Mathematical modeling of crown fire initiation in three-dimensional setting

Valeriy Perminov, Alexander Goudov

Abstract**—**Setting up and numerical solution of crown forest fire initiation and spread are presented with the use of a general mathematical model of forest fire. In this paper the theoretical investigations of the problems of forest fire initiation was carried out. Mathematical model of forest fire was based on an analysis of experimental data and using concept and methods from reactive media mechanics. The research was based on numerical solution of three dimensional Reynolds equations. The boundary-value problem is solved numerically using the method of splitting according to physical processes.

Keywords**—** control volume, discrete analogue, forest fire, ignition, mathematical model.

I. Introduction

Annually in Russia more than 1 million of hectares of the forest area are burnt as a result of a forest fire. The most dangerous form of the forest fire is crown fire which amounts to 70 % of burnt area and causes the largest damage. Great interest of the problem concerned is also explained by the influence of forest fires on the ground level layer of the atmosphere, which causes medium temperature drop due to the area smoke screen and leads to the damage or delay of agricultural plant ripening and different ecological disasters. Considering that, natural investigations of these problems are merely impossible, methods of mathematical modeling are urgent. Mathematical model of forest fire was based on an analysis of known experimental data and using concept and methods from reactive media mechanics. In this paper the assignment and theoretical investigations of the problems of crown forest fire spread in windy condition were carried out. In this context, a study - mathematical modeling - of the conditions of forest fire spreading that would make it possible to obtain a detailed picture of the change in the velocity, temperature and component concentration fields with time, and determine as well as the limiting conditions of forest fire propagation is of interest. One of the objectives of these studies is the improvement of knowledge on the fundamental physical mechanisms that control forest fire initiation and spread. A great deal of work has been done on the theoretical problem of how forest fire initiation. Crown fires are initiated by convective and radiative heat transfer from surface fires. However, convection is the main heat transfer mechanism. Theoretical explanation of this process was given by Van Wagner [1]. This theory proposed there depends on three based forest properties: crown base height, bulk density and moisture content of forest fuel. Also crown forest fire initiation have been studied in details (Alexander [2], Van Wagner [3], Xanthopoulos, [4], Rothermel [5,6], Van Wagner, [7], Cruz [8], Albini [9], Scott, J. H. and Reinhardt, E. D. [10] and others). The more complete consideration of this problem is provided by group of co-workers at Tomsk University (Grishin [11], Grishin and Perminov [12], Perminov [13,14] and others). A general mathematical model of forest fires was obtained by Grishin [11] based on an analysis of known and original experimental data [11,15] and using concepts and methods of reactive media mechanics. Two-phase model used in [16-17] may be considered as a continuation and extension of the formulation proposed by Grishin and Perminov [12-14]. At present doesn't take into account the mutual interaction of the forest fires and boundary layer of atmosphere.

II. Physical and Mathematical Formulation

During the deduction of equations and boundary and initial conditions adopt the next assumptions: 1) the forest represents a multi-phase, multistoried, spatially heterogeneous medium; 2) in the fire zone the forest is a porous-dispersed, two-temperature, single-velocity, reactive medium; 3) the forest canopy is supposed to be non - deformed medium (trunks, large branches, small twigs and needles), i.e. the medium is assumed to be quasi-solid (almost non-deformable during wind gusts); 4) let there be a so-called "ventilated" forest massif, in which the volume of fractions of condensed forest fuel phases, consisting of dry organic matter, water in liquid state, solid pyrolysis products, and ash, can be neglected compared to the volume fraction of gas phase (components of air and gaseous pyrolysis products); 5) the flow has a developed turbulent nature and molecular transfer is neglected; 6) gaseous phase density doesn't depend on the pressure because of the low velocities of the flow in comparison with the velocity of the sound. Let the coordinate reference point x_1, x_2 , x_3= 0 be situated at the center of the surface forest fire source at the height of the roughness level, axis $0x_1$ directed parallel to the Earth's surface to the right in the direction of the unperturbed

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wind speed, axis $0x_2$ directed perpendicular to $0x_1$ and axis $0x_3$ directed upward (Fig. 1).

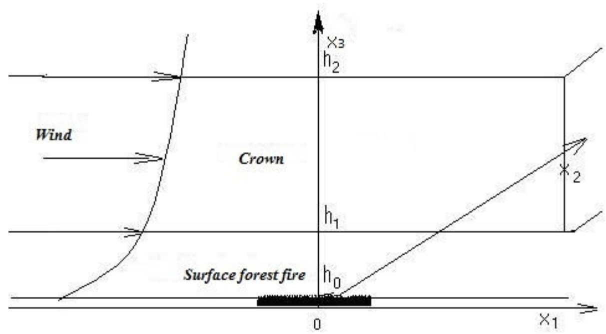

Fig.1. Crown fire initiation

Using the results of [11-14] and known experimental data [15] we have the following sufficiently general equations, which define the state of the medium in the forest fire zone, written using tensor notation:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_j}(\rho v_j) = \dot m, \quad j = 1,2,3, \quad i = 1,2,3; \tag{1}$$

$$\rho \frac{dv_i}{dt} = -\frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_j}(-\rho \overline{v_i' v_j'}) - \rho s c_d v_i |\vec v| - \rho g_i - \dot m v_i; \tag{2}$$

$$\rho c_p \frac{dT}{dt} = \frac{\partial}{\partial x_j}(-\rho c_p \overline{v_j' T'}) + q_5 R_5 - \alpha_v (T - T_s) + + k_g (c U_R - 4\sigma T^4); \tag{3}$$

$$\rho \frac{dc_\alpha}{dt} = \frac{\partial}{\partial x_j}(-\rho \overline{v_j' c_\alpha'}) + R_{5\alpha} - \dot m c_\alpha, \ \alpha = 1,5; \tag{4}$$

$$\frac{\partial}{\partial x_j}\left(\frac{c}{3k}\frac{\partial U_R}{\partial x_j}\right) - k c U_R + 4 k_S \sigma T_S^4 + 4 k_g \sigma T^4 = 0, \tag{5}$$
$$k = k_g + k_S;$$

$$\sum_{i=1}^{4} \rho_i c_{pi} \varphi_i \frac{\partial T_S}{\partial t} = q_3 R_3 - q_2 R_2 + k_s (c U_R - 4\sigma T_S^4) + + \alpha_V (T - T_S); \tag{6}$$

$$\rho_1 \frac{\partial \varphi_1}{\partial t} = -R_1, \rho_2 \frac{\partial \varphi_2}{\partial t} = -R_2, \rho_3 \frac{\partial \varphi_3}{\partial t} = \alpha_C R_1 - \frac{M_C}{M_1} R_3, \rho_4 \frac{\partial \varphi_4}{\partial t} = 0; \tag{7}$$

$$\sum_{\alpha=1}^{5} c_\alpha = 1, \ p_e = \rho R T \sum_{\alpha=1}^{5} \frac{c_\alpha}{M_\alpha}, \vec v = (v_1, v_2, v_3), \vec g = (0,0,g)$$

$$\dot m = (1 - \alpha_c) R_1 + R_2 + \frac{M_c}{M_1} R_3 + R_{54} + R_{55},$$

$$R_{51} = -R_3 - \frac{M_1}{2M_2} R_5, R_{52} = \nu_g (1 - \alpha_c) R_1 - R_5,$$

$$R_{53} = 0, R_{54} = \alpha_4 R_1, R_{55} = \frac{\alpha_5 \nu_3}{\nu_3 + \nu_{3*}} R_3.$$

Reaction rates of these various contributions (pyrolysis, evaporation, combustion of coke and volatile combustible products of pyrolysis) are approximated by Arrhenius laws whose parameters (pre-exponential constant k_i and activation energy E_i) are evaluated using data for mathematical models [11,13].

$$R_1 = k_1 \rho_1 \varphi_1 \exp\left(-\frac{E_1}{RT_s}\right), R_2 = k_2 \rho_2 \varphi_2 T_s^{-0.5} \exp\left(-\frac{E_2}{RT_s}\right),$$

$$R_3 = k_3 \rho \varphi_3 s_\sigma c_1 \exp\left(-\frac{E_3}{RT_s}\right), R_5 = k_5 M_2 \left(\frac{c_1 M}{M_1}\right)^{0.25} \frac{c_2 M}{M_2} T^{-2.25} \exp\left(-\frac{E_5}{RT}\right).$$

The system of equations (1)–(7) must be solved taking into account the initial and boundary conditions [11-14]:

$$x_1 = -x_{1e}: v_1 = V_e, v_2 = 0, \frac{\partial v_3}{\partial x_1} = 0, T = T_e, c_\alpha = c_{\alpha e}, -\frac{c}{3k}\frac{\partial U_R}{\partial x_1} + c U_R / 2 = 0; \tag{8}$$

$$x_1 = x_{1e}: \frac{\partial v_1}{\partial x_1} = 0, \frac{\partial v_2}{\partial x_1} = 0, \frac{\partial v_3}{\partial x_1} = 0, \frac{\partial c_\alpha}{\partial x_1} = 0, \frac{\partial T}{\partial x_1} = 0, \frac{c}{3k}\frac{\partial U_R}{\partial x_1} + \frac{c}{2} U_R = 0; \tag{9}$$

$$x_2 = x_{20}: \frac{\partial v_1}{\partial x_2} = 0, \frac{\partial v_2}{\partial x_2} = 0, \frac{\partial v_3}{\partial x_2} = 0, \frac{\partial c_\alpha}{\partial x_2} = 0, \frac{\partial T}{\partial x_2} = 0, -\frac{c}{3k}\frac{\partial U_R}{\partial x_2} + \frac{c}{2} U_R = 0; \tag{10}$$

$$x_2 = x_{2e}: \frac{\partial v_1}{\partial x_2} = 0, \frac{\partial v_2}{\partial x_2} = 0, \frac{\partial v_3}{\partial x_2} = 0, \frac{\partial c_\alpha}{\partial x_2} = 0, \frac{\partial T}{\partial x_2} = 0, \frac{c}{3k}\frac{\partial U_R}{\partial x_2} + \frac{c}{2} U_R = 0. \tag{11}$$

$$x_3 = 0: v_1 = 0, v_2 = 0, \frac{\partial c_\alpha}{\partial x_3} = 0, -\frac{c}{3k}\frac{\partial U_R}{\partial x_3} + \frac{c}{2} U_R = 0, \tag{12}$$
$$v_3 = v_{30}, T = T_g, |x_1| \le \Delta, |x_2| \le \Delta$$
$$v_3 = 0, \ T = T_e, |x_1| > \Delta, |x_2| > \Delta;$$

$$x_3 = x_{3e}: \frac{\partial v_1}{\partial x_3} = 0, \frac{\partial v_2}{\partial x_3} = 0, \frac{\partial v_3}{\partial x_3} = 0, \frac{\partial c_\alpha}{\partial x_3} = 0, \frac{\partial T}{\partial x_3} = 0, \frac{c}{3k}\frac{\partial U_R}{\partial x_3} + \frac{c}{2} U_R = 0. \tag{13}$$

where $\frac{d}{dt}$ is the symbol of the total (substantial) derivative; α_V is the coefficient of phase exchange; ρ - density of gas – dispersed phase, t is time; v_i - the velocity components; T, T_S, - temperatures of gas and solid phases, U_R - density of radiation energy, k - coefficient of radiation attenuation, P - pressure; c_p – constant pressure specific heat of the gas phase, c_{pi}, ρ_i, φ_i – specific heat, density and volume of fraction of condensed phase (1 – dry organic substance, 2 – moisture, 3 – condensed pyrolysis products, 4 – mineral part of forest fuel), R_i – the mass rates of chemical reactions, q_i – thermal effects of chemical reactions; k_g , k_S - radiation absorption coefficients for gas and condensed phases; T_g –temperature of ignition, T_e - the ambient temperature; c_α - mass concentrations of α - component of gas - dispersed medium, index α=1,2,...,5, where 1 corresponds to the density of oxygen, 2 - to carbon monoxide CO, 3 - to carbon dioxide and inert components of air, 4 - to particles of black, 5 - to particles of smoke; R – universal gas constant; M_α , M_C, and M molecular mass of α - components of the gas phase, carbon and air mixture; g is the gravity acceleration; c_d is an empirical coefficient of the resistance of the vegetation, s is the specific surface of the forest fuel in the given forest stratum, $_g$ – mass fraction of gas combustible products of pyrolysis, α_4 and α_5 – empirical constants. To define source terms which characterize inflow (outflow of mass) in a volume unit of the gas-dispersed phase were used the following formulae for the rate of formulation of the gas-dispersed mixture $\dot{m}$, outflow of oxygen R_{51} , changing carbon monoxide R_{52} , generation of black R_{54} and smoke particles R_{55} . Coefficients of multiphase (gas and solid phase) heat and mass exchange are defined

$$\alpha_V = \alpha S - \gamma C_P \dot{m}, S = 4\varphi_S / d_S .$$

Here α = $Nu\lambda/d_S$ – coefficient of heat exchange for sample of forest combustible material (for example needle), Nu – Nusselt number for cylinder, φ_s- volume fraction of condensed phase, λ – coefficient of heat conductivity for pine needle; γ – parameter, which characterize relation between molecular masses of ambient and inflow gases.

It is supposed that the optical properties of a medium are independent of radiation wavelength (the assumption that the medium is “grey”), and the so-called diffusion approximation for radiation flux density were used for a mathematical description of radiation transport during forest fires. To close the system (1)–(7), the components of the tensor of turbulent stresses, and the turbulent heat and mass fluxes are determined using the local-equilibrium model of turbulence (Grishin, [11]). The system of equations (1)–(7) contains terms associated with turbulent diffusion, thermal conduction, and convection, and needs to be closed. The components of the tensor of turbulent stresses $\rho\overline{v_i' v_j'}$, as well as the turbulent fluxes of heat and mass $\overline{\rho v_j' c_p T'}$, $\overline{\rho v_j' c_\alpha'}$ are written in terms of the gradients of the average flow properties using the formulas

$$-\rho\overline{v_i' v_j'} = \mu_t \left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right) - \frac{2}{3} K \delta_{ij},$$

$$-\rho\overline{v_j c_p T'} = \lambda_t \frac{\partial T}{\partial x_j}, \ -\rho\overline{v_j c_\alpha'} = \rho D_t \frac{\partial c_\alpha}{\partial x_j},$$

$$\lambda_t = \mu_t c_p / \Pr_t, \rho D_t = \mu_t / Sc_t, \mu_t = c_\mu \rho K^2 / \varepsilon,$$

where μ_t, λ_t, D_t are the coefficients of turbulent viscosity, thermal conductivity, and diffusion, respectively; Pr_t, Sc_t are the turbulent Prandtl and Schmidt numbers, which were assumed to be equal to 1. In dimensional form, the coefficient of dynamic turbulent viscosity is determined using local equilibrium model of turbulence [9]. It should be noted that this system of equations describes processes of transfer within the entire region of the forest massif, which includes the space between the underlying surface and the base of the forest canopy, the forest canopy and the space above it, while the appropriate components of the data base are used to calculate the specific properties of the various forest strata and the near-ground layer of atmosphere. This approach substantially simplifies the technology of solving problems of predicting the state of the medium in the fire zone numerically. The thermodynamic, thermophysical and structural characteristics correspond to the forest fuels in the canopy of a different (for example pine forest [11], [13]) type of forest. The conditions of symmetry are used because of the patterns of flow and distributions of all scalar functions are symmetrical relatively to the axes Ox_2.

III. Method of Calculation and Results

The boundary-value problem (1)–(7) we solve numerically using the method of splitting according to physical processes [13]. In the first stage, the hydrodynamic pattern of flow and distribution of scalar functions was calculated. The system of ordinary differential equations of chemical kinetics obtained as a result of splitting [13] was then integrated. A discrete analog was obtained by means of the control volume method using the SIMPLE like algorithm [13], [18]. The accuracy of the program was checked by the method of inserted analytical solutions. The time step was selected automatically. Fields of temperature, velocity, component mass fractions, and volume fractions of phases were obtained numerically. Within the framework of the problem assignments mentioned above the process of crown forest fire initiation was studied at the stages of inert warming up, drying, pyrolysis and ignition of products of pyrolysis (Fig.2). At the moment of ignition the gas combustible products of pyrolysis burns away, and the concentration of oxygen is rapidly reduced. The temperatures of both phases reach a maximum value at the point of ignition. The ignition processes are of a gas - phase nature, i.e. initially heating of solid and gaseous phases occurs, moisture is evaporated. Then decomposition process into condensed and volatile pyrolysis products starts, the latter being ignited in the forest canopy. Note also that the transfer of energy from the fire source takes place due to radiation; the value of radiation heat flux density is small compared to that of the convective heat flux. As a result of heating of forest fuel elements, moisture evaporates, and pyrolysis occurs accompanied by the release of gaseous products, which then ignite. The effect of the wind on the zone of forest fire initiation is shown in Figures 3-5 present the space distribution of field of temperature for gas phase for different instants of time when a wind velocity V_e= 7 m/s.

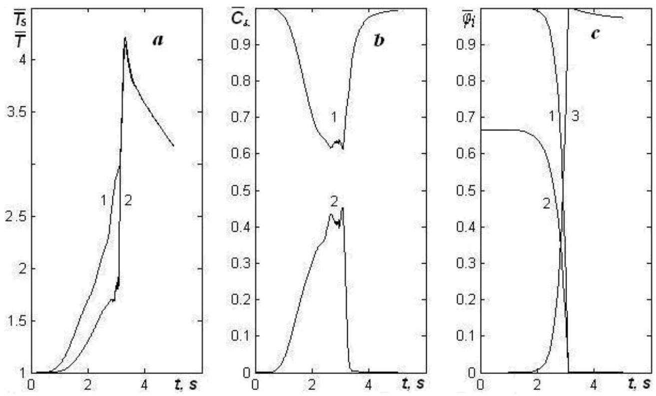

Fig. 2. Relationships of dimensionless temperatures, concentrations and volume fractions in the lower boundary of the forest canopy:
$a)(1-\overline{T}=T/T_e, 2-\overline{T}_s=T_s/T_e, T_e=300K)$, $b)(1-\overline{C}_1=C_1/C_{1e}, 2-\overline{C}_2=C_2/C_{1e}$, $C_{1e}=0.23)$, $c)(1-\overline{\varphi}_1=\varphi_1/\varphi_{1e}, 2-\overline{\varphi}_2=\varphi_2/\varphi_{2e}, 2-\overline{\varphi}_3=\varphi_3/\varphi_{3e})$.

We can note that the isosurfaces are deformed by the action of wind. The isosurfaces of the temperature of gas phase 1, 2, 3 and 4 correspond to the temperatures $\overline{T}$ = 1.2., 2, 3. and 4. In the vicinity of the source of heat and mass release, heated air masses and products of pyrolysis and combustion float up. The wind field in the forest canopy interacts with the gas-jet obstacle that forms from the surface forest fire source and from the ignited forest canopy base. Recirculating flow forms beyond the zone of heat and mass release, while on the windward side the movement of the air flowing past the ignition region accelerates. Under the influence of the wind the tilt angle of the flame is increased. As a result, this part of the forest canopy, which is shifted in the direction of the wind from the center of the surface forest fire source, is subjected to a more intensive warming up. The isosurfaces of the gas phase are deformed in the direction of the wind. Figures 3 and 4 present the distribution of the velocity and isosurfaces of the temperature at the different instants of time when a wind velocity V_e= 7 m/s.

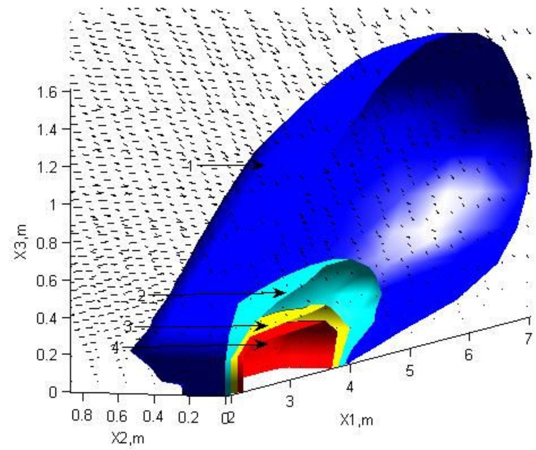

Fig. 3. The vectorial field of velocity and temperature at *t=3.3 s.*

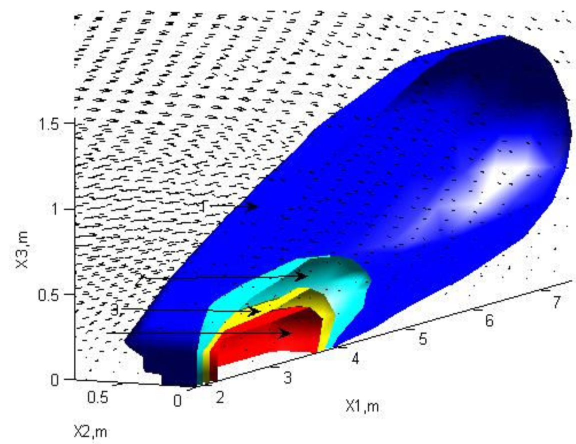

Fig. 4. The vectorial field of velocity and temperature at *t=3.8 s.*

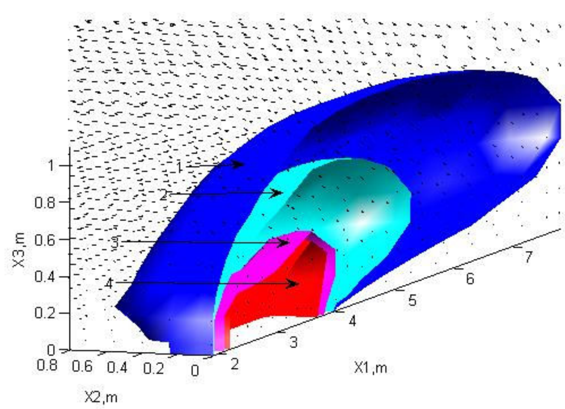

Fig. 5. The vectorial field of velocity and temperature at *t=4.8 s.*

IV. Conclusion

Mathematical model and the result of the calculation give an opportunity to evaluate critical condition of the forest fire initiation and spread which allows applying the given model for preventing fires. The model overestimates the conditions of the crown forest fires initiations. From an analysis of calculations and experimental data it was found that for the cases in question the minimum total incendiary heat pulse is 2600 kJ/m^2 (Grishin [11]).· Calculations demonstrated that the value of the radiant heat flux for both problems is considerably less than the convective one, therefore radiation has a weak effect on local and integral characteristics of the problem discoursed above. The results obtained agree with the laws of physics and experimental data [11, 13, 15].

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