Structural Characteristics of Granulated Ferromagnetic Materials and their Average Magnetic Properties

Sandulyak A. A., Sandulyak A. V., Ershova V. A., Sandulyak D.A.

Abstract **–** The new model of granular bed consisted of fractional cells is used to obtain parameters of its porosity.

Results of researches (of magnetic permeability) of "elementary" effective magnetic channel in porous are given. Such results permit to calculate a relative intensity of field between the grains.

The formulae for calculating relative level of magnetizing cylindrical filter-matrix samples of various lengths are obtained. A considerable influence of the samples relative dimensions on this level is shown.

Keywords – granulated material, demagnetizing factor, relative level of magnetization.

I. Introduction

THE simulation of the structure of granular materials (and their pore volumes) is recognized as a difficult problem [1-5], for solving of which the particular solutions are usually used. The obtaining of data of magnetic properties for such materials is another difficult task [6, 7], though such data is in great demand, especially by using of magnetic separators.

For example, matrix magnetic separators of filter type [8 – 11] are used to remove ferro-impurities from various technological media in order to conform to the standard quality indicators. A specific operating element, viz. a magnetized matrix performs the targeted capture of ferro-impurities in such apparatuses; to be more exact, the loading of granules in the working volume of the separator serves this function. However, there is little attention paid to the issue of evaluating the extent to which potential capabilities of a granular matrix used as a magnetic are employed. Thus, there are errors in identifying the operating regimes, inconsistencies in factual and expected results of such apparatuses, incomplete and not always objective (including comparative) estimates of the results obtained.

For instance, when choosing particular values of the volume and the corresponding matrix sizes, people often overlook the following important condition. Magnetic properties of a matrix-body, e.g. such fundamental magnetic (and therefore, magnetic-sorption) parameters as a mean magnetic induction B_b and/or magnetic permeability μ_b, except for magnetic properties of the material (matter) of the granules and their concentration in the volume given, greatly depend on the form of this body, this holds true for a solid magnetic as well. In particular, for a cylindrical body these parameters vary with its length L, to be more specific, they vary with the relation of this length to the cylinder diameter D, i.e. a relative dimension of L/D as a peculiar form factor. It is this circumstance, ignored as a rule, which causes the so-called demagnetizing factor N worsening magnetic properties of a matrix in comparison with potential values of induction B and/or magnetic permeability μ, these properties are also inherent for a quite long or classic toroidal sample of the same matrix.

The issue of obtaining such vitally necessary information on the demagnetizing factor of non-homogeneous (granular, grain and other) magnets as distinct from solid ones have not been sufficiently elaborated until recently. Actually, a systematic study of the matter was initiated in [12]. Thus, as applied to the sample bodies of these magnets (a loading of ball-bearing balls) of a cylindrical form of various lengths, i.e. with different relative dimension L/D, field dependences of B_b induction have been obtained experimentally. They have really demonstrated quite strong influence of parameter L/D on a relative level of induction $B_b/B=\Lambda$ and permeability $\mu_b/\mu=\Lambda$, especially at relatively low (let us note, intrinsic to filter-separators) values of L/D [13], when Λ value can amount to just Λ =0.5-0.6 (50-60%) and less. At that, equation $\mu_b/\mu= B_b/B=\Lambda$ holds true on the ground of $B_b= \mu_0\mu_bH$ and $B=\mu_0\mu H$, where μ_0 is a magnetic constant, H is the intensity of a magnetizing field; while the relations μ_b/μ and B_b/B per se can be conveniently named by a relative level of magnetization Λ as an indicator of fractional usage of potential magnetic properties of a magnet.

In connection with the objectively arising need for a routine accounting of the matrix relative dimension L/D real role in the level of its magnetization Λ in a particular case, it becomes necessary to obtain the corresponding calculating dependencies. With their help basing on a given and/or assumed geometry of the filter-matrix, we could provide fair and accurate operational information about factual and anticipated working efficiency (in terms of filter-matrix magnetization) of the existing and newly created separators. We can also solve a number of problems related to magnetic separation, direct and inverse ones.

II. A Contribution of the Model of Granulated Material Structure

THE structure of a granular material could be specially simulated by fractional cells.

For well-known ordered packings of balls, these fractional cells are represented by parallelepipeds conditionally cut from the granular bed (in the most general case, they are skewed except for the cubic cell), in which the vertices are located at the centers of eight neighboring balls (Fig. 1). In this case, the length of the edge of the parallelepipedic cell is equal to the diameter of the round grain d. The cell includes eight fractional segments of the cut balls and, re-

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markably, the total volume of these segments is equal to the volume of the entire ball.

Consequently, the necessary geometric parameters of any of these cells can be simply and accurately determined and such key characteristics as the volume fraction of the granular material in the cell (ratio of the ball volume to the volume of the parallelepipedic cell) and the volume fraction of pores in the cell can be easily found. This implies that we can easily obtain the information about the packing density of grains γ and porosity ω not only for individual cells but also for the whole bed. These parameters characterize the granular bed as a whole because each of these fractional cells actually plays the role of an elementary ministructural unit of the entire bed made up of these cells using the principle of strictly multiple "modules".

It should be noted that the approach itself, which is based on the simulation of a granular bed (with an ordered packing of balls) using fractional cells only, is quite justified. It is dictated by the necessity of observing the fundamental principles of modeling the structure of discrete media with spatially repeated elements, when there are short- and long-range orders in the arrangement of its constituents. The cell of any discrete medium should be such an elementary unit of the cell–modular arrangement of this medium

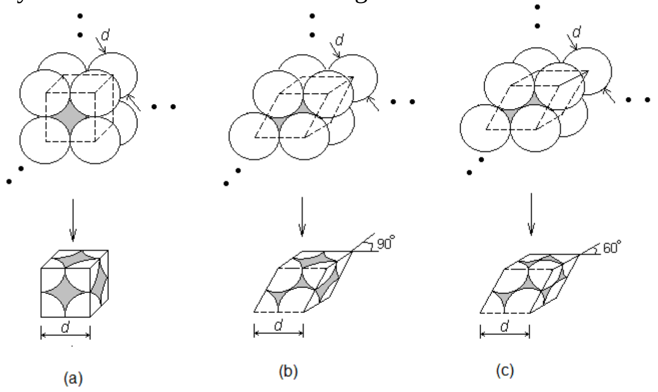

Fig. 1. Illustrative configurations of (a) cubic, (b) square-rhombic, and (c) rhombic cells cut from the corresponding ordered structures of balls.

that all cells in it should be not only strictly identical but also ideally "match" each other in all directions without gaps and overlaps. This implies that it is impossible to divide a multiball medium into structurally sound cells based on the whole ball or group of whole balls, which are usually prone to the "overlapping" staggered arrangement, even when the balls are the same and packed using any known ordered packing pattern. The only exception is the cubic packing, for which the "whole-ball" method can be easily used because of the strictly "corridor-like" arrangement of balls. By the way, this simulation method is widely used in solving numerous problems as a first approximation to the actual granular bed.

Figure 1 illustrates three types of fractional cells: cubic, square–rhombic, and rhombic, which are cut from the corresponding ordered structures of balls. It is reasonable to name these fractional parallelepipedic cells (except for the cubic cell, which is traditionally used in similar cases) using their geometric distinctive features, such as square edge, rhombic edge, and "skewness" direction. The remaining two types of cells, such as rhombic-diagonal and square-rhombic-diagonal, can be readily obtained from the rhombic (Fig. 1c) and cubic (Fig. 1a) cells by "displacing" them toward the diagonal plane of the parallelepipedic cell. In this case, the "top" ball of the cell of the corresponding structure of balls falls into a potential well between three and four "bottom" balls, respectively.

Such approach permits to easily obtain the well-known parameters of porosity ω for cells (and for granulated medium in whole): cubic – ω=0.477, square-rhombic – ω=0.396, rhombic – ω=0.302, rhombic-diagonal – ω=0.26.

III. A CONTRIBUTION OF THE MODEL OF MAGNETIZATION OF A GRANULATED MATERIAL

THE idea of selective (some kind of channel-by-channel, or "shot") magnetization is developed in papers [6, 7] where elementary effective magnetic channels formed (in the rope form) along magnetization direction are responsible for magnetization of such a medium (Fig. 2). Self-organization of these channels is possible due to granules' chains (mainly sinuous chains) that are always really manifested among a lot of skeletal granulated structure granules-chains chaotically located in the medium.

Thereat each of these magnetic channels likened to quasi-continuous channel, although it may be characterized by average (by volume) permeability values [6, 7], nevertheless, is not equivalent by cross section. Thus, as far as the radius $r=r_c$ of its core and the radius $r=r_p$ of "growing" tubular layer increase, their magnetic resistance increases because of growing porous interlayer between R radius granules in the chain. It makes us considering new key characteristics such as "tube" permeability $\tilde{\mu}$ and core permeability

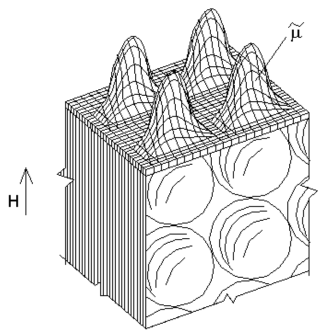

Fig. 2: Illustration of selective (channel-by-channel) magnetization model of granulated medium with expressed shape (profile in section) of magnetic permeability $\tilde{\mu}$ for each effective (in the "rope") magnetic channel.

$\langle\tilde{\mu}\rangle$ (certainly, the permeability of granules' material μ is known).

Expressions for $\tilde{\mu}$ can be easily obtained from equivalence between magnetic resistance of quasi-continuous tube and total resistance of its actual parts (in granules and interlayer) [6]:

$$\tilde{\mu} = \frac{\mu}{\mu - \sqrt{1-(r/R)^2}\,(\mu-1)} = \frac{h}{H}, \tag{1}$$

where H – magnetization field intensity.

One of the expressions for $\langle\tilde{\mu}\rangle$, namely

$$\langle\tilde{\mu}\rangle = \frac{2\mu}{(r/R)^2(\mu-1)}\left\{\frac{\mu}{\mu-1}\ln\left[\mu-(\mu-1)\sqrt{1-\left(\frac{r}{R}\right)^2}\right]+\sqrt{1-\left(\frac{r}{R}\right)^2}-1\right\}, \tag{2}$$

follows from averaging profile permeability $\tilde{\mu}$, particularly, from integration of the first expression (1) for $\tilde{\mu}$.

Design data (*dashed lines*) obtained according to (1, 2) and experimental data (*dots*) on $\langle\tilde{\mu}\rangle$ and $\tilde{\mu}$ are gathered in figures 3, 4. Noting good agreement of design data with experiment, one should note that $\tilde{\mu}$ has extreme profile (by analogy with the profile of liquid velocity in the tube) which is bell-shaped, in outward appearance similar to Gauss distribution (Fig. 4).

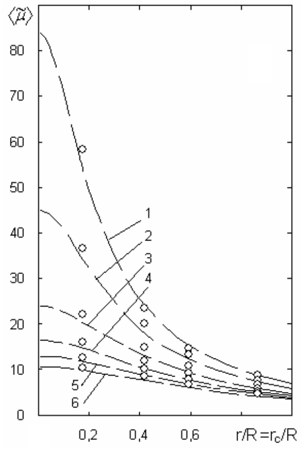

Fig. 3: Permeability of quasi-continuous cores (in the magnetic channel) with radius r=rc; dots correspond to experimental data, lines was calculated according to expression (2), 1 – H=18 kA/m, 2 – 36, 3 – 70, 4 – 105, 5 – 140, 6 – 175.

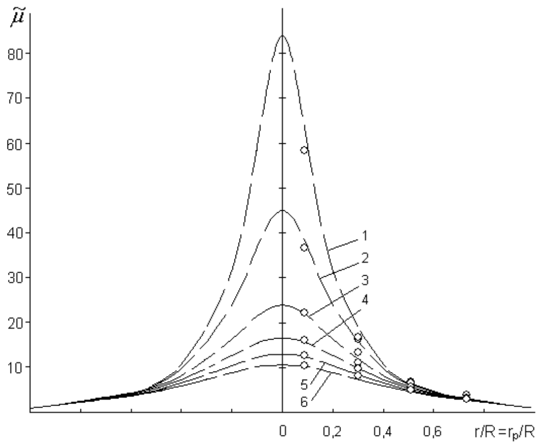

Fig. 4: Permeability of quasi-continuous tubes (in the magnetic channel) with radius $r=r_p$; dots correspond to experimental data, lines was calculated according to expression (1).

IV. IMPLEMENTATION OF APPROACH TO OBTAINING FORMULE FOR CALCULATING RELATIVE LEVEL OF FILTER-MATRIX MAGNETIZATION

THE same fundamental connection between the demagnetization factor (a demagnetizing coefficient) N and the magnetic permeability of the matter μ and the body μ_b [14-18] can be applied to a granular (formally – a quasi-solid) magnet, as to a solid one:

$$N = \frac{1}{\mu_b - 1} - \frac{1}{\mu - 1}. \quad (3)$$

Thereat, it is easy to get a calculating formula from this connection to define a relative level of filter-matrix magnetization; just to remind you – it is a formula for a relative level of magnetic permeability and respective magnetic induction $\mu_b/\mu= B_b/B=\Lambda$ [19]:

$$\Lambda = \frac{1}{\mu}\left[\frac{\mu-1}{(\mu-1)N+1}+1\right], \quad (4)$$

needless to say, the formula can be obtained for a given sample of a filter-matrix with known values of magnetic permeability of its quasi-solid matter μ and a demagnetization factor N.

Note that in contrast to the afore-considered phenomenological approach where the demagnetizing factor N of the sample-body was a kind of a shadow parameter, exhibiting itself latently as an externally observed phenomenon of sample-body magnetization level reduction, here N appears as a quantitative parameter. Its influence on Λ is evaluated quite definitely (Fig. 5), namely, in the range μ= 5-10 covering a characteristic for granular matrixes range of μ [12] dependencies of Λ on N considerably decrease with N increase (Fig.5) [19].

As to the data necessary for calculating N values by formula (4) as applied to the specific samples of granular medium, they are amply given in paper [7] for the samples of a cylindrical granular magnet (balls loading). Thus, following the definition (3) and basing on the aforementioned experimental data in [11] and in paper [7] there have been performed calculations for

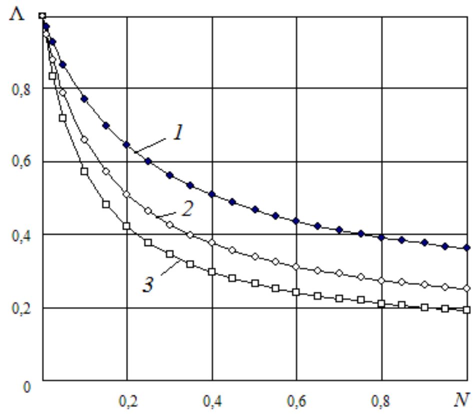

Fig. 5. Relative level of magnetic body magnetizing depending on its demagnetizing factor 1 – μ=5; 2 – μ=7.5; 3 – μ=10.

specific values of parameter N for various values of L/D of such samples. Moreover, appropriate processing of these data manifested, that the dependence of the demagnetization factor N on the sample relative dimension L/D has an exponential view, but with a quite peculiar argument, viz. a radical of the relative dimension [7], i.e.

$$N = \exp\left(-k_N\sqrt{L/D}\right), \quad (5)$$

with the defined value of coefficient $k_N \cong 1.5$ in (5) true for the studied quasi-solid (granular) samples in the range μ= 6.9-8.5 [12] characteristic for the magnetizing separator matrixes.

Consequently, taking into account relation (5), expression (4) written in the expanded view as ([19]):

$$\Lambda = \frac{1}{\mu}\left[\frac{\mu-1}{(\mu-1)\exp\left(-1{,}5\sqrt{L/D}\right)+1}+1\right], \quad (6)$$

becomes a basic calculating formula which fully characterises the influence of immediate ‘initiator’ of parameter N, a relative dimension of filter-matrix L/D on the relative level of its magnetization Λ (Fig.6).

Expression (4) following from the classical relation (3) and facilitating the elaboration of the calculating formula (6) is naturally true for the entire range of possible varying of

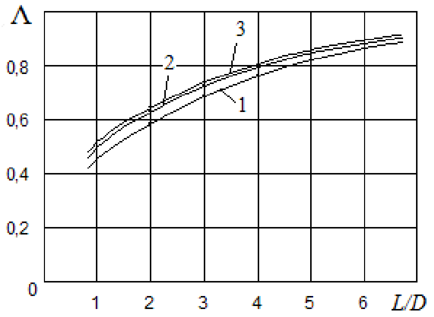

Fig. 6. Calculation data obtained according to (6), the data of relative magnetization level of filter-matrix samples in terms of their relative size, 1–μ=8.5, 2–μ=7.4, 3– μ=6.9.

the demagnetization factor: N=0-1. Alongside with that, for filter-matrixes of industrial and experimental magnetic separators relative dimension *L/D* values of which are mainly in the order of unity and more, the real range of N values is relatively narrow: N < 0.2-0.25. That is why it seems reasonable to try to obtain its particular, simplified phenomenological variants for the narrowed range of N, basing on expression (4).

To do this, we should refer to the earlier studied family of dependencies Λ on N (Fig.5) constructed according to expression (4), but as the argument it is rational to use not N but $N^{0,7}$ (Fig.7).

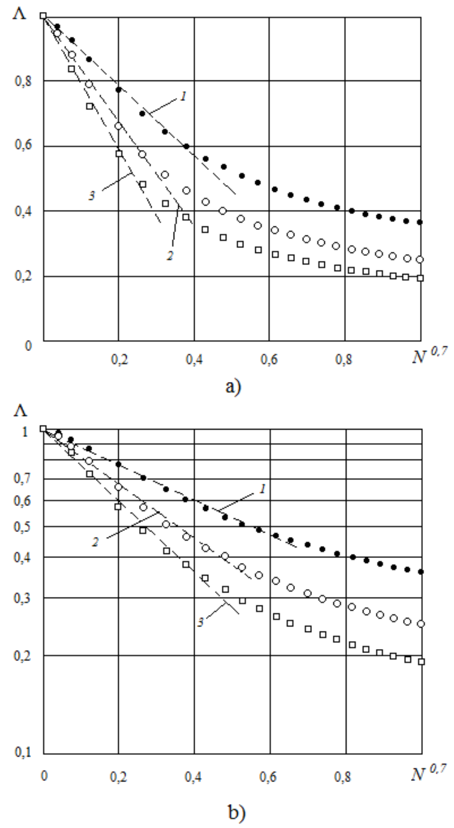

Fig. 7. On the possibility of fractional linearization of data in Fig. 5 in common (*a)* and semi-logarithmic (*b*) coordinates, the argument is a power function of the demagnetizing factor.

In these coordinates and in the range of μ=5-10, which is still of great interest for us, the aforementioned dependencies are linearised quite well, as is seen in Fig. 7*a*, but only in the range of $N^{0,7}$< 0.25-0.4 (Fig.7*a*, dashed lines) which corresponds to somewhat reduced values of N, namely N< 0.14-0.27. It means that in this case the calculating formula has the following view:

$$\Lambda = 1 - k_\mu N^{0,7}, \tag{7}$$

with that parameter k_μ depending on the magnetic permeability μ of the quasi-solid magnetic 'matter' (Fig. 7*a*, dashed lines) complies to the linear connection with μ (Fig. 8, curve 1):

$$k_\mu = 0{,}21\mu, \tag{8}$$

which allows us, basing on (7), to put down the following version of the expression for the relative level of filter-matrix magnetization (for the ranges N<0.14-0.27 and μ=5-10):

$$\Lambda = 1 - 0{,}21\mu N^{0,7}. \tag{9}$$

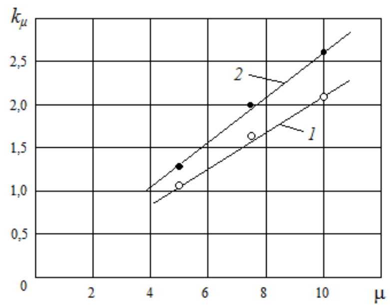

Fig. 8. Illustration of linear dependence of k_μ coefficient introduced in formulae (7) and (11), lines 1 and 2 respectively, on magnetic permeability of the quasi-solid filter-matrix 'matter'.

Subsequently, inserting here expression (5) for N we may obtain, alternatively to expression (6), the following calculating formula for the stated ranges of N and μ:

$$\Lambda = 1 - \frac{0{,}21\mu}{\exp\sqrt{L/D}}. \tag{10}$$

From the point of view of considerable expansion of N values range, i.e. expansion to the values of $N^{0,7}$< 0.45-0.65, or up to a quite acceptable range of N< 0.32-0.54, it is more preferable to employ the option of semi-logarithmic coordinates used for representing in them the same data (shown in Fig. 7*a*). The fact of fractional (for the mentioned range of N) linearization of these data used in semi-logarithmic coordinates (Fig. 7*b*, dashed lines) indicates the possibility of using the calculating formula of an exponential type:

$$\Lambda = \exp\left(-k_\mu N^{0,7}\right), \tag{11}$$

whereas parameter k_μ, depending on the magnetic permeability μ of the quasi-solid magnetic 'matter' (Fig. 7*a*, dashed lines) also complies to the linear but slightly different than before connection with μ (Fig. 8, curve 2):

$$k_\mu = 0{,}26\mu. \tag{12}$$

After substituting (12) in (11) there follows one more version of the expression for the relative level of filter-matrix magnetization, here it is for the stated above expanded range of N< 0.32-0.54 and the previous range of μ=5-10:

$$\Lambda = \exp\left(-0{,}26\mu N^{0,7}\right). \tag{13}$$

And substitution of (5) into (13) yields, parallel with expressions (6) and (10), the following calculating (formal) formula for the above stated ranges of N and μ:

$$\Lambda = \exp\left(-\frac{0{,}26\mu}{\exp\sqrt{L/D}}\right). \tag{14}$$

The obtained formulae (10) and (14) may be used for practical calculations when solving direct and inverse problems connected with identifying actual level of magnetization Λ of a specific (by L/D) filter-matrix of the magnetic separator, and which is nonetheless important, the tasks related to defining the necessary value of the filter-matrix relative dimension L/D:

$$\frac{L}{D}=\left(\ln\frac{0{,}21\mu}{1-\Lambda}\right)^2, \quad \frac{L}{D}=\left[\ln\left(-\frac{0{,}26\mu}{\ln\Lambda}\right)\right]^2, \qquad (15)$$

proceeding from the pre-set level of its magnetization Λ.

For example, when creating an apparatus (experimental, pilot production, or industrial one) there is a decision taken to allow the reduction of the filter-matrix magnetization level by no more than say 20%, i.e. the acceptable value of Λ for this case is Λ=0.8. Then the calculation by formula (15), let's say for the case of μ =7.5, manifests the necessity to procure obligatory relative dimension of the working member of this apparatus, i.e. filter-matrix, with L/D=4.1-4.7 dimension. By the way, similar result (L/D=4.2) is received when performing calculations with the control formula (6) after turning L/D parameter from an argument into a function.

III. Conclusion

THE new model of granular bed consisted of fractional cells is used to obtain parameters of its porosity.

Results of researches (of magnetic permeability) of "elementary" effective magnetic channel in porous are given. Such results permit to calculate a relative intensity of field between the grains.

Judging by the data of real relative level of magnetizing granular material samples with various values of such parameter as their relative dimension L/D, we have to acknowledge the form factor role and dependant on it demagnetizing factor of samples (as quasi-solid magnets) to be essential and compulsory to be taken into account. Factual data on this parameter of magnetic separators filter-matrixes geometry bespeak of apparent under-employment of their magnetic properties. For instance for a filter-matrix the length of which is commensurable with its diameter, a relative level of its magnetization is no less than 50 – 60%.

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