Assessment and Application of 3D Galerkin Finite Volume Explicit Solver for Computation of Seepage and Uplift in Dam Foundation

S.R. Sabbagh-Yazdi, N.E. Mastorakis, and B. Bayat

***Abstract*—** In this paper, development of a Galerkin finite volume three-dimensional seepage solver on mesh of tetrahedral is described. The numerical analyzer is utilized for solving the seepage in porous media and uplift under gravity dams with upstream cut off wall. The results of numerical solver in terms of uplift pressure in natural foundation of a gravity dam with upstream cut off wall are compared with analytical solutions obtained by application of conformal mapping technique for a constant unit ratio of foundation depth over half of dam base ($T/b=1$). The accuracy of the results computed uplift pressure for homogeneous and isotropic condition present acceptable agreements with the analytical solutions for various ratios of cut off wall depth over half of dam base (s/b). Having assessed the accuracy of the model, it is applied to evaluate the quality of the results of the common empirical relations for uplift pressure estimation. In order to present the ability of the verified model to cop with real world problems, it is applied to solve seepage through a natural porous foundation of a gravity dam with three incline layers with different coefficients of permeability.

***Keywords*—** Galerkin Finite Volume Method, Seepage and Uplift, Multilayer Dam Foundation

I. Introduction

THE problem of seepage flow underneath of gravity dams can be formulated in terms of a non-linear partial differential equation. The equation describes a constant density fluid flow in a heterogeneous and isotropic porous media [1].

Although empirical formulations are suggested for simple cases, due to the inherently complex boundary conditions and intricate physical geometries in any practical problem, an analytical solution is not possible for complicated dam foundations [2].

Many researchers have worked on seepage problem analysis and solving its governing equations. These works differ by the technique used to solve the problem.

Jie et al. (2004) presented a finite difference method (FDM) based on boundary-fitted coordinate (BFC) transformation. The curvilinear grid system, with computational boundary being coincident with the physical boundary, is numerically obtained by solving the Poisson equation. Seepage analysis can then be done by FDM in a uniform transformed orthogonal coordinate system [3]. Serafim et al. (1985) proposed a finite difference method to study three-dimensional seepage in anisotropic heterogeneous foundations especially for earth dams [4].

Kiousis (2002) presented a least-squares implementation of the finite-element method to evaluate stream functions in the solution of field problems. The method is programmatically similar to the solution of the Laplace equation, and is based on the development of a stream field that is orthogonal to an already calculated potential field [5]. Griffiths and Fenton (1997) brought together random field generation and finite element techniques to model steady seepage through a three-dimensional soil domain in which the permeability is random distributed in space [6]. Griffiths and Fenton (1998) also combined random field theory and finite element techniques with Monte-Carlo simulations to study the statistics of exit gradient predictions as a function of soil permeability variance and spatial correlation [7]. Boufadel et al. (1999) investigated steady seepage from two-dimensional domains using a dimensionless formulation for variably saturated media [8].

Li et al. (2003) presented an Element-free method (EFM) for seepage analysis with a free surface based on the moving least square method which needs only the information at nodes. It avoids troublesome modifications of the mesh as in the finite element method [9].

Plizzari (1998) studied uplift pressure effects in cracked concrete gravity dams. A parametric study on the influence of uplift pressure on stress intensity factors and crack-propagation angle is performed 10]. Dewey et al. (1994) reviewed and compared the procedures for including uplift pressures in the hand-calculation methods. They proposed three separate models for including uplift pressures in a finite element analysis [11].

This paper presents a Galerkin finite volume method for modeling water flow in a saturated homogeneous porous media with complex boundary systems. The solution domain

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is discretized with tetrahedral cells and each Control Volume (CV) is constructed around the tetrahedral vertices. Using this strategy the partial differential of fluid volume conservation equations are discretized into a system of differential/algebraic equations. These equations are then resolved in time. These methods are suitable for intricate physical geometries and flow through three dimensional saturated porous media with constant volume. Simulation results for the case of homogeneous and isotropic porous media underneath of a gravity dam with upstream cut off is presented and compared with analytical solutions obtained by application of conformal mapping technique for a constant unit ratio of foundation depth over half of dam base ($T/b=1$). The accuracy of the results computed uplift pressure are assessed by comparison of computed results for various ratios of cut off wall depth over half of dam base length (*s/b*) with the analytical solutions obtained using conformal mapping technique by Pavlovsky,1956 [12].

Then, the verified model is applied to evaluate the quality of the results of the common empirical relations for uplift pressure estimation and their errors are investigated.

Finally, the ability of the developed Galerkin finite volume solver to cop with real world heterogeneous problems is demonstrated by its application to solve seepage through a natural porous foundation of a gravity dam with three isotropic incline layers. In order to provide better understanding from the solution results color coded surfaces of the computed pressure head, velocity vectors and flow net are presented and the computed uplift pressure underneath of the dam is compared with the results of common empirical relations.

II. Modeling Algorithm

A. Mathematical Model

The problem of seawater seepage is governed by a partial differential equation for the ground water flow that describes the head distribution in the heterogeneous zone of interest underneath of a gravity dam. The general form of flow equation for a confined saturated heterogeneous and anisotropic porous media can be written as [1]:

$$\frac{\partial}{\partial x_i}\left(k_i \frac{\partial h}{\partial x_i}\right)=S_s\frac{\partial h}{\partial t} \quad (i=1,2,3) \tag{1}$$

Where, h is the reference hydraulic head referred to as the freshwater head, k_i is a component of the hydraulic conductivity tensor, S_s is the specific storage and t is time.

If head gradient flux in direction *i* (secondary variable) is defined as,

$$F_i^d = k_i \,\partial h \big/ \partial x_i \quad (i=1,2,3) \tag{2}$$

and hence, the equation takes the form:

$$S_s\frac{\partial h}{\partial t}-\left(\frac{\partial}{\partial x_i}F_i^d\right)=0 \quad (i=1,2,3) \tag{3}$$

The boundary conditions for this equation may be stated as follows [1]:

- *Dirichlet boundary condition:*

$$h(x_b,z_b;t)=h_d(x,z;t) \ \text{ in } \ B_d \tag{4}$$

- *Neumann boundary condition:*

$$V.n_i=V_n(x_b,z_b;t) \ \ in \ \ B_n \tag{5}$$

where n_i is the outward unit vector normal to the boundary, (x_b,z_b) is a spatial coordinate on the boundary, h_d and V_n are the Dirichlet functional value and Neumann flux, respectively.

It should be noted that, for a homogeneous and isotropic porous media the following relations are valid.

$$k_x=k_y=k_z=k \qquad \frac{\partial k_x}{\partial x}=0,\ \ \frac{\partial k_y}{\partial y}=0,\ \ \frac{\partial k_z}{\partial z}=0 \tag{6}$$

While, for a homogeneous and anisotropic porous media the following relations are valid.

$$k_x\neq k_y\neq k_z \qquad \frac{\partial k_x}{\partial x}=0,\ \ \frac{\partial k_y}{\partial y}=0,\ \ \frac{\partial k_z}{\partial z}=0 \tag{7}$$

Furthermore, for a heterogeneous and anisotropic porous media the following relations are valid.

$$k_x\neq k_y\neq k_z \qquad \frac{\partial k_x}{\partial x}\neq 0,\ \ \frac{\partial k_y}{\partial y}\neq 0,\ \ \frac{\partial k_z}{\partial z}\neq 0 \tag{8}$$

B. Numerical Model

During the last twenty years there has been a strong focus upon the utilization of the Finite Volume methods for solving fluid flow and heat transfer problems or, as it is more generally known, problems in Computational Fluid Dynamics (CFD). This success is mostly due to the conservative nature of the scheme and the fact that the terms appearing in the resulting algebraic equations have a specific physical interpretation. In fact, the straightforward formulation and low computational cost compared with other methods have made Finite Volume Method the preferred choice for most CFD practitioners [13].

Over the last ten years, several Finite Volume methods based on Unstructured Mesh (FVUM) have in many ways overcome the structured nature of the original CV method. In general, the FVUM methods can be categorized into two approaches, namely, vertex-centered or cell-centered. The classification of the approach is based on the relationship between the CV and the finite element like unstructured mesh. The approach described here is the vertex-centered, which uses linear shape function of tetrahedral elements as the interpolation function within the CVs formed by gathering all the elements sharing a nodal point. This approach is very similar to the Galerkin Finite Element Method with linear elements [14,15].

In a finite element mesh, the sub-regions are called elements, with the vertices of the elements being the nodal locations. For the vertex-centered approach only the basic three dimensional elements, tetrahedrons with four nodes are considered [16].

Therefore, each node in the solution domain is associated with one CV. Consequently, each CV consists of some tetrahedral elements, as illustrated in Fig.1 . The CV can be assembled in a straightforward and efficient manner at the element level. The flow across each control surface must be determined by an integral.

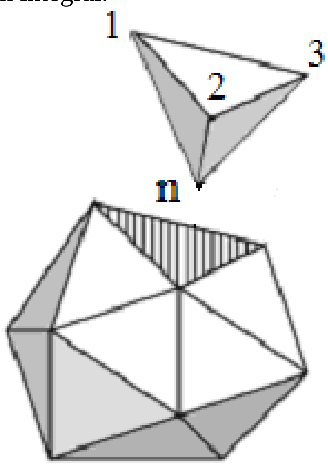

Fig.1: sub-domain Ω associated with node *n* of the computational field

The FVUM discretization process is initiated by utilizing the integrated form of the equation (1). By application of the Variational Method, after multiplying the residual of the above equation by the test function ϕ and integrating over a sub-domain Ω, we have:

$$S_s \int_\Omega \frac{\partial h}{\partial t}\phi \, d\Omega + \int_\Omega \frac{\partial F_i^d}{\partial x_i}\phi \, d\Omega = 0 \quad (i=1,2,3) \tag{9}$$

The terms containing spatial derivatives can be integrated by part over the sub-domain Ω and then equation (5) may be written as:

$$S_s \int_\Omega \frac{\partial h}{\partial t}\phi \, d\Omega + \int_\Omega F_i^d \phi \, d\Omega - \int_\Omega F_i^d \frac{\partial \phi}{\partial x_i} d\Omega = 0 \quad (i=1,2,3) \tag{10}$$

Using gauss divergence theorem the equation takes the form:

$$S_s \int_\Omega \frac{\partial h}{\partial t}\phi \, d\Omega + k_i \oint_\Gamma \phi \, h(n.d\Gamma) - \int_\Omega F_i^d \frac{\partial \phi}{\partial x_i} d\Omega = 0 \quad (i=1,2,3) \tag{11}$$

where Γ is the boundary of domain Ω.

Following the concept of weighted residual methods, by considering the test function equal to the weighting function, the dependent variable inside the domain Ω can be approximated by application of a linear combination, such as $h = \sum_{k=1}^{N_{nodes}} h_k \varphi_k$ [17].

According to the Galerkin method, the weighting function ϕ can be chosen equal to the interpolation function φ. In finite element methods this function is systematically computed for desired element type and called the shape function. For a tetrahedral type element (with four nodes), the linear shape functions, φ_k, take the value of unity at desired node *n*, and zero at other neighboring nodes *k* of each triangular element ($k \neq n$) [17].

Extending the concept to a sub-domain to the CV formed by the elements meeting node *n* (Fig.1), the interpolation function φ_n takes the value of unity at the center node n of CV Ω and zero at other neighboring nodes m (at the boundary of the CV Γ). Noteworthy that, this is an essential property of weight function, φ, which should satisfy homogeneous boundary condition on T at boundary of sub-domain [12]. That is why the integration of the linear combination $h = \sum_{k=1}^{N_{nodes}} h_k \varphi_k$ (as approximation) over elements of sub-domain Ω takes the value of h_n (the value of the dependent variable in central node n). By this property of the shape function φ ($\varphi_n = 0$ on boundary Γ of the sub-domain Ω), the boundary integral term in equation (9) takes zero value for a CV which the values of T assumed known at boundary nodes.

After omitting zero term, the equation (9) takes the following form:

$$S_s \frac{\partial}{\partial t}\int_\Omega h\varphi \, d\Omega - \int_\Omega F_i^d \frac{\partial \varphi}{\partial x_i} d\Omega = 0 \quad (i=1,2,3) \tag{12}$$

In order to drive the algebraic formulation, every single term of the above equation first is manipulated for each element then the integration over the CV is performed. The resulting formulation is valid for the central node of the CV.

For the terms with no derivatives of the shape function φ, an exact integration formula is used as, $\int_\Lambda \varphi_1^a \varphi_2^b \varphi_3^c \varphi_4^d = 6\Lambda(a!b!c!d!)/(a+b+c+d+3) = \Lambda/4$ (for *a*=1 and *b*=*c*=*d*=*0*), where Λ is the volume of the tetrahedral element [15]. This volume can be computed by the integration formula as,

$$\Lambda = \int_\Lambda x_i (d\Lambda)_i \approx \sum_k^4 [\bar{x}_i \, \delta\ell_i]_k \tag{13}$$

where $\bar{x}_i$ and $\delta\ell_i$ are the average direction *i* coordinates and projected area (normal to direction *i*) for every side face opposite to node *k* of the element.

Therefore, the transient term $\partial/\partial t \int_\Omega \phi \, h \, d\Omega$ for each tetrahedral element Λ (inside the sub-domain) can be written as:

$$\partial/\partial t \int_\Lambda \phi \; h \; d\Lambda = (\Lambda/4)\, dh/dt \tag{14}$$

Consequently, the transient term of equation (12) for the sub-domain Ω (with central node *n*) takes the following form:

$$S_s \frac{\partial}{\partial t}\int_\Omega \varphi \, h \, d\Omega = S_s \frac{\Omega_n}{4}\frac{dh_n}{dt} \tag{15}$$

Now we try to discretize the terms containing spatial derivative, $\int_\Omega F_i^d \,(\partial\phi / \partial x_i)\, d\Omega$ in equation (10). Since the only unknown dependent variable is $h = \sum_k^4 h_k \varphi_k$ and the shape functions, φ_k, are chosen linear piecewise in every tetrahedral element, the heat gradient flux (F_i^d which is formed by first derivative) is constant over each element and can be taken out of the integration. On the other hand, the integration of the shape function spatial derivation over tetrahedral element can be converted to boundary integral using Gauss divergence theorem [18], and hence: $\int_\Lambda \partial\varphi / \partial x_i \, d\mathrm{A} = -\oint_\Lambda \varphi (d\mathrm{A})_i$. Here, A is component of the side face element normal to the direction i. The discrete form of the line integral can be written as: $\oint_\Delta \varphi.(d\mathrm{A})_i \approx 1/\Lambda \sum_k^4 [\bar{\varphi}\,\delta\ell_i]_k$, where $[\bar{\varphi}\,\delta\ell_i]_k$ is formed by considering the side of the element opposite to the node k, and then, multiplication of its component perpendicular to the direction i by $\bar{\varphi}$ the average shape function value of its three end nodes.

Hence, the term $\int_\Omega F_i^d \,(\partial\phi / \partial x_i)\, d\Omega \approx -\sum_1^N [F_i^d / \Lambda \sum_k^4 (\bar{\varphi}\,\delta\ell_i)_k]$ for a CV Ω (containing N elements sharing its central node). Since the shape function φ takes the value of unity only at central node of CV and is zero at the nodes located at the boundary of CV, $\bar{\varphi} = 1/3$ for the faces connected to the central node of CV and $\bar{\varphi} = 0$ for the boundary faces of the CV. On the other hand the sum of the projected area (normal to direction i) of three side faces of every tetrahedral element equates to the projected area of the fourth side face; hence the term containing spatial derivatives in direction i of the equation (10), can be written as:

$$\int_\Omega F_i^d \,\frac{\partial\varphi}{\partial x_i}\, d\Omega = -\frac{1}{3}\sum_{m=1}^{M}\left[F_i^d\,\delta\ell_i\right]_m \tag{16}$$

where $[\delta\ell_i]_m$ is the component of the boundary face m (opposite to the central node of the CV Ω) perpendicular to direction i. Note that, F_i^d is computed at the center of tetrahedral element of the CV, which is associated with side m. The head gradient flux in direction i, $F_i^d = k_i\, \partial h / \partial x_i$, at each tetrahedral element can be calculated using Gauss divergence theorem, $\int_\Omega F_i^d \, d\Omega = k_i \int_\Lambda \partial h / \partial x_i \, d\Lambda = -k_i \oint_\Delta T\,(d\Delta)_i$, where $(d\Delta)_i$ is the projection of side faces of the element perpendicular to direction i. By expressing the boundary integral in discrete form as, $\oint_\Delta h(d\Delta)_i \approx \sum_k^3 (\bar{h}\,\delta\ell_i)_k$, for each element inside the CV Ω. Therefore, we have,

$$[F_i^d]_m = -\frac{k_i}{\Lambda_m}\sum_{k=1}^{3}\left(\bar{h}\,\delta\ell_i\right)_k \tag{17}$$

where, $\delta\ell_i$ is the component of k^{th} face of a tetrahedral element (perpendicular to the direction i) and $\bar{h}$ is the average head of that face and Λ is the volume of the element.

Noteworthy that for CVs at the boundary of the computational domain, central node n of the CV Ω locates at its own boundary. For the boundary sides connected to the node n there are no neighboring elements to cancel the contribution. Hence, their contributions remain and they act as the boundary sides of the sub-domain. Therefore, there is no change to described procedure for computation of the spatial derivative terms $\int_\Omega F_i^d \,(\partial\varphi / \partial x_i)\, d\Omega$.

Finally, using expressions (15) and (16) the equation (12) can be written for a CV Ω (with center node n) as:

$$S_s \frac{dh_n}{dt}\,\Omega_n = -\frac{4}{3}\sum_{m=1}^{M}\left[F_i^d\,\delta\ell_i\right]_m \quad (i=1,2,3) \tag{18}$$

The volume of CV Ω can be computed by summation of the volume of the elements associated with node n.

The resulted numerical model, which is similar to Non-Overlapping Scheme of the Cell-Vertex FVUM, can explicitly be solved for every node n (the center of the sub-domain Ω which is formed by gathering elements sharing node n). The explicit solution of head at every node of the domain of interest can be modeled as,

$$h_n^{t+\Delta t} = h_n^t - \frac{\Delta t}{S_s}\left[\frac{4}{3\Omega_n}\Big(\sum_{k=1}^{N} F_i^d \Delta l_i\Big)_n\right] \quad (i=1,2) \tag{19}$$

C. Computational Steps

Now we need to define a limit for the explicit time step, δt. Considering thermal diffusivity as $\alpha = \kappa/\rho C$ with the unit (m^2/s), the criterion for measuring the ability of a material for head change. Hence the rate of head change can be expressed as, $\Omega_n / \Delta t \approx k$. Therefore, the appropriate size for local time stepping can be considered as,

$$\Delta t = \beta\, \Omega_n / k \qquad (\beta \le 1) \tag{20}$$

β is considered as a proportionality constant coefficient, which its magnitude is less than unity. For the steady state problems this limit can be viewed as the limit of local computational step toward steady state.

However, there are different sizes of CVs in unstructured meshes. This fact implies that the minimum magnitude of the above relation be considered. Hence, to maintain the stability of the explicit time stepping, the global minimum time step of the computational field should be considered. So,

$$\Delta t = \beta\left(\frac{\Omega_n}{k}\right)_{\min} \quad (\beta \le 1) \tag{21}$$

Noteworthy that for the solution of steady state problems on suitable fine unstructured meshes, the use of local computational step instead of global minimum time step may considerably reduce the computational efforts.In order to stabilizing the numerical solution, time step is restricted by:

$$\Delta t = \left(S_s \, \Omega_n \big/ \max(k_i) \right)_{\min} \qquad (22)$$

where Ω_n is area of each CV and k_i (*i=1,2,3*), is hydraulic conductivity in direction *i*.

III. Model Verification

In the following sections, the accuracy of the results of the developed model is verified by comparison of the computed seepage and uplift pressure in natural foundation of a gravity dam with a cut off wall with analytical solutions obtained by application of conformal mapping technique. The plots of computed pressure head, velocity vectors and flow net are used to provide better understanding of the numerical solution results.

To verify the above described numerical model, a test case considered, for which analytical solution is available. The analytical solutions of the seepage and uplift pressure through the homogeneous and isotropic dam foundation are obtained for a number of ratios of cut off wall over half of dam base length (*s/b*) using conformal mapping technique. The parameters were chosen so that the analyzed cases correspond to those analytically solved by Pavlovsky, 1956 [12].

The geometry of the dam foundation with an upstream cut off wall at the dam base test case is schematically described in Fig.2. The boundary conditions employed in present numerical simulation are also illustrated. The foundation region considered to be as homogeneous and isotropic sand with $k_x = k_y = k_z = 5\times10^{-5}\ m/Sec$ and $S_s = 8\times10^{-5}\ 1/m$. The model is represented in a discrete form by a three-dimensional tetrahedral mesh for a cubic dam foundation in Fig.3.

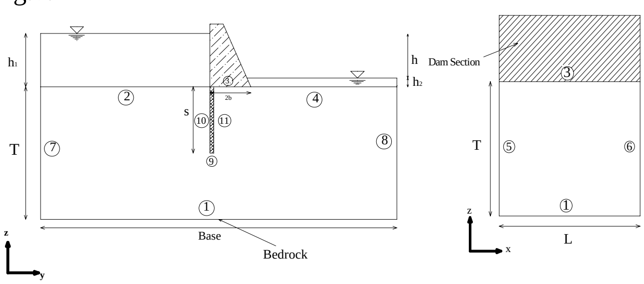

Fig.2 : Description of verification case

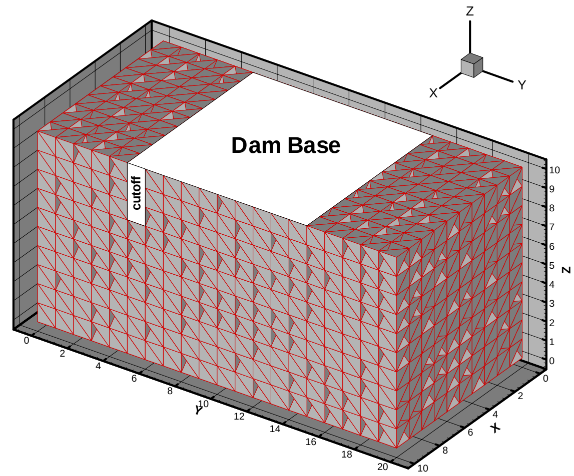

Fig.3 : A three-dimensional tetrahedral mesh for dam foundation

Figure 4 shows a typical computed color coded surfaces of head in the homogeneous and isotropic sand foundation of dam with upstream cutoff.

Figures 5 and 6, present typical computed color coded velocity vectors and flow net, respectively, in a homogeneous and isotropic sand foundation of dam with upstream cutoff.

Figure 7 presents plots of uplift pressure distribution underneath of dam with upstream cutoff for various ratios of cutoff wall depth over half of dam base length (*s/b*) for a constant unit ratio of foundation depth over half of dam base ($T/b = 1$).

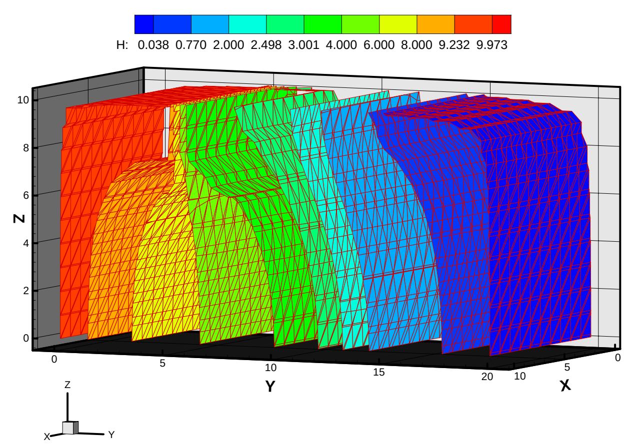

Fig.4 : Typical computed color coded surfaces of head in the homogeneous and isotropic sand foundation of dam

Figure 8 presents plots of uplift pressure drop $D_p = (h_L - h_R)/h \times 100$ underneath of dam with upstream cutoff for various ratios of cutoff wall depth for a range of *s/b* for $T/b = 1$. In this relation h_L and h_R are pressure heads upstream and downstream of the cutoff wall and *h* is the difference of water heads at upstream and downstream of dam. The average error between numerical results and analytical solution is 0.56%, while the maximum error is computed as 7% .

As can be seen, the accuracy of the results computed by present matrix free Galerkin finite volume model for solution of seepage flow and computation of uplift pressure are quite acceptable.

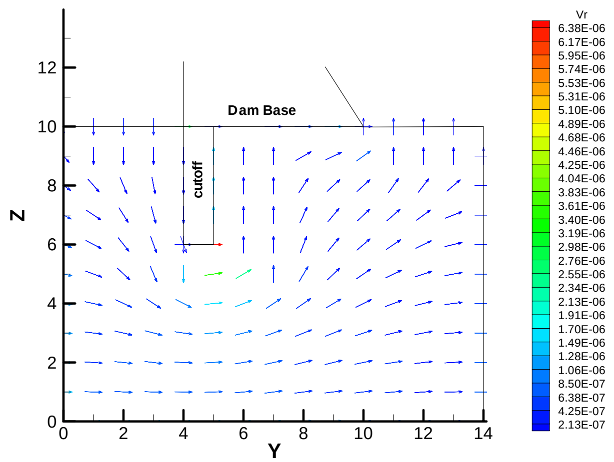

Fig.5 : Typical computed velocity vectors in the homogeneous and isotropic sand foundation of dam

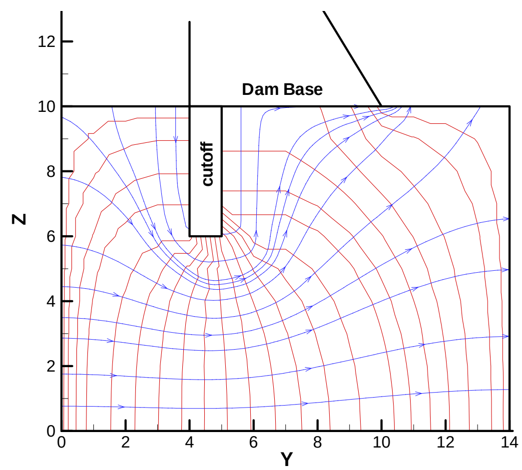

Fig.6 : Typical computed flow net in the homogeneous and isotropic sand foundation of dam

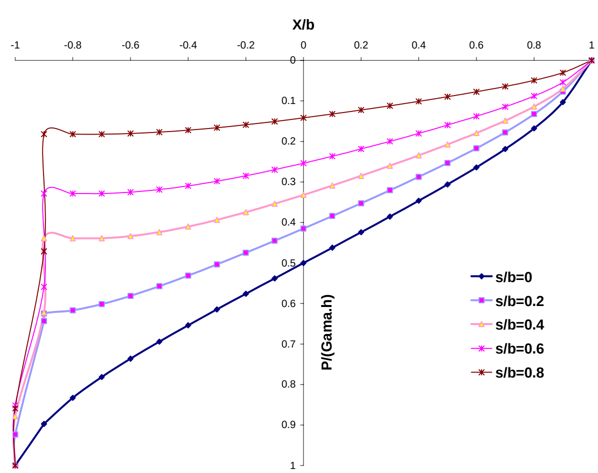

Fig.7 : Uplift pressure distribution underneath of dam with upstream cutoff for different *s/b* $(T/b = 1)$

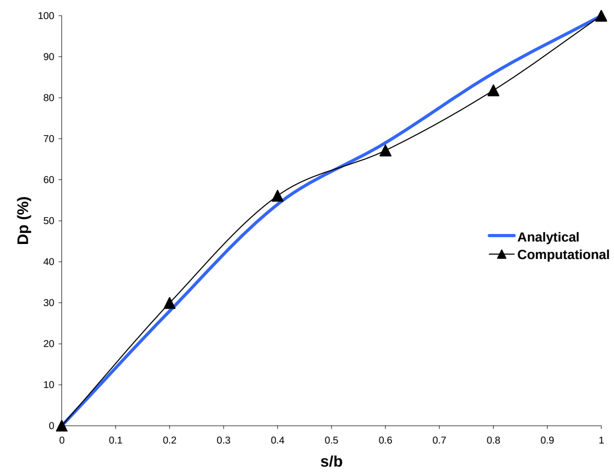

Fig.8 : The comparison of the computed results for various (*s/b*) with the analytical solution of Pavlovsky,1956 [3]

IV. Errors of Empirical Relations

There are some empirical relations which globally estimate the value of uplift force underneath of the dam base as functions of pore pressure in dam foundation. These relations define some criterion for considering equivalent seepage length L_e. Then, uplift pressure at any point underneath of the dam base can be estimated by considering linear variation of pressure head from H_{up} to H_{down} along L_e.

One of the popular relations for calculation of equivalent seepage length L_e is proposed by Lane [3].

$$(L_e)_{Lane} = \sum L_{vertical} + \sum L_{Horizontal} \tag{23}$$

Another popular relation for calculation of equivalent seepage length L_e is proposed by Bligh [3].

$$(L_e)_{Bligh} = \sum L_{vertical} + \frac{1}{3}\sum L_{Horizontal} \tag{24}$$

In this section the estimated uplift pressure foundation of a dam (specified in the previous section) using above mentioned relations are compared with the uplift pressure computed by introduced numerical solver (Fig.9).

Following table presents the uplift pressure errors caused by using the two empirical relations for estimating L_e are presented.

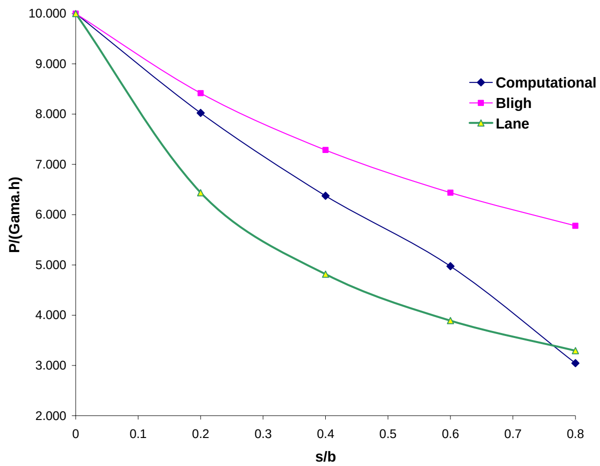

Fig.9 : Uplift pressure variation underneath of dam with upstream cutoff for different *s/b* ($T/b = 1$)

Table 1 : Uplift pressure errors using empirical relations for L_e

s/b / ***$P_c/(g_w h)$***	***0***	***0.2***	***0.4***	***0.6***	***0.8***
Computed	*10.002*	*8.053*	*6.376*	*4.975*	*3.046*
Bligh	*10.0*	*8.417*	*7.286*	*6.438*	*5.778*
Error(%)	*0.02*	*-4.9*	*-14.3*	*-29.4*	*-89.7*
Lane	*10.0*	*6.438*	*4.818*	*3.893*	*3.294*
Error(%)	*0.02*	*19.78*	*24.43*	*21.76*	*-8.14*

V. Model Application

In this section, the three layers foundation region considered to be as heterogeneous foundation case with following coefficients of permeability:

$$k_{x_1} = k_{y_1} = k_{z_1} = 1\times10^{-5}\ m/Sec$$

$$k_{x_2} = k_{y_2} = k_{z_2} = 5\times10^{-5}\ m/Sec$$

$$k_{x_3} = k_{y_3} = k_{z_3} = 1\times10^{-5}\ m/Sec$$

The model is represented in a discrete form by a three-dimensional tetrahedral mesh for a cubic dam foundation. The geometrical features of the case are described in Fig.10.

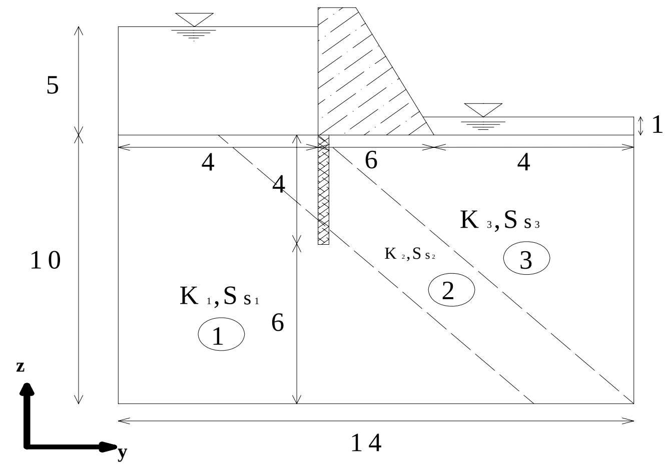

Fig.7 : Description of the application problem

Figure 11 shows a typical computed color coded surfaces of head in the isotropic sand foundation of dam with upstream cutoff. Figure 12 and Figure 13, present typical computed color coded velocity vectors and flow net, respectively, in a heterogeneous foundation of dam with upstream cutoff. Figure 14 presents plots of uplift pressure distribution underneath of dam with upstream cutoff in the heterogeneous foundation.

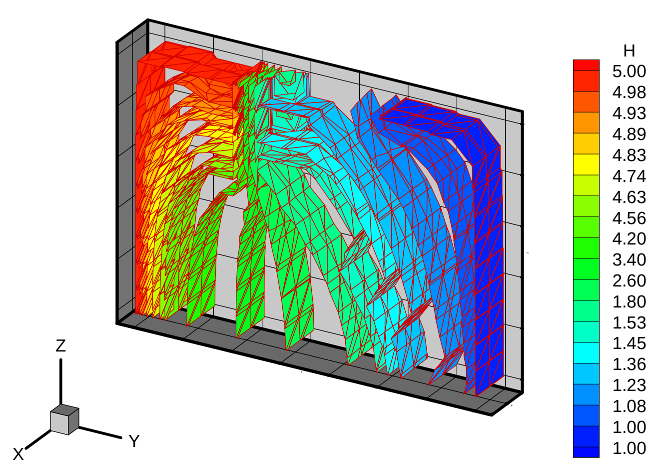

Fig.11 : Typical computed 3D color coded surfaces of head in the isotropic sand foundation of dam

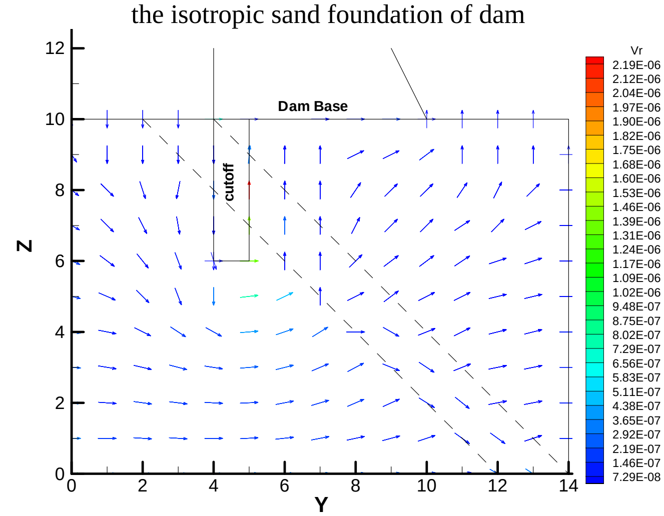

Fig.12 : Typical computed velocity vectors in a heterogeneous foundation of dam

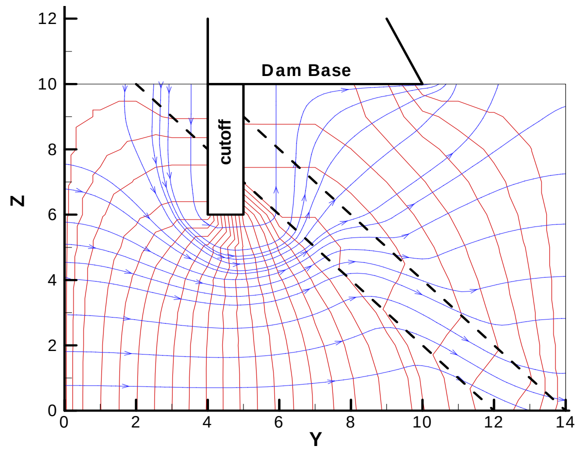

Fig.13 : Typical computed flow net in a heterogeneous foundation of dam

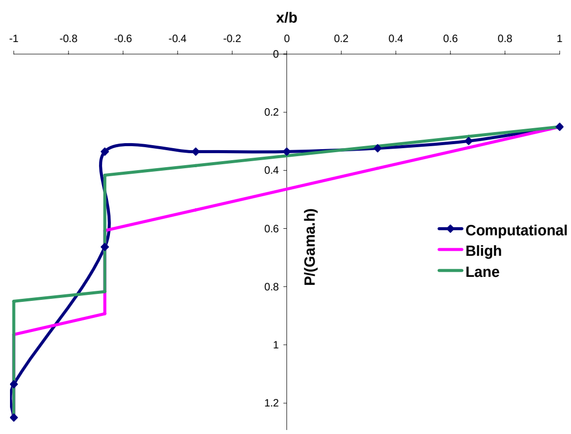

Fig.14 : Uplift pressure distribution underneath of dam with upstream cutoff in A HETEROGENEOUS foundation

VI. CONCLUSION

In present paper, a three-dimensional numerical model is developed for computing the seepage flow and uplift pressure under gravity dams with cutoff wall. This model explicitly solves the equation of ground water flow on the three dimensional unstructured mesh using Galerkin Finite Volume Method developed for linear tetrahedral elements. Since there is no shape function in the final formulation of the model, the introduced matrix free nodal base method consumes very light computational overhead. The proposed algorithm suits the unstructured meshes and therefore, the developed model can predict seepage flow and pressure head distribution in geometrical complex porous media. In order to verify the accuracy of model results, the seepage flow through a homogeneous and isotropic sand dam foundation is solved for various ratios of upstream cutoff wall depth over half of dam base length (s/b) for a constant unit ratio of foundation depth over half of dam base (*T/b=1*). The computed results of uplift pressure distribution are compared with the analytical solutions obtained by application of conformal mapping technique by Pavlovsky, 1956. The agreements between the computed results obtained from described modeling algorithm and analytical solutions are promising.

The comparison of the results of present numerical solver with the empirical relations shows that the uplift pressure calculated using the equivalent seepage lengths suggested by Lane and Bligh are close to the computed values only for small *s/b*. For large values of *s/b* the equivalent seepage length suggested by Bligh will produce over estimated uplift pressure values.

The good performance and acceptable results of the introduced modeling technique encouraged application of the model for solving seepage flow in a real world multilayer porous foundation. Hence, the ability of the developed model for solving seepage flow through a natural heterogeneous foundation of a gravity dam with three isotropic incline layers is examined.

In order to provide better understanding from the solution results color coded surfaces of the computed pressure head, velocity vectors and flow net are presented and the computed uplift pressure underneath of the dam is compared with the results of common empirical relations.

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