Calculation of cost functions by leather fatliquoring for its optimization

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Abstract**—** The paper deals with the economic aspects of the tannery operation of leather fatliquoring. It describes a mathematical model of the studied process and its use in computing of the cost function depending on the operating conditions. The results can be used to determine the optimum process realization to the purpose of saving energy and raw materials.

Keywords**—**Fatliquoring of leathers, mathematical model, minimazing of operating costs, computer simulation, Maple.

I. INTRODUCTION

PROCESSING of raw hides is a sequence of many operations, which produce a number of liquid and solid waste. As a result of using different chemicals may be those wastes often harmful. Therefore, it is necessary to find methods that lead to the minimization of waste and also find ways to remove unwanted substances from tanning waste for subsequent use [1].

In the following text we describe possibility to the use of indirect modeling methods for optimization of leather fatliquoring with respect to minimazing of the operating costs.

II. MATHEMATICAL DESCRIPTION OF LUBRICANT SORPTION INTO THE LEATHER MATERIAL

Fatliquoring is one of the processes in tannery technology. Its purpose is transport of lubricant into internal structures of the leather material in order to slipperiness and achieves ductility, flexibility and resilience finished to mechanical stress. Therefore, it is important to deal with the proposal diffusion model describing the transport of lubricant into the internal structure of the treated leathers.

This process allows the diffusion transport of lubricant into the inner porous structure of the leather. Suppose diffusion mechanism of the components transport in the internal structure of leather material wherein all deviations from Flick's diffusion are included in the value of the effective diffusion coefficient.

It is assumption of the closed system that is most similar to the real case, realized by tanning barrel. Mixing in a barrel will be perfect. Temperature during the entire process is constant.

Properties of the solid phase (leather) are given by following constant and variable parameters.

a) Constant parameters:

- total area *S*,
- total volume *V*, or total weight *m*,
- thickness 2*b*,
- porosity ε,
- sorption parameters of lubricant, i.e. kinetic and thermodynamic parameters – in the limiting case of linear sorption and infinitely fast sorption kinetics, it is only a sorption constant of lubricant in the leather *A*,
- transport parameters - in the limiting case and perfect mixing of the liquid phase, it is only diffusion coefficient of lubricant in liquid dispersion *D*.

b) Variable parameters:

- concentration of bound (adsorbed) lubricant in the leather $c_A(x,\tau)$,
- concentration of unbound lubricant in the leather $c(x,\tau)$.

Properties of the liquid phase are given by following constant and variable parameters.

a) Constant parameters:

- volume of liquid in the tanning barrel V_0,
- initial concentration of lubricant in the liquid phase c_{0p}.

b) Variable parameters:

- concentration of lubricant in the liquid phase $c_0(\tau)$.

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Other parameters are time τ and space coordinate x. Due to geometrical properties, the solid material (leather) is considered as a plane plate with isotropic properties with respect to the sorption characteristics of a lubricant. We also suppose that all geometry parameters are constant.

Diffusion of lubricant in the leather can be then described by equation (1) [2], [4]:

$$\frac{\partial c(x,\tau)}{\partial \tau} = D\frac{\partial^2 c(x,\tau)}{\partial x^2} - \frac{\partial c_A(x,\tau)}{\partial \tau}, \quad (0 \le x \le b, \tau > 0). \qquad (1)$$

Limiting case assumes a linear sorption of lubricant:

$$c_A(x,\tau) = Kc(x,\tau), \qquad (2)$$

$$\frac{\partial c_A(x,\tau)}{\partial \tau} = K\frac{\partial c(x,\tau)}{\partial \tau}. \qquad (3)$$

Substituting of the linear case into equation (1) is:

$$\frac{\partial c\ (x,\tau)}{\partial \tau} = K\frac{\partial^2 c(x,\tau)}{\partial x^2}, \quad (0 \le x \le b, \tau > 0), \qquad (4)$$

where

$$K = \frac{D}{1+A}. \qquad (5)$$

Providing isotropic properties can be applied the condition of symmetry (6):

$$\frac{\partial c(0,\tau)}{\partial x} = 0. \qquad (6)$$

If we relate the concentration of lubricant in leather to the total volume of the leather and the concentration of lubricant in the bath to the total volume of the bath, then the reminder of perfect mixing bath is given by equation (7):

$$c(b,\tau) = \varepsilon c_0(\tau). \qquad (7)$$

At stationary conditions on the surface of the leather, the loss of lubricant in the bath is equal to the volume increment of lubricant in the leather. Lubricant enters into the leather through its surface. Increase of mass for a given time is determined by the diffusion flux density Jx multiplied by the size of the total surface. The diffusion flux density is given by Flick's second law [2], [3]. Then the other boundary condition is:

$$V_0\frac{dc_0(\tau)}{dt} = -SD\frac{\partial c(b,\tau)}{\partial x}. \qquad (8)$$

Initial concentration of lubricant in the leather is equal to zero (9):

$$c(x,0) = 0. \qquad (9)$$

Initial concentration of lubricant in the leather is given by equation (10):

$$c_0(0) = c_{0p}. \qquad (10)$$

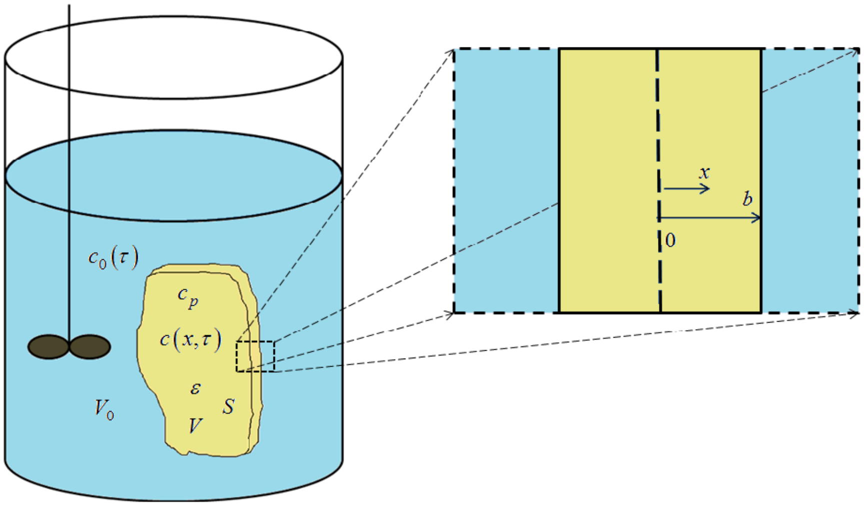

Fig. 1. Sketch of the fatliquoring model

By Laplace transformation we’ve got analytical solution of the model, which describes dimensionless concentration fields in the leather $C(X,Fo)$:

$$C(X,Fo)=1-\left[\frac{\varepsilon(1+A)}{\varepsilon(1+A)+Na}-\right.$$

$$\left.2\,Na\sum_{n=1}^{\infty}\frac{\cos(q_n X)\,e^{-\left(q_n^2 Fo\right)}}{\varepsilon(1+A)-\dfrac{\varepsilon(1+A)}{q_n}\sin(q_n)-Na\,q_n\sin(q_n)}\right], \tag{11}$$

where q are roots of transcendent equation (12):

$$C=\frac{c}{c_r}, \tag{13}$$

c_r is equilibrium concentration of lubricant in the solid phase. It is computed from balance equation:

$$c_r=\frac{c_{0p}Na}{\dfrac{Na}{\varepsilon}+1+A}, \tag{13}$$

X is dimensionless space coordinate:

$$X=\frac{x}{b}, \tag{14}$$

Fo is Fourier number (dimensionless time):

$$Fo=\frac{K\cdot\tau}{b^2}, \tag{15}$$

Na is a ratio of volume of liquid in the bath to the volume of the leather:

$$Na=\frac{V_0}{V}. \tag{16}$$

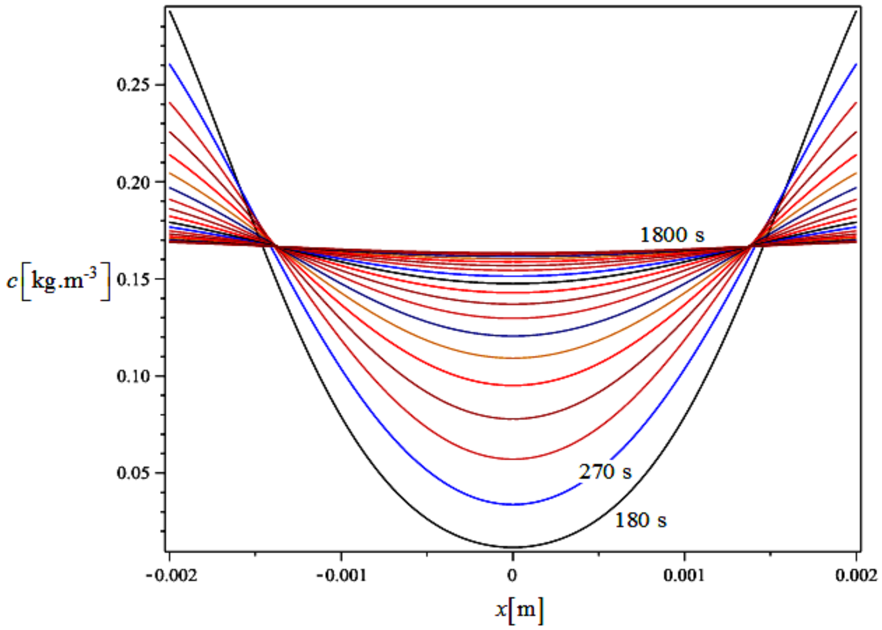

Fig. 2 Concentration of lubricant in the leather during fatliquoring.
Parameters: Thickness of leather 4 mm, ratio of volume of liquid in the bath to the volume of the leather 4, , sorption coefficient *A* is 15, porosity of the leather 0.5, initial concentration of lubricant in the bath 0.5 kg.m^{-3}.

It is evident that course of fatliquoring depends on many parameters of both solid and liquid phase. This parameters influence time course, effectivity and also processing costs. To the main factors depend diffusion coefficiet and sorption parameters of lubricant. In the limiting case of linear sorption is it the sorption coefficient *A*.

The Fig. 3 shows concentration fields in the specific times of fatliquoring for different values of the sorption coefficient *A*. Solid lines are concentration fields computed for *A* = 2, dashed lines are concentration fields for *A* = 4 and dotted lines are concentration fields for *A* = 6. It is evident that higher value of the sorption coefficient *A* causes a slowing down the process and also decrease of the final concentration of lubricant in the leather.

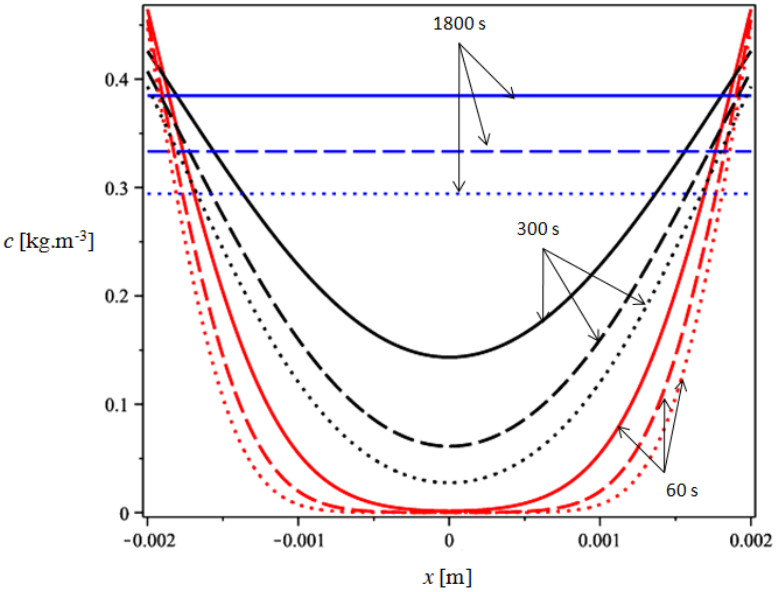

Fig. 3 Assessment of concentration fields of lubricant in the leather for various sorption constants *A*. *A* = 2 (solid lines), *A* = 4 (dashed lines), *A* = 6 (dotted lines).

III. COMPUTATION OF THE COST FUNCTIONS FOR FATLIQUORING

Determining of the optimal dosage of the lubricant due to the leather which is subjected to the fatliquoring, it is necessary to automate and optimize the process. The optimal dosage of lubricant can be determined by the cost function which specifies the dependence of the proportion of the operating costs on ratio of lubricant mixture volume V_0 .

The main operating costs N_T are the sum of costs for lubricant *NL* and the consumed electric energy N_E [6]:

$$N_T = N_L + N_E \, . \tag{17}$$

Costs of lubricant mixture are given by the product of the unit price K_L and the volume of the lubricant mixture V_0:

$$N_L = V_0 K_L \, . \tag{18}$$

Costs of electrical energy are given by the product of the unit price of electrical energy K_E, electric power for the propulsion device *P* and the duration of the process τ:

$$N_E = K_E \cdot P \cdot \tau \, . \tag{19}$$

The optimal duration of the process depends on the desired lubrication efficiency of the process, which can be determined based on the concentration of lubricant in the liquid phase relative to its initial concentration.

IV. COMPUTER CALCULATION OF THE COST FUNCTIONS

For computer calculation of the cost functions we used a software Maple user interface. The programmed source code can compute and visualize cost functions for required input parameters.

As an example,we show the results obtained under these conditions: effectivity of process 0.5, thickness of leather 3 mm, sorption coeficient of lubricant in the leather 3, electric power for the propulsion device 10 kW, unit price of electric energy 0.5 Euro/kWh, unit price of lubricant 10 Euro/m^3.

The Fig. 4 shows computed cost function. It is evident that the optimum volume of the lubricant is 2.3 m^3 . To this point the operating costs decrease and after reaching the minimum the costs rise.

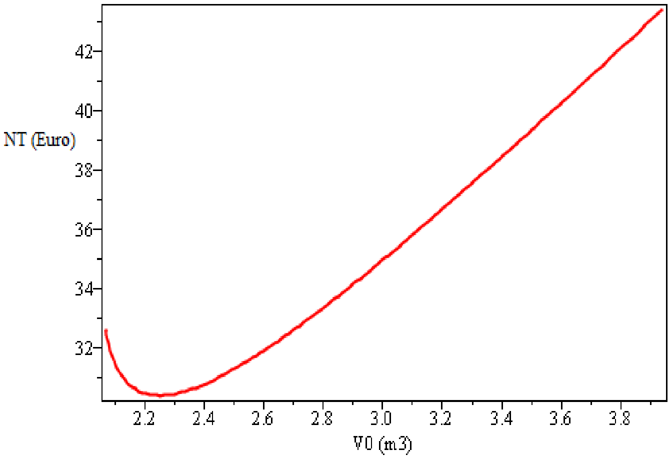

Fig. 4 Concentration of lubricant in the leather during fatliquoring

The Fig. 5 shows computed concentration field of lubricant in the leather for the optimal volume of the lubricant 2.3 m^3. The Fig. 6 depicts computed concentration field of lubricant in the liquid phase under the same condnitions. It is evident that for these conditions can be reached the maximum concentration of lubricant in the skin C = 0.4. This corresponds to the time of process Fo = 1.5 (ie 25 minutes).

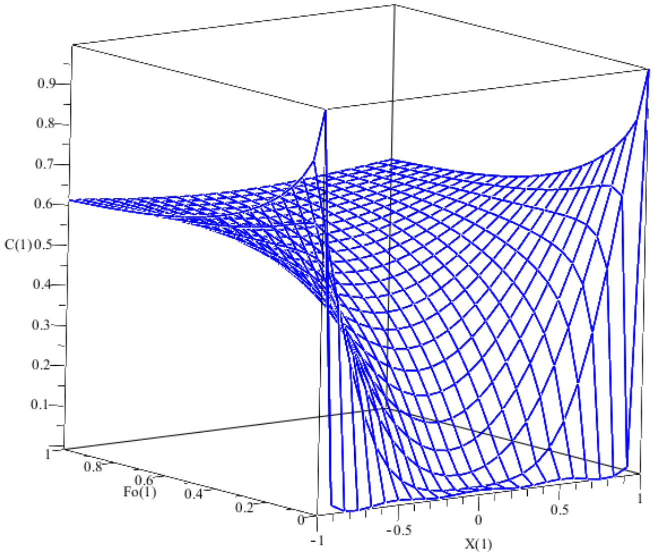

Fig. 5 Concentration field of lubricant in the leather for V_o = 2.3 m^3.

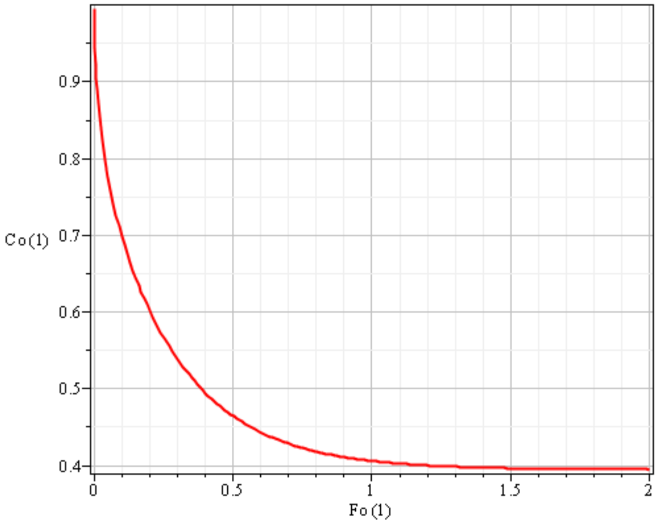

Fig. 6 Concentration field of lubricant in the liquid phase for V_o = 2.3 m^3.

As we described in the previous section, the operating costs for lubrification strongly depend on fixing power of lubricant in the leather. The Fig. 7 depicts the shift of cost functions optimum and increase of the costs for the sorption coefficient values from the A = 3 to A = 4.

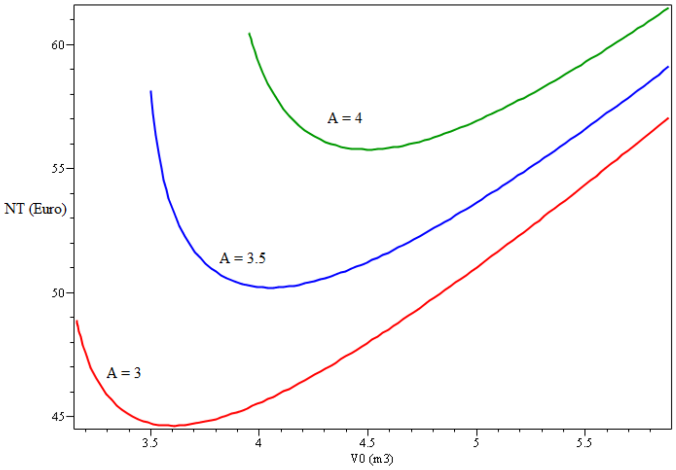

Fig. 7 Shift of cost functions optimum and increase of the costs for the sorption coefficient values from the A = 3 to A = 4. *Parameters*: effectivity of process 0.5, thickness of leather 3 mm, electric power for the propulsion device 10 kW, unit price of electric energy 0.92 Euro/kWh, unit price of lubricant 8 Euro/m^3.

V. CONCLUSION

The formulated mathematical model describing diffusion transport of lubricant into the inner porous structure of the leather enabled us to simulate the process by computer andcompute operating costs . The computes cost functions for the required input conditions can be used to determine the optimum process to the purpose of saving energy and raw materials.

VI. LIST OF SYMBOLS

A- sorption coefficient of lubricant to the leather, [1];
b - half thickness of the leather, [m];
c - concentration of unbound lubricant in the leather, [kg.m^{-3}];
C- dimensionless concentration of unbound lubricant in the leather, [1];
c_A - concentration of bound lubricant in the leather, [kg.m^{-3}];
c_p - initial concentration of lubricant in the leather, [kg.m^{-3}];
c_p - initial concentration of lubricant in the leather, [kg.m^{-3}];
c_r - equilibrium concentration of lubricant, [kg.m^{-3}];
c_0 - concentration of lubricant in the liquid phase, [kg.m^{-3}];
c_{0p} - initial concentration of lubricant in the liquid phase, [kg.m^{-3}];
D – diffusion coefficient, [m^2.s^{-1}];
Fo - Fourier number, [1];
K - modified diffusion coefficient, [m^2.s^{-1}];
K_E- unit costs for electrical energy, [Euro/kWh];
K_L - unit costs for lubrificant mixture, [Euro/m^3];
N_E - costs for electrical energy, [Euro];
N_L - costs for lubrificant mixture, [Euro];
N_T - total operating costs, [Euro];

Na - ratio V_0/V, [1];
P - electric power for the propulsion device, [kWh];
S - area of the leather, [m^2];
V - volume of the leather, [m^3];
V_0 - volume of the liquid phase, [m^3];
x - space coordinate, [m];
X - dimensionless space coordinate, [1];
ε - porosity of the leather, [1];
τ - time, [s].

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