Electromagnetic approach to nondestructive testing of fibre concrete

Leonard Hobst, Tereza Komárková and Jiří Vala

***Abstract*—Mechanical, thermal, etc. behaviour of fibre concrete structures, including (most frequently steel) fibres and a concrete matrix, is conditioned namely by i) the amount of fibres and ii) their directional distribution. Fibres should prevent quasi-brittle crack damage caused by tensile stresses and other deterioration processes, thus reliable nondestructive testing methods for i) and ii) are required. This paper presents an original electromagnetic approach based on different electromagnetic material characteristics, as magnetic permeability, dielectric permittivity and electric conductivity, of both applied materials in a composite, i. e. of (most frequently steel) fibres and concrete matrix, occurring in a special class of Maxwell equations, to the identification of i) and ii).**

***Index Terms*—Fibre concrete, nondestructive testing, electromagnetic measurements, identification problems.**

I. INTRODUCTION

CEMENTITIOUS composites for building structures need improvement of their mechanical, thermal, etc. properties to remove or (at least) suppress formation of macro- and macro-cracks caused by tensile stresses (because of quasi-brittle behaviour of a cementitious matrix) and other processes of material deterioration, as discussed by [3] and [11]. One widely used alternative of such improvement is the application of short strengthening metal (most frequently steel) fibers. Resulting properties of a composite are then conditioned by i) the volume fraction and ii) the distribution of directions of fibres. Since classical destructive approaches are not allowed in many cases and us cannot handle ii) properly, the development of nondestructive approaches is required. However, apart from advertisement of various producers of measurement technique, no sufficiently general, robust, accurate, inexpensive and reliable method for nondestructive identification of both i) and ii) is available, which can be seen as motivation for the increasing number of relevant research papers in last 2 decades; more historical comments can be found in [8].

Some nondestructive approaches rely on the sequences of planar images, typically from the X-ray analysis, as discussed in [7], often supplied by the two-dimensional fast Fourier transform, or some other nontrivial numerical technique (e. g. the edge detection from the level set method) – cf. [9] and [24]. New (seemingly exact) 3-dimensional results come from the CT-scanning – see [14] and [25]. However, such approach is rather expensive, in particular in the case of 3-dimensional images, and works only with carefully prepared samples, not with real building structures in situ.

Nevertheless, all other approaches of identification of i) and ii) are indirect, rely on the identification of some effective (macroscopic) material characteristics (hiding the available knowledge of material structure) and work with certain calibration procedures, sometimes with the tricky ones, in the optimal case with those explainable by appropriate physical and mathematical considerations. Unfortunately, more advanced theoretical studies usually lead to the design of vague and complicated identification algorithms, whose practical implementation is difficult or quite impossible because of the gaps in the theory. From this point of view, [6], [10], [13] and [27] work with various generalizations of the Maxwell - Garnett rule, well-known from the theory of mixtures, formulated in its original form for spherical particles. Other studies come from some mathematical homogenization approach (physically motivated in some cases), e. g. from the two-scale convergence, understood as certain new convergence type between the strong and weak one, studied for periodic material structures in [2] in large details, for linearized problems with the results of [12] instead of the Maxwell - Garnett rule, applicable even to slightly non-periodic material models – cf. [18] and [21]. However, substantial loss of periodicity in deterministic models, as well as implementation of advanced stochastic approaches, referring to Monte Carlo or similar computational simulations. lead to very complicated mathematical formulations like [15], without relevant support of identification software.

Up to now, the most successful indirect measurement approaches seem to be based on measurements of some magnetic or electromagnetic quantities, making use of the different values of such material characteristics, namely of the dielectric permittivity ε, the electric conductivity σ and the magnetic permeability μ, sometimes also of the magnetic susceptiblity χ. Such time-independent characteristics can be considered as constant just for (macroscopically) homogeneous materials and as scalar just for isotropic ones, which brings technical difficulties namely to the identification of ii). The purely magnetic approach, described (as an alternative) in [24], leads to only one Laplace equation where an unknown characteristics μ comes just from boundary equations; however, its application requires to make circular holes for the Hall probe and to choose measurement positions carefully to obtain reasonable differences of (rapidly decreasing) magnetic fluxes for the identification procedure.

The solvability of the complete evolutionary system of Maxwell equations, even of the direct one (i. e. for the a priori known material characteristics) as well as the convergence of needed sequences of approximate solutions, hidden in computational algorithms, involves serious open questions – see [1] and [20]. Consequently all reasonable electromagnetic experiments must be based on the very special physical and geometrical setting. Various approaches of such type have been suggested in [4], [5] and [26] in the last several years. In this paper we shall sketch another original methodology, based on the measurements of complex impedance, which seems (both theoretically and from first practical experiments) to be able to cover both i) and ii) naturally.

II. Physical background and mathematical description

The analysis of electromagnetic fields works with a set of scalar and vector quantities, introduced on $\Omega \times I$ where Ω is a domain in the (in general) 3-dimensional Euclidean space, supplied by Cartesian coordinates, and I means a finite time interval; dots refer to partial time derivatives. Using the notation of [17], pp. 1 and 4, such basic quantities on $\Omega \times I$ are the scalar volume charge density ρ, the electric current density (charge flux) J, the electric field intensity E, the magnetic field intensity H, the electric flux density (electric displacement) D, the magnetic flux density (magnetic induction) B, the electric current density J and the magnetization (average magnetic moment per unit volume) M, all vectors.

The obvious charge conservation principle reads

$$\dot{\rho} + \nabla \cdot J = 0\,. \tag{1}$$

The important relations between the remaining quantities

$$\nabla \cdot D = \rho\,, \qquad \nabla \cdot B = 0\,, \tag{2}$$

well-known as the Gauss laws for electric and magnetic fields, and

$$\dot{D} - \nabla \times H + J = 0\,, \qquad \dot{B} + \nabla \times E = 0\,, \tag{3}$$

referenced as the the Ampère and Faraday laws. Following [17], p. 11, we are allowed to consider the linear constitutive equations

$$J = \sigma E\,, \quad D = \varepsilon E\,, \quad B = \mu H\,, \quad M = \chi H\,. \tag{4}$$

Moreover, we can introduce the energy density

$$w = \frac{1}{2}\left(D \cdot E + B \cdot H\right) \tag{5}$$

and the total energy flux (Poynting vector) $P = E \times H$; then we receive, in addition to (1), inserting (3) into (5),

$$\begin{aligned}
&\dot{w} + \nabla \cdot P + J \cdot E \\
&= \dot{D} \cdot E + \dot{B} \cdot H + \nabla \cdot (E \times H) + J \cdot E \\
&= \dot{D} \cdot E + \dot{B} \cdot H + H \cdot \nabla \times E - E \cdot \nabla \times H + J \cdot E \\
&= (\dot{D} - \nabla \times H + J) \cdot E + (\dot{B} + \nabla \times E) \cdot H = 0\,,
\end{aligned} \tag{6}$$

which can be seen as the energy conservation principle.

Moreover, from (3) and (2) for homogeneous materials (with zero derivatives of ε, σ and μ) we obtain

$$\begin{aligned}
&\nabla \times \nabla \times H = \nabla J + \nabla \dot{D} = \sigma \nabla \times E + \varepsilon \nabla \times \dot{E} \\
&= -\sigma \dot{B} - \varepsilon \ddot{B} = -\sigma\mu \dot{H} - \varepsilon\mu \ddot{H}\,, \\
&\nabla \times \nabla \times E = -\nabla \times \dot{B} = -\mu \nabla \times \dot{H} \\
&= -\mu \dot{J} - \mu \ddot{D} = -\mu\sigma \dot{E} - \mu\varepsilon \ddot{E}\,.
\end{aligned} \tag{7}$$

Applying the mathematical formula $\nabla \times \nabla \times S = \nabla(\nabla S) - \nabla \cdot \nabla S$, for the choice both $S = H$ and $S = E$ its left side degenerates, thanks to (4), to $\Delta S = \nabla \cdot \nabla S$, and (7) gets the simple form

$$\begin{aligned}
\Delta H &= \sigma\mu \dot{H} + \varepsilon\mu \ddot{H}\,, \\
\Delta E &= \mu\sigma \dot{E} + \mu\varepsilon \ddot{E}\,.
\end{aligned} \tag{8}$$

A more complicated form of (8) and (7) can be derived in the same way without any homogeneity assumption. In particular, for isotropic materials ε, σ and μ can be considered as scalar constants, thus $\sigma\mu = \mu\sigma$ and $\varepsilon\mu = \mu\varepsilon$.

Especially for the stationary pure magnetic field the second equation and the right side of the first one in (8) vanish, which results in the homogeneous Laplace equation $\Delta H = 0$ (with no explicit μ). Then the interface boundary condition of type $(S - S^{\times}) \cdot \nu = 0$, with $S^{\times}$ in the role of some scalar variable S coming from a domain adjacent to S (or from external environment) where nu is the local (formally outward) unit normal vector to the boundary of Ω, can be implemented. This configuration with the natural choice $S = B$ contains μ, thus enables us to exploit some rather simple semi-implicit identification formulae, at least those obtained from the mixture theory by [6]; for more detailed discussion see [24] (cf. Fig. 1 lower with another choice $S = M$, too). Moreover, it seems to be reasonable, analogously to [23], to convert more complicated problems to some similar semi-stationary form, as will be demonstrated here for the special case of harmonic time dependence.

Through the inverse Fourier transform, general solutions of Maxwell equations can be built, following [17], p. 13 as linear combinations of single-frequency solutions

$$S(r,t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{S}(r,\omega) \exp(-i\omega t)\,\mathrm{d}\omega \tag{9}$$

with $S = E$, $S = H$, etc., where r denotes the distance from some selected fixed point from Ω in R^3 and the time t is transformed to the frequency ω between 0 and 2π; the phasor amplitudes $\tilde{E}$, $\tilde{H}$, etc., are complex-valued. Consequently, using the notation * for complex conjugates, (5), (3) and (2) get the form

$$\begin{aligned}
&w = \frac{1}{2} \operatorname{Re}\left(\tilde{D} \cdot \tilde{E}^* + \tilde{B} \cdot \tilde{H}^*\right)\,, \\
&\nabla \times \tilde{H} = \tilde{J} - i\omega \tilde{D}\,, \qquad \nabla \times \tilde{E} = i\omega \tilde{B}\,, \\
&\nabla \cdot \tilde{D} = \tilde{\rho}\,, \qquad\qquad\quad\;\; \nabla \cdot \tilde{B} = 0\,;
\end{aligned} \tag{10}$$

this can be derived even without (9), for particular values of ω, also with $-i\omega t + \varphi$ containing different real additive constants φ instead of $-i\omega t$. In addition to (10), under the same assumptions on material characteristics, using the identity matrix I, then (8) reads

$$\begin{aligned}
&\Delta\widetilde{H} + \left(I + \sigma\varepsilon^{-1}\frac{i}{\omega}\right)\omega^2 \varepsilon\mu \widetilde{H} = 0\,, \\
&\Delta\widetilde{E} + \omega^2 \mu\varepsilon \left(I + \varepsilon^{-1}\sigma\frac{i}{\omega}\right)\widetilde{E} = 0\,,
\end{aligned} \tag{11}$$

which are (formally) two separated complex Hemholtz equation (instead of the original real Laplace one), with a (seemingly free) real parameter ω, useful for special settings.

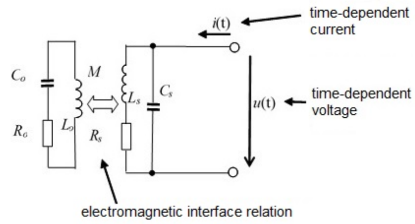

Fig. 1. Simplified electric scheme of the electromagnetic testing method.

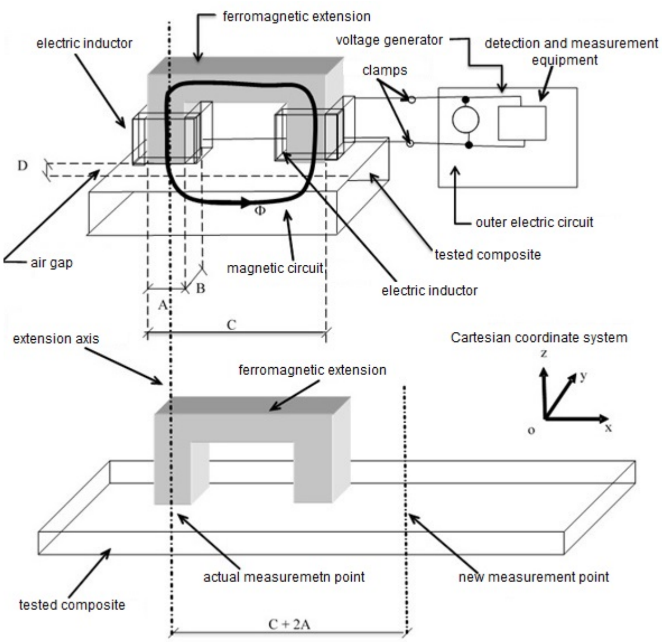

Fig. 2. Global scheme of the experimental device.

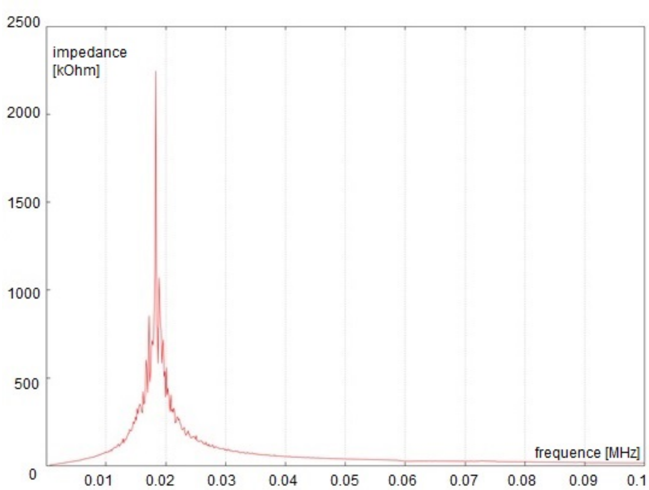

Fig. 3. Impedance as an experimentally detected function of frequency.

III. Experimental setup and illustrative example

The announced electromagnetic measurement and identification system relies on the analysis coming from (11), making use of some special choices and simplifications. In the case for

Fig. 4. Various types of applied inductors.

Fig. 5. Preparation of reference composite specimens.

one-dimensional modelling, following [16], Chap. 7, open to various generalizations, a RLC circuit consisting of (parallel or serial) capacitors, inductors and resistors can be characterized by their impedances R, $-i\omega L$ and $(-i\omega C)^{-1}$ where R is proportional to σ^{-1}, L to μ and C to ε (in our general notation). In such sense, for the first introduction of the simultaneous nondestructive testing approach of i) and ii), developed at the Faculty of Civil Engineering of the Brno University of Technology (BUT) in the intensive collaboration with its Faculty of Electrical Engineering and Communication, simplified scheme of corresponding circuits at Fig. 1 can be useful. The more detailed information to the design of the whole measurement system can be obtained from Fig. 2; the principal quantity for such methodology is then the complex impedance $|\tilde{E}|/|\tilde{H}|$, which is a function of ω; this can be analyzed experimentally, as demonstrated by Fig. 3. To handle 2- and 3-dimensional analysis of i) and ii), the complete methodology assumes several steps with a moving and rotating

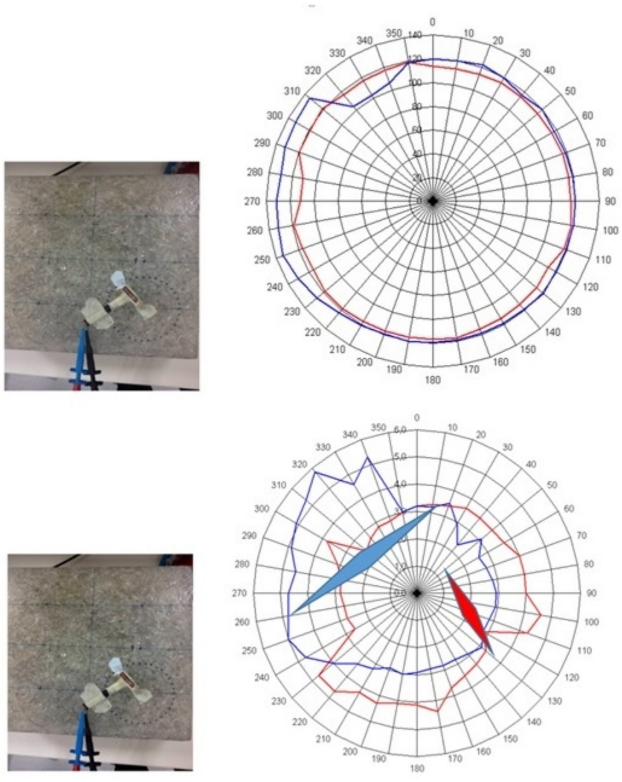

Fig. 6. Two cases of resulting planar roses of fibre directions.

specimen, under resonance and other significant conditions. Some applied inductors for the exploitation under laboratory conditions and in situ are shown at Fig. 4. However, the system is still in development, involving the process of its industrial certification, thus we may only sketch its capabilities here, together with some illustrative identification results.

Practical validation of formal homogenization (or similar) results can be performed using reference composite specimens with a priori known properties; preparation of 4 such special specimens is presented by Fig. 5; the transparency of these specimens should support their simple visual inspection. The identification results for i) and ii) in the planar case, well-known as the "roses of directions", discussed in [7], for a practically homogeneous and isotropic material in various depths from the wall surface (the upper graph) and for a quite other fibre distribution (the lower graph) is demonstrated at Fig. 6. The attached photographs document the applicability of such identification approach in situ, unlike the (rather expensive) comparative laboratory CT scanning from Fig. 7, using the commercial equipment GE phoenix v!tome!c L240, available at BUT in CEITEC (Central European Institute of Technology, in collaboration with the Masaryk University in Brno): the couple of upper images shoe the informative surface view and the view to fibres inside the specimen, whereas the lower image highlights required results of directional analysis. Nevertheless, the (seemingly exact) results from CT scanning are very useful as a relevant database for the development of numerical identification procedures including their software implementation.

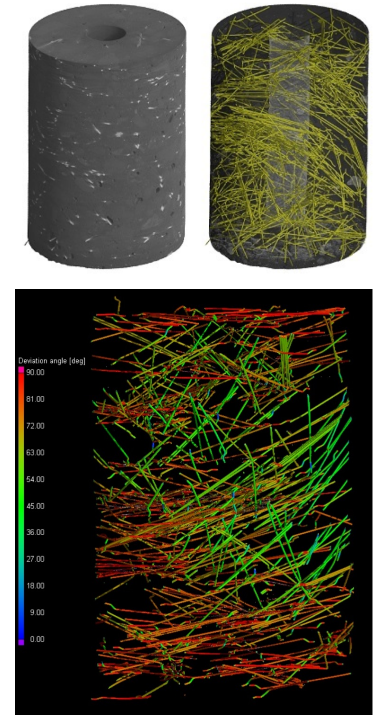

Fig. 7. CT directional analysis of fibres for a special cylindric specimen.

IV. Conclusion

First experience with the new methodology of identification of i) and ii), based on the design of an original measurement system and on the nontrivial physical, mathematical and computational analysis, validates its applicability to real concrete (and similar) structures, which has been the principal motivation for its development. However, it cannot be still seen as a closed research project: in addition to the need to overcome some both technical and theoretical difficulties (from the optimal choice of inductors to the convergence of numerical algorithms for ill-posed inverse problems), the relation between i), ii) and real mechanical (and similar) material properties, conditioned by the micro- and macro-crack formation and development, together with cohesion between metal fibres and a cementitious matrix (as analyzed in [19]), should be studied in much more details.

Another research direction has been opened using the CT data together with advanced numerical simulations and practical measurement results, with the substantial contribution of the above sketched approach. This can be seen as a potential bridge between the micro- and macrostructural analysis, avoiding artificial correlation functions, as well as between the study of material samples and whole engineering structures in the near future.

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