

Numerical study of MHD flow and heat transfer through porous medium between two parallel plates with Hall and ion slip effects

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Abstract— This paper studies the effects of Hall and ion slip on two dimensional incompressible flow and heat transfer of an electrically conducting viscous fluid in a porous medium between two parallel plates, generated due to periodic suction and injection at the plates. The flow field, temperature and pressure are assumed to be periodic functions in $e^{i\omega t}$ and the plates are kept at different but constant temperatures. A numerical solution for the governing nonlinear ordinary differential equations is obtained using quasilinearization method. The graphs for velocity, temperature distribution and skin friction are presented for different values of the fluid and geometric parameters.

Keywords— *Unsteady; Porous medium; Temperature distribution; Skin friction; Hall effect; ion slip effect*

I. INTRODUCTION

The flow and temperature distribution through porous channels is of great importance in technological and biological flows, for example motion of water waves over a shallow beach, the mechanics of cochlea in the human ear, transpiration cooling, filtration, food preservation, petroleum industry, cosmetic industry, polymer technology, the flow of blood in the arteries and artificial dialysis, magnetohydrodynamic (MHD) generators, etc.,. The problem of two dimensional incompressible laminar flow between parallel plates with either suction or injection was considered by many researchers. Berman [1] investigated the flow of two dimensional steady incompressible laminar viscous fluid through a porous channel where both channel plates have equal permeability and the flow at the center line of the channel attains maximum. Later Yuan [2] extended the problem for different values of suction and injection Reynolds numbers. Terril and Shresta [3] considered the incompressible viscous fluid through porous walls when suction normal velocities are distinct. Gersten and Gross [4] have examined a three dimensional incompressible laminar viscous fluid flow and temperature distribution with transverse sinusoidal suction velocity. Alpher [5] examined an incompressible laminar flow and convection heat transfer between parallel plates through a

transverse magnetic field. Cox [6] discussed a two dimensional incompressible viscous fluid between two parallel porous walls with symmetric and asymmetric suction. Singh and Sharma [7] solved numerically the steady incompressible laminar flow of a non-Newtonian fluid and temperature distribution in a porous annulus. Ramanamurthy et al. [8] studied the two dimensional incompressible viscous fluid between porous parallel plates due to periodic suction and injection at the plates, the problem is analyzed up to second order. Vidhya and Kesavan [9] considered an incompressible viscous fluid flow and temperature distribution in a porous medium between two vertical parallel plates and the problem is analyzed analytically. Attia and Kotb[10] considered incompressible steady and unsteady MHD fluid flow and temperature distribution between porous parallel plates with suction or injection. Attia [11] considered an unsteady transient MHD flow and heat transfer between parallel plates with Hall and ion slip effects and obtained the analytical and numerical solutions for velocity and temperature distribution respectively. Bhat and Mittal [12] analyzed the effects of an electrically conducting convective heat transfer in a uniform wall with Hall and ion slip. Jha and Apere [13] examined the effects of Hall and ion slip on unsteady MHD Couette flow of rotating fluid and obtained an analytical solution by Laplace transform. Ram [14] studied the effects of Hall and ion slip on steady incompressible electrically conducting rotating fluid with temperature distribution and obtained an analytical solution. Ghara et. al [15] investigated an unsteady electrically conducting Couette flow between porous parallel plates with Hall and ion slip effects and the solution of flow field equations obtained by Laplace transform technique.

In this paper we consider the unsteady two dimensional flow and temperature distribution of an electrically conducting viscous fluid between two parallel horizontal plates with Hall and ion slip currents. The solution is obtained numerically using quasilinearization technique and the effects of suction Reynolds number, suction and injection ratio, Hartman number, inverse Darcy parameter, frequency parameter, Hall and ion slip parameters on velocity,

temperature distribution and skin friction are studied and shown graphically.

Nomenclature

h	Distance between parallel plates
V_2	Suction velocity
V_1	Injection velocity
D^{-1}	Inverse Darcy parameter, $\frac{h^2}{k_1}$
a	Suction and injection ratio, $1 - \frac{V_1}{V_2}$
p	Fluid pressure
\vec{Q}	Velocity vector
c	Specific heat at constant temperature
E	Eckert number, $\frac{\mu V_2}{\rho h c (\theta_2 - \theta_1)}$
k	Thermal conductivity
k_1	Permeability parameter
t	Time
u	Velocity component in x- direction
v	Velocity component in y-direction
U_0	Entrance velocity
Pr	Prandtl number, $\frac{\mu c}{k}$
R	Suction Reynolds number, $\frac{\rho V_2 h}{\mu}$
J	Current density
\vec{B}	Total magnetic field.
b	Induced magnetic field
B_0	Magnetic flux density
\vec{E}	Electric field
Ha	Hartmann number, $B_0 h \sqrt{\frac{\sigma}{\mu}}$

Greek Letters

ξ	Dimensionless y coordinate, $\frac{y}{h}$
τ	Force stress tensor
γ	Dimensionless axial variable, $\left(\frac{U_0}{aV_2} - \frac{x}{h} \right)$
ρ	Fluid density
μ	Fluid viscosity
ω	Frequency
σ	Electric conductivity
α	Non dimensional frequency parameter, ωt
θ	Temperature
θ_1	Temperature at the lower plate
θ_2	Temperature at the upper plate
θ^*	Dimensionless temperature, $\frac{\theta - \theta_1}{\theta_2 - \theta_1}$
β_i	Ion slip parameter
β_e	Hall parameter
α_e	Hall and ion-slip parameter, $1 + \beta_i \beta_e$

II. FORMULATION OF THE PROBLEM

The unsteady laminar incompressible electrically conducting viscous fluid is assumed to be flowing between two parallel porous plates located at the $y=0$ and $y=h$ as shown in the fig.1. The two plates are kept at two different constant temperatures θ_1 for the lower plate and θ_2 for the upper plate. Let there be a periodic suction at the upper plate and periodic injection at the lower plate with velocities $\text{Re}al(V_2 e^{i\omega t})$ and $\text{Re}al(V_1 e^{i\omega t})$ respectively.

The governing equations of an incompressible electrically conducting viscous fluid flow and energy in a porous medium are

$$\nabla \cdot \vec{Q} = 0 \quad (1)$$

$$\rho \left[\frac{\partial \vec{Q}}{\partial t} + (\vec{Q} \cdot \nabla) \vec{Q} \right] = -\nabla p - \mu \nabla \times \nabla \times \vec{Q}$$

$$-\frac{\mu}{k_1} \bar{Q} + J \times \bar{B} \quad (2)$$

$$\rho c \left[\frac{\partial \theta}{\partial t} + (\bar{Q} \cdot \nabla) \theta \right] = \mu [(\nabla \bar{Q}) : (\nabla \bar{Q})^T + (\nabla \bar{Q}) : (\nabla \bar{Q})] +$$

$$k \nabla^2 \theta + \frac{\mu}{k_1} \bar{Q}^2 + \frac{1}{\sigma} J^2 \quad (3)$$

where \bar{Q} is the velocity vector, ρ is the density, p is the fluid pressure, k is the thermal conductivity, μ is the fluid viscosity, c is the specific at constant temperature, θ is the temperature, k_1 is the permeability parameter, J is current density, \bar{B} is total magnetic field and σ is the electric conductivity.

Since the flow is along x - direction, the flow variables are assumed to be independent of the coordinate z . We choose the velocity vector as $\bar{Q} = u(x,y,t) \hat{i} + v(x,y,t) \hat{j}$ and temperature function as $\theta(x,y,t)$ and neglecting the displacement current, the Maxwell equations and the generalized Ohm's law are

$$\nabla \cdot \bar{B} = 0, \nabla \times \bar{B} = \mu_m \bar{J}, \nabla \times \bar{E} = \frac{\partial \bar{B}}{\partial t},$$

$$\bar{J} = \sigma (\bar{E} + \bar{Q} \times \bar{B}) + \frac{\beta_e}{B_0} (\bar{J} \times \bar{B}) - \frac{\beta_e \beta_i}{B_0^2} (\bar{J} \times \bar{B}) \times \bar{B} \quad (4)$$

Where μ_m is the magnetic permeability, \bar{E} is the electric field, and $\bar{B} = B_0 + b$, b is induced magnetic field, B_0 is magnetic flux density, β_e is the Hall parameter, β_i is the ion slip parameter.

Under the assumptions that the induced magnetic field is negligible compared to the applied magnetic field so that magnetic Reynolds number is small, the electric field is zero and magnetic permeability is constant throughout the flow field.

We take the velocity components in x and y directions as (1965)

$$u = \left(\frac{U_0}{a} - \frac{V_2 x}{h} \right) f^1(\xi) e^{i\omega t}, v = V_2 f(\xi) e^{i\omega t},$$

$$p = p(x, \xi) e^{i\omega t} \text{ and}$$

$$\theta = \theta_1 + \frac{\mu V_2}{\rho h c} \left[g_1(\xi) + \left(\frac{U_0}{a V_2} - \frac{x}{h} \right)^2 g_2(\xi) \right] e^{i\omega t} \quad (5)$$

The boundary conditions on the velocity and temperature distribution are

$$u(x, \xi, t) = 0, v(x, \xi, t) = V_1, \theta(x, \xi, t) = \theta_1 \text{ at } \xi = 0$$

$$u(x, \xi, t) = 0, v(x, \xi, t) = V_2, \theta(x, \xi, t) = \theta_2 \text{ at } \xi = 1 \quad (6)$$

Where $\xi = \frac{y}{h}$.

From equations (2) & (3), we get the following non dimensional equations

$$\text{Re} [f f^{111} - f^1 f^{11}] \cos \alpha - f^{1V} + D^{-1} f^{11} + \frac{Ha^2 \alpha_e}{\alpha_e^2 + \beta_e^2} f^{11} = 0 \quad (7)$$

$$g_1^{11} + \text{Re Pr} (4 f^{12} - f g_1^1 + D^{-1} f^2 + \frac{Ha^2}{\alpha_e^2 + \beta_e^2} f^2) \cos \alpha = 0 \quad (8)$$

$$g_2^{11} + \text{Re Pr} (f^{112} + 2 f^1 g_2 - f g_2^1 + D^{-1} f^2 + \frac{Ha^2}{\alpha_e^2 + \beta_e^2} f^2) \cos \alpha = 0 \quad (9)$$

Where prime denotes differentiation with respect to ξ ,

$\text{Re} = \frac{\rho h V_2}{\mu}$ is Reynolds number, $\text{Pr} = \frac{\mu c}{k}$ is the Prandtl

number, $D^{-1} = \frac{h^2}{k_1}$ is inverse Darcy's parameter,

$\alpha_e = 1 + \beta_e \beta_i$ is Hall and ion slip parameter and

$Ha = B_0 h \sqrt{\frac{\sigma}{\mu}}$ is Hartmann number.

The dimensionless form of temperature from equation (5) can be written as

$$\theta^* = \frac{\theta - \theta_1}{\theta_2 - \theta_1} = E (g_1 + \gamma^2 g_2)$$

Where E is the Eckert number, $E = \frac{\mu V_2}{\rho h c (\theta_2 - \theta_1)}$

The boundary conditions from eq. (9) in terms of f, g_1 and g_2 are

$$f(0) = 1 - a, f(1) = 0, f^1(0) = 0, f^1(1) = 0,$$

$$g_1(0) = 0, g_1(1) = w, g_2(0) = 0, g_2(1) = 0 \quad (10)$$

III. SOLUTION OF THE PROBLEM

The following system of first order linear differential equations are obtained from the non linear equations (7), (8), and (8) by the substitution

$$(f, f^1, f^{11}, f^{111}, f^{1v}, g_1, g_1^1, g_2, g_2^1) =$$

$$(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8)$$

$$\frac{dx_1}{d\xi} = x_2, \quad \frac{dx_2}{d\xi} = x_3, \quad \frac{dx_3}{d\xi} = x_4,$$

$$\frac{dx_4}{d\xi} = D^{-1} x_3 + \frac{Ha^2 \alpha_e}{\alpha_e^2 + \beta_e^2} x_3 + \text{Re}(x_1 x_4 - x_2 x_3) \cos \alpha,$$

$$\frac{dx_5}{d\xi} = x_6,$$

$$\frac{dx_6}{d\xi} = -2 x_7 + \text{Re Pr}(-D^{-1} x_1^2 - \frac{Ha^2}{\alpha_e^2 + \beta_e^2} x_1^2 - 4 x_2^2 + x_1 x_6) \cos \alpha, \quad \frac{dx_7}{d\xi} = x_8,$$

$$\frac{dx_8}{d\xi} = \text{Re Pr}(-D^{-1} x_2^2 - x_3^2 - 2 x_2 x_7 + x_1 x_8 - \frac{Ha^2}{\alpha_e^2 + \beta_e^2} x_2^2) \cos \alpha \quad (11)$$

the boundary conditions in terms of

$x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8$ are

$$x_1(0) = 1 - a, \quad x_2(0) = 0, \quad x_5(0) = 0, \quad x_7(0) = 0$$

$$x_1(1) = 1, \quad x_2(1) = 0, \quad x_5(1) = w, \quad x_7(1) = 0 \quad (12)$$

Using quasilinearization technique (1965) the system of equations (11) is solved numerically subject to the boundary conditions (12).

IV. RESULTS AND DISCUSSIONS

To understand the flow characteristics in better way, the numerical results for the axial velocity u , radial velocity v , temperature distribution θ and skin friction coefficient S_f are

calculated correct to six places of decimal for different values of Hartmann number Ha , suction Reynolds number Re , suction and injection ratio a , frequency parameter α , Inverse Darcy parameter D^{-1} and Prandtl number Pr in the domain $[0,1]$.

It is examined that from Fig. 1(a) and 1(b), when the values of Re increases the velocity decreased and the temperature distribution is increased from lower plate to upper plate. It is observed that from Fig.2(a), 2(b) and 8, when the values of 'a' increases the velocity is decreased where as the temperature distribution is increased towards the upper plate and the skin friction at lower plate is increased with respect to Re . It is analyzed that from Fig. 3(a), 3(b) and 9 when the values of α increases the velocity and temperature distribution are decreased from lower plate to upper plate and the skin friction at the lower plate is also decreased with respect to Re . It is noted that from Fig. 4(a), 4(b) and 10 when the values of Ha increases the velocity is increased gradually up to the centre of the plates then decreased symmetrically and where as the temperature distribution is decreased from the lower plate to upper plate and the skin friction at the lower plate is increased with respect Re . It examined that from Fig. 5(a), 5(b) and 12 when the values of β_e increases the velocity is increased where as the temperature is decreased from lower plate to upper plate and the skin friction at the lower plate is also decreased with respect to Re . It examined that from Fig. 6(a), 6(b) and 11 when the values of α_e increases the velocity is increased where as the temperature is decreased towards the upper plate and the skin friction at the lower plate is also decreased with respect to Re . It is noticed that From Fig.7(a), 7(b) and 13 when the values of D^{-1} increases the velocity is decreased where as the temperature distribution is increased from the lower plate to upper plate and the skin friction at the lower plate is increased with respect to Re . It is observed that from Fig.14 when the values of Pr increases the temperature distribution is increased towards the upper plate.

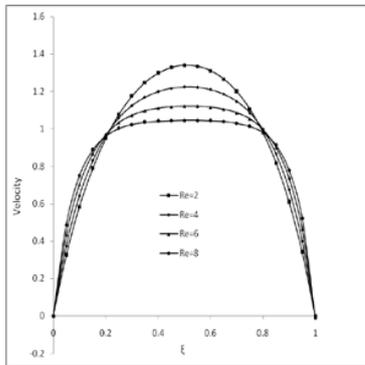


Fig.1(a) Effect of Re on Velocity

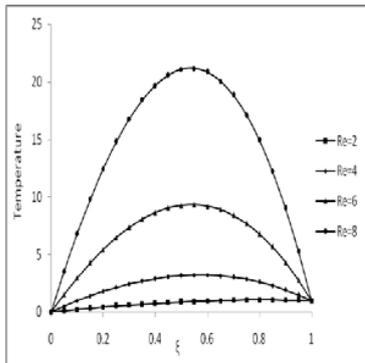


Fig.1(b) Effect of Re on Temperature

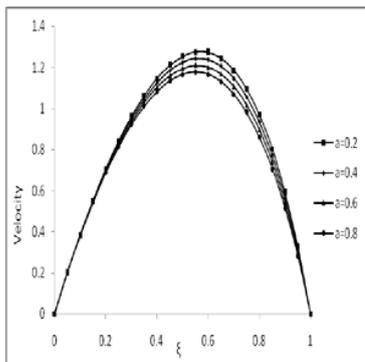


Fig.2(a) Effect of 'a' on Velocity

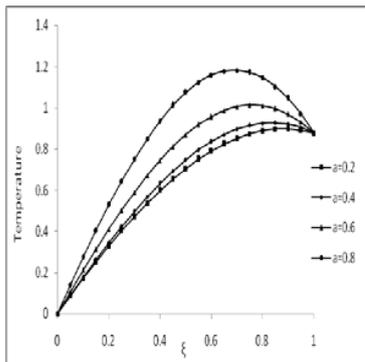


Fig.2(b) Effect of 'a' on Temperature

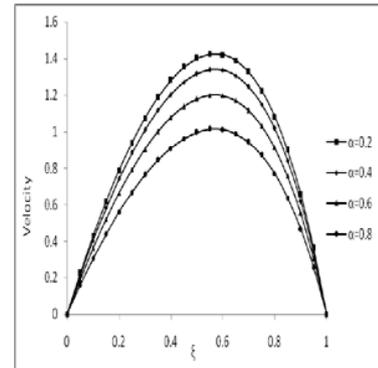


Fig.3(a) Effect of α on Velocity

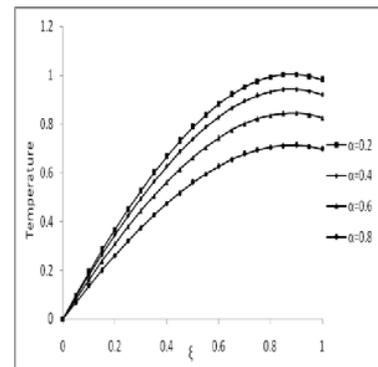


Fig.3(b) Effect of α on Temperature

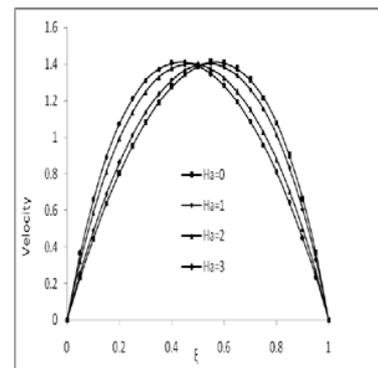


Fig.4(a) Effect of Ha on Velocity

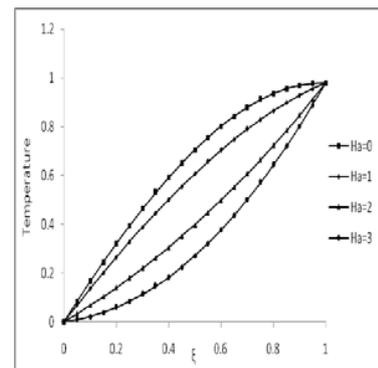


Fig.4(b) Effect of Ha on Temperature

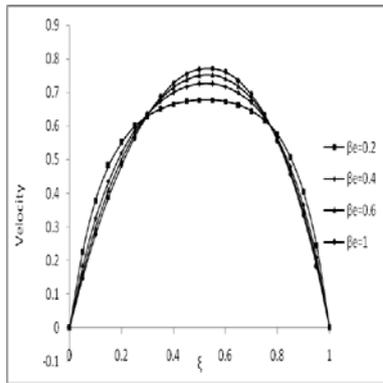


Fig.5(a) Effect of β_e on Velocity

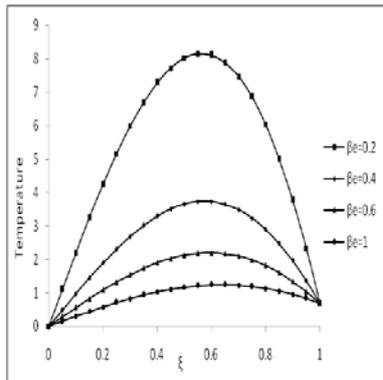


Fig.5(b) Effect of β_e on Temperature

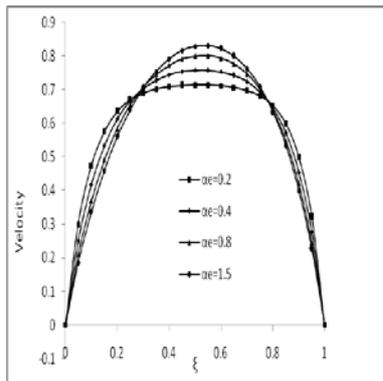


Fig.6(a) Effect of α_e on Velocity

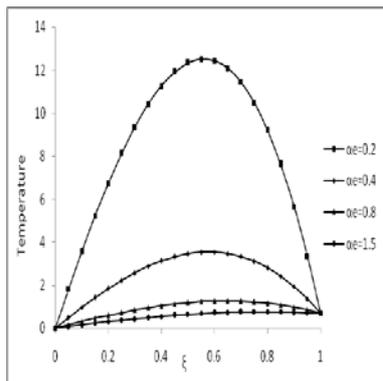


Fig.6(b) Effect of α_e on Temperature

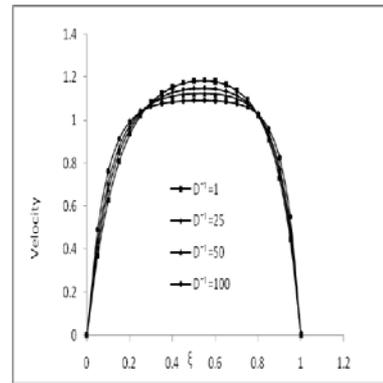


Fig.7(a) Effect of D^{-1} on Velocity

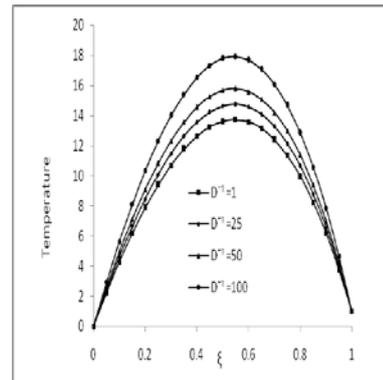


Fig.7(b) Effect of D^{-1} on Temperature

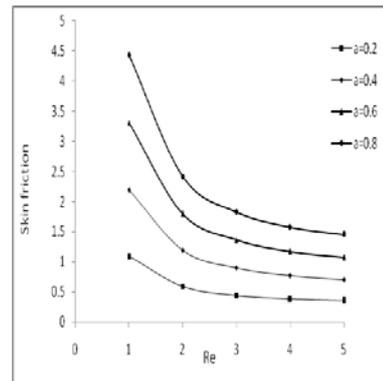


Fig.8 Effect of 'a' on Skin friction

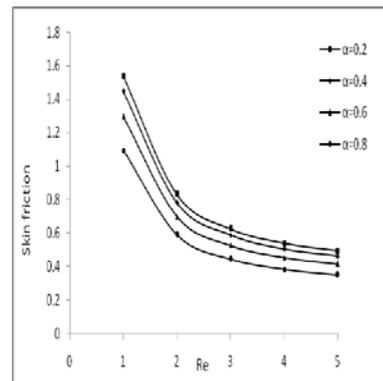


Fig.9 Effect of α on Skin friction

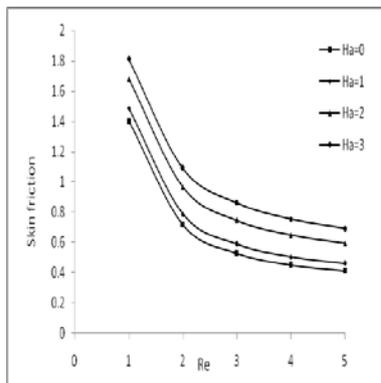


Fig.10 Effect of Ha on Skin friction

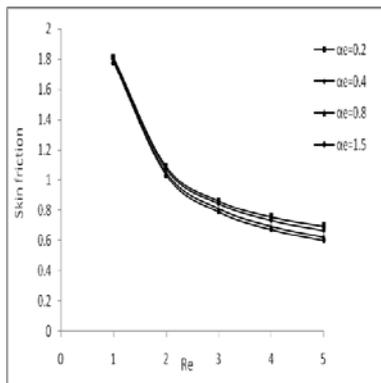


Fig.11 Effect of α_e on Skin friction

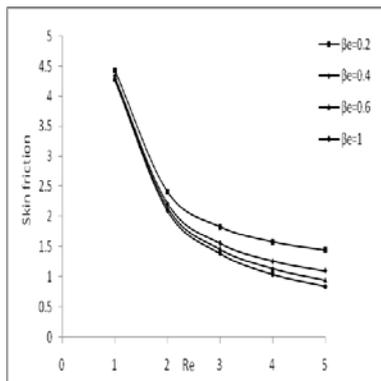


Fig.12 Effect of β_e on Skin friction

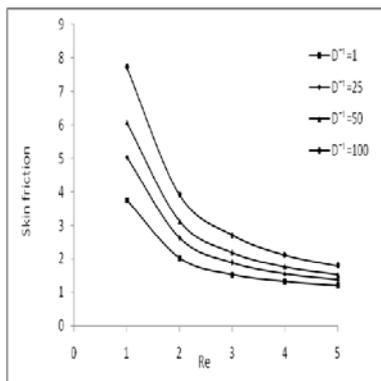


Fig. 13 Effect of D^{-1} on Skin friction

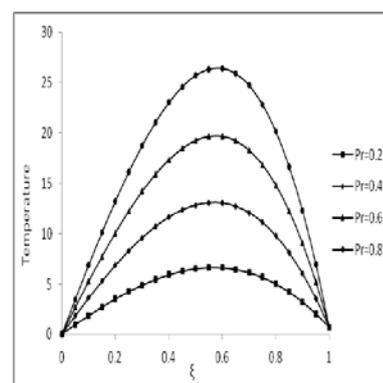


Fig. 14 Effect of Pr on Temperature

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