Identification of Systems with Friction via Distributions using the simplified Dahl model

Radu Zglimbea, Virginia Finca, Constantin Marin

Abstract**—**This paper extends the identification procedures based on distributions theory to continuous time systems with friction. There are defined the so called generalized friction dynamic systems (GFDS) as a closed loop structure around a smooth system with discontinuous feedback loops representing friction reaction vectors. Both GFDS with static friction models (SFM) and dynamic friction models (DFM), also simplified Dahl model are analyzed. The identification problem is formulated as a condition of vanishing the existence relation of the system. Then, this relation is represented by functionals using techniques from distribution theory based on testing function from a finite dimensional fundamental space. The advantage es of representing information by distributions are pointed out when special evolutions as sliding mode, or limit cycle can appear. The proposed method does not require the derivatives of measured signals for its implementation. Some experimental results are presented to illuminate further its advantages and practical use.

Keywords**—**Distribution theory; Friction; Identification; The simplified Dahl model.

I. INTRODUCTION

Motion in many mechanical, hydraulic or pneumatic systems is influenced by the so-called friction forces because of interactions with the environment or of the interaction between their components.

Friction is a complex phenomenon, not yet completely known, with many different physical causes, so it is a difficult task to model it. Such models contain some specific nonlinearity such as stiction, hysteretic, Stribeck effect, stick-slip, depending on velocity [1], [2], [3], [4]. These models depend on many parameters whose values can change during the system evolution or are influenced by some other causes as external temperature, quality of materials etc. In literature there are accepted a large variety of friction models as Coulomb friction model [5], Dahl model [5], [6], exponential model [7], bristle model [8], state variable model [9].

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Ignoring friction in controlling such systems can lead to tracking errors, limit cycles, undesired stick-slip motion [2].

To avoid these difficulties, adaptive control strategies, named model-based friction compensation techniques [2], are recommended.

A survey of models, analysis tools and compensation methods for the control of machines with friction is presented in [11]. Furthermore, the application of classical identification methods for continuous time friction models requires the acceleration measurement that is not an easy task. A frequency domain approach to identification of mechanical systems with friction is developed in [12], which does not require the acceleration information but the procedure is available for periodic excitation input only. Good results on continuous time system identification based on distribution theory are reported in [13], for linear systems, or in [14], for nonlinear systems.

Because of their discontinuities, identification of systems with frictions is much more difficult. One way is to perform continuous time domain identification transforming the system differential equations to an algebraic system that reveals the unknown parameters [15], [16], [17]. This can be done by using some modulating functions to generate functional to avoid the direct computation of the input-output data derivatives [18], [19]. From computational point of view many advantages are obtained by using the classic methods based on orthogonal functions.

This paper extends the procedures of [13], [20],[14], based on distributions, for parametric identification in continuous time systems with friction. By this method, it is possible to perform identification of these systems, processing only information on position and the sign of the velocity in any consistent transient response. The proposed method is a batch on-line identification method because identification results are obtained during the system evolution after some time intervals but not in any time moment. Even if it is based on the input-output measurements only, the method is insensitive to the initial state of any transient.

The paper is organized as follows: After introduction in the first section, Section II presents the structure of generalised friction dynamic systems GFDS. Section III, presents some aspects regarding the problem of continuous time system identification based on distributions. Applications of the identification methods for different types of systems with frictions take the space of Section IV. Some experimental results and implementation aspects are presented in Section V, and conclusions in Section VI.

II. GENERALISED FRICTION DYNAMIC SYSTEMS

A generalised dynamic friction system (GFDS) is a system characterised by the state equation of the form

$$\dot{x} = f(x,u,r_1,..,r_i,..,r_p) \quad (1)$$

where $x(t) \in \mathbf{X} \subseteq \mathbb{R}^{n}, \forall t \geq t_0$, is the state vector and $u(t) \in \mathbf{U} \subseteq \mathbb{R}^{q}, \forall t \geq t_0$ is the input vector. The vectors

$$r_i(t) \in \mathbf{R}_i \subseteq \mathbb{R}^{n}, \forall t \geq t_0, i = 1:p \quad (2)$$

are called friction reaction vectors. They depend on x and u through a specific operator $\Psi_i\{\}$, called friction operator,

$$r_i = \Psi_i\{x,u\}, i = 1:p \quad (3)$$

There are two categories of friction models: static friction models (SFM) and dynamic friction models.(DFM).

For SFM, the operator (3) is a non dynamic mapping

$$r_i = F_i(x,u): \mathbf{X} \times \mathbf{U} \to \mathbf{R}_i \subseteq \mathbb{R}^{m_i}, i = 1:p \quad (4)$$

with a specific structure as follows.
For any $i = 1:p$, there are two functions

$$v_i = \nu_i(x,u): \mathbf{X} \times \mathbf{U} \to \mathbf{V}_i \subseteq \mathbb{R}^{m_i}, i = 1:p, \quad (5)$$

which determines the so called generalized velocity vector v_i, and

$$a_i = \alpha_i(x,u): \mathbf{X} \times \mathbf{U} \to \mathbf{A}_i \subseteq \mathbb{R}^{m_i}, i = 1:p, \quad (6)$$

expressing the so called active component of the velocity vector v_i. In SFM, the non dynamic mapping (4) can be expressed as a function of v_i, and a_i only, that means,

$$r_i = \rho_i(v_i,a_i) = F_i(x,u), i = 1:p \quad (7)$$

Inspired from mechanical systems /11/, the expression of the function ρ_i from (7) is explicitly defined for $v_i = 0$ and for $v_i \neq 0$. As a result, two components of the friction reaction vectors $r_i, i = 1:p$ can be defined: static friction reaction r_i^s and cinematic friction reaction r_i^c, where,

$$r_i^s = \rho_i^s(v_i,a_i) = F_i^s(x,u) = \begin{cases} \rho_i(v_i,a_i)\big|_{v_i=0}, & v_i = 0 \\ 0, & v_i \neq 0 \end{cases} \quad (8)$$

$$r_i^c = \rho_i^c(v_i,a_i) = F_i^c(x,u) = \begin{cases} \rho_i(v_i,a_i)\big|_{v_i \neq 0}, & v_i \neq 0 \\ 0, & v_i = 0 \end{cases} \quad (9)$$

A particular structure gives

$$r_i = r_i^s + r_i^c \quad (10)$$

which is equivalent with

$$\rho_i(v_i,a_i) = \rho_i^s(v_i,a_i) + \rho_i^c(v_i,a_i), \forall v_i \in \mathbf{V}_i, \forall a_i \in \mathbf{A}_i. \quad (11)$$

The adjective static and dynamic, for the friction reaction vectors $r_i, i = 1:p$, must be understood with respect to the velocity vector v_i only. Also, for a vector $v_i \in \mathbb{R}^{m_i}, i = 1:p$, it is defined the function $sgn(v_i)$ as

$$\mathrm{sgn}(v_i) = v_i / \|v_i\| \quad (12)$$

where $\|v_i\|$ is the Euclidian norm of $\mathbb{R}^{m_i}$. In this norm the function $\mathrm{sgn}(v_i)$ is a discontinuous function in the point $v_i = 0$. It is observed that

$$\|\mathrm{sgn}(v_i)\| = \mathrm{sgn}(\|v_i\|) = 1,\ v_i \neq 0,\ \mathrm{sgn}(\|0\|) = 0. \quad (13)$$

If $m_i = 1$, v_i is a scalar variable, then (12) can be presented by using inequalities as

$$\mathrm{sgn}(v_i) = -1 \; if \; v_i < 0;\ 0 \; if \; v_i = 0;\ 1 \; if \; v_i > 0. \quad (14)$$

Because of (8) and (9), the system state vector evolution $x(t)$ is characterized by a status of two values, related to each friction reaction vectors $r_i, i = 1:p$,

1. Evolution inside a surface characterized by zero value of the velocity vector v_i, $x(t) \in S_i$, where

$$S_i = \{x \in \mathbf{X}, v_i = 0\} = \{x \in \mathbf{X}, \nu_i(x) = 0\}. \quad (15)$$

2. Evolution with nonzero value of the velocity vector v_i, that means outside the surface S_i, $x(t) \notin S_i$.

There are different specific expressions for the functions $\rho_i^s(v_i,a_i)$ and $\rho_i^c(v_i,a_i)$ considering (10) and (11), but for all of them three conditions must be accomplished:

a. Outside the surface S_i, r_i is a vector opposite to $v_i \neq 0$

$$r_i = -\lambda_i \cdot v_i,\ \lambda_i > 0, v_i \neq 0 \quad (16)$$

b. Inside the surface S_i, r_i is a vector opposite to a_i

$$r_i = -\gamma_i \cdot a_i,\ \gamma_i > 0, v_i = 0 \quad (17)$$

c. There is a closed subset $S_i^0(u) \subseteq S_i$, called sticky area (SA), which keeps the system state inside . This means

$$\tfrac{d}{dt} v_i(x(t)) = [\tfrac{d}{dx} v_i(x)]^T \cdot \dot{x}(t) = 0, \forall x \in S_i^0(u).$$

Inside the SA $r_i = -a_i$. Because the input u can change the SA position the state x can be forced to be out of $S_i^0(u)$, crossing its border. For any admissible u, the function $r_i = F_i(x,u)$ is continuous with respect to $\forall x \in S_i^0(u)$. Because of this, when the system state $x(t)$ arrives on or leaves out $S_i^0(u)$ the friction reaction $r_i(t)$ is a continuous time function. Condition c, is called the smooth sticky condition (SSC). However, when $x(t) \in S_i \setminus S_i^0(u)$, $r_i(t)$ has a discontinuity and $\tfrac{d}{dt} v_i(x(t)) \neq 0$. In this case $x(t)$ passes from one side to other of $S_i \setminus S_i^0(u)$, as a switching mode or as a sliding mode. For example, expressions as (18) and (19) of (8) and (9) respectively, satisfy conditions a, b, c, where by a_i it must understand $a_i = a_i(x,u)$,

$$r_i^s = \rho_i^s(v_i,a_i) = -\max\{Q_i, \|a_i\|\} \cdot \mathrm{sgn}(a_i) \cdot [1 - \mathrm{sgn}(\|v_i\|)] \quad (18)$$

$$r_i^c = \rho_i^c(v_i,a_i) = -[Q_i + K_{vi} \cdot \|v_i\| + B_i \cdot (e^{\beta_i \cdot \|v_i\|} - 1)] \cdot \mathrm{sgn}(v_i) \quad (19)$$

As it can be observed, the cinematic reaction r_i^c is a sum of three components, $r_i^{cc}, r_i^{cv}, r_i^{cs}$ expressing respectively Coulomb friction, viscous friction and the so called Stribeck effect, /4/, /11/,

$$r_i^c = r_i^{cc} + r_i^{cv} + r_i^{cs}. \quad (20)$$

From (18) and (19), considering (10), a friction reaction vector r_i on the surface $\nu_i(x) = 0$ takes the form

$$r_i = \rho_i(v_i,a_i)\big|_{v_i=0} = \rho_i(0,a_i) = \begin{cases} -a_i, & \|a_i\| \le Q_i \\ -a_i^Q, & \|a_i\| > Q_i \end{cases} \tag{21}$$

where

$$a_i^Q \in A_i^Q = \{ a_i \, , \|a_i\| = Q_i \} \tag{22}$$

and

$$\|\rho_i(0,a_i)\| = Q_i \ \text{ if } \ \|a_i\| \ge Q_i \, , \tag{23}$$

but

$$\nu_i(x) \ne 0 \;\Rightarrow\; \rho_i(v_i,a_i) = \rho_i^c(v_i,a_i) \tag{24}$$

$$\lim_{\|v_i\|\to 0} \|\rho_i(v_i,a_i)\| = Q_i \, , \ \forall a_i \in \mathbf{A}_i \, . \tag{25}$$

so, the smooth sticky condition is assured.

For $m_i = 1$, all r_i, a_i, v_i are scalar variables so the static reaction (18), r_i^s, is illustrated as in Fig. 1.a. and the cinematic reaction (19), r_i^c, as in Fig. 1.b.

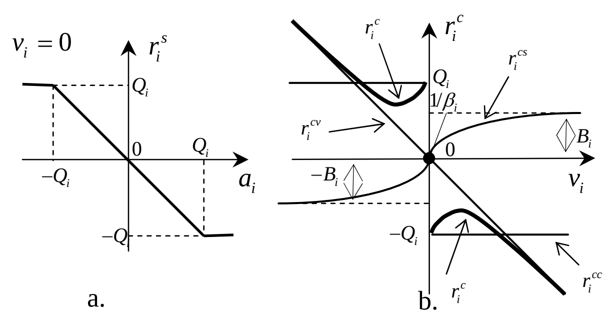

Fig. 1. Static and cinematic components of a scalar friction reaction

A friction reaction vector r_i, as above defined, has a sticky characteristic which means there is a subset $S_i^S(u) \subseteq S_i$, called sticky set (SS), such way

$$\dot v_i(t) = d/dt\{v_i(x(t))\} = 0, \forall x(t) \in S_i^S(u(t)) \subseteq S_i \, . \tag{26}$$

The position of SS depends on input vector u. When the system state $x(t)$ approaches $S_i^S(u)$, generated by a vector r_i, it remains inside of that SS till the input $u(t)$ changes the position of $S_i^S(u)$, forcing $x(t)$ to be outside of it. Substituting (4) into (1) and denoting

$$\mathbf{f}(x,u) = f(x,u,F_1(x,u)_1,..,F_i(x,u),..,F_p(x,u)) \tag{27}$$

the GDFS takes the compact form

$$\dot x = \mathbf{f}(x,u), \ x(t_0) = x_0 \, , t \ge t_0 \, . \tag{28}$$

This is a differential system with a discontinuous function on right side so for its analytical description, special mathematical approaches are necessary. For example approaches describing the solution in the Charatheodory sense [4], using the Filippov approach [23], or differential inclusions and differential inequalities [22]. However, for the identification it is supposed a solution exist for (19) and are available as measurements the input variable u and the output variable y where

$$y = h(x,u) \tag{29}$$

The structure of a GDFS with SFM is illustrated in Fig. 2.

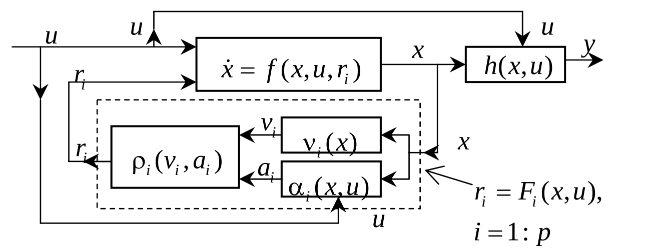

Fig. 2. The feedback structure of a GDFS with SFM.

For dynamic friction models, (DFM) the operator (3) is a dynamic system characterized by an additional state vector z_i, expressing internal changes in some surfaces of relative movements. The friction reaction vector r_i is the output of a dynamic system

$$r_i = h_i(z_i,x,u) \tag{30}$$

$$\dot z_i = f_i(z_i,x,u) \tag{31}$$

There are many types of DFM but we shall present a simplified Dhal model [2], [5], for $m_i = 1$, characterized by a first order nonlinear dynamic system

$$\dot z_i = -a_i \cdot \|v_i\| \cdot z_i + v_i \tag{32}$$

$$r_i = c_i \cdot z_i \tag{33}$$

where the velocity v_i is $\quad v_i = \nu_i(x) \in \mathbb{R}$. (34)

This system can be expressed as a single differential equation with respect to the friction reaction variable

$$\dot r_i = c_i \cdot [1 - (a_i / c_i) \cdot \mathrm{sgn}(v_i) \cdot r_i \,] \cdot v_i \tag{35}$$

III. CONTINUOUS TIME SYSTEM IDENTIFICATION BASED ON DISTRIBUTIONS

This section presents the main results on continuous time system identification based on distribution, as have been presented in [13]. Let Φ_n be the fundamental space from distribution theory [21] of the real testing functions, $\varphi : \mathbb{R} \to \mathbb{R}, t \to \varphi(t)$, having continuous derivatives at least up to the order n, with compact support T for any of the above derivative. The linear space Φ_n is organized as a topological space considering a specific norm [21]. A distribution is a linear, continuous real functional on Φ_n, $F : \Phi_n \to \mathbb{R}, \varphi \to F(\varphi) \in \mathbb{R}$. Let $q : \mathbb{R} \to \mathbb{R}, t \to q(t)$ be a function that admits a Riemann integral on any compact interval T from $\mathbb{R}$. Using this function, a unique distribution $F_q : \Phi_n \to \mathbb{R}, \ \varphi \to F_q(\varphi) \in \mathbb{R}$ can be build by the relation $F_q(\varphi) = \int_{\mathbb{R}} q(t) \cdot \varphi(t) \cdot dt$, $\forall \varphi \in \Phi_n$.

Considering, at least, $q \in C^0(\mathbb{R})$, the following important equivalence take place [22],

$$F_q(\varphi) = 0, \forall \varphi \in \Phi_n \Leftrightarrow q(t) = 0, \forall t \in \mathbb{R} \, . \tag{36}$$

The m-order derivative of a distribution is a new distribution, $F_q^{(m)} \in \Phi_n^/$ uniquely defined by the relations,

$$F_q^{(m)}(\varphi) = (-1)^m \cdot F_q(\varphi^{(m)}), \forall \varphi \in \Phi_n \tag{37}$$

When $q \in C^m(\mathbb{R})$, then $F_q^{(m)}(\varphi) = F_{q^{(m)}}(\varphi) = \int_{\mathbb{R}} q^{(m)}(t)\varphi(t)dt$

that means the k-order derivative of a distribution generated by a function $q \in \mathrm{C}^m(\mathbb{R})$ equals to the distribution generated by $q^{(m)}$, the k-order time derivative of the function q. If $q \in \mathrm{C}^m(\mathbb{R})$, from (26), (29) one can write, $\forall \varphi \in \Phi_n$

$$F_q^{(m)}(\varphi) = \int_{\mathbb{R}} q^{(m)}(t)\varphi(t)dt = (-1)^m \int_{\mathbb{R}} q(t) \cdot \varphi^{(m)}(t) \cdot dt \tag{38}$$

Let us consider a dynamical continuous time system expressed by a differential operator, $q_{\theta/(u,y)} = F_C(u, y, \theta)$ (39) whose expression depends on a vector of parameters

$$\theta = [\theta_1 \ ... \ \theta_i \ ... \ \theta_p]^T . \tag{40}$$

It represents a family of models with a given structure in constant parameters. A special case is the model expressing a linear relation in the parameters

$$q_{\theta/(u,y)} = F_C(u, y, \theta) = \sum_{i=1}^{p} w_i \cdot \theta - v = w^T \cdot \theta - v , \tag{41}$$

where w_i and v represent a sum of the derivatives of some known, possible nonlinear, functions ψ_i^j, ψ_0^j, with respect to the input and output variables,

$$w_i = \sum_{j=1}^{p_i} [\psi_i^j(u, y)]^{(n_i^j)}, i = 1:p , \tag{42}$$

$$v = \sum_{j=1}^{p_0} [\psi_0^j(u, y)]^{(n_0^j)} . \tag{43}$$

where parameters p_i, n_i^j, p_0, n_0^j are given integer numbers. In [13] the existence and uniqueness conditions for a problem of distribution based continuous time system identification are presented.

Suppose that it is possible to record the functions (u, y) in an the time interval $T \subset \mathbb{R}$, called observation time interval or just time window. The restriction of the functions (u, y) to the time interval T is denoted by (u_T, y_T) respectively. If no confusion would appear, then we may drop the subscript T.

An identification problem means to determine the parameter $\theta = \widehat{\theta}$, given the priori information on the model structure F_C, (30), and a set of observed input-output pairs (u_T, y_T), $\widehat{\theta} = \widehat{\theta}(u_T, y_T, F_C)$ in a such a way that,

$$q_{\widehat{\theta}/(u_T, y_T)}(t) = 0, \forall t \in \mathbb{R} \tag{44}$$

This condition involves,

$$q_{\widehat{\theta}/(u,y)}(t) = 0, \forall t \in \mathbb{R}, \forall (u, y) \in \Omega \times \Gamma \tag{45}$$

for any input-output pair (u, y) observed to that system.

Let us consider two families of regular distributions, $F_{w_i}, i = 1:p$, and $F_v(\varphi)$ created based on the functions (42), (43), (46)

$$F_{w_i}(\varphi) = \sum_{j=1}^{p_i} \int_{\mathrm{R}} [\psi_i^j(t)]^{(n_i^j)} \varphi(t)dt = \sum_{j=1}^{p_i} (-1)^{n_i^j} \int_{\mathrm{R}} [\psi_i^j(t)]\varphi^{(n_i^j)}(t)dt$$

which determines the row vector,

$$F_w^T(\varphi) = [F_{w_1}(\varphi), ..., F_{w_i}(\varphi), ..., F_{w_p}(\varphi)] \in \mathbb{R}^p . \tag{47}$$

$$F_v(\varphi) = \sum_{j=1}^{p_0} \int_{\mathrm{R}} [\psi_0^j(t)]^{(n_0^j)} \varphi(t)dt = \sum_{j=1}^{p_0} (-1)^{n_0^j} \int_{\mathrm{R}} [\psi_0^j(t)]\varphi^{(n_0^j)}(t)dt . \tag{48}$$

Any input-output pair (u, y) observed from the system (41) is described by a pair of regular distribution (F_w, F_v) for any $\varphi \in \Phi_n$, [13]. The problem of the system (39) parameter identification can be represented now by distributions. For example, the regular distribution generated by the continuous function $q_{\theta/(u,y)}$ from (39), is related to the parameter vector θ, $\forall \varphi \in \Phi_n$ as (49)

$$F_{q_\theta}(\varphi) = F_{q_\theta/(u,y)}(\varphi) = \sum_{i=1}^{p} F_{w_i}(\varphi)\theta_i - F_v(\varphi) = F_w^T(\varphi)\theta - F_y(\varphi)$$

If a triple (u^*, y^*, θ^*) is a realization of the model (39), then the identity (61) takes place,

$$F_{q_{\theta^*}}(\varphi) = F_{q_{\theta^*}/(u^*,y^*)}(\varphi) = 0, \forall \varphi \in \Phi_n \tag{50}$$

and vice versa, if an input-output pair (u^*, y^*) of the family of models (49), with unknown parameter θ, generates a distribution

$$F_{q_\theta}(\varphi) = F_{q_\theta/(u^*,y^*)}(\varphi) = \sum_{i=1}^{p} F_{w_i}(\varphi) \cdot \theta_i - F_v(\varphi) \tag{51}$$

which satisfies

$$F_{q_\theta}(\varphi) = F_{q_\theta/(u^*,y^*)}(\varphi) = 0, \forall \varphi \in \Phi_n \Rightarrow \theta = \theta^* , \tag{52}$$

As θ has p components it is enough a chose (utilize) a finite number $N \geq p$ of fundamental function $\varphi_i, i = 1:N$ and to build an algebraic equation, $\mathbf{F}_w \cdot \theta = \mathbf{F}_v$ (53)

where $\mathbf{F_w}$ is an $(N \times p)$ matrix of real numbers

$$\mathbf{F}_w = [F_w^T(\varphi_1);...;F_w^T(\varphi_k);...;F_w^T(\varphi_N)]^T \tag{54}$$

where k-th row $F_w^T(\varphi_k)$ is given by (47). The symbol $\mathbf{F}_v$ denotes an N -column real vector built from (48),

$$\mathbf{F}_v = [F_v(\varphi_1), ..., F_v(\varphi_k), ..., F_v(\varphi_N)]^T . \tag{55}$$

When only the restriction (u_T, y_T) of the pair (u, y) on the time interval T is available, any φ_i must have for its k derivative $\varphi_k^{(m)}(t), m = 1:n$ the same compact support T_k,

$$\mathrm{supp}\{\varphi_k^{(m)}(t)\} = T_i = [t_a^k, t_b^k] \subseteq T, \ \forall m = 1:n, \ k = 1:N \tag{56}$$

Below there are some simple testing functions $\varphi_k \in \Phi_n$,

$$\varphi_k(t) = \alpha_k \cdot \beta_k(t_a^k, t_b^k) \cdot \Psi_k(t, t_a^k, t_b^k) \tag{57}$$

$$\Psi(t, t_a^k, t_b^k) = \begin{cases} \sin^{n_k}[\pi \cdot (t - t_b^k) / (t_b^k - t_a^k)], \ \forall k, n_k \geq n \\ 0, \quad \forall t \in (-\infty, t_a^k] \cup [t_b^k, \infty) \end{cases} \tag{58}$$

where α_i is a scaling factor and β_i normalizes the area

$$\beta_k(t_a^k, t_b^k) = 1 / \int_{t_a^k}^{t_b^k} \Psi_k(t, t_a^k, t_b^k), \ \forall t_a^k < t_b^k . \tag{59}$$

If $r = rank(\mathbf{F}_w) = p$, then a unique solution is obtained.

$$\widehat{\theta} = (\mathbf{F}_w^T \cdot \mathbf{F}_w)^{-1} \cdot \mathbf{F}_w^T \cdot \mathbf{F}_v = \theta^* \tag{60}$$

IV. APPLICATION FOR A FRICTION MECHANICAL SYSTEM IDENTIFICATION USING THE SIMPLIFIED DAHL MODEL

Let us consider the simplest system with a single friction, as in Fig. 3, represented by a mass m attached to a spring with stiffness K_P and viscosity K_V, moving on a horizontal surface. The end of the spring is a fixed point. A horizontal force u acts on the mass.

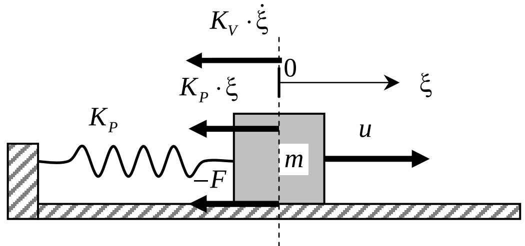

Fig. 3. Principle diagram of the friction mechanical system

Originally the mass is at rest in a position expressed by the variable $\xi = 0$. The equation of motion is

$$m \cdot \ddot{\xi} + K_V \cdot \dot{\xi} + K_P \cdot \xi + F = u; \tag{61}$$

where is friction of force F is given by the relation:

$$F = \sigma_0 \cdot z; \tag{62}$$

σ_0 represents the stiffness of contact, again z represents the displacement at speed 0 and is modeled by:

$$\frac{dz}{dt} = \dot{\xi} - \frac{\sigma_0 \cdot |\dot{\xi}|}{F_C} \cdot z; \tag{63}$$

The state x of (1) has three components, $x_1 = \xi;\ x_2 = \dot{\xi};\ x_3 = \ddot{\xi}$, so

$$\dot{x}_1 = x_2;\ \ddot{x}_2 = x_3;\ m \cdot \dot{x}_3 = u - K_P \cdot x_1 - K_V \cdot x_2; \tag{64}$$

Using the simplified Dahl:

$$\dot{F} = \sigma_0 \cdot \dot{\xi} \cdot [1 - \mathrm{sgn}(\dot{\xi}) \cdot \frac{F}{F_C}], \tag{65}$$

the relation (62) and (63), deriving the relation (61), this becomes:

$$m\dddot{\xi} + K_V \ddot{\xi} + (K_P + \sigma_0)\dot{\xi} - \frac{\sigma_0}{F_C}|\dot{\xi}|u + m\frac{\sigma_0}{F_C}|\dot{\xi}|\ddot{\xi} + K_V \frac{\sigma_0}{F_C}|\dot{\xi}|\dot{\xi}$$

$$+K_P \frac{\sigma_0}{F_C}\xi|\dot{\xi}| = \dot{u}\ . \tag{66}$$

Denoting

$$\theta_1 = m;\ \theta_2 = K_v;\ \theta_3 = K_P + \sigma_0;\ \theta_4 = \frac{\sigma_0}{F_C};\ \theta_5 = m\frac{\sigma_0}{F_C};\theta_6 = K_V \frac{\sigma_0}{F_C};$$

$$\theta_7 = K_P \frac{\sigma_0}{F_C}. \tag{67}$$

the parameters that have to be identified, (66) is expressed as the operator (41), where $p = 7$.

$$w_1 = \dddot{\xi}; w_2 = \ddot{\xi}; w_3 = \dot{\xi}; w_4 = u\,\mathrm{sgn}(\dot{\xi})\dot{\xi}; w_5 = \ddot{\xi}\,\mathrm{sgn}(\dot{\xi})\dot{\xi}; w_6 = \dot{\xi}\,\mathrm{sgn}(\dot{\xi})\dot{\xi};$$

$$w_7 = \xi\,\mathrm{sgn}(\dot{\xi})\dot{\xi}; v = \dot{u}\ . \tag{68}$$

The distribution image (51) of this differential operator, evaluated for a testing function φ_k on the time interval $T_k = [t_a^k, t_b^k] \subseteq T$, contains the elements given by (46), (47), (48) of the form $F_{w_i}(\varphi_k) = \int_{t_a^k}^{t_b^k} w_i(t) \cdot \varphi_k(t)dt$ where, (69) results

$$F_{w_1}(\varphi_k) = -\int_{t_a^k}^{t_b^k} \xi(t) \cdot \dddot{\varphi}_k(t)dt\ ;\ \ F_{w_2}(\varphi_k) = \int_{t_a^k}^{t_b^k} \xi(t) \cdot \ddot{\varphi}_k(t)dt$$

$$F_{w_3}(\varphi_k) = -\int_{t_a^k}^{t_b^k} \xi(t) \cdot \dot{\varphi}_k(t)dt\ \ F_{w_4}(\varphi_k) = \int_{t_a^k}^{t_b^k} [\mathrm{sgn}(\dot{\xi}(t))\dot{\xi}(t)u(t)]\varphi_k(t)dt$$

$$F_{w_5}(\varphi_k) = \int_{t_a^k}^{t_b^k} [\ddot{\xi}(t)\mathrm{sgn}(\dot{\xi}(t))\dot{\xi}(t)]\varphi_k(t)dt; F_{w_6}(\varphi_k) = \int_{t_a^k}^{t_b^k} [\dot{\xi}(t)\mathrm{sgn}(\dot{\xi}(t))\dot{\xi}(t)]\varphi_k(t)dt$$

$$F_{w_7}(\varphi_k) = \int_{t_a^k}^{t_b^k} [\xi(t)\mathrm{sgn}(\dot{\xi}(t))\dot{\xi}(t)]\varphi_k(t)dt\ ; F_v(\varphi_k) = -\int_{t_a^k}^{t_b^k} u(t)\dot{\varphi}_k(t)dt$$

For the evaluation of these integrals only input-output pair (ξ, u) and the sign of $\dot{\xi}$ are necessary in the case of friction without Stribeck effect . Otherwise also the speed $\dot{\xi}$ and the acceleration $\ddot{\xi}$ have to be measured. Integrals (69) are utilized to build the system (53), (54), (55), whose solution is (60).

V. EXPERIMENTAL RESULTS

To implement in Simulink the feedback structure of a system described by differential equation (61):

$$m \cdot \ddot{\xi} + K_v \cdot \dot{\xi} + K_p \cdot \xi + F = u\ ;$$

with $A = [0\ \ 1; \dfrac{-K_P}{m}\ \ \dfrac{-K_V}{m}];\ B = [0\ \ \dfrac{1}{m}];\ C = [1\ \ 0];\ D = 0$

which represents the linear part of a system, described via state-space and $\dot{F} = \sigma_0 \cdot \dfrac{dz}{dt} = \sigma_0 \cdot \dot{\xi} \cdot [1 - \mathrm{sgn}(\dot{\xi}) \cdot \dfrac{F}{F_C}]$ (65)

which represents the simplified Dahl model, is the nonlinear part of a system.

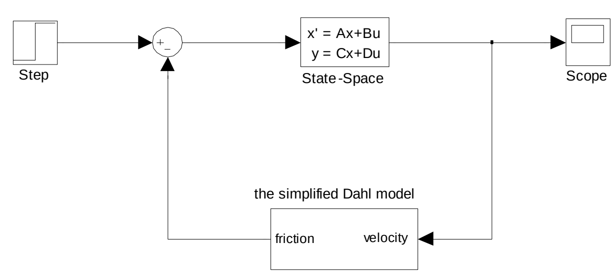

Fig. 4. Block diagram of the friction mechanical system using the simplified Dahl model

A step input u(t)=2·**1**(t) is applied from initial state x(0)=[2 6 3] considering A=[0 1.0000 ; -0.8000 –0.1000]; B=[0 ; 0.2000]; C=[1 0]; D=0 in which $m = 5$; $K_P = 4$ and $K_V = 0.5$;

The result of these simulation is:

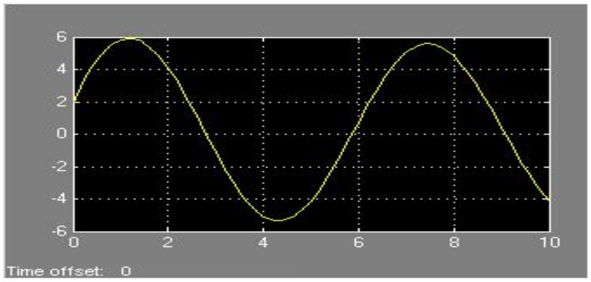

Fig. 5. Block diagram of simulation from fig. 4.

To implement distribution based identification methods, an experimental platform (DBI) has been developed. It allows creating testing functions with settable parameters, automatically to create and solve the system (53). The input-output data for identification are obtained from an external source (a data file) or internally by simulation.

Many examples and types of friction systems have been implemented for identification but, because of limited space in this paper, only one example is analysed, based on the application presented in section IV.

In the first example, the measured signals, as indicated in Fig. 6., are generated by a step input $u(t) = 2 \cdot \mathbf{1}(t)$ with initial state x(0)=[2 6 3] and considering $m,\ K_P + \sigma_0,\ K_V, \dfrac{\sigma_0}{F_C},\ m \cdot \dfrac{\sigma_0}{F_C},\ K_V \cdot \dfrac{\sigma_0}{F_C},\ K_P \cdot \dfrac{\sigma_0}{F_C}$ as parameters for identification. Seven testing functions φ_k on T_K , as (57), with n_k=4 and T_1=[0,3];T_2=[3,6]; T_3=[6,9]; T_4=[9,12] are utilized.

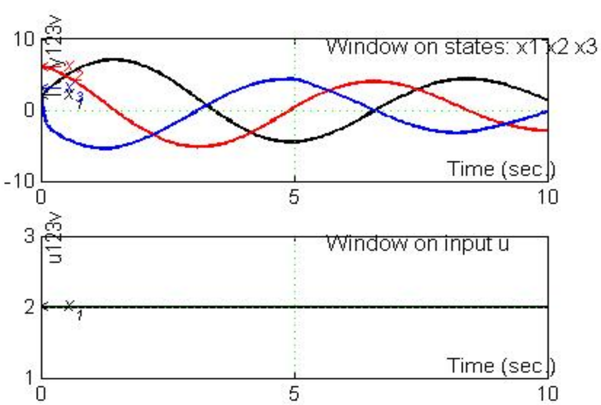

Fig. 6. Measured variables for the friction system using the simplified Dahl model and initial state x(0)=[2 6 3]

The matrices F_W , and $F_V{}^T$, respectively are:

-10.4502	-12.3516	15.7123	-10.1908	11.6045	-12.6103	4.3498
12.7302	-11.8704	-10.3976	-11.9654	12.9043	13.4010	13.8743
- 9.2890	10.2423	12.6523	12.3490	10.3087	10.3067	11.4280
5.9832	-12.3446	14.5378	-13.5967	12.8376	11.5482	12.7601
12.4782	4.3098	13.8941	10.8281	16.3167	13.8213	16.2810
13.6754	5.8956	7.7602	-10.5032	15.2784	15.3462	13.2903
6.7653	-1.3405	10.8903	11.7598	13.3452	12.8368	12.6587
14.3752	14.752	14.3752	14.3752	14.3752	14.3752	14.3752

The real and identified parameter values are respectively:

m : **5.00** **4.9980479519**

$K_P + \sigma_0$: **7.00** **6.9977692527**

K_V : **0.50** **0.5084353753**

$\dfrac{\sigma_0}{F_C}$: **3.00** **2.9964871357**

$m \cdot \dfrac{\sigma_0}{F_C}$: **15.00** **14.9883563673**

$K_V \cdot \dfrac{\sigma_0}{F_C}$: **1.50** **1.4966571237**

$K_P \cdot \dfrac{\sigma_0}{F_C}$: **12.00** **11.9967343671**

and the conditioning number of F_w, cond(F_w)=30.4328.

For the same input but with x(0)=[1 2 3] and T_1=[0,5]; T_2=[5,10];T_3=[10,15];T_4=[15,20];

The matrices F_W and $F_V{}^T$, are:

-8.5827	-8.1706	12.6403	-6.3093	12.7543	-4.5341	7.2567
10.1132	-4.4704	-10.6064	-8.6526	10.7543	5.5213	10.7345
-5.5172	10.5623	9.8666	10.0882	12.1908	10.5678	6.7841
5.0847	-10.2452	8.7684	-4.1249	10.4562	12.7891	5.4567
13.0847	8.3452	4.7834	3.9871	5.4562	10.6523	3.3452
12.5643	10.7656	6.5432	-10.3526	8.8743	4.5622	12.0945
10.4507	-10.0734	10.3024	9.6734	3.0944	10.4508	10.5921
12.6721	12.6721	12.6721	12.6721	12.6721	12.6721	12.6721

cond(F_w)=28.4618.,

The identification results are:

m : 5.00 4.9970578919

$K_P + \sigma_0$: 7.00 6.9356772327

K_V : 0.50 0.4934355752

$\dfrac{\sigma_0}{F_C}$: 3.00 2.9855876132

$m \cdot \dfrac{\sigma_0}{F_C}$: 15.00 14.9986723781

$K_V \cdot \dfrac{\sigma_0}{F_C}$: 1.50 1.4921549274

$K_P \cdot \dfrac{\sigma_0}{F_C}$: 12.00 11.9851204581.

The measured variables for the second example are illustrated in Fig.7.

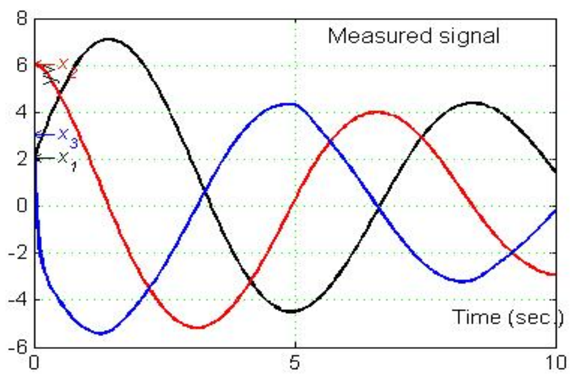

Fig. 7. Measured variables for the friction system using the simplified Dahl model and initial state x(0)=[1 2 3]

The third example refers to the same conditions as in the second example but considering errors in the measurement of both input and output. A zoom of these measurements containing error is shown in Fig,8.

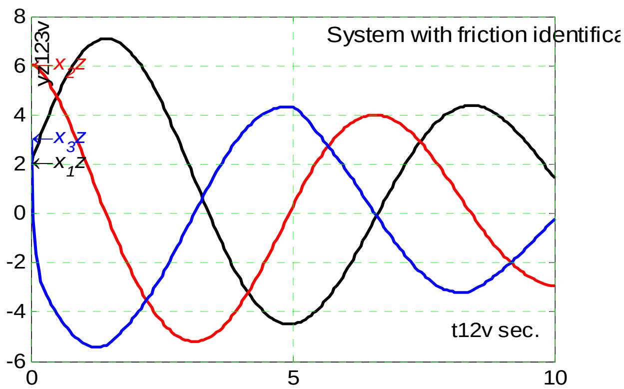

Fig. 8. Measured variables for the friction system without Stribeck effect and initial state x(0)=[1 2 3]

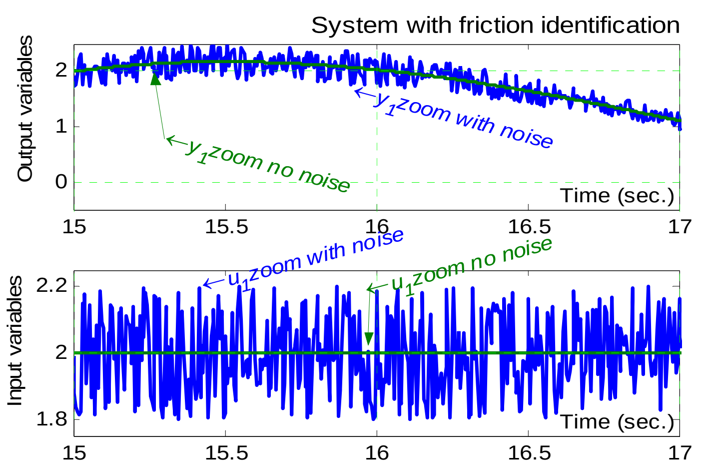

Fig. 9. A zoom representation of measurements containing error for the friction system without Stribeck effect and initial state x(0)=[1 2 3]

VI. CONCLUSIONS

The above results illustrates the advantages of distribution based identification for systems with discontinuities on the right side. Description by functionals allows to enlarge the area of systems to which identification procedures can be applied. This paper is development of a paper "Identification of systems with friction via distributions".

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