Comparison of Full versus Fractional Factorial Experimental Design for the prediction of Cutting Forces in Turning of a Titanium alloy: A case study

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Abstract— The present paper concerns with the analysis and the optimization of the main cutting force (Fc) during turning of Ti-6Al-4V ELI titanium alloy under dry cutting condition by applying either full or fractional experimental design. The main cutting variables (spindle speed, feed rate and depth of cut) were treated as inputs in whilst the main cutting force (Fc) was considered as the machinability output (quality (target). Therefore, a three parameter design was selected with each parameter having three levels. For the full factorial design, the complete combination array was selected consisted of 27 experiments. For the fractional factorial design only nine (9) experiments according to the L9 orthogonal array proposed by Taguchi's DOE were used. The results obtained by both methodologies were further analyzed by applying ANOM and ANOVA techniques and compared in order to examine the suitability of the proposed experimental designs for machinability studies.

Keywords— Full / fractional factorial design, comparison study, titanium alloy, turning, cutting force.

I. Introduction

Design of experiments (DOE) methodology provides four different approaches, for experimental data analysis namely the "best guess", the "one factor at a time", the "full factorial" and the "fractional factorial". In general, experiments are designed by adopting one of the available orthogonal arrays (OAs) from which experimental runs will be determined aiming at collecting the necessary results [1, 2].

The present paper is focused on a comparison study between full and fractional factorial design of experiments (DOE) method when they applied on the outputs of a material removal process i.e., turning of a titanium alloy. Note that numerous authors have published studies aimed at evaluating the effects of the cutting parameter variations on the resulted cutting forces. In planning the experimentation, some authors have used full factorial designs; others used fractional ones [3-5].

Full factorial DOE method is selected many times of the experimenters versus the fractional factorial design and vice versa [6-20]. At this point, a crucial question arises. Which one is better or appropriate in the case of predicting cutting forces during turning of difficult-to-cut materials like titanium alloys?

Usually, the DOE method can be divided in full factorial and fractional factorial design [3, 4]. Full factorial design means that after parameter design (selection of tested parameters and their levels) all combination of the parameter levels should be tested in order to analyze the results. In the other hand, using fractional factorial design, only the statistically important experiments should be used in order to analyze the results. Robust design utilizes Taguchi orthogonal arrays in order to perform fractional factorial design of experiments [21]. Taguchi method is especially suitable for industrial use, but can also be used for scientific research. Note that the basic elements of Taguchi's "quality philosophy" as well as a recent bibliography on Taguchi's approach to DoE may be found in Taguchi et al.'s Quality Engineering Handbook [22].

In recent decades, considerable improvements have been achieved in turning, enhancing machining of difficult-to-cut materials and resulting in improved machinability (better surface finish and smaller cutting forces). The forces acting on the tool are an important aspect of machining. Knowledge of the cutting forces is needed for estimation of power requirements and for the design of machine tool elements, tool-holders and fixtures, adequately rigid and free from vibration. Cutting force calculation and modeling are two of the major aspects of metal cutting theory. The large number of interrelated parameters that influence the cutting forces makes the development of a proper model a very difficult task [19].

The prediction of the main cutting force developed during longitudinal turning of Ti-6Al-4V ELI titanium alloy by applying the above mentioned DOE methods (full vs fractional) are presented here. The experimental data were extracted from a previous study concerning the machinability of the same material by applying a feed forward back propagation (FFBP) artificial neural network (ANN) [23]. All

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turning experiments followed the kinematics of longitudinal turning and a 3D cutting force system was considered according to standard theory of oblique cutting; see Fig. 1 [24].

It was revealed that fractional DOE is quite sufficient in analyzing cutting forces. Titanium was selected as a typical difficult-to-cut advanced material; therefore the results of the study can be generalized to other alloys with better machinability characteristics.

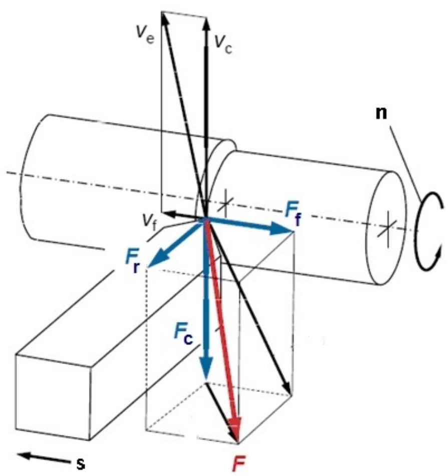

Fig. 1. Kinematics of longitudinal turning and cutting forces system.

II. PARAMETER DESIGN

The comparative study is performed in a three level three parameter design (3^3). Table I present the levels and the parameters of the 3^3 design.

Table I. Parameter design.

Parameters		Levels		
		1	**2**	**3**
Speed	**n (rpm)**	420	600	850
Feed	**s (mm/rev)**	0.1	0.18	0.33
Depth of Cut	**a (mm)**	0.5	1	1.5

Table II presents all combinations of the parameter design 3^3; thus twenty-seven (27) experiments.

Table III presents the statistically important combinations of the parameter design 3^3; nine (9) experiments. This array was taken by Taguchi and called L9(3^4) orthogonal array [21]. Orthogonality means that each per of columns have all level combinations equal times each one.

Table II. Full factorial experimental array.

	n (rpm)	**s (mm/rev)**	**a (mm)**	**Fc (N)**
1	420	0.1	0.5	140
2	420	0.1	1	258
3	420	0.1	1.5	370
4	420	0.18	0.5	236
5	420	0.18	1	410
6	420	0.18	1.5	570
7	420	0.33	0.5	284
8	420	0.33	1	564
9	420	0.33	1.5	840
10	600	0.1	0.5	120
11	600	0.1	1	226
12	600	0.1	1.5	318
13	600	0.18	0.5	182
14	600	0.18	1	350
15	600	0.18	1.5	502
16	600	0.33	0.5	270
17	600	0.33	1	538
18	600	0.33	1.5	760
19	850	0.1	0.5	132
20	850	0.1	1	240
21	850	0.1	1.5	330
22	850	0.18	0.5	200
23	850	0.18	1	352
24	850	0.18	1.5	500
25	850	0.33	0.5	288
26	850	0.33	1	562
27	850	0.33	1.5	800
Average (μ)				**383**

Table III. Fractional factorial experimental array L9.

	n (rpm)	**s (mm/rev)**	**a (mm)**	**Empty**	**Fc (N)**
1	420	0.1	0.5	1	140
2	420	0.18	1	2	410
3	420	0.33	1.5	3	840
4	600	0.1	1	3	226
5	600	0.18	1.5	1	502
6	600	0.33	0.5	2	270
7	850	0.1	1.5	2	330
8	850	0.18	0.5	3	200
9	850	0.33	1	1	562
Average (m)					**386.7**

III. ANALYSES OF MEANS ANOM

Analysis of means (ANOM analysis) is the procedure of estimating the means of each parameter level [19, 21]. The calculated 'mean values' are tabulated in Tables IV and V for full and fractional design, correspondingly.

Table IV. Mean values - Full factorial design

Mean parameter value	Level 1	Level 2	Level 3
m_{ni}	408.0	362.9	378.2
m_{si}	237.1	366.9	545.1
m_{ai}	205.8	388.9	554.4

Table V. Mean values - Fractional factorial design

Mean parameter value	Level 1	Level 2	Level 3
m_{ni}	463.3	332.7	364.0
m_{si}	232.0	370.7	557.3
m_{ai}	203.3	318.0	557.3

In a similar manner, the plots of mean values are presented in Figs 2 and 3 for full and fractional approaches, correspondingly.

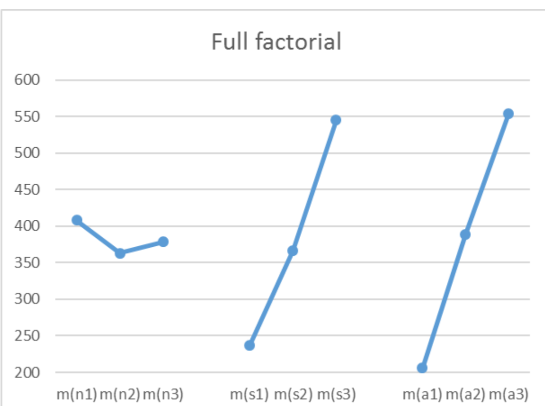

Fig. 2. Plot of means – Full factorial design.

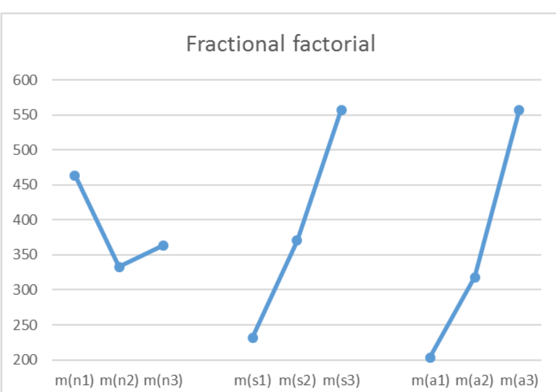

Fig.3. Plot of means – Fractional factorial design.

IV. PREDICTION OF OPTIMUM CUTTING FORCE USING FRACTIONAL DESIGN

Based on the ANOM analysis plot of means were obtained; see Fig. 3. The optimum level of a parameter is the level that results in the minimum force (Fc). Using fractional factorial approach, the best parameter values to minimize the cutting force are: speed (600rpm; Level 2), feed (0.1 mm/rev; Level 1), and depth of Cut (0.5mm Level 1). This combination is not appeared in L9 orthogonal array (Table III) However, it is included in the full factorial approach (Table II) and it is the best combination for all the twenty-seven (27) experiments (Fc=120N; minimum for the whole range of experiments).

V. ANALYSES OF VARIANCES (ANOVA)

Analysis of variances (ANOVA) is an additive data decomposition statistical method using sum of squares which indicates the variance of each parameter onto the experimental area [2, 21]. The symbols used read as follows:

- DoF: Degree of freedom
- SoS: Sum of squares
- MS: mean square
- F: F ratio used for only quantitative understanding (in general if F is smaller than 1, means that the factor or parameter is not important)
- %: Shows the impact of its parameter on total error
- Error: due to parameter levels

Total error: $$\sum_{1}^{n}(n_i-\mu)^2 \quad (1)$$

Table VI. ANOVA - Full factorial design

	DoF	SoS	MS	F	%
n	2	9,471	4,735	0.1	0.9%
s	2	430,408	215,20 4	5.3	40.4 %
a	2	547,520	273,76 0	6.7	51.4 %
Error	6	987,399			
Total Error	26	1,065,53 1	40,982		

Table VII. ANOVA - Fractional factorial design

	DoF	SoS	MS	F	%
n	2	27,923	13,961	0.3	6.9%
s	2	159,915	79,957	1.6	39.8%
a	2	202,360	101,18 0	2.0	50.3%
*e	2	11,891	5,945	0.1	3.0%
Error	8	402,088	50,261		
*Total Error	2	11,891	5,945		

*error due to empty column.

VI. CONCLUSIONS

From the results presented above it is concluded that the use of fractional factorial design for analyzing cutting force in turning of titanium alloys leads to quite accurate results.

ANOM analysis of the two methods indicated the same trends of parameters levels; compare Figs 2 and 3.

Prediction of the best combination parameter values using the fractional factorial design is confirmed (the combination speed: 600rpm; feed: 0.1mm/rev and depth of Cut: 0.5mm results in the minimum cutting force, Fc=120N).

ANOVA analysis gave for both approaches the same results.

- Depth of cut is the most important parameter given an impact about 50%
- Feed is the second most important factor given an impact about 40%
- Speed is not an important parameter for cutting forces inside the experimental region.

As future perspectives it could be mentioned the investigation of the parameter interactions and the identification of criteria for selecting between additive, regression or artificial neural network models.

Note that more materials will be tested in order to investigate the impact of hardness of the test material on the results.

The same approach can be implemented for analyzing other performance indicators such as surface roughness and/or tool wear measures.

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