Optimal Design of an Impact Damper for a Nonlinear Friction-Driven Oscillator

E. Ehsan Maani Miandoab, A. Yousefi-Koma, D.Ehyaei

Abstract: - In the present study a Friction-Driven oscillator is investigated analytically using perturbation method and numerically with Runge–Kutta's integration procedure. The analytical method is also used to investigate damping performance of a single-particle-impact damper on amplitude and frequency of system over a wide range of particle-to-structure mass ratios, clearance, and coefficients of restitution. Considering sensitivity of the system to variation of mass ratio and coefficient of restitution, optimal values are obtained for these parameters. These optimal values are plotted as a function of the other two parameters. The frequency of the system has low sensitivity to the variation of coefficient of restitution but varies with clearance and mass ratio changes.

Key- words: - Friction-Driven oscillator, Impact damper, Perturbation, Optimal coefficient of restitution.

I. INTRODUCTION

Friction-driven oscillation is a kind of self-excited vibration in which the variable friction force generates periodic motion. The vibration of dry friction damped systems has been of considerable interest to researchers for a long time, for it occurs frequently in everyday life as well as in engineering systems such as creaking doors, squeaking chalks, and rattling turbine blade joints The characteristics of the friction force are quite complex and depend on the normal pressure, slip velocity, surface and material properties [1]. A single particle impact damper is a common vibration-damping device consisting of a single particle enclosed within a container.

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The container can either be mounted directly to the structure to be damped [2] or can be designed as a part of the structure, often as holes drilled directly into the structure [3]. The advantages of impact dampers over traditional damping devices are that impact dampers are inexpensive, have simple designs that provide effective damping performance over a range of accelerations and frequencies [4,5]. In addition, impact dampers are robust and operate in environments that are too harsh for other traditional damping methods [6]. This damper can also be used to damp harmonic excitation system [7]. Also the impact damper can be used for two-degrees-of-freedom system [8]. Vibration damping with impact dampers has been used in a wide variety of applications including vibration attenuation of cutting tools [9] television aerials [10], turbine blades [11,12] structures [13] and plates [2], tubing, and shafts [14,15,16].

In a recent paper, the control of friction-driven oscillation by using impact damper has been studied [17]. An experimental setup of a single degree-of-freedom friction-driven oscillator with an attached impact damper was designed and an approximate solution of general steady-state response was then derived analytically using a piecewise equivalent linearization approach.

In this paper experimental results, reported in literature were used to find the equation of motion. Using dimensionless variables and choosing the order of the magnitude of the different elements of the system it was found that the problem depends on a few parameters, so perturbation method is used to solve this problem [18]. The behavior of this system is analyzed over a wide range of particle-to-structure mass ratios, non-dimensional clearance, d, and coefficient of restitution to derive optimal values for these parameters.

II. PROBLEM FORMULATION

Figure (1) shows the standard mass on a moving belt model of a friction oscillator. The frictional force is a function of relative velocity, thus the equation of the motion of the mass can be written as equation (1).

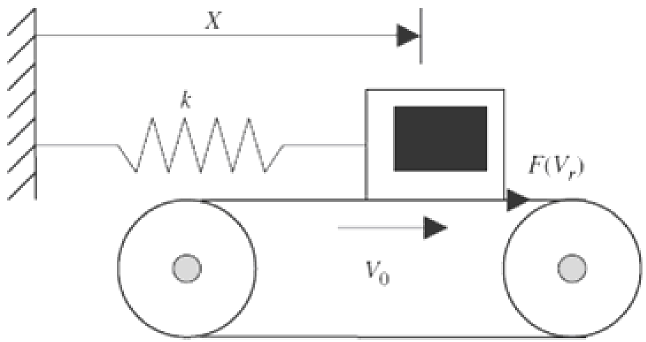

Fig.1
Mass on the moving belt system

$$M\ddot{X}(t)+KX(t)=F(V_0-\dot{X}) \quad as\ \dot{X}<V_0 \tag{1}$$

Where F is the friction force, K the linear spring stiffness and V_0 is the constant belt speed.

It is convenient to introduce a new variable x (t), replacing X (t) as

$$x(t)=X(t)-F(V_0/K) \tag{2}$$

Therefore Equation (1) becomes

$$M\ddot{x}(t)+Kx(t)=F(V_0-\dot{x})-F(V_0) \tag{3}$$

It is assumed that the frictional force can be adequately expressed as a Taylor's series. Assuming $\dot{x}(t)$ to be small and the Taylor's series converges rapidly enough to justify the use of the first three terms only,

$$M\ddot{x}(t)+Kx(t)=F(V_0)-\frac{dF}{dV}\dot{x}(t)+\frac{1}{2}\frac{d^2F}{d^2V}\dot{x}(t)^2-\frac{1}{6}\frac{d^3F}{d^3V}\dot{x}(t)^3-F(V_0) \tag{4}$$

By putting

$$\frac{dF}{dV}==F_V^{(1)}\ ,\ \frac{d^2F}{d^2V}=\frac{F_V^{(2)}}{2}\ ,\ \frac{d^3F}{d^3V}=\frac{F_v^{(3)}}{6} \tag{5}$$

Equation (4) can be written as

$$M\ddot{x}(t)+Kx(t)=-F_V\ \dot{x}(t)+\frac{F_V^{\ 2}}{2}\dot{x}(t)^2-\frac{F_v^{\ 3}}{6}\dot{x}(t)^3 \tag{6}$$

The non-dimensional form of the above equation is

$$y''(\tau)+y(\tau)+ay'(\tau)-by'^2(\tau)+cy'^3(\tau)=0 \tag{7}$$

Where the prime denotes differentiation with respect to τ and

$$\omega=\sqrt{\frac{k}{M}};\ \tau=\omega t;\ x_0=\frac{F(V_0)}{K};\ \ y(\tau)=\frac{x(t)}{x_0};$$

$$a=\frac{F_V^{(1)}}{\sqrt{MK}};\ b=\frac{F_V^{(2)}}{2MK};\ c=\frac{F_V^{(3)}}{6(MK)^{1.5}}F^2(V_0)$$

For M=.56 (kg) and K=763 (N/m) the values of parameters ($a, b\ and\ c$) are obtained experimentally [17] as below:

$a=-0.04 \qquad b=-0.0099 \qquad c=0.0088$

Thus the Equation (7) can be written as

$$y''(\tau)+y(\tau)=\varepsilon(y'(\tau)-0.2475y'^2(\tau)-0.22y'^3(\tau)) \tag{8}$$

Where $\varepsilon=0.04$.

Multiple Scale method is used to obtain the solution of present system. Solving Equation (8) it follows

$$y(\tau)=2.46(\exp(-.04\tau+c_1)+1)^{-.5}\cos(\tau+c_2) \tag{9}$$

Equation (9) shows that

$$y(\tau)\to 2.46\cos t+O(\varepsilon) \quad as\ \tau\to\infty \tag{10}$$

The answer is independent from the values of c_1 as long as it is not zero. This result is in complete agreement with the numerical solution. Figure (2) shows Numerical and Analytical solution of Equation (8). The initial conditions are found by putting c_1 and c_2 equal to zero.

Figure (3) shows the peaks of numerical and analytical solutions as a function of dimensionless time. For the numerical analysis, Runge–Kutta's integration procedure is employed. This figure indicates a complete agreement between numerical and analytical solutions.

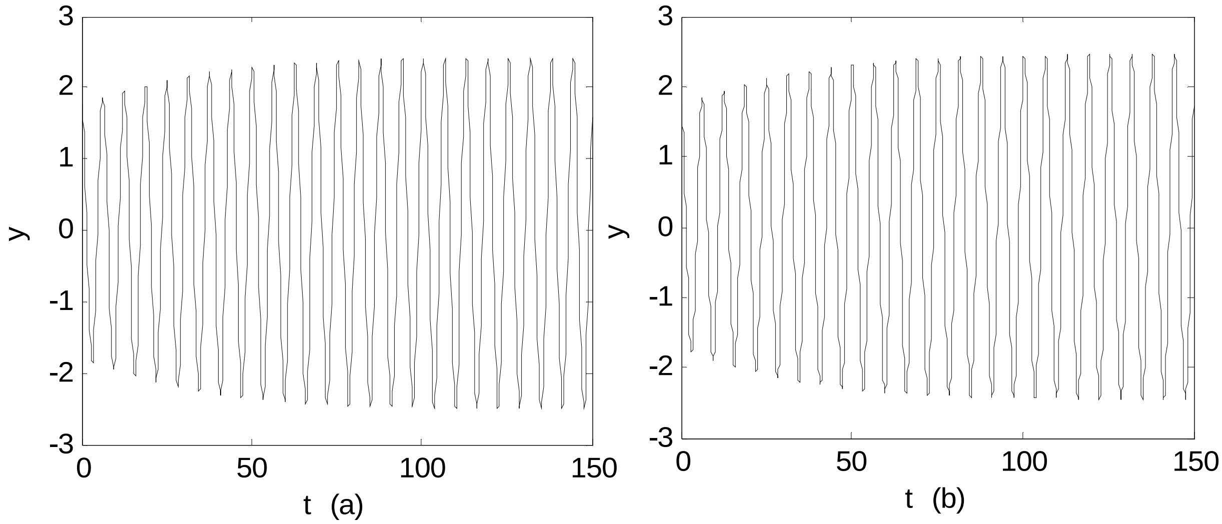

Fig.2
Comparison of the numerical solution (b) with the analytical solution (a) for $y(0) = 1.7536$ and $\dot{y}(0) = 0.1096$
($c_1 = c_2 = 0$)

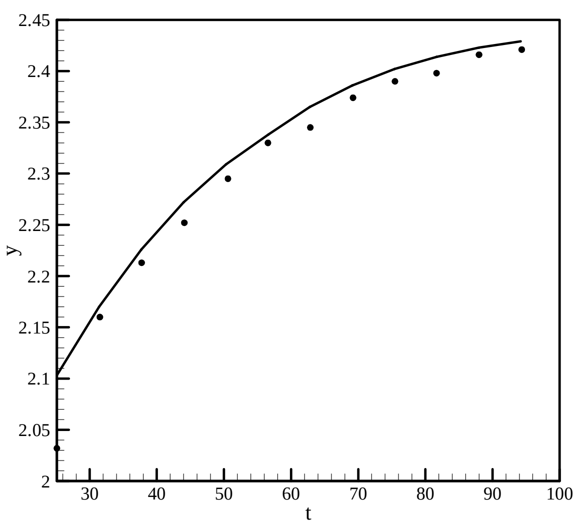

Fig.3
The peaks of numerical and analytical solutions

- *Solid line indicates analytical data,*
- *Symbols indicate numerical data.*

Moreover, the effect of an impact damper is also investigated. A single-particle-impact damper is a common vibration-damping device consisting of a single particle enclose within a container shows the friction-driven oscillator with an impact damper (Figure 4).

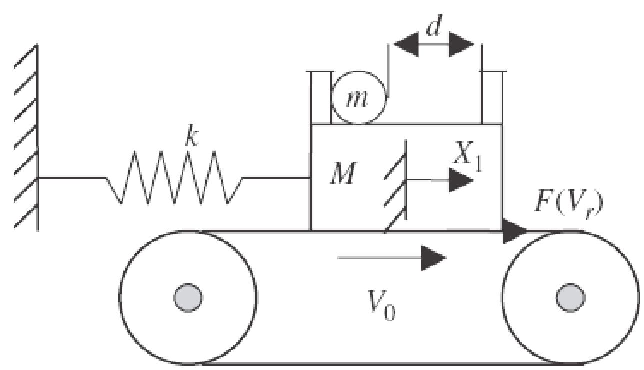

Fig.4
Mass on a moving belt system with an impact damper

Considering the steady-state motion of the system, a periodic solution with two non-symmetric impacts per cycle is derived. Without any loss of generality, the time origin is arbitrarily set $\tau = 0$ at the instant of an impact with the right-hand side wall. Now, the next two consecutive impacts occur at $\tau = \tau_1$ and $\tau = \tau_2$.

The differential equations describing the motion of the system between two consecutive impacts can be written in the following non-dimensional form:

$$y_1(\tau) = 2.46(\exp(-.04\tau + c_1) + 1)^{-0.5} \cos(\tau + c_2)$$
$$u'' = 0. \; or \; u' = v_1$$
$$for \; |y_1 - u| < d \quad and \quad 0^+ < \tau < \tau_1^- \tag{11}$$

And

$$y_1(\tau) = 2.46((\exp(-.04\tau + c_3) + 1)^{-0.5} \cos(\tau + c_4)$$
$$u'' = 0. \ or \ u' = v_2$$
$$for \ |y_1 - u| < d \ \ and \ \ \tau_1^{+} < \tau < \tau_2 \tag{12}$$

The superscript "+" and "-"indicate conditions just after and prior to the impact.

It can be written:

$$x_1(t) = X_1(t) - F(v_0)/k \ , d \ = \frac{d_0}{2x_0} \tag{13}$$

Where M is the mass of structure and u is the displacement of particle.

Since there is no force acting on the secondary mass "m" between impacts, its velocity during the intervals $0^{+} < \tau < \tau^{-}{}_1$ and $\tau_1^{+} < \tau < \tau_2$ remains constant and is assumed as v_1 and v_2 , in respective intervals.

Now, consider the displacement continuity at the impact as

$$y_1(0^{+}) = y_1(\tau_2) \tag{14}$$

$$y_1(\tau_1^{+}) = y_1(\tau_1^{-})$$
(15)

The definition of the coefficient of restitution ε yields

$$\varepsilon = -\frac{y_1(0^{+}) - v_1}{y_1(\tau_2) - v_2} \tag{16}$$

It is used when the impact occurs on the right-hand side wall, and

$$\varepsilon = -\frac{y_1(\tau_1^{+}) - v_2}{y_1(\tau_1^{-}) - v_1} \tag{17}$$

When the impact occurs on the left-hand side wall.

Using the conservation of momentum principle, we get

$$My_1(0^{+}) + mv_1 \ = My_1(\tau_2) + mv_2 \tag{18}$$

$$My_1(\tau_1^{+}) + mv_2 \ = My_1(\tau_1^{-}) + mv_1 \tag{19}$$

From the kinematics of motion of the secondary mass between impacts, it may be written as follows

$$y_1(0^{+}) - y_1(\tau_1^{-}) + 2d \ = -v_1\tau_1 \tag{20}$$

$$y_1(\tau_1) - y_1(\tau_1^{+}) + 2d \ \ = v_2(\tau_2 - \tau_1) \tag{21}$$

Now we have total 8 unknown variables, $c_1, c_2, c_3, c_4, \tau_1, \tau_2, v_1, v_2$ and 8 nonlinear equations. Using these equations we obtain the values of these variables for the given values of a, b, c, M, m, d and ε . System behavior is studied over a wide range of particle-to-structure mass ratio and coefficients of restitution.

In this study the parameters h and r are quantified by

$$h = \frac{A_{damped}}{A_{undamped}} \qquad\qquad r = \frac{m}{M}$$

Where A_{damped} is the amplitude of structure with impact damper and $A_{undamped}$ is the amplitude of the structure without impact damper. It is obvious that the effect of impact damper on the amplitude of system has inverse relation with h.

III. PROBLEM SOLUTION

A system of friction-driven oscillations with an impact damper is solved analytically for the particular range of d. In Figure (5) the non-dimensional amplitude of the controlled system, h, is plotted as a function of dimensionless clearance *d*, for different values of mass ratio *r*= 0.05, 0.10, 0.15 with e=0.8. It is observed that dimensionless amplitude of the controlled system decreases by increasing non-dimensional distance, d.

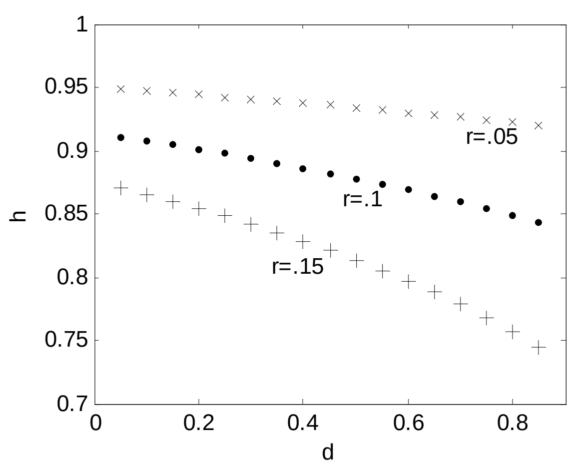

Fig.5
Analytical results for different values of r, with e=0. 8

In Figures 6(a) and 6(b), dimensionless amplitude h, is plotted against e for two different values of *r* and *d*. It is observed that the non-dimensional amplitude of the controlled system, h, increases by increasing restitution coefficient. Because dissipation of energy approaches zero in this condition, when $\varepsilon \to 1$ the amplitude of the controlled system approaches the amplitude of the uncontrolled system. Consequently the performance of impact damper is weak in higher limits of e.

Figures (6) and (7) show that dimensionless amplitude *h* decrease when e decreases because of increasing the energy dissipation. From the overall observation of all results, it is noticed that there is an infinite restitution coefficient, *e*, that decreasing restitution coefficients less than this value, result in small influence on the amplitude of the system. On the other hand very low restitution coefficients are practically unattainable and problematic, because of material deformation. Thus we determine an optimal restitution coefficient that depends on *r* and *d*. Another important conclusion is that the response of the system in large coefficient of restitution is independent of *r*, and there is a lower limit for *r* by which the amplitude converges to a specific value.

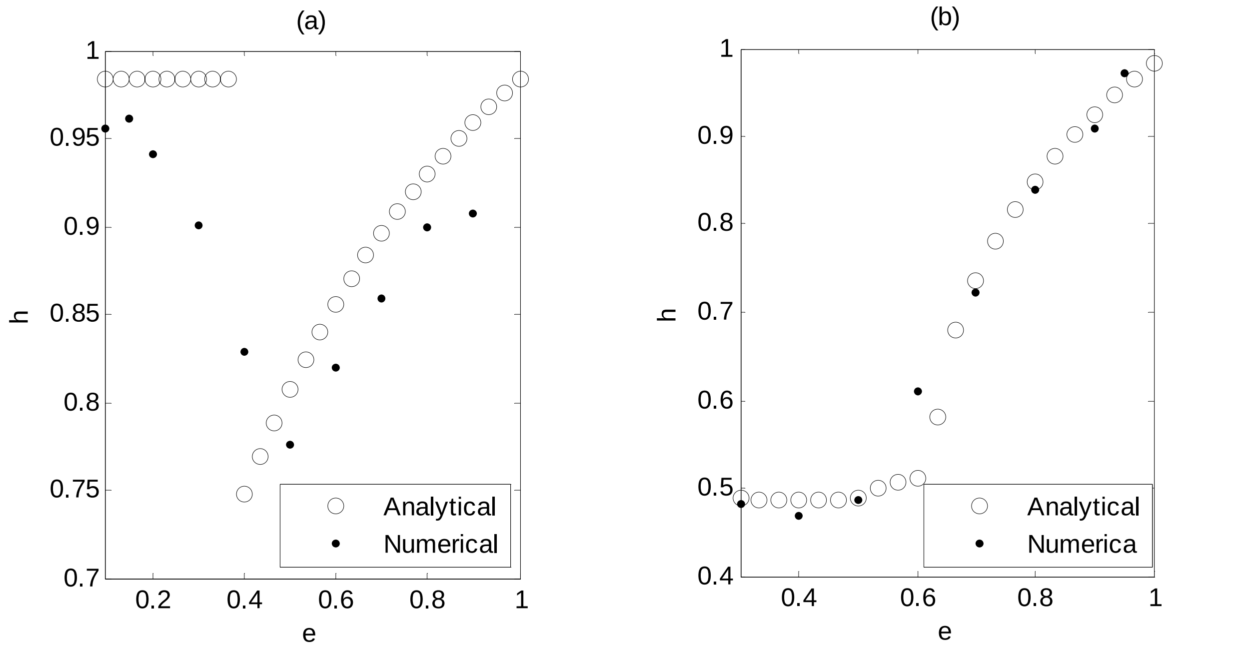

Fig.6
Non-dimensional amplitude of a controlled system, h, as a function of the coefficient of restitution for (a) d=0.6, r=0.05; (b) d=0.8 r=0.1

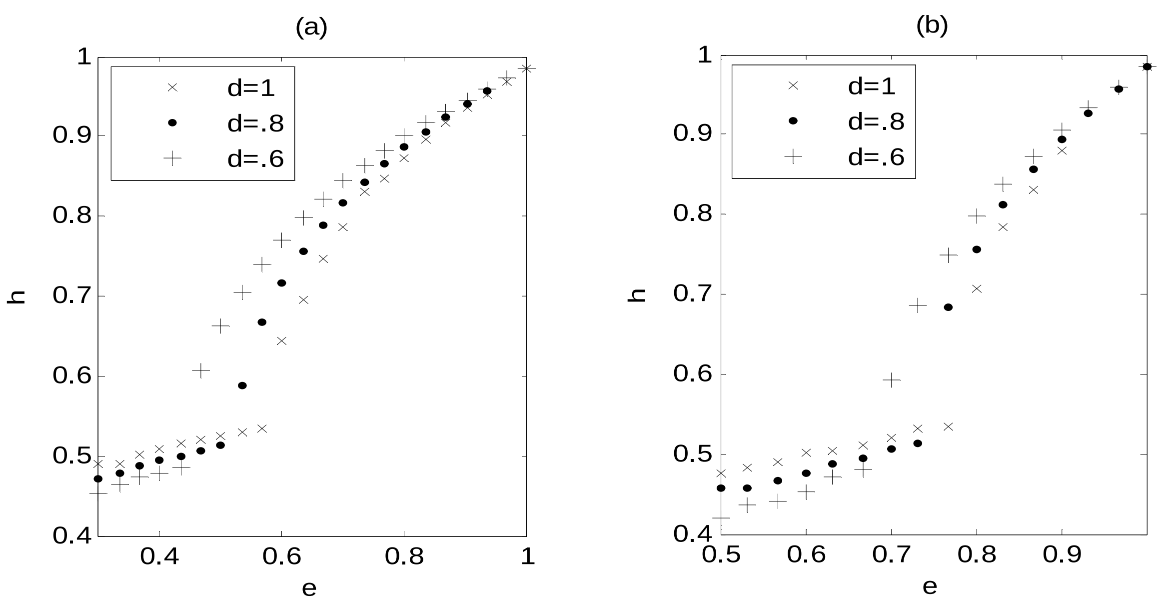

Fig.7
Analytical results for different values of d with (a) r=0.075 (b) r=0.1

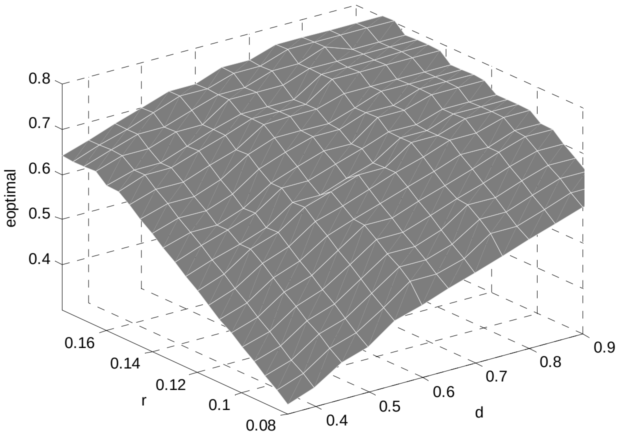

Fig.8
Optimal coefficient of restitution as a function of r and d

The optimal coefficient of restitution is shown in Figure (8) for a wide range of mass ratio and clearance.

As indicated in Figure (8) the optimal coefficient of restitution increases when *r* and *d* decrease.

Figure (9) shows the effect of mass ratio on *h* for constant *e* and *d*. The results presented in this figure show that the influence of impact damper on amplitude of the system increases when mass ratio increases, because momentum transfer between the structure and particle increases by increasing particle mass.

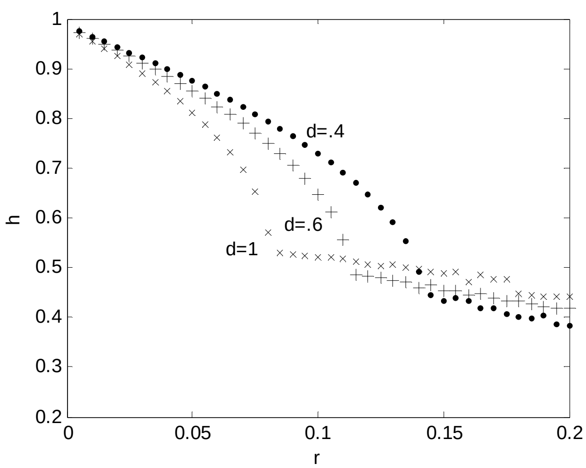

Fig.9
Analytical results for e=0.8
and with d=0. 4, 0.6 and 1

From the overall observation of all results, it is noticed that there is an infinite mass ratio that increasing mass ratio more than this value results in small influence on the amplitude of the system. Moreover, a much higher mass ratio increases the impulse transfer. This results in unwanted excessive vibration at the resonance of the main structure. Consequently we determine the optimal mass ratio that depends on *e* and *d*. Another important conclusion is that the response of the system in large mass ratios is independent of *d*, and there is a higher limit for *r* by which the amplitude converges to a specific value. A much smaller mass ratio will however reduce the momentum transfer between the impact damper and the structure, reducing thereby the effectiveness of the impact damper.

The optimal mass ratio is plotted as a function of *e* and *d* in Figure (10).

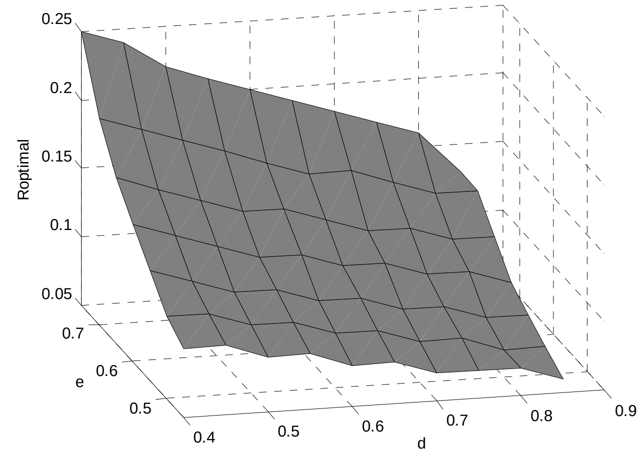

Fig.10
Optimal mass ratio as a function of *d* and *r*

From Figure (10) it can be seen that optimal mass ratio increases by increasing restitution coefficient and decreasing clearance.

The two consecutive impacts occur at $\tau = \tau_1$ and $\tau = \tau_2$, and as it is mentioned earlier $\tau_2 \neq 2\tau_1$. The non-dimensional period of system τ_2 is examined analytically and the frequency of the system $f = 1/\tau_2$ is plotted as a function of variables *r* and *d*.

Figures (11) and (12) show that by decreasing *d*, system frequency decreases. Results presented in Figure (12) shows a relatively low sensitivity to a variation of the restitution coefficient. It is also indicated that the frequency decreases at large values of mass ratio.

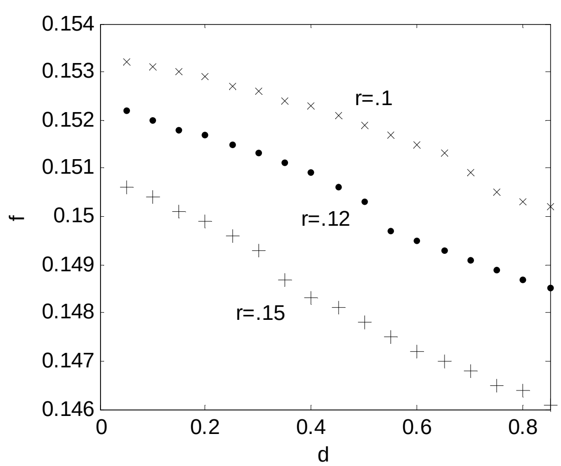

Fig.11
Effect of *d* on the system frequency
for different values of *r* and *e*=0.8

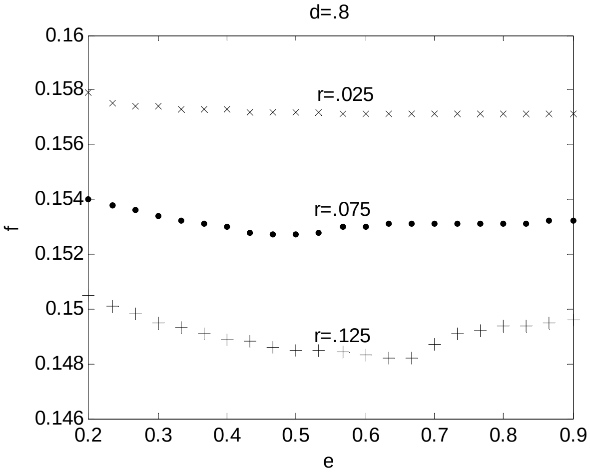

Fig.12

Sensitivity of the system to *e* and *r* on the frequency of system for *d*=0.8

IV. Conclusion

The amplitude of the friction-driven oscillator without impact damper was investigated numerically and analytically. The perturbation method was employed in the analytical solution and results were very close to the numerical results. The control of a single degree-of-freedom friction-driven oscillator by using an impact damper was studied analytically and numerically for different values of clearance, mass ratio and restitution coefficients. Results were compared with numerical results. These conclusions are interpreted:

- The amplitude of the controlled system decreases by decreasing restitution coefficient as a result of mounting energy dissipation. It has been observed that decreasing restitution coefficient, less than the specific value, results in small influence on the system amplitude, on the other hand very low restitution coefficients are practically unattainable and problematic, because of material deformation, and thus this specific value is defined as the optimal restitution coefficient which depends on the mass ratio and clearance.
- Optimal coefficient of restitution increases by increasing *r* and *d*.
- The response of the system in large coefficient of restitution is independent of *r*, and there is a lower limit for *r* by which the amplitude converges to a specific value.
- Momentum transfer between impact damper and structure increases with increasing mass ratio, having more influence on amplitude. However when mass ratio reaches specific value, the system has low sensitivity to its varied values. This value is optimal for the mass ratio because much higher mass ratio will increase the impulse transfer. This would result in unexpected excessive vibration at the resonance at the main structure
- Optimal mass ratio increases by increasing restitution coefficient and decreasing clearance.
- Variation of the restitution coefficient has low influence on the system frequency.

The frequency of the system decreases by increasing mass ratio and clearance.

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