Application of the Hotelling's and F statistical for the determination of defects in the dental enamel

Cortez José Italo, González Flores Marcos, Perea González Gloria Patricia,

Vega Galina Victor Javier, Cortez Liliana, Cortez Ernest Italovich.

Abstract—The present work presents the experiment that was made to verify and to corroborate the physical changes in the dental enamel. The data were obtained in voltage terms in four stages: without treatment, dental paste, acid engraver and adhesive. The covariance matrices of the samples were then compared with each other to obtain an estimation of every pair of matrices. The inverse estimated matrix was obtained, and finally the Hottelling's statistical was calculated for the multivaried case. We show that the selected treatments have differences in their averages. This allows us to conclude that there are physical changes in the dental enamel.

Keywords—rugosity of the dental enamel, multivariate data analysis, covariance, Hottelling's and F statistical, inverse matrix, data matrix.

I. Introduction

In the "Development of prototypes for the detection of defects in materials", instrumentation for the diagnosis of physical defects in dental enamel is being developed [1], which consists of a system with the following devices: 1.- data acquisition card based on the 8032 microcontroller, 2.- a sending- receiving infrared spectrum tract, 3.- a mechanism for the positioning and scanning of the dental samples studied, 4.- computer for the storage and processing of the data acquired from the surface of the tooth enamel, 5.- design and implementation of software for the processing of the data acquired, [2,3].

Manuscript received February 5, 2007; Revised received April 8, 2007.

This work was supported by the DAGU (Dirección de Atención y Gestión Universitaria)

J.I.C. Author is with the Faculty of Computing Science of the Autonomous University of Puebla, Mexico. (corresponding author to provide phone: 52-222-229-5500 ext.7205; e-mail: jitalo@siu.buap.mx).

C.L. S. B. Author is with Faculty of Sciences of the Electronics of the University Autonomous of Puebla, Mexico. (corresponding author to provide phone: 52-222-229-5500 ext.7405; e-mail: lcortez@ece.buap.mx).

F.G.M Author is with Faculty of Computing Science of the Autonomous University of Puebla, Mexico. (corresponding author to provide phone: 52-222-229-5500 ext.7405; e-mail: mgonzalez@cs.buap.mx).

P.G.G.P. Author is with Faculty of Odontology of the Autonomous University of Puebla, Mexico. (corresponding author to provide phone: 52-222-229-5500 ext.640005; e-mail:pattypere@hotmail.com).

V.G.J. Author is with Faculty of Odontology of the Autonomous University of Puebla, Mexico. (corresponding author to provide phone: 52-222-229-5500 ext.640005; e-mail:vega40@hotmail.com).

C.E.I. Faculty of Electronic Sciences of the Autonomous University of Puebla, Mexico. E-mail: italovich@hotmail.com).

The data acquired are subjected to a multivariate statistical processing, which consists of proving the hypothesis that the mean vectors of the 2 samples are equal $H_o : \mu_1 = \mu_2$; that is, that there are no changes between one treatment and the other, contrary to that there are, that is to say $H_1 : \mu_1 \neq \mu_2$ this will allow the determination if there are differences in the mean vectors of the samples coming from an orthodontic treatment, which will be used for the Hottelling's statistical test for the multivariate case.

The Hottelling's test T^2 is a generalization of the statistic t (1):

$$t = \left(\bar{x}_1 - \bar{x}_2\right) \Big/ \left\{ s\sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \right\} \tag{1}$$

For the general case, p variables will be had, which are $x_1, x_2, x_3 \ldots x_p$ and 2 samples with sizes n_1 and n_2. Therefore, there will be 2 vectors of sampling means x_1 and x_2 which are calculated using (2) and (3).

$$\bar{x}_j = \sum_{i=1}^{n} x_{ij} \big/ n \tag{2}$$

$$\bar{x} = \begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \\ \vdots \\ \bar{x}_p \end{bmatrix} \tag{3}$$

Also, 2 covariance matrices will be used, C_1 and C_2 where each one will be calculated using the formulas (4), (5) and (6).

$$s_j^2 = \sum_{i=1}^{n} \left(x_{ij} - \bar{x}_j\right)^2 \big/ (n-1) \tag{4}$$

$$c_{jk} = \sum \left(x_{ij} - \bar{x}_j \right)\left(x_{ik} - \bar{x}_k \right) \Big/ (n-1) \quad (5)$$

$$C = \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{ip} \\ c_{21} & c_{22} & \cdots & c_{2p} \\ \vdots & \vdots & \cdots & \vdots \\ c_{p1} & c_{p2} & \cdots & c_{pp} \end{bmatrix} \quad (6)$$

An estimation of the covariance matrix of the population is obtained by (7):

$$C = \{(n_1 - 1)C_1 + (n_2 - 1)C_2\} / (n_1 + n_2 - 2) \quad (7)$$

Therefore, the statistic defined by the Hottelling test T^2 is:

$$T^2 = n_1 n_2 \left(\bar{x}_1 - \bar{x}_2 \right)' C^{-1} \left(\bar{x}_1 - \bar{x}_2 \right) \Big/ (n_1 + n_2) \quad (8)$$

A significantly large value for this statistic will be evidence that the mean vectors are different for the 2 sampling populations. The significance or lack of significance of T^2 is simply determined by transforming the statistic of the formula (8) into (9).

$$F = (n_1 + n_2 - p - 1)^2 T^2 \Big/ \{(n_1 + n_2 - 2)p\}, \quad (9)$$

which has a distribution F with p and $(n_1 + n_2 - p - 1)$ degrees of freedom.

Given that T^2 is a quadratic form can be rewritten as shown in (10).

$$T^2 = \frac{n_1 n_2}{n_1 + n_2} \sum_{i=1}^{p} \sum_{k=1}^{p} \left(\bar{x}_{1i} - \bar{x}_{2i} \right) c^{ik} \left(\bar{x}_{1k} - \bar{x}_{2k} \right). \quad (10)$$

Where $\bar{x}_{jl}$ is the mean of the variable x_l in the l-th sample and c^{ik} is the element in the i-th row and k-th column of the inverse matrix C^{-1} [4, 5].

The program was developed in pascal language, due to the facility with which the matrices can be handled; first the inverse of a covariance matrix is obtained which is calculated using matlab. This matrix is imported to the program that will use it to calculate the Hottelling statistic [6].

II. Description of the Problem

During the orthodontic treatment, physical changes are produced in the structure of the tooth enamel, which continues to be subject of study since it may be the cause of possible afflictions in the dental organs.

The test of significance with multivariate data will allow comparison of the mean of 2 samples to know if there are differences in them, for which we will only analyze one zone of the tooth.

III. Experiment

The measurement of the rugosity is done carrying out a scanning of the surface of the tooth enamel with intervals of 50 micras to an entire tooth, which generated a data matrix of 75x40 (the decision was made to analyze a third of the dental organ, the region to research was limited to a square shape due to the irregular surface of the tooth) which represents the population of said tooth. Each column of the matrix is represented by a variable of said matrix. Four different measurements were carried out generating 4 files (data matrices) corresponding to each of the stages of the orthodontic treatment. In the first stage, the tooth is washed with running water. In the second stage, the tooth was cleaned using a low speed motor with a cup brush, using Oral-B prophylactic paste during 15 seconds in order to eliminate any residue adhered to the tooth surface. It was then washed with running water and the tooth surface was dried with a stream of air. The third stage consists of applying phosphoric acid at 37% during 60 seconds in order to create micro pores (demineralization) on the enamel surface, and in the fourth stage the adhesive was applied and it was photophlymerized during 20 seconds. The significance test was applied to each sample taking them in 2's.

IV. Development

Once the data acquisition stage had been carried out, we proceeded to identify the zone of the tooth to analyze to carry out the significance tests. The criteria used are the same the orthodontist uses for the placement of braces, which consists of dividing the tooth in 3 parts: mesial third, middle third, and distal third [7]. To determine which variables represent each third, 75 was divided by 3, given that the population was of 75x40 for each stage, which leads to assigning 25 variables for each third. The development of the research was concrete to compare the middle third, since it is the position in which the brace is placed. The variables $x_1 \ldots x_{25}$ are those which are used for said tests.

Next, the algorhythms to calculate the mean vector, variance vector, and covariance matrix are presented.

```
Input n,p,A(i,j)

for i=1 to p
     sum ← 0
       for j=1 to n do
          sum ← sum +A(i,j)
           x(i) ← sum/n
Output x(i)

Input A(i,j),n,p,x(i)

  for i=1 to p do
    sum ← 0
      for j=1 to n do
         sum ← sum + √(a(i, j) − x(i))
```

```
    s ⟵ sum / (n-1)
output: s(i)
```

where n is the size of the sample, p number of variables and A(i,j) the data matrix.

```
Input: A(i,j),p,n,x(i)

for  j=1 to p do
  for j=1 to p do
     sum ⟵ 0
       for i=to n do
         sum ⟵ sum+(A(i,j)- x(i))*(A(i,k)-x(k))
       Cov(j,k) ⟵ sum/(n-1)
Output: Cov(i,j)
```

The algorhythms used to calculate the statistic T^2 and the statistic F are [4]:

Input : n_1, n_2, (x_i), p, (a_{ij}^{-1})

$T_1 \leftarrow 0.0$

for i = 1, 2,…,p do

for k = 1,2,…,p do

$$T_1 \leftarrow T_1 + ((x_{1i} - x_{2i}) * a^{ik} * (x_{1k} - x_{2k}))$$

$$T \leftarrow (n_1 * n_2) / (n_1 + n_2) * T_1$$

Output : T (Statistic T)

Input: n_1, n_2, T, p

$$F \leftarrow ((n_1 + n_2 - p - 1)^2 * T) / ((n_1 + n_2 - 2) * p)$$

Output: F (Statistic F).

where: n_1 y n_2 are the size of the sample

$x(i)$ the mean vectors

p the number of variables

a_i^{-1} the inverse matrix of the covariance .

V. Results

The experiment carried out allowed corroboration that the orthodontic treatment generates physical changes in the tooth enamel, so that the 4 different structures of data obtained by the orthodontic treatment are applied [8], which are: sample without treatment, samples with application of toothpaste, with application of etching acid and finally with application of an adhesive, which were compared in 2's; that is, the sample without treatment with application of tooth paste, sample without treatment with application of etching acid, sample without treatment with application of adhesive, sample of application of prophylactic paste with etching acid, sample of application of toothpaste with adhesive and finally application of etching acid with adhesive were compared. The first phase consisted of calculating the covariance matrices of each sample with its corresponding mean vectors using the software created for this test, results of which are seen in Table 1 and 2.

After carrying out the calculation of the covariance matrixes and mean vectors of the samples, we proceeded to calculate an estimation of the covariance matrix, for which the formula was used (7) and the inverse matrix was calculated using matlab [9] with the instruction "inv", generating a file which will save the calculation of said matrix. All this for the sample without treatment and application of toothpaste (graph 1), sample without treatment and application of etching acid (graph 2), sample without treatment and application of adhesive (graph 3), sample of toothpaste with application of etching acid (graph 4), sample of toothpaste with application of adhesive (graph 5) and sample of application of etching acid with adhesive (graph 6).

Finally, it is shown that the tests of hypothesis applied to the surface of the tooth enamel show the rejection that the mean vectors are equal, which is information that allows one to conclude that there are physical changes in the tooth enamel in each stage of the orthodontic treatment. In the next table I we show the estimated matrix of the sample without treatment with application of toothpaste.

Table I (a)
Estimated matrix (column 1 to 5)

Symbol	Quantity				
X_1	0.3587	0.0475	-0.0288	0.0079	0.0617
X_2	0.0475	0.1048	0.0059	0.0228	0.0035
X_3	-0.0288	0.0059	0.1089	-0.0023	-0.0042
X_4	0.0079	0.0228	-0.0023	0.1644	0.0205
X_5	0.0617	0.0035	-0.0042	0.0205	0.0860
X_6	-0.0110	0.0005	-0.0022	-0.0084	0.0012
X_7	-0.0024	-0.0058	-0.0061	0.0388	0.0145
X_8	0.0031	-0.0095	0.0020	0.0031	-0.0121
X_9	0.0111	-0.0044	0.0067	0.0159	-0.0091
X_{10}	0.0166	0.0002	-0.0093	0.0047	0.0135
X_{11}	0.0447	0.0016	0.0100	0.0082	0.0052
X_{12}	0.0185	0.0101	-0.0107	-0.0007	0.0050
X_{13}	0.0270	-0.0045	-0.0051	0.0166	-0.0063
X_{14}	0.0599	0.0053	-0.0123	0.0207	0.0123
X_{15}	0.0100	-0.0076	-0.0057	0.0276	0.0197
X_{16}	0.0249	0.0085	0.0092	0.0324	0.0177
X_{17}	0.0004	-0.0034	-0.0054	0.0160	0.0047
X_{18}	0.0268	0.0007	-0.0044	0.0367	0.0157
X_{19}	0.0270	-0.0026	0.0106	0.0161	0.0076
X_{20}	0.0939	0.0036	0.0022	0.0063	0.0150
X_{21}	0.0328	0.0036	0.0019	0.0492	0.0260
X_{22}	0.0457	0.0070	0.0037	0.0203	0.0163
X_{23}	0.0550	0.0121	0.0122	0.0338	0.0372
X_{24}	0.0325	0.0020	0	0.0234	0.0146
X_{25}	0.0319	-0.0056	-0.0017	0.0268	0.0166

Table I (b)
Estimated matrix (column 6 to 10)

Symbol	Quantity				
X_1	-0.0110	-0.0024	0.0031	0.0111	0.0166
X_2	0.0005	-0.0058	-0.0095	-0.0044	0.0002

X_3	-0.0022	-0.0061	0.0020	0.0067	-0.0093
X_4	-0.0084	0.0388	0.0031	0.0159	0.0047
X_5	0.0012	0.0145	-0.0121	-0.0091	0.0135
X_6	0.0760	-0.0032	0.0028	0.0098	0.0039
X_7	-0.0032	0.0901	-0.0060	-0.0071	0.0090
X_8	0.0028	-0.0060	0.0755	0.0554	0.0011
X_9	0.0098	-0.0071	0.0554	0.0738	0.0071
X_{10}	0.0039	0.0090	0.0011	0.0071	0.0839
X_{11}	0.0033	0.0086	0.0060	0.0118	0.0176
X_{12}	-0.0046	0.0048	0.0001	0.0044	0.0046
X_{13}	0.0122	0.0146	0.0170	0.0225	0.0244
X_{14}	-0.0190	0.0314	-0.0041	0.0033	0.0289
X_{15}	0.0061	0.0291	0.0074	0.0082	0.0109
X_{16}	0.0027	0.0285	-0.0005	0.0089	0.0163
X_{17}	0.0019	0.0194	0.0060	0.0018	0.0023
X_{18}	0.0077	0.0304	0	0.0114	0.0105
X_{19}	0.0056	0.0076	0.0094	0.0202	0.0101
X_{20}	0.0063	0.0115	0.0127	0.0209	0.0225
X_{21}	0.0032	0.0375	0.0090	0.0205	0.0354
X_{22}	0.0026	0.0209	-0.0020	0.0127	0.0158
X_{23}	0.0139	0.0286	-0.0033	0.0180	0.0167
X_{24}	-0.0005	0.0282	0.0002	0.0095	0.0229
X_{25}	0.0049	0.0260	0.0017	0.0081	0.0200

Table I (c)
Estimated matrix (column 11 to 15)

Symbol	Quantity				
X_1	0.0447	0.0185	0.0270	0.0599	0.0100
X_2	0.0016	0.0101	-0.0045	0.0053	-0.0076
X_3	0.0100	-0.0107	-0.0051	-0.0123	-0.0057
X_4	0.0082	-0.0007	0.0166	0.0207	0.0276
X_5	0.0052	0.0050	-0.0063	0.0123	0.0197
X_6	0.0033	-0.0046	0.0122	-0.0190	0.0061
X_7	0.0086	0.0048	0.0146	0.0314	0.0291
X_8	0.0060	0.0001	0.0170	-0.0041	0.0074
X_9	0.0118	0.0044	0.0225	0.0033	0.0082
X_{10}	0.0176	0.0046	0.0244	0.0289	0.0109
X_{11}	0.0726	0.0025	0.0240	0.0233	0.0167
X_{12}	0.0025	0.0359	0.0076	0.0083	0.0061
X_{13}	0.0240	0.0076	0.1066	0.0378	0.0272
X_{14}	0.0233	0.0083	0.0378	0.1179	0.0258
X_{15}	0.0167	0.0061	0.0272	0.0258	0.0514
X_{16}	0.0240	-0.0045	0.0266	0.0492	0.0266
X_{17}	0.0030	0.0039	0.0184	0.0277	0.0207
X_{18}	0.0169	0.0060	0.0268	0.0323	0.0284
X_{19}	0.0267	0.0057	0.0268	0.0274	0.0187
X_{20}	0.0441	0.0093	0.0492	0.0512	0.0383
X_{21}	0.0345	0.0151	0.0486	0.0591	0.0511
X_{22}	0.0230	0.0046	0.0192	0.0341	0.0202
X_{23}	0.0223	0.0047	0.0149	0.0419	0.0316
X_{24}	0.0264	0.0017	0.0321	0.0445	0.0281
X_{25}	0.0176	0.0025	0.0440	0.0567	0.0317

Table I (d)
Estimated matrix (column 16 to 20)

Symbol	Quantity				
X_1	0.0249	0.0004	0.0268	0.0270	0.0939
X_2	0.0085	-0.0034	0.0007	-0.0026	0.0036
X_3	0.0092	-0.0054	-0.0044	0.0106	0.0022
X_4	0.0324	0.0160	0.0367	0.0161	0.0063
X_5	0.0177	0.0047	0.0157	0.0076	0.0150
X_6	0.0027	0.0019	0.0077	0.0056	0.0063
X_7	0.0285	0.0194	0.0304	0.0076	0.0115
X_8	-0.0005	0.0060	0	0.0094	0.0127
X_9	0.0089	0.0018	0.0114	0.0202	0.0209
X_{10}	0.0163	0.0023	0.0105	0.0101	0.0225
X_{11}	0.0240	0.0030	0.0169	0.0267	0.0441
X_{12}	-0.0045	0.0039	0.0060	0.0057	0.0093
X_{13}	0.0266	0.0184	0.0268	0.0268	0.0492
X_{14}	0.0492	0.0277	0.0323	0.0274	0.0512
X_{15}	0.0266	0.0207	0.0284	0.0187	0.0383
X_{16}	0.0740	0.0064	0.0342	0.0244	0.0304
X_{17}	0.0064	0.0426	0.0180	0.0072	0.0231
X_{18}	0.0342	0.0180	0.0664	0.0216	0.0287
X_{19}	0.0244	0.0072	0.0216	0.0365	0.0383
X_{20}	0.0304	0.0231	0.0287	0.0383	0.1008
X_{21}	0.0468	0.0345	0.0577	0.0365	0.0689
X_{22}	0.0305	0.0050	0.0241	0.0239	0.0412
X_{23}	0.0570	0.0125	0.0457	0.0279	0.0517
X_{24}	0.0420	0.0141	0.0357	0.0230	0.0419
X_{25}	0.0386	0.0214	0.0341	0.0245	0.0457

Table I (e)
Estimated matrix (column 21 to 25)

Symbol	Quantity				
X_1	0.0328	0.0457	0.0550	0.0325	0.0319
X_2	0.0036	0.0070	0.0121	0.0020	-0.0056
X_3	0.0019	0.0037	0.0122	0	-0.0017
X_4	0.0492	0.0203	0.0338	0.0234	0.0268
X_5	0.0260	0.0163	0.0372	0.0146	0.0166
X_6	0.0032	0.0026	0.0139	-0.0005	0.0049
X_7	0.0375	0.0209	0.0286	0.0282	0.0260
X_8	0.0090	-0.0020	-0.0033	0.0002	0.0017
X_9	0.0205	0.0127	0.0180	0.0095	0.0081
X_{10}	0.0354	0.0158	0.0167	0.0229	0.0200
X_{11}	0.0345	0.0230	0.0223	0.0264	0.0176
X_{12}	0.0151	0.0046	0.0047	0.0017	0.0025
X_{13}	0.0486	0.0192	0.0149	0.0321	0.0440
X_{14}	0.0591	0.0341	0.0419	0.0445	0.0567
X_{15}	0.0511	0.0202	0.0316	0.0281	0.0317

X_{16}	0.0468	0.0305	0.0570	0.0420	0.0386
X_{17}	0.0345	0.0050	0.0125	0.0141	0.0214
X_{18}	0.0577	0.0241	0.0457	0.0357	0.0341
X_{19}	0.0365	0.0239	0.0279	0.0230	0.0245
X_{20}	0.0689	0.0412	0.0517	0.0419	0.0457
X_{21}	0.1227	0.0417	0.0619	0.0546	0.0482
X_{22}	0.0417	0.0390	0.0433	0.0312	0.0275
X_{23}	0.0619	0.0433	0.1006	0.0500	0.0463
X_{24}	0.0546	0.0312	0.0500	0.0529	0.0368
X_{25}	0.0482	0.0275	0.0463	0.0368	0.0628

The inverse matrix (table I) was then calculated using matlab and the next matrix was obtained (table II):

Table II (a)
Inverse matrix (column 1 to 5)

Symbol	Quantity				
X_1	6.3329	-1.7606	1.4440	-0.5754	-4.7338
X_2	-1.7606	12.8041	-1.7302	-2.9728	1.5115
X_3	1.4440	-1.7302	12.0384	1.1403	-0.0222
X_4	-0.5754	-2.9728	1.1403	9.6909	-0.3162
X_5	-4.7338	1.5115	-0.0222	-0.3162	20.0244
X_6	0.6989	-1.8595	2.4868	2.7431	0.5063
X_7	0.3427	2.0841	-0.5700	-2.2019	0.6428
X_8	-1.6118	-0.0649	0.3839	2.0341	-0.0189
X_9	0.5914	1.9836	-0.3588	-3.6262	4.6339
X_{10}	0.6803	-0.1327	1.2720	0.6373	-2.9058
X_{11}	-0.9718	0.4082	-1.5629	-0.5265	0.6460
X_{12}	-1.0855	-5.2268	4.2650	3.7304	0.1012
X_{13}	-0.6719	-0.3293	-0.0662	-0.1907	1.9761
X_{14}	-2.2309	-0.9588	2.8155	2.4228	2.5231
X_{15}	3.9908	2.1778	3.2309	-2.3509	-8.1031
X_{16}	0.3919	-3.0691	-0.9166	-0.4938	1.0480
X_{17}	2.1012	-0.9979	-0.1300	-1.7257	-1.2252
X_{18}	-3.3913	0.7230	2.3043	-0.3953	3.2427
X_{19}	2.6870	4.8782	-6.5323	-3.2691	-3.8731
X_{20}	-8.3767	-0.2747	0.0554	5.8227	8.2898
X_{21}	3.6855	0.0373	-2.0960	-3.7562	-3.4075
X_{22}	-2.1331	-2.9195	0.8359	-1.2484	-1.2757
X_{23}	0.0616	-1.0138	-3.4103	-0.1859	-7.7510
X_{24}	0.6672	-0.0845	2.9404	2.6693	3.7331
X_{25}	2.1035	4.1970	-1.3297	-3.8078	-2.1622

Table II (b)
Inverse matrix (column 6 to 10)

Symbol	Quantity				
X_1	0.6989	0.3427	-1.6118	0.5914	0.6803
X_2	-1.8595	2.0841	-0.0649	1.9836	-0.1327
X_3	2.4868	-0.5700	0.3839	-0.3588	1.2720
X_4	2.7431	-2.2019	2.0341	-3.6262	0.6373
X_5	0.5063	0.6428	-0.0189	4.6339	-2.9058
X_6	18.1568	-0.0998	2.2806	-2.3928	-1.7872
X_7	-0.0998	19.4798	-3.5185	5.9952	-0.0724
X_8	2.2806	-3.5185	39.4906	-33.6955	0.6542
X_9	-2.3928	5.9952	-33.6955	48.0669	-2.0136
X_{10}	-1.7872	-0.0724	0.6542	-2.0136	16.5660
X_{11}	-1.1297	-0.9601	-0.2527	1.0830	-1.9900
X_{12}	4.6982	-3.5114	3.4518	-3.6549	-0.2964
X_{13}	-3.1086	-0.5477	1.8062	-4.9974	-1.0838
X_{14}	6.0747	-1.0516	3.2216	-1.7340	-2.1833
X_{15}	-1.7896	-6.2669	-3.9585	1.7652	3.1705
X_{16}	-0.0256	-1.4924	-4.5816	5.4166	1.2218
X_{17}	-3.3343	-5.6317	-3.6789	2.3515	4.8012
X_{18}	-1.6399	-3.0218	4.0433	-3.0156	3.3652
X_{19}	-4.2693	5.9217	1.8519	-11.2018	3.8088
X_{20}	1.2894	5.9034	-3.6709	4.9466	0.4247
X_{21}	0.6946	2.0337	-2.4446	1.6881	-5.7932
X_{22}	-0.4602	-13.6672	15.8727	-15.8217	-0.4281
X_{23}	-5.9442	0.4265	7.1110	-12.0547	2.5388
X_{24}	7.2922	-3.3786	-0.9470	3.7284	-5.4216
X_{25}	-2.0436	0.9199	-3.8739	7.8782	-3.0618

Table II (c)
Inverse matrix (column 11 to 15)

Symbol	Quantity				
X_1	-0.9718	-1.0855	-0.6719	-2.2309	3.9908
X_2	0.4082	-5.2268	-0.3293	-0.9588	2.1778
X_3	-1.5629	4.2650	-0.0662	2.8155	3.2309
X_4	-0.5265	3.7304	-0.1907	2.4228	-2.3509
X_5	0.6460	0.1012	1.9761	2.5231	-8.1031
X_6	-1.1297	4.6982	-3.1086	6.0747	-1.7896
X_7	-0.9601	-3.5114	-0.5477	-1.0516	-6.2669
X_8	-0.2527	3.4518	1.8062	3.2216	-3.9585
X_9	1.0830	-3.6549	-4.9974	-1.7340	1.7652
X_{10}	-1.9900	-0.2964	-1.0838	-2.1833	3.1705
X_{11}	23.5711	-0.2662	1.4028	1.2162	0.6883
X_{12}	-0.2662	39.1840	-3.8201	-2.3068	-5.1451
X_{13}	1.4028	-3.8201	20.7308	3.8194	1.9774
X_{14}	1.2162	-2.3068	3.8194	26.2102	6.8196
X_{15}	0.6883	-5.1451	1.9774	6.8196	52.3030
X_{16}	-3.8319	10.6719	-5.2362	-11.0243	-8.4346
X_{17}	2.6788	2.0805	0.1156	-10.4037	-4.1814
X_{18}	-0.6574	0.4706	-0.2986	3.4171	-1.5946
X_{19}	-9.7141	-10.4048	-1.8269	-4.6116	1.0042
X_{20}	-8.0469	5.0181	-5.9216	0.9508	-15.5601
X_{21}	0.2864	-6.9346	-2.3836	-3.8352	-7.3721
X_{22}	0.5744	3.4401	4.4683	-5.4470	1.7347
X_{23}	5.4275	-7.1996	13.1077	7.3988	7.0967
X_{24}	-7.1250	8.9436	-8.6381	-3.7160	-4.9122
X_{25}	3.6517	2.7794	-14.2992	-18.1909	-9.9771

Table II (d)
Inverse matrix (column 16 to 20)

Symbol	Quantity				
X_1	0.3919	2.1012	-3.3913	2.6870	-8.3767
X_2	-3.0691	-0.9979	0.7230	4.8782	-0.2747
X_3	-0.9166	-0.1300	2.3043	-6.5323	0.0554
X_4	-0.4938	-1.7257	-0.3953	-3.2691	5.8227
X_5	1.0480	-1.2252	3.2427	-3.8731	8.2898
X_6	-0.0256	-3.3343	-1.6399	-4.2693	1.2894
X_7	-1.4924	-5.6317	-3.0218	5.9217	5.9034
X_8	-4.5816	-3.6789	4.0433	1.8519	-3.6709
X_9	5.4166	2.3515	-3.0156	-11.2018	4.9466
X_{10}	1.2218	4.8012	3.3652	3.8088	0.4247
X_{11}	-3.8319	2.6788	-0.6574	-9.7141	-8.0469
X_{12}	10.6719	2.0805	0.4706	-10.4048	5.0181
X_{13}	-5.2362	0.1156	-0.2986	-1.8269	-5.9216
X_{14}	-11.0243	-10.4037	3.4171	-4.6116	0.9508
X_{15}	-8.4346	-4.1814	-1.5946	1.0042	-15.5601
X_{16}	42.6864	10.5649	0.1435	-8.7232	11.3597
X_{17}	10.5649	47.4090	-2.5939	3.0675	-9.6601
X_{18}	0.1435	-2.5939	37.1708	-8.1972	11.7323
X_{19}	-8.7232	3.0675	-8.1972	79.7846	-14.6535
X_{20}	11.3597	-9.6601	11.7323	-14.6535	51.1988
X_{21}	-1.3369	-9.0284	-13.2050	1.0170	-11.4387
X_{22}	0.7620	19.1948	4.7815	-21.2973	-18.1461
X_{23}	-17.7546	-1.1563	-5.9340	9.3478	-11.4516
X_{24}	-7.3140	-1.3124	-7.8948	0.8653	1.0128
X_{25}	0.9106	-3.9132	-7.8429	-3.0998	-7.4895

Table II (e)
Inverse matrix (column 21 to 25)

Symbol	Quantity				
X_1	3.6855	-2.1331	0.0616	0.6672	2.1035
X_2	0.0373	-2.9195	-1.0138	-0.0845	4.1970
X_3	-2.0960	0.8359	-3.4103	2.9404	-1.3297
X_4	-3.7562	-1.2484	-0.1859	2.6693	-3.8078
X_5	-3.4075	-1.2757	-7.7510	3.7331	-2.1622
X_6	0.6946	-0.4602	-5.9442	7.2922	-2.0436
X_7	2.0337	-13.6672	0.4265	-3.3786	0.9199
X_8	-2.4446	15.8727	7.1110	-0.9470	-3.8739
X_9	1.6881	-15.8217	-12.0547	3.7284	7.8782
X_{10}	-5.7932	-0.4281	2.5388	-5.4216	-3.0618
X_{11}	0.2864	0.5744	5.4275	-7.1250	3.6517
X_{12}	-6.9346	3.4401	-7.1996	8.9436	2.7794
X_{13}	-2.3836	4.4683	13.1077	-8.6381	-14.2992
X_{14}	-3.8352	-5.4470	7.3988	-3.7160	-18.1909
X_{15}	-7.3721	1.7347	7.0967	-4.9122	-9.9771
X_{16}	-1.3369	0.7620	-17.7546	-7.3140	0.9106
X_{17}	-9.0284	19.1948	-1.1563	-1.3124	-3.9132
X_{18}	-13.2050	4.7815	-5.9340	-7.8948	-7.8429
X_{19}	1.0170	-21.2973	9.3478	0.8653	-3.0998
X_{20}	-11.4387	-18.1461	-11.4516	1.0128	-7.4895
X_{21}	33.7203	-7.1854	-1.0196	-6.1420	11.0374
X_{22}	-7.1854	102.8730	-12.0145	-13.2248	-1.2464
X_{23}	-1.0196	-12.0145	46.6472	-18.8422	-16.0161
X_{24}	-6.1420	-13.2248	-18.8422	75.0974	1.7818
X_{25}	11.0374	-1.2464	-16.0161	1.7818	62.1072

Table III
Results

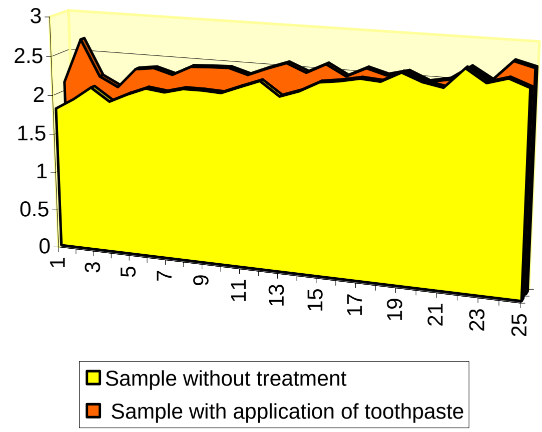
Stages corresponding to an orthodontic process.	Hottelling's statistic T^2	Statistic F
Sample without treatment and application tooth paste.	345.8153	9.5764
Sample without treatment and application of etching acid.	473.7985	13.1206
Sample without treatment with application of ionomer cement.	1170.2313	32.4064
Sample with application of toothpaste and etching acid	789.5596	21.8647
Sample with application of toothpaste and ionomer cement	1263.6704	34.9939
Sample with application of etching acid with ionomer cement.	1368.1251	37.8865

The values obtained in each comparison show significantly large values which supply an indication of difference in the mean vectors in each one of the stages compared (Table III, column T^2), then one can see in the last column of Table III that the values obtained by the statistic F (obtained by formula 9) have a value greater than 1, which according to [2] is indicative that there is significance, that is, the mean vectors show differences. This can be corroborated if we create graphs of the mean vectors of each stage of the orthodontic process (Graphs 1, 2, 3, 4, 5, and 6) as is shown in Graph 7. Table IV shows the mean vectors of the samples in each stage of the orthodontic treatment.

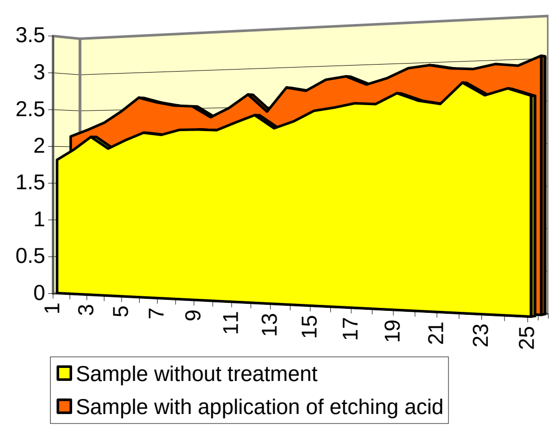
Table IV

Mean vectors of the samples

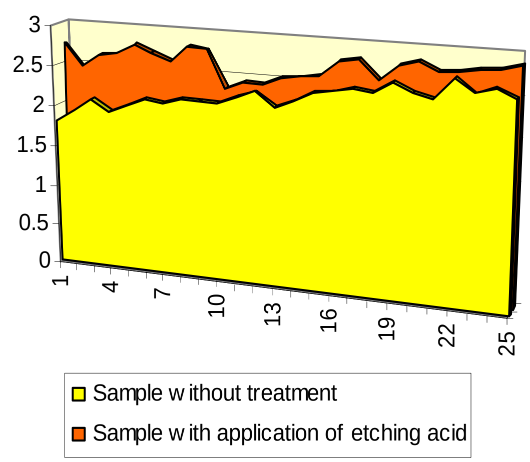
Mean vector Sample without treatment	Mean vector Sample with application of toothpaste	Mean vector Sample with application of etching acid	Mean vector Sample with application of adhesive
1.8174	2.1079	2.1406	2.7324
1.9565	2.6738	2.2251	2.457
2.1259	2.2182	2.3227	2.625
1.9668	2.0986	2.4731	2.6494
2.0781	2.3471	2.6508	2.7895
2.1718	2.3794	2.582	2.7046
2.1426	2.3125	2.5361	2.6074
2.205	2.4311	2.52	2.8012
2.205	2.4458	2.3711	2.7822
2.1933	2.4526	2.4882	2.3281
2.2885	2.3842	2.6562	2.4174
2.3794	2.4819	2.4301	2.4111
2.209	2.5669	2.7314	2.5117
2.2939	2.4682	2.6855	2.5424
2.4204	2.5883	2.8173	2.5639
2.4531	2.459	2.8515	2.7578
2.5029	2.5771	2.7431	2.8066
2.4863	2.5088	2.8134	2.5688
2.6128	2.5713	2.9272	2.7763
2.5171	2.4677	2.9565	2.8349
2.4721	2.5146	2.9091	2.7241
2.7182	2.6811	2.8925	2.7436
2.5634	2.5395	2.9433	2.7924
2.6362	2.7876	2.9155	2.8071
2.5464	2.7246	3.0214	2.8686



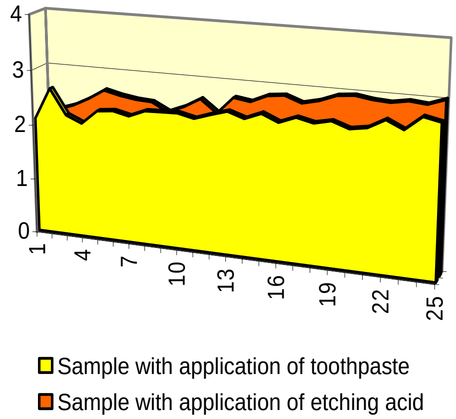
Graph 1: Mean vectors of the samples without treatment and application of toothpaste.



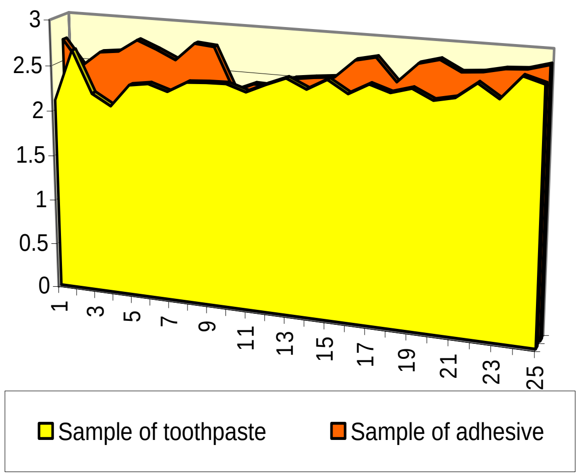
Graph 2: Mean vectors of the samples without treatment and with application of etching acid.



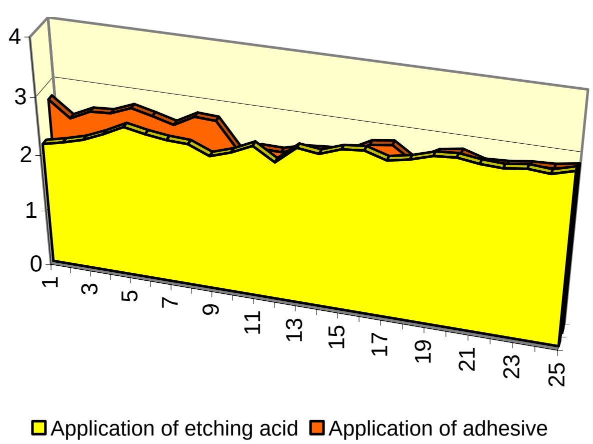
Graph 3: Mean vectors of the samples without treatment and application of adhesive.



Graph 4: Mean vectors of the samples with application of toothpaste and etching acid

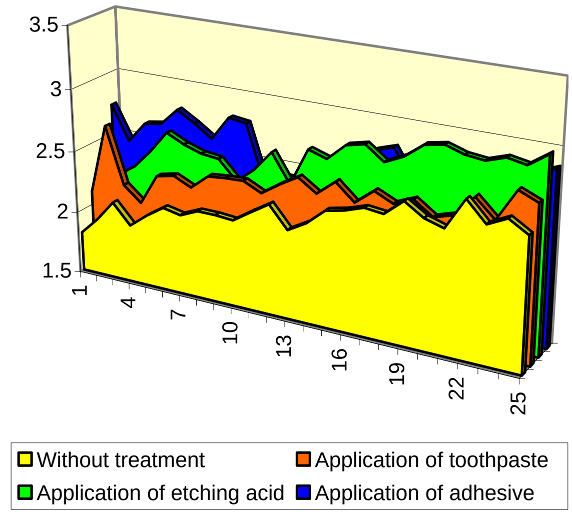


Graph 5: Mean vectors of the samples with application of toothpaste and adhesive.



Graph 6: Mean vectors of the samples with application of etching acid and adhesive

Graph 7 shows a comparison of the 4 stages of an orthodontic treatment, where we can see the differences caused upon applying said process, graph 7, said difference shows that there is a loss of enamel in each stage.



Graph 7: Mean vector of the 4 stages of an orthodontic process.

VI. Conclusion

Comparing the data obtained in the middle third of the samples without treatment and application of toothpaste, without treatment and application of etching acid, without treatment and application of adhesive, with toothpaste and etching acid, with toothpaste and adhesive and finally that of application of etching acid with adhesive, in order to prove the hypothesis that the mean vectors are equal or not; that it, to show if there are physical changes in the application of each stage of orthodontic treatment, situation that would be proven if the hypothesis that the mean vectors are equal were rejected. The results shown The results shown in Table 1 allowed the hypothesis that the mean vectors are equal to be rejected, this makes it more evident as the stages for an orthodontic treatment are applied, all this together considering the 25 variables belonging to the mentioned third. This was proved upon applying the significance test of T^2 together with the F for multivariate data. The result of this work has the purpose of giving information to the orthodontist, given that it indicates that upon carrying out said treatment, physical changes are occurring in the surface of the tooth enamel using the technology developed in the laboratory.

Up to this moment, only the loss of enamel in the treatment has been qualified and the quantification of the loss of enamel will continue to be worked on using the described technology. The results obtained are satisfactory and this technique will continue to be worked on for future analysis in different zones of the tooth.

ACKNOWLEDGMENT

The authors thank the financial support of DAGU-BUAP. In particular the Damian Hernandez and to the Carmen Santiago Diaz for all the attention.

REFERENCES

[1] J.I.Cortez, *Methods and devices for the optical infrared defectoscopia in materials dielectric.* Ph.D. Dissertation. Russia, 2001, pp. 35-70.

[2] J. Italo, M. Bustillo, C. Pérez, V. De León,” Analysis of the influence of the rugosity of surfaces using the method of” in *Proc. SOMI XVII National Conf. of Instrumentation*, Mexico, 2002, pp.82-88.

[3] D.V.Kiesewetter, V.I. Maliugin, J. Italo Cortez, “Researches of the enamel of dent in coherent light” in *Proc. Laser for measurement and information transfer 2005.* SPIE, v.6251, Russia, july2006.

[4] Bryan F. J. Manly, *Multivariate Statical Methods*, Ed. Chapman and Hall, University of Otago, New Zeland, 1986, pp.60-145

[5] Alvin C. Rencher, *Multivariate Statistical Inference and Applications*, Ed. John Wiley & Son, Brigham Young University, 1998, pp.34-87.

[6] William H, *Numerical Recipes in Pascal, The art of scientific computing,* Ed. Springer-Verlag. Cambridge, 1992, pp.45-67.

[7] Murray,J.J, *The Prevention of Dental Disease*. Oxford University Press, Oxford, 1989, pp.23-39.

[8] Dietrich P, *Enamel alterations from bracket bonding and debonding: a study with the scanning microscope*. AJO.May 1981

[9] John H. Mathews, Kurtis D. Fink , *Numerical Method with MATLAB*, Ed. Prentice Hall, USA, 1999, pp.67-85.

Cortez Jose Italo, he made studies of Electrical Engineering and Masters of Sciences in power stations in the Polytechnical Institute of Kiev, Ukraine and Ph.D in Technical Sciences in the Severe Zapadni State Technical University of Saint Petersburg, Russia. At the moment it works as titular professor in the Faculty of Computing Science of the Autonomous University of Puebla, Mexico.

Gonzalez Flores Marcos made studies of Computing Science in the Autonomous University of Puebla, Mexico; and the Master in Computing Science in the Technological of Pachuca, Mexico. He's a titular professor in the Faculty of Computing Science of the Autonomous University of Puebla, Mexico.

Cortez Liliana made studies of Electrical Engineering and Masters of Sciences in power stations Polytechnical Institute of Kiev, Ukraine and Ph.D in Technical Sciences in the Polytechnical State University of Saint Petersburg, Russia. At the moment it works as titular professor in the Faculty of Sciences of the Electronics of the University Autonomous of Puebla, Mexico.

Perea González Gloria Patricia studied a Degree in the Faculty of Odontolgy of the Autonomous University of Puebla, Mexico. She made studies of Masters and Ph.D in Education Sciences. At themoment she is titular professor of the Faculty of Odontology of the Autonomous University of Puebla, Mexico.

Vega Galina Victor Javier studied a Degree in the Faculty of Chemical Sciences and the Masters in the Institute of Physiology, both in the Autonomous University of Puebla, Mexico. At the present he is a titular professor of the Faculty of Odontology of the Autonomous University of Puebla, Mexico.

Cortez Ernest Italovich is student of the Faculty of Electronic Sciences of the Autonomous University of Puebla, Mexico.