# Holomorphic generic families of singular systems under feedback and derivative feedback

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*Abstract:-*
Following Arnold techniques, in this paper we obtain a canonical reduced form for regularizable singular systems and we describe generic holomorphic families with respect feedback and derivative feedback, that permit us, to analyze the neighborhood of a given system.

*Key-Words:-* Singular systems, Feedback equivalence, Canonical form, Miniversal deformation.

## 1 Introduction

Let $M$ be the smooth manifold of triples of matrices $(E, A, B)$ where $E, A \in M_n(\mathbb{C})$, $B \in M_{n\times m}(\mathbb{C})$, which represent singular time-invariant linear systems in the form

$$E\dot{x}(t) = Ax(t) + Bu(t) \qquad (1)$$

(that we call triple or system indistinctly).

These equations, arise in a natural way when modelling different set-ups, for instance, when modelling mechanical multibody systems and electrical circuits, largely studied by different authors (Dai [3], García-Planas [4], P. Kunkel, V. Mehrmann [7], for example).

It is well known that a system $E\dot{x} = Ax + Bu$ is called regular if and only if $\det(\alpha E - \beta A) \neq 0$ for some $(\alpha, \beta) \in \mathbb{C}^2$. Remember that the regularity of the system guarantees the existence and uniqueness of classical solutions.

For no regular systems one can ask for whether the close loop system is uniquely solvable for all consistent initial solution, when this is possible the system will be called regularizable by proportional and derivative feedback. That is to say, the system is regularizable if and only if, there exist matrices $F_E, F_A \in M_{m\times n}(\mathbb{C})$ such that the system $(E + BF_E)\dot{x} = (A + BF_A)x + Bu$ is regular. A special subset of regularizable systems is the subset of standardizable systems. That is to say, the set of systems $(E, A, B)$ for which there exist a derivative feedback $F_E$ such that $E + BF_E$ is invertible, so after to apply the derivative feedback $F_E$ and premultiplying the equation by $(E + BF_E)^{-1}$, the system being standard.

Notice that, the set $M_R$, consisting in all regularizable systems is an open and dense set in the space of all systems.

In order to obtain a simple description of systems, we consider an equivalence relation in the space $M$ of singular systems that preserves regularizability character, consisting in to apply one or more of the following elementary transformations: basis change in the space state, basis change in the input space, proportional feedback, derivative feedback and premultiplication by an invertible

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matrix.

The central goal of this paper is to obtain a canonical reduced form for regularizable systems under the equivalence relation defined. The Arnold technique of constructing a local canonical form, called versal deformation, of a holomorphic family of square matrices under conjugation (see [1]), can be generalized to this case obtaining a local canonical form for holomorphic families of regularizable holomorphic systems. Remember that a holomorphic family $(E(\lambda), A(\lambda), B(\lambda))$ $\lambda = (\lambda_1, \dots, \lambda_k)$ at a point $p = (0, \dots, 0)$ are families of triples of matrices whose entries are convergent in the power series expansion of complex parameters $\lambda_1, \dots, \lambda_k$ in a neighborhood of $p$. (The germ of a family $(E(\lambda), A(\lambda), B(\lambda))$ at $p$ is called a deformation of the triple $(E(0), A(0), B(0))$, (see [1], [2]).

The results obtained in this paper are important for application in which one has matrices that arise from physical measurements, which means that their entries are known only approximately.

## 2 Equivalence relation and canonical forms

For every integers $p, q$, we will denote by $M_{p\times q}(\mathbb{C})$ the space of $p$-rows and $q$-columns complex matrices, and if $p = q$ we will write only $M_p(\mathbb{C})$, and by $Gl(n;\mathbb{C})$ the linear group formed by the invertible matrices of $M_p(\mathbb{C})$. In all the paper, $M$ denotes the space of triples of matrices $(E, A, B)$ with $E, A \in M_n(\mathbb{C})$, $B \in M_{n\times m}(\mathbb{C})$ and $M_R$ denotes the open and dense space of regularizable systems.

In order to classify systems preserving regularizability character, we consider the following equivalence relation.

**Definition 1** *The triples $(E, A, B)$ and $(E', A', B')$ in $M$, are said to be equivalent if and only if*

$$(E', A', B') = (QEP+QBF_E, QAP+QBF_A, QBR)$$

*for some $Q, P \in Gl(n;\mathbb{C})$, $R \in Gl(m;\mathbb{C})$, $F_E, F_A \in M_{m\times n}(\mathbb{C})$. In a matrix form:*

$$\begin{pmatrix} E' & A' & B' \end{pmatrix} = Q \begin{pmatrix} E & A & B \end{pmatrix} \begin{pmatrix} P & 0 & 0 \\ 0 & P & 0 \\ F_E & F_A & R \end{pmatrix}.$$

That is to say, the triples $(E, A, B)$ and $(E', A', B')$ are equivalent if and only if $(E', A', B')$ can be obtained from $(E, A, B)$ by means of one or more of the following elementary transformations

i) Basis change in the space state,

ii) Basis change in the inputs space,

iii) Feedback,

iv) Derivative feedback,

v) Premultiplication by an invertible matrix.

It is immediate that the equivalence relation generalizes the feedback equivalence between standard linear systems.

Loiseau, Ölçadiram and Malabre in [8] consider the restricted pencil $s\pi E - \pi A$ where $\pi$ is the projection of state space over $\operatorname{Im} B$, and they prove that two triples are equivalent if and only if the associated restricted pencils are strictly equivalent, consequently a singular system $(E, A, B)$, can be reduced to

$$\left( \begin{pmatrix} 0 \\ E_1' \end{pmatrix}, \begin{pmatrix} 0 \\ A_1' \end{pmatrix}, \begin{pmatrix} I_r & 0 \\ 0 & 0 \end{pmatrix} \right)$$

where $(E_1', A_1')$ is the Kronecker canonical reduced form of the pencil $s\pi E + \pi A$. García-Planas and Magret in [6] obtain the same result using polynomial matrices.

For regularizable systems we obtain a most useful reduced form in the following manner.

**Proposition 1** *Let $(E, A, B)$ be a $n$-dimensional $m$-input regularizable system. Then, it can be reduced to $\left( \begin{pmatrix} I_r & \\ & N \end{pmatrix}, \begin{pmatrix} A_1 & \\ & I_{n-r} \end{pmatrix}, \begin{pmatrix} B_1 \\ 0 \end{pmatrix} \right)$ where $(A_1, B_1)$ is a pair in its Kronecker canonical form, and $N$ is a nilpotent matrix in its canonical reduced form.*

**Proof.** Let $(E, A, B)$ be a regularizable triple, making a proportional and derivative feedback in such a way that the standardizable subsystem being maximal. We consider the equivalent triple where the pair $(A, B)$ is in its Weirstass form

$$\left(\begin{pmatrix} I_1 & \\ & N \end{pmatrix}, \begin{pmatrix} A_c & \\ & I_2 \end{pmatrix}, \begin{pmatrix} B_1 \\ B_2 \end{pmatrix}\right)$$

(Observe that if the triple is standardizable then $I_1 = I_n$).

It suffices to prove that in this case $B_2 = 0$. For that we consider the following equivalent triple

$$\begin{pmatrix} I_1 & \\ & P^{-1} \end{pmatrix} \begin{pmatrix} I_1 & 0 & A_c & 0 & B_1 \\ 0 & N & 0 & I_2 & B_2 \end{pmatrix} \begin{pmatrix} I_1 & & & & \\ & P & & & \\ & & I_2 & & \\ & & & P & \\ & F & & & R \end{pmatrix} =$$

$$\begin{pmatrix} I_1 & 0 & A_c & 0 & B_1 R \\ 0 & P^{-1}NP+PB_2F & 0 & I_2 & P^{-1}B_2R \end{pmatrix}$$

where and if $B_2 \neq 0$, $P^{-1}NP + PB_2F = \begin{pmatrix} I_3 & \\ & N \end{pmatrix}$, $P^{-1}BR = \begin{pmatrix} B_{21} \\ 0 \end{pmatrix}$, so the standardizable part is not maximal.

Finally, it suffices to reduce the system $(A_c, B_1)$ in its Kronecker canonical reduced form. $\square$

## 2 Miniversal deformations

The equivalence relation may be seen as induced by Lie group action. Let us consider the following Lie group $\mathcal{G} = Gl(n;\mathbb{C}) \times Gl(n;\mathbb{C}) \times Gl(m;\mathbb{C}) \times M_{m\times n}(\mathbb{C}) \times M_{m\times n}(\mathbb{C})$ acting on $M$.

The action $\alpha : \mathcal{G} \times M \longrightarrow M$ is defined as follows:

$$\begin{array}{l} \alpha((P,Q,R,F_E,F_A),(E,A,B)) = \\ (QEP + QBF_E, QAP + QBF_A, QBR) \end{array} \tag{2}$$

So, the orbits are equivalence classes of triples of matrices under the equivalence relation considered.

$$\begin{array}{l} \mathcal{O}(E,A,B) = \\ \{(QEP + QBF_E, QAP + QBF_A, QBR)\} \end{array}$$

$\forall Q, P \in Gl(n;\mathbb{C})$, $R \in Gl(m;\mathbb{C})$, $F_E, F_A \in M_{m\times n}(\mathbb{C})$.

For a triple $(E, A, B) \in M$, we denote by

$$\begin{array}{l} T_{(E,A,B)}\mathcal{O}(E,A,B) = \\ \{(EP + QE + BF_E, AP + QA + BF_A, BR + QB)\} \end{array}$$

for all $P, Q \in M_n(\mathbb{C})$, $R \in M_m(\mathbb{C})$, $F_E, F_A \in M_{m\times n}(\mathbb{C})$, the tangent space at $(E, A, B)$ to the orbit through $(E, A, B)$.

Now, we will use the description of the orthogonal complementary subspace to the tangent space to the orbit to explicit miniversal deformations.

First, we recall the definition of versal deformations. Let $H$ be a smooth manifold.

Two families $(E(\lambda), A(\lambda), B(\lambda))$ and $(E'(\lambda), A(\lambda), B'(\lambda))$ are called equivalent if there exist matrices $Q(\lambda)$, $P(\lambda)$, $R(\lambda)$, $F_E(\lambda)$, $F_A(\lambda)$ holomorphic at the origen $p$ such that

$$\begin{array}{l} \begin{pmatrix} E'(\lambda) & A'(\lambda) & B'(\lambda) \end{pmatrix} = \\ Q(\lambda)\begin{pmatrix} E(\lambda) & A(\lambda) & B(\lambda) \end{pmatrix}\begin{pmatrix} P(\lambda) & & \\ & P(\lambda) & \\ F_E(\lambda) & F_A(\lambda) & R(\lambda) \end{pmatrix} \end{array}$$

in a neighborhood of the origen $p$.

**Definition 2** *Let $\Lambda$ be a neighborhood of the origin of $\mathbb{C}^\ell$. A deformation $\varphi(\lambda)$ of $x_0$ is a smooth mapping*

$$\varphi : \Lambda \longrightarrow H$$

*such that $\varphi(0) = x_0$. The vector $\lambda = (\lambda_1, \ldots, \lambda_\ell) \in \Lambda$ is called the parameter vector.*

The deformation $\varphi(\lambda)$ is also called *differentiable family* of elements of $H$.

Let $G$ be a Lie group acting smoothly on $H$. We denote the action of $g \in G$ on $x \in H$ by $g \circ x$.

**Definition 3** *The deformation $\varphi(\lambda)$ of $x_0$ is called versal if any deformation $\varphi'(\xi)$ of $x_0$, where $\xi = (\xi_1, \ldots, \xi_k) \in \Lambda' \subset \mathbb{C}^k$ is the parameter vector, can be represented in some neighborhood of the origin as*

$$\varphi'(\xi) = g(\xi) \circ \varphi(\phi(\xi)), \quad \xi \in \Lambda'' \subset \Lambda', \tag{3}$$

*where $\phi : \Lambda'' \longrightarrow \mathbb{C}^\ell$ and $g : \Lambda'' \longrightarrow G$ are differentiable mappings such that $\phi(0) = 0$ and $g(0)$ is the identity element of $G$. Expression 3 means that any deformation $\varphi'(\xi)$ of $x_0$ can be* *obtained from the versal deformation $\varphi(\lambda)$ of $x_0$ by an appropriate smooth change of parameters $\lambda = \phi(\xi)$ and an equivalence transformation $g(\xi)$ smoothly depending on parameters.*

A versal deformation having minimal number of parameters is called *miniversal*.

The following result was proved by Arnold [1], in the case where $\mathrm{Gl}(n;\mathbb{C})$ acts on $M_n(\mathbb{C})$, and was generalized by Tannenbaum [9], in the case where a Lie group acts on a complex manifold. It provides the relationship between a versal deformation of $x_0$ and the local structure of the orbit.

**Theorem 1** ([9])

1. *A deformation $\varphi(\lambda)$ of $x_0$ is versal if and only if it is transversal to the orbit $\mathcal{O}(x_0)$ at $x_0$.*
2. *Minimal number of parameters of a versal deformation is equal to the codimension of the orbit of $x_0$ in $M$, $\ell = \operatorname{codim}\mathcal{O}(x_0)$.*

Let $\{v_1,\dots,v_\ell\}$ be a basis of any arbitrary complementary subspace $(T_{x_0}\mathcal{O}(x_0))^c$ to $T_{x_0}\mathcal{O}(x_0)$ (for example, $(T_{x_0}\mathcal{O}(x_0))^\perp$).

**Corollary 1** *The deformation*

$$x:\Lambda\subset\mathbb{C}^\ell \longrightarrow H, \quad x(\lambda)=x_0+\sum_{i=1}^{\ell}\lambda_i v_i \quad (4)$$

*is a miniversal deformation.*

In order to describe a complementary subspace of $T_{(E,A,B)}\mathcal{O}(E,A,B)$, we consider the following standard hermitian product in the space $M$

$$\langle x_1,x_2\rangle=\mathrm{tr}(E_1E_2^*)+\mathrm{tr}(A_1A_2^*)+\mathrm{tr}(B_1B_2^*),$$

where $x_i=(E_i,A_i,B_i)\in M,$, $A^*$ denotes the conjugate transpose of a matrix $A$ and tr denotes the trace of the matrices.

**Proposition 2**

$$T_{(E,A,B)}\mathcal{O}(E,A,B)^\perp = \{(X,Y,Z)\mid X^*B=0, Y^*B=0, Z^*B=0, EX^*+AY^*+BZ^*=0, X^*E+Y^*A=0.\}$$

**Proof.** Let $(X,Y,Z)$ be in $T_{(E,A,B)}\mathcal{O}(E,A,B)^\perp$ equivalently

$$\langle (EP+QE+BU, AP+QA+BV, BR+QB),(X,Y,Z)\rangle=0$$

that is to say

$$\begin{array}{l}\mathrm{tr}((EP+QE)X^*)+\mathrm{tr}((AP+QA)Y^*)+\mathrm{tr}(QBZ^*)\\ +\mathrm{tr}((BU)X^*)+\mathrm{tr}((BV)Y^*)+\mathrm{tr}((BR)Z^*)=\\ \mathrm{tr}((EX^*)Q)+\mathrm{tr}((AY^*)Q)+\mathrm{tr}((BZ^*)Q)\\ +\mathrm{tr}((X^*E)P)+\mathrm{tr}((Y^*A)P)+\\ +\mathrm{tr}((X^*B)U)+\mathrm{tr}((Y^*B)V)+\mathrm{tr}((Z^*B)R)=0.\end{array}$$

$\forall\,(P,Q,R,U,V)\in T_e\mathcal{G}$. That is to say

$$\mathbf{AB}=0$$

where

$$\mathbf{A}=\begin{pmatrix} EX^*+AY^*+BZ^* & 0 & 0 & 0 & 0\\ 0 & X^*E+Y^*A & 0 & 0 & 0\\ 0 & 0 & X^*B & 0 & 0\\ 0 & 0 & 0 & Y^*B & 0\\ 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & Z^*B\end{pmatrix}$$

and

$$\mathbf{B}=\begin{pmatrix} Q&0&0&0&0\\ 0&P&0&0&0\\ 0&0&U&0&0\\ 0&0&0&V&0\\ 0&0&0&0&R\end{pmatrix}=0.$$

We observe that this condition is equivalent to

$$\mathbf{AX}=0$$

where

$$\mathbf{X}=\begin{pmatrix} Q&*&*&*&*\\ *&P&*&*&*\\ *&*&U&*&*\\ *&*&*&V&*\\ *&*&*&*&R\end{pmatrix}=0,$$

where $*$ are arbitrary matrices in adequate size. So, taking into account that the product is hermitian product we have

$$\mathbf{A}=0$$

and the proof is concluded. □

**Corollary 2** *Let $(E,A,B)$ be a standardizable triple in its canonical reduced form. Then a miniversal deformation is given by*

$$(E,A,B)+\{(0,Y,Z)\}$$

*where $(A,B)+\{(Y,Z)\}$ is a miniversal deformation of the pair $(A,B)$ under block-similarity equivalence.*

**Proof.** Following proposition 2, we have $(X,Y,Z)\in T_{(E,A,B)}\mathcal{O}((E,A,B))^{\perp}$ if and only if

$$\left\{\begin{aligned} EX^*+AY^*+BZ^* &= 0\\ X^*E+Y^*A &= 0\\ X^*B &= 0\\ Y^*B &= 0\\ Z^*B &= 0\end{aligned}\right. \tag{5}$$

Taking into account that $E=I_n$

$$\left\{\begin{aligned} X^*-AY^*-BZ^* &= 0\\ X^*+Y^*A &= 0\\ X^*B &= 0\\ Y^*B &= 0\\ Z^*B &= 0\end{aligned}\right. \tag{6}$$

Observe that if $X^*=-AY^*-BZ^*$, $Y^*B=0$, $Z^*B=0$, then $X^*B=0$, so the system is equivalent to

$$\left\{\begin{aligned} X^*-AY^*-BZ^* &= 0\\ -AY^*-BZ^*+Y^*A &= 0\\ Y^*B &= 0\\ Z^*B &= 0\end{aligned}\right. \tag{7}$$

The last three equations describe the miniversal orthogonal deformation of the pair $(A,B)$ (see [5]) and the first equation inform us that all equation the parameters of the matrix $X$ are depending on the parameters of $Y$ and $Z$. So, if we want a minimal miniversal deformation we can take $X=0$. $\square$

## 2 Holomorphic canonical form

Now, we are going to explicit the miniversal orthogonal deformation for regularizable triples. First of all and taking into account the homogeneity of the orbits, we observe that we can consider the triple in its canonical reduced form. So, partitioning the matrices $X^*=\begin{pmatrix}X_1 & X_2\\ X_3 & X_4\end{pmatrix}$, $Y^*=\begin{pmatrix}Y_1 & Y_2\\ Y_3 & Y_4\end{pmatrix}$, $Z^*=\begin{pmatrix}Z_1 & Z_2\end{pmatrix}$ following the blocks on the matrices $E$, $A$, $B$ in its canonical reduced form, we obtain the following independent systems:

i)
$$\left.\begin{aligned} X_1+A_cY_1+B_1Z_1 &= 0\\ X_1+Y_1A_c &= 0\\ X_1B_1 &= 0\\ Y_1B_1 &= 0\\ Z_1B_1 &= 0\end{aligned}\right\}$$

according remark, this system corresponds to the miniversal orthogonal deformation to the standard system $(I,A_2,B_1)$ (see [5] for a solution).

ii)
$$\left.\begin{aligned} -NX_4 &= Y_4\\ X_4N-NX_4 &= 0\end{aligned}\right\}$$

this system corresponds to the miniversal orthogonal deformation to the square matrix $N_1$ (see [1] for a solution).

iii)
$$\left.\begin{aligned} NX_3+Y_3 &= 0\\ X_3+Y_3A_c &= 0\\ X_3B_1 &= 0\\ Y_3B_1 &= 0\end{aligned}\right\}$$

having zero-solution, and

iv)
$$\left.\begin{aligned} X_2+A_cY_2+B_1Z_2 &= 0\\ X_2N+Y_2 &= 0\end{aligned}\right\}.$$

To solve system iv), we partition the system into independent subsystems corresponding to the blocks in the matrix $A_c=\begin{pmatrix}N_1 & \\ & J\end{pmatrix}$, so $B_1=\begin{pmatrix}B'\\ 0\end{pmatrix}$, obtaining

$$X_1^2-N_1X_1^2N+B'Z_2=0\}$$

and

$$X_2^2-JX_2^2N=0\}$$

with solutions

$$X_2^1=\begin{pmatrix}X_{11} & \dots & X_{1r}\\ \vdots & & \vdots\\ X_{s1} & \dots & X_{sr}\end{pmatrix},$$

$$X_{ij}=\begin{pmatrix}0 & \dots & 0 & x_1 & \dots & x_\nu\\ 0 & \dots & x_1 & x_2 & \dots & x_{\nu+1}\\ & \iddots & & \iddots & \iddots & \\ x_1 & \dots & & & \dots & x_\ell\end{pmatrix}.$$

$$X_{ij} = \begin{pmatrix} 0 & \dots & 0 & x_1 \\ 0 & \dots & x_1 & x_2 \\ & \iddots & & \\ x_1 & \dots & x_{\ell-1} & x_\ell \end{pmatrix},$$

or

$$X_{ij} = \begin{pmatrix} 0 & \dots & 0 & 0 \\ \vdots & & & \vdots \\ 0 & \dots & 0 & 0 \\ 0 & \dots & 0 & x_1 \\ 0 & \dots & x_1 & x_2 \\ & \iddots & \iddots & \\ x_1 & \dots & x_{\ell-1} & x_\ell \end{pmatrix},$$

depending on the size of the nilpotent submatrices in $N$ and $N_1$, and $Z_2 = \begin{pmatrix} -x_1 & \dots & -x_\ell \end{pmatrix}$.
And $X_2^2 = 0$.

Finally, we describe a simplest holomorphic canonical form.

**Theorem 2** *Given a triple $(E, A, B) \in M_R$ in its canonical reduced form and the orthogonal miniversal deformation, we can consider a minimal miniversal deformation $(E + X, A + Y, B + Z)$ with $X = \begin{pmatrix} 0 & 0 \\ X_3 & X_4 \end{pmatrix}$, $Y = \begin{pmatrix} Y_1 & 0 \\ 0 & 0 \end{pmatrix}$, $Z = \begin{pmatrix} Z_1 & 0 \end{pmatrix}$. $Y_1$, $Z_1$ in such away that $(A_2 + Y_1, B_1 + Z_1)$ being a minimal deformation of the pair $(A_1, B_1)$. Concretely, $Y_1 = \begin{pmatrix} 0 & 0 \\ Y_1^2 & Y_2^2 \end{pmatrix}$, $Z_1 = \begin{pmatrix} Z_1^1 & Z_2^1 \\ 0 & Z_2^2 \end{pmatrix}$ where the block-decomposition correspond to that of $(A_1, B_1)$ and*

*i) all the entries in $Y_1^2$ are zero except $y_i^{p+1}, \dots, y_i^n$, $\quad i = 1, k_1 + 1, \dots, k_1 + \dots + k_{p-1} + 1$,*

*ii) the matrices $Y_2^2$ are such that $J + Y_2^2$ is the miniversal deformation of $J$ given by Arnold [1],*

*iii) all the entries in $Z_1^1$ are zero except $z_i^j$, $\quad 2 \le i \le p$, $\quad k_1 + \dots + k_{i-2} + k_i + 1 \le j \le k_1 + \dots + k_{i-2} + k_{i-1} - 1$ (provided that $k_i \le k_{i-1} + 2$,*

*iv) $Z_2^1$ is such that $z_{p+1}^i = \dots = z_m^i = 0$, $\quad i = k_1, k_1 + k_2, \dots, k_1 + \dots + k_p$,*

*v) all the entries in $Z_2^2$ are arbitrary.*

*$N_1 + X_4$ is a miniversal deformation of the square matrix $N_1$ given by Arnold (see [1]), and $X_3 = (X_{ij})$ with*

$$X_{ij} = \begin{pmatrix} 0 & \dots & 0 \\ \vdots & & \vdots \\ 0 & \dots & 0 \\ x_1 & \dots & x_\ell \end{pmatrix},$$

$$X_{ij} = \begin{pmatrix} 0 & \dots & 0 & \dots & 0 \\ \vdots & & \vdots & & \vdots \\ 0 & \dots & 0 & \dots & 0 \\ 0 & \dots & x_1 & \dots & x_\ell \end{pmatrix},$$

*corresponding to size in the nilpotent submatrices $N_1$ and $N_2$.*

## 3 Conclusion

It is well know that computing the fine canonical structure elements of triples of matrices $(E, A, B) \in M_n(\mathbb{C}) \times M_n(\times M_{n\times m}(\mathbb{C})$ under feedback and derivative feedback corresponding to the singular systems $E\dot{x} = Ax + Bu$ are ill-posed problem because of arbitrary small perturbations in the entries may drastically change the canonical structure. The knowledge of holomorphic canonical forms permit us to know the canonical structures what are nearby of a fixed triple.

## References


[1] V. I. Arnold, *On matrices depending on parameters*, Russian Math. Surveys, **26**, 2 (1971), pp. 29–43.

[2] V. I. Arnold, *Geometrical methods in the theory of ordinary differential equations.* Springer, New York. 1988.

[3] L. Dai “Singular Control Systems”. Springer Verlag. New York (1989).

[4] Mª I. García-Planas, A. Díaz, *Canonical forms for multi-input reparable singular systems.* Wseas Transactions on Mathematics. **6** (4), pp. 601-608, (2007).

[5] Mª I. García Planas, *Estudio geométrico de familias diferenciables de parejas de matrices* Edicions UPC, Barcelona, (1994).

[6] Mª I. García-Planas, M. D. Magret, *Polynomial Matrices and generalized linear multivariable dynamical systems.* IRBS of Wseas Press, pp. 17-22 (2000).

[7] P. Kunkel, V. Mehrmann “Differential-Algebraic Equations. Analysis and Numerical Solution” EMS, Zurich, (2006).

[8] J. J. Loiseau, K. Ölçadiram, M. Malabre, *Feedback canonical forms of singular systems.* Kybernetica 27, (4), pp289-305, (1991).

[9] A. Tannenbaum, *Invariance and System Theory: Algabraic and geometric Aspects*, Lecture Notes in Math. 845, Springer-Verlag (1981).