic coverage factor, and on the other hand, between the PFD value of a system without PST and a system with PST. Finally we will present an approximation in order to calculate the PFD value, if the ratio between the PST interval and the Proof Test interval is very small.

***Keywords*—**1oo1-system, Diagnostic Coverage Factor, Partial Stroke Test, Probability of Failure on Demand, Proof Test, Relative Probability of Failure Reduction

I. INTRODUCTION

For any developers of safety-related systems, it is a challenge to extend the Proof Test interval for safety-related applications and to get, at the same time, an identical or, even better, a smaller Probability of Failure on Demand (*PFD*). In the standard IEC/EN 61508 a Proof Test is defined as a "periodic test performed to detect failures in a safety-related system so that, if necessary, the system can be restored to an "as new" condition or as close as practical to this condition" [1]. One possibility to extend the Proof Test interval is the use of Partial Stroke Tests (PST). These tests may be performed between two Proof Tests, manually or automatically, sometimes or frequently. In the scientific literature there are only a few approaches to describe mathematically the *PFD* of safety-related Systems using the PST [2], [3], [4]. The parameters in Table I are necessary to understand the equations shown in this paper.

TABLE I
PARAMETERS

PFD_{avg}	Probability of Failure on Demand, average
λ_{DU}	Rate for dangerous, undetectable failures
DC_{PST}	Diagnostic coverage factor for a PST
t_{PST}	Time of the PST
t_{PT}	Time of the proof-test

In [2] the average *PFD*, $PFD_{avg1}(t)$, of a system with PST according to (1) is calculated.

$$PFD_{avg1}(t) = \frac{1}{2} \cdot DC_{PST} \cdot \lambda_{DU} \cdot t_{1.PST} + \frac{1}{2} \cdot (1 - DC_{PST}) \cdot \lambda_{DU} \cdot t_{PT} \tag{1}$$

In [3] and [4] it is additionally kept in mind that the *PFD* value after a successful PST only depends on failures that have not been detected by the PST. Therefore, two equations are necessary to describe by mathematics this issue, see (2) and (3).

$$PFD_{avg1,w.PST}(t) = PFD_{avg1}(t) \tag{2}$$

$$PFD_{avg1,a.PST}(t) = \frac{1}{2} \cdot (1 - DC_{PST}) \cdot \lambda_{DU} \cdot t_{PT} \tag{3}$$

So the average *PFD* can only be described by an average band of probability, Fig. 1.

The calculation of a *PFD* for a 1oo1 system without a PST is described in e. g. [4] - [8].

For a 1oo1 system with PST in this paper we specify a unique mathematic function without a helping probability band. Doing so, we get, on the one hand, additional correlations between the reduction of *PFD* and the diagnostic coverage factor, and on the other hand, between the *PFD* value of a system without PST and a system with PST.

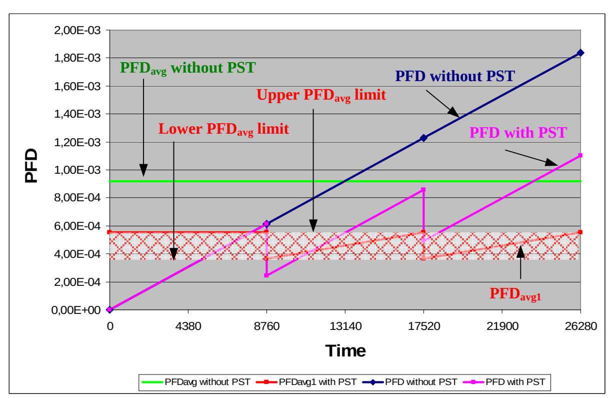

Fig. 1 PFD_{avg1} with λ_{DU} = 7*10^{-8} 1/h, t_{PT} = 3 years, t_{PST} = 12 months, DC_{PST} = 60 %

II. 1OO1-SYSTEM WITH PST, $PFD_{AVG3,w.PST}$

With the method of calculation presented here, which as far we know, hasn't been considered yet, one can give a constant value as the average value for the *PFD* value. The principle is, that at first an average value will be appraised for each PST interval following the well known method

$$E(t)=\frac{1}{t_1-t_0}\cdot\int_{t_0}^{t_1} f(t)\,dt \tag{1}$$

Finally the average value will be generated via all appraised single average value.

The *PFD* value between two PSTs will be appraised using the following equation and will occur in sections.

$$PFD=\begin{cases}\lambda_{DU}\cdot t & 0\le t<t_{1.\,PST}\\ \lambda_{DU}\cdot t-DC_{PST}\cdot\lambda_{DU}\cdot t_{1.\,PST} & t_{1.\,PST}\le t<t_{2.\,PST}\\ \lambda_{DU}\cdot t-DC_{PST}\cdot\lambda_{DU}\cdot t_{2.\,PST} & t_{2.\,PST}\le t<t_{3.\,PST}\\ \vdots & \vdots\\ \lambda_{DU}\cdot t-DC_{PST}\cdot\lambda_{DU}\cdot t_{last\ PST} & t_{last\ PST}\le t<t_{PT}\end{cases}$$

The average value for the functions defined in sections will be calculated with (4). The result is:

For the interval $0\le t<t_{1.\,PST}$:

$$PFD_{avg\,3}=\frac{1}{2}\cdot\lambda_{DU}\cdot t_{1.PST} \tag{5}$$

For the interval $t_{1.\,PST}\le t<t_{2.\,PST}$:

$$\begin{aligned}PFD_{avg\,3}&=\frac{1}{2}\cdot\lambda_{DU}\cdot\frac{1}{t_{1.PST}}\cdot\left(t_{2.PST}^{\;2}-t_{1.PST}^{\;2}\right)\\&\quad-DC_{PST}\cdot\lambda_{DU}\cdot\left(t_{2.PST}-t_{1.PST}\right)\\&=\frac{1}{2}\cdot\lambda_{DU}\cdot\frac{t_{2.PST}^{\;2}}{t_{1.PST}}-\lambda_{DU}\cdot t_{1.PST}\cdot\left(\frac{1}{2}+DC_{PST}\right)\end{aligned} \tag{6}$$

For the interval $t_{2.\,PST}\le t<t_{3.\,PST}$:

$$\begin{aligned}PFD_{avg\,3}&=\frac{1}{2}\cdot\lambda_{DU}\cdot\frac{t_{3.PST}^{\;2}}{t_{1.PST}}\\&\quad-\lambda_{DU}\cdot\left(\frac{1}{2}\cdot\frac{t_{2.PST}^{\;2}}{t_{1.PST}}+DC_{PST}\cdot t_{2.PST}\right)\end{aligned} \tag{7}$$

For the following PST interval, the corresponding (7) will be used.

For the last interval $t_{last\ PST}\le t<t_{PT}$, which ends with the Proof-Test time t_{PT}:

$$\begin{aligned}PFD_{avg\,3}&=\frac{1}{2}\cdot\lambda_{DU}\cdot\frac{t_{PT}^{\;2}}{t_{1.PST}}\\&\quad-\lambda_{DU}\cdot\left(\frac{1}{2}\cdot\frac{t_{last\ PST}^{\;2}}{t_{1.PST}}+DC_{PST}\cdot t_{last\ PST}\right)\end{aligned} \tag{8}$$

Once all PFD_{avg3} have been appraised in sections, the $PFD_{avg3,\ w.PST}$ will be defined via the average value of all PFD_{avg3}:

$$PFD_{avg\,3,\,w.PST}=\frac{1}{\text{Number of PST}}\cdot\sum PFD_{avg\,3} \tag{9}$$

With the same parameters as used for Fig. 1 we get the $PFD_{avg3,w.PST}$ value and the trajectory as shown in Fig. 2.

III. COHERENCE BETWEEN $PFD_{w.PST}$ AND $PFD_{AVG3,\ w.PST}$

The approach presented up to now to appraise the $PFD_{avg3,\ w.PST}$ consists at first to appraise the single PFD_{avg} value between two PSTs and then to calculate the average value via all PFD_{avg}.

Underneath, it should be attested, as far as we know for the first time, that the coherence between *PFD* and $PFD_{avg3,\ w.PST}$

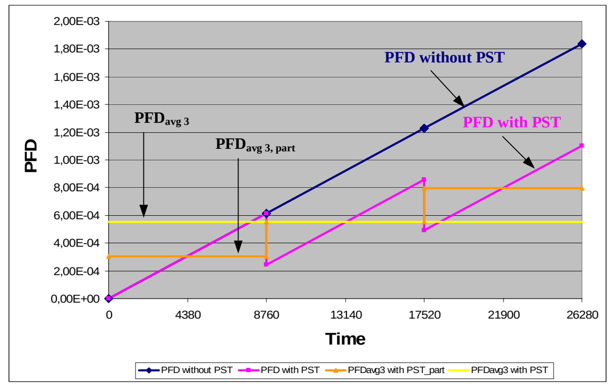

Fig. 2 PFD_{avg3} with λ_{DU} = 7*10^{-8} 1/h, t_{PT} = 3 years, t_{PST} = 12 months, DC_{PST} = 60 %

exists. Thereby it will be provided that the intervals between two PSTs are identical, though this is not necessary, as one can easily demonstrate, e. g. with the help of the Riemann's integrable criteria [9]. This attests that each defined and limited function $f(x)$ in $[a, b]$ is than exactly integrated via $[a, b]$, if this one has an endless number of discontinuity on $[a, b]$. Than the integral will be calculated via the function $f(x)$ through the separation of the interval $[a, b]$ into endless small intervals [9].

Generally the following equation counts for the PFD_{avg}, successively written with $PFD_{part\ av}$, in *n*-ten PST-interval (generalization of (7))

$$\begin{aligned}PFD_{part\ av}&=\frac{1}{2}\cdot\frac{\lambda_{DU}}{t_{1.PST}}\cdot\left[t_{n.PST}^{\;2}-t_{(n-1).PST}^{\;2}\right]\\&\quad-\lambda_{DU}\cdot DC_{PST}\cdot t_{\,(n-1).PST}\end{aligned} \tag{10}$$

with

$$t_{(n-1).PST}=t_{n.PST}-t_{1.PST} \tag{11}$$

follows

$$PFD_{part\ av}=\left(1-DC_{PST}\right)\cdot\lambda_{DU}\cdot t_{n.PST}+A \tag{12}$$

whereas

$$A=\left(DC_{PST}-\frac{1}{2}\right)\cdot\lambda_{DU}\cdot t_{1.PST} \tag{13}$$

is. A detailed derivation of these equations can be found in

[10].

In order to calculate the average value ($PFD_{avg3,\,w.PST}$) via all $PFD_{part\ av}$, it will be provided that n intervals exist. Thereby the following equation should count, what would otherwise be a limit of the demonstration:

$k = 1\ldots n$

$t_{n.PST} = t_{PT}$,

It means, that the time of the *n*. PSTs coincides with the time of the Proof Test, and

$t_{0.PST} = 0$, the time of the process to be defined.

Firstly the single $PFD_{part\ av}$ of all *n* intervals will be calculated.

With $t_k = k \cdot t_{1.PST}$ (14)

it results in

$$\begin{aligned} k = 1: \quad PFD_{part\ av} &= (1 - DC_{PST}) \cdot \lambda_{DU} \cdot 1 \cdot t_{1.PST} + A \\ &= (1 - DC_{PST}) \cdot \lambda_{DU} \cdot 1 \cdot t_{1.PST} + \\ &\quad \left(DC_{PST} - \frac{1}{2}\right) \cdot \lambda_{DU} \cdot t_{1.PST} \\ &= \frac{1}{2} \cdot \lambda_{DU} \cdot t_{1.PST} \end{aligned}$$

which corresponds to (5)! And for

$$\begin{aligned} k = 2: \quad PFD_{part\ av} &= (1 - DC_{PST}) \cdot \lambda_{DU} \cdot 2 \cdot t_{1.PST} + A \\ k = 3: \quad PFD_{part\ av} &= (1 - DC_{PST}) \cdot \lambda_{DU} \cdot 3 \cdot t_{1.PST} + A \\ k = 4: \quad PFD_{part\ av} &= (1 - DC_{PST}) \cdot \lambda_{DU} \cdot 4 \cdot t_{1.PST} + A \end{aligned}$$

etc. up to

$$\begin{aligned} k = n: \quad PFD_{part\ av} &= (1 - DC_{PST}) \cdot \lambda_{DU} \cdot n \cdot t_{1.PST} + A \\ &= \ldots \\ &= \lambda_{DU} \cdot t_{n.PST} - \frac{1}{2} \cdot \lambda_{DU} \cdot t_{1.PST} \\ &\quad - DC_{PST} \cdot \lambda_{DU} \left(t_{n.PST} - t_{1.PST}\right) \end{aligned}$$

This equation is the same as (8) for the PST-interval of $t_{last\ PST} \le t < t_{PT}$, as shown in the following calculation:

$$\begin{aligned} k = n: \quad PFD_{part\ av} &= (1 - DC_{PST}) \cdot \lambda_{DU} \cdot n \cdot t_{1.PST} + A \\ &= (1 - DC_{PST}) \cdot \lambda_{DU} \cdot t_{n.PST} + \\ &\quad \left(DC_{PST} - \frac{1}{2}\right) \cdot \lambda_{DU} \cdot t_{1.PST} \\ &= \lambda_{DU} \cdot t_{n.PST} - \frac{1}{2} \cdot \lambda_{DU} \cdot t_{1.PST} - \\ &\quad DC_{PST} \cdot \lambda_{DU} \left(t_{n.PST} - t_{1.PST}\right) \end{aligned}$$

with (12) and with $t_{n.PST} = t_{PT}$ and

$t_{n.PST} - t_{1.PST} = t_{last\ PST}$ results for (see [10])

$$\begin{aligned} k = n: \quad PFD_{part\ av} &= \frac{1}{2} \cdot \frac{\lambda_{DU}}{t_{1.PST}} \cdot \left[t_{PT}{}^2 - t_{last\ PST}{}^2 \right] \\ &\quad - \lambda_{DU} \cdot DC_{PST} \cdot t_{last\ PST} \end{aligned}$$

which is the same as in (8).

When the sum is made via all $PFD_{part\ av}$, i. e from $k = 1$ up to $k = n$, so results in:

$$\begin{aligned} \Sigma PFD_{part\ av} &= (1 - DC_{PST}) \cdot \lambda_{DU} \cdot \\ &\quad (1 + 2 + 3 + 4 + \ldots + n) \cdot t_{1.PST} + n \cdot A \\ &= (1 - DC_{PST}) \cdot \lambda_{DU} \cdot \frac{n \cdot (n+1)}{2} \cdot t_{1.PST} \\ &\quad + n \cdot A \end{aligned} \tag{15}$$

To calculate the average value of the $PFD_{avg3,\,w.PST}$ from (15), this equation must still be divided over the amount of PST intervals:

$$\begin{aligned} PFD_{avg3,\,w.PST} &= \frac{1}{n} \cdot \Sigma PFD_{part\ av} \\ &= (1 - DC_{PST}) \cdot \lambda_{DU} \cdot \frac{(n+1)}{2} \cdot t_{1.PST} + A \\ &= \frac{1}{2} \cdot (1 - DC_{PST}) \cdot \lambda_{DU} \cdot (n+1) \cdot t_{1.PST} + A \end{aligned} \tag{16}$$

With $(n+1) \cdot t_{1.PST} = t_{PT} + t_{1.PST}$ and (13)

$A = \left(DC_{PST} - \frac{1}{2}\right) \cdot \lambda_{DU} \cdot t_{1.PST}$

it results in

$$\begin{aligned} PFD_{avg3,\,w.PST} &= \frac{1}{2} \cdot (1 - DC_{PST}) \cdot \lambda_{DU} \cdot (t_{PT} + t_{1.PST}) + \\ &\quad \left(DC_{PST} - \frac{1}{2}\right) \cdot \lambda_{DU} \cdot t_{1.PST} \\ &= \ldots \\ &= \frac{1}{2} \cdot [(1 - DC_{PST}) \cdot \lambda_{DU} \cdot t_{PT} + \\ &\quad DC_{PST} \cdot \lambda_{DU} \cdot t_{1.PST}] \end{aligned} \tag{17}$$

Compare this equation with

$$\begin{aligned} PFD_{w.\,PST}(t) &= DC_{PST} \cdot \lambda_{DU} \cdot t_{1.\,PST} + \\ &\quad (1 - DC_{PST}) \cdot \lambda_{DU} \cdot t \\ &= \lambda_{DU} \cdot [DC_{PST} \cdot t_{1.\,PST} + \\ &\quad (1 - DC_{PST}) \cdot t] \end{aligned} \tag{18}$$

to time $t = t_{PT}$:

$$PFD_{w.\,PST}(t = t_{PT}) = (1 - DC_{PST}) \cdot \lambda_{DU} \cdot t_{PT} + DC_{PST} \cdot \lambda_{DU} \cdot t_{1.\,PST} ,$$

so one finds, that counts:

$$PFD_{avg3,\,w.PST} = \frac{1}{2} \cdot PFD_{w.\,PST}(t = t_{PT}) . \tag{19}$$

IV. Coherence between the relative probability of failure reduction B_{1REL} and the DC_{PST} factor

Assuming that all PST intervals have the same length, i. e. $t_{n.PST} = t_{1.PST}$, and so further the inequation $t_{1.PST} \ll t_{PT}$ counts, a coherence between the relative probability of failure reduction B_{1rel} and the DC_{PST} factor should be appraised in the following. For the probability of failure reduction B_1 counts at time t_{PT} of the Proof Test the equation, see [3] ($PFD_{wo.PST}$: *PFD* value for a system without a PST; $PFD_{w.PST}$: *PFD* value for a system with a PST):

$$\begin{aligned} B_1(t = t_{PT}) &= PFD_{wo.\,PST}(t = t_{PT}) - PFD_{w.\,PST}(t = t_{PT}) \\ &= \lambda_{DU} \cdot DC_{PST} \cdot (t_{PT} - t_{1.\,PST}) \end{aligned} \tag{20}$$

A relative probability of failure reduction B_{1rel} can therefore at time t_{PT} be defined as follow:

$$\begin{aligned} B_{1rel}(t = t_{PT}) &= \frac{B_1(t = t_{PT})}{PFD_{wo.\,PST}(t = t_{PT})} \\ &= \frac{\lambda_{DU} \cdot DC_{PST} \cdot (t_{PT} - t_{1.\,PST})}{\lambda_{DU} \cdot t_{PT}} \\ &= DC_{PST} \cdot \left(1 - \frac{t_{1.\,PST}}{t_{PT}}\right) \end{aligned} \tag{21}$$

Provided that $t_{1.PST} << t_{PT}$ one obtains the approximation

$$B_{1rel}(t = t_{PT}) \approx DC_{PST}\,. \tag{22}$$

If one dissolves (21) according to $PFD_{w.\,PST}(t = t_{PT})$ this would mean for practical application, provided that $t_{1.PST} << t_{PT}$ counts, the following:

$$\begin{aligned} PFD_{w.\,PST}(t = t_{PT}) &= PFD_{wo.\,PST}(t = t_{PT}) - \\ &\quad B_{1rel}(t = t_{PT}) \cdot PFD_{wo.\,PST}(t = t_{PT}) \\ &= PFD_{wo.\,PST}(t = t_{PT}) \cdot \\ &\quad (1 - B_{1rel}(t = t_{PT})) \\ &= PFD_{wo.\,PST}(t = t_{PT}) \cdot (1 - DC_{PST}) \end{aligned} \tag{23}$$

It means that the probability of failure of a system with PST at time $t = t_{PT}$ depends only, provided that, $t_{1.PST} << t_{PT}$ counts, on the probability of failure of a system without PST and the DC_{PST} factor!

To underline the validity of this statement, the results achieved in this paper will be compared with each other. In a first work following parameters are given:

$t_{PT} = 3\ Jahre = 26280\ h$ and

$t_{1.PST} = 4380\ h = 0{,}5$ years

$// = 8760\ h = 1$ year

$// = 13140\ h = 1{,}5$ years

The chosen failure rate is

$$\lambda_D = \lambda_{DU,\,PT} = 7 \cdot 10^{-8}\,\frac{1}{h}\,.$$

The results for the relative probability of failure reduction $B_{1rel.}$ according to (21) are shown in Table II.

To check the validity of (23), the *PFD* value for the exact value for $PFD_{w.\,PST}(t = t_{PT})$ according to (18) as well as the one from (23) calculated value and the relative difference of both values are given in Table III. The ratio between $t_{1.PST}$ and t_{PT} averages there

for $t_{1.PST} = 4380$ h: $\dfrac{t_{1.PST}}{t_{PT}} = \dfrac{4380\ h}{26280\ h} = 0{,}1666$

for $t_{1.PST} = 8760$ h: $\dfrac{t_{1.PST}}{t_{PT}} = \dfrac{8760\ h}{26280\ h} = 0{,}3333$

for $t_{1.PST} = 13140$ h: $\dfrac{t_{1.PST}}{t_{PT}} = \dfrac{13140\ h}{26280\ h} = 0{,}5$

As one can see on the values in Table III a ratio $\dfrac{t_{1.PST}}{t_{PT}} = \dfrac{4380\ h}{26280\ h} = 0{,}1666$ is not satisfying to become an adequate small difference between the exact *PFD* and the approximated *PFD*. In this case it means that the *PFD* value must be calculated with the exact formula for a system with PST.

To check the validity of (23), the *PFD* value for the exact value of $PFD_{w.\,PST}(t = t_{PT})$ according to (18) and the ones from (23) calculated values as well as the relative difference of both values are given in Table V. The ratio between $t_{1.PST}$ and t_{PT} averages there:

TABLE II
TABLE OF VALUE FOR THE REL. PROBABILITY OF FAILURE REDUCTION B_{1rel} WITH $t_{PT} = 3$ *years*

$t_{1.PST}$ \ DC_{PST}	60%	70%	80%	90%
4380	50,00%	58,33%	66,67%	75,00%
8760	40,00%	46,67%	53,33%	60,00%
13140	30,00%	35,00%	40,00%	45,00%

for $t_{1.PST} = 24$ h: $\dfrac{t_{1.PST}}{t_{PT}} = \dfrac{24\ h}{8760\ h} = 0{,}00274$

for $t_{1.PST} = 48$ h: $\dfrac{t_{1.PST}}{t_{PT}} = \dfrac{48\ h}{8760\ h} = 0{,}00548$

The result would be different if one changes the parameters as shown here:

$t_{PT} = 1\ Jahr = 8760\ h$

$t_{1.PST} = 24\ h = \dfrac{1}{365}$ years

$// = 48\ h = \dfrac{2}{365}$ years

$// = 168\ h = \dfrac{7}{365}$ years

The chosen failure rate is

$$\lambda_D = \lambda_{DU,\,PT} = 3{,}8 \cdot 10^{-7}\,\frac{1}{h}\,.$$

In Table IV we can see the result for the relative probability of failure reduction B_{1rel} with these parameter values. It may be assessed that the values of B_{1rel} is nearly equal to the chosen DC_{PST} factor.

for $t_{1.PST} = 168$ h: $\dfrac{t_{1.PST}}{t_{PT}} = \dfrac{168\ h}{8760\ h} = 0{,}01918$

As one can see on the values in Table V, a ratio $\dfrac{t_{1.PST}}{t_{PT}} < 0{,}02$ is satisfying to become an adequate small difference between the exact *PFD* and the approximated *PFD*.

TABLE III
TABLE OF VALUE FOR THE REL. PROBABILITY OF FAILURE REDUCTION B_{1rel} WITH $t_{PT} = 3$ *year*

DC_{PST} / $t_{1.PST}$	60%			70%		
	exac. PFD	app. PFD	rel. Δ in %	exac. PFD	app. PFD	rel. Δ in %
4380	9,20E-04	7,36E-04	20,00	7,67E-04	5,52E-04	28,00
8760	1,10E-03	7,36E-04	33,33	9,81E-04	5,52E-04	43,75
13140	1,29E-03	7,36E-04	42,86	1,20E-03	5,52E-04	53,85
DC_{PST} / $t_{1.PST}$	80%			90%		
	exac. PFD	app. PFD	rel. Δ in %	exac. PFD	app. PFD	rel. Δ in %
4380	6,13E-04	3,68E-04	40,00	4,60E-04	1,84E-04	60,00
8760	8,58E-04	3,68E-04	57,14	7,63E-04	1,84E-04	75,00
13140	1,10E-03	3,68E-04	66,67	1,01E-03	1,84E-04	81,82

It should be observed that the approximation is optimally adapted when the DC_{PST} factor is also small. It means in this case that it is easier to calculate the *PFD* value for a System with PST using the approximation formula.

TABLE IV
TABLE OF VALUE FOR THE REL. FAILURE PROBABILITY REDUCTION B_{1rel} WITH $t_{PT} = 1$ *year*

DC_{PST} / $t_{1.PST}$	50%	65%	75%	85%
24	49,86%	64,82%	74,79%	84,77%
48	49,73%	64,64%	74,59%	84,54%
168	49,04%	63,75%	73,56%	83,37%

TABLE V
TABLE OF VALUE FOR THE REL. FAILURE PROBABILITY REDUCTION B_{1rel} WITH $t_{PT} = 1$ *year*

DC_{PST} / $t_{1.PST}$	50%			65%		
	exac. PFD	app. PFD	rel. Δ in %	exac. PFD	app. PFD	rel. Δ in %
24	1,67E-03	1,66E-03	0,27	1,17E-03	1,17E-03	0,51
48	1,68E-03	1,66E-03	0,82	1,18E-03	1,17E-03	1,00
168	1,69E-03	1,66E-03	1,89	1,20E-03	1,17E-03	2,92
DC_{PST} / $t_{1.PST}$	75%			85%		
	exac. PFD	app. PFD	rel. Δ in %	exac. PFD	app. PFD	rel. Δ in %
24	8,39E-04	8,32E-04	0,82	5,07E-04	4,99E-04	1,53
48	8,48E-04	8,32E-04	1,88	5,16E-04	4,99E-04	3,27
168	8,78E-04	8,32E-04	5,19	5,52E-04	4,99E-04	9,58

V. COHERENCE BETWEEN $PFD_{WO.PST}$ AND $PFD_{AVG3,\,w.PST}$

From both previous chapters III and IV a light coherence between the *PFD* values $PFD_{wo.PST}$, i. e. for a system without PST, and the average *PFD* value, $PFD_{avg3,\ w.PST}$, i. e. for a System with PST, can be established. Provided that, all PST intervals have the same length, i. e. $t_{n.PST} = t_{1.PST}$, and that the inequation $t_{1.PST} << t_{PT}$ counts. It results then from (19) and (23).

$$\begin{aligned} PFD_{avg3,\,w.PST} &= \frac{1}{2}\cdot PFD_{w.\,PST}(t = t_{PT}) \\ &= \frac{1}{2}\cdot PFD_{wo.\,PST}(t = t_{PT})\cdot(1 - DC_{PST}) \end{aligned} \tag{24}$$

To check the validity of this equation the same parameters as mentioned before are used again:

$t_{PT} = 1\ Jahr = 8760\ h$

$t_{1.PST} = 24\ h = \frac{1}{365}$ years

$// = \quad 48\ h = \frac{2}{365}$ years

$// = \quad 168\ h = \frac{7}{365}$ years

The chosen failure rate is

$\lambda_D = \lambda_{DU,\,PT} = 3{,}8\cdot 10^{-7}\,\frac{1}{h}$.

In Table VI the *PFD* values with exact values for $PFD_{avg3,mittel}$ according to (5) up to (9), in (25) are presented

TABLE VI
TABLE OF COMPARISON TO THE EXACTLY PFD VALUE AND THE APPROXIMATED PFD VALUES WITH $t_{PT} = 1$ *year*

DC_{PST} / $t_{1.PST}$	50%			65%		
	exac. $PFD_{avg3,w}$	app. $PFD_{avg3,w}$	rel. Δ in %	exac. $PFD_{avg3,w}$	app. $PFD_{avg3,w}$	rel. Δ in %
24	8,34E-04	8,32E-04	0,27	5,86E-04	5,83E-04	0,51
48	8,39E-04	8,32E-04	0,82	5,90E-04	5,83E-04	1,28
168	8,46E-04	8,32E-04	1,62	6,02E-04	5,83E-04	3,18
DC_{PST} / $t_{1.PST}$	75%			85%		
	exac. $PFD_{avg3,w}$	app. $PFD_{avg3,w}$	rel. Δ in %	exac. PFD	app. PFD	rel. Δ in %
24	4,20E-04	4,16E-04	0,82	2,54E-04	2,50E-04	1,53
48	4,24E-04	4,16E-04	1,88	2,58E-04	2,50E-04	3,27
168	4,39E-04	4,16E-04	5,19	2,76E-04	2,50E-04	9,58

generally,

$$\begin{aligned} PFD_{avg3,\,mittel} &= \frac{1}{n}\cdot\Sigma PFD_{part\ av} \\ &= \frac{1}{n}\cdot\sum_n\Bigg(\frac{1}{2}\cdot\frac{\lambda_{DU}}{t_{1.PST}}\cdot\left[t_{n.PST}{}^2 - t_{(n-1).PST}{}^2\right] \\ &\quad - \lambda_{DU}\cdot DC_{PST}\cdot t_{(n-1).PST}\Bigg) \end{aligned} \tag{25}$$

and the ones from (24) calculated values and the relative difference of both values is given.

As one can see on the values in Table VI, a ratio of

$\frac{t_{1.PST}}{t_{PT}} < 0{,}02$ is satisfying to become an adequate small difference between the exact and the approximate $PFD_{avg3,w.PST}$ value.

It should be observed that the approximation is optimally adapted when the DC_{PST} factor is also small. It means in this case, that it is very easy to calculate the *PFD* value for a system with PST using the approximation formula (24).

VI. Conclusion

In this paper the mathematical coherence between the *PFD* value of a 1oo1 system without PST and the average *PFD* value of a 1oo1 system with PST has been presented. If the relative probability of failure reduction is approximately the DC_{PST} factor or if the ratio between the PST interval and the Proof Test interval is sufficiently small, then for this calculation we can use for calculation of the *PFD* value a simple approximation, see (24).

Advanced studies may deal with other architecture models like 1oo2 or 2oo3 systems. We assume similar coherence between the different *PFD* parameters as examined for the 1oo1 system in this paper.

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