Robust Autocorrelation Testing in Multiple Linear Regression

Lim Hock Ann, Habshah Midi

Abstract —It is very essential to detect the autocorrelation problem due to its responsibility for ruining the important properties of Ordinary Least Squares (OLS) estimates. The Breusch-Godfrey test is the most commonly used method for autocorrelation detection. However, not many statistics practitioners aware that this test is easily affected by high leverage points. In this paper, we proposed a new robust Breusch-Godfrey test which is resistant to the high leverage points. The results of the study signify that the robustified Breusch-Godfrey test is very powerful in the detection of autocorrelation problem with and without the presence of high leverage points.

Keywords — Autocorrelation, High Leverage Points, Robust Breusch-Godfrey Test.

I. INTRODUCTION

THE Ordinary Least Squares (OLS) method is often used to estimate the parameters of linear regression model because of tradition and ease of computation. The OLS estimates have an optimum properties if all the underlying model assumptions are met. In practice, practitioners do not check the underlying assumptions especially the assumptions of random and uncorrelated errors. When errors are correlated with the previous errors such that $E(u_i, u_j) \neq 0$ for $i \neq j$, we say that errors are autocorrelated.

Autocorrelation violates the important properties of the OLS (see [25]). The OLS estimates become less efficient when the autocorrelation problems exist. Hence, the detection of autocorrelation is very important because the consequences of autocorrelation problem lead to misleading conclusion about the statistical significance of the estimated regression coefficients. (see [8], [18]).

There are quite a number of written articles related to autocorrelation testing procedures (Breusch [2], Durbin and Watson [3], Geary [6], Godfrey [7], Hosking [11] and [12]). Among them, the Breusch-Godfrey (BG) is the most widely used test to detect the presence of autocorrelation. We suspect that this test will be affected by high leverage point since this test is based on the OLS which is known to be easily affected by outliers. High leverage point which is an outlying observation in the X direction have an unduly effect on the OLS estimates (see [4], [5],[9],[15]-[17], [20]-[22]).

In this paper, we propose a robust Breusch-Godfrey test which is not much affected by outliers for the detection of autocorrelation problem in multiple linear regression. The proposed test incorporates the high efficient and high breakdown point MM-estimator (Yohai [24]) in the Breusch-Godfrey procedure. We call this new test as the Modified Breusch-Godfrey test (MBG). The performances of the MBG and BG tests are investigated by using numerical examples and simulation study.

II. THE PROPOSED BREUSCH-GODFREY TEST

Breusch-Godfrey Test was developed by Breusch [2] and Godfrey [7]. There are many practical points of this autocorrelation test.

Firstly, it allows for nonstochastic regressors, such as the lagged values of regressand in the model. Secondly, it allows the lagged values of regressand Y_{t-1}, Y_{t-2}, etc to appear as explanatory variables in the model. The lagged values of the regressand can follow higher-order autoregressive scheme such as AR(1), AR(2), etc. Thirdly, it takes into account correlations among disturbances lagged more than once. This makes Breusch-Godfrey test a powerful test to detect autocorrelation problem in time series data if there is a seasonal autocorrelation in which $E(u_t, u_{t-j})$ is significance for some j other than 1. Fourthly, it allows simple or higher-order moving averages of white noise error terms ε_t such as MA(1), MA(2), etc in the regression model. Lastly, this test is applicable for both time series and cross sectional data. Other existing tests do not have these practical features.

Let consider multiple linear regression with autocorrelated errors

$$Y_t = X_t \underset{\sim}{\beta} + u_t \qquad (1)$$

If the error term u_t follows the pth-order autoregressive, AR(p), then

$$u_t = \rho_1 u_{t-1} + \rho_2 u_{t-2} + ... + \rho_p u_{t-p} + \varepsilon_t \qquad (2)$$

where ε_t is a white noise error term that satisfies all the classical assumptions.

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The null hypothesis H_0 to be tested, is

$$H_0 : \rho_1 = \rho_2 = ... = \rho_p = 0 \qquad (3)$$

that is, there is no serial correlation of any order.

To formulate a modified Breusch-Godfrey test, we first identify the components of the BG test which are affected by high leverage points and then replace these by robust alternative. The BG test requires two times of minimizing sum of squares residuals in the development of the test statistic $(n-p)R^2$. Firstly, it involves the original regression and secondly involves the auxiliary regression. Harter [10] confirmed that squaring of the residual causes the least square becomes extremely vulnerable to the presence of high leverage points. Hence, the BG test statistic is very sensitive and easily affected by high leverage points. To rectify this problem, the robust MM-estimators introduced by Yohai [24] is integrated into the BG test to formulate a new robust BG test. We call this test as Modified Breusch-Godfrey test denoted as MBG.

The proposed MBG test is summarized as follows:

Step 1: Estimate the coefficients of (1) by using MM-estimator and get the residuals, $\hat{u}_t$.

Step 2: Regress $\hat{u}_t$ on the original X_t and its *p*th-order autoregressive by the MM-estimator.

Step 3: Obtain the R^2 from the auxiliary regression in Step 2. The R^2 for MBG test is then formulated as:

$$R^2 = \frac{SSR}{(SSE + SSR)} \qquad (4)$$

where SSE is the sum of squared errors and SSR is the sum of squared regression of the auxiliary regression.

When the sample size is large, the statistic $(n-p)R^2$ is asymptotically following Chi-squared distribution with the degree of freedom of *p*, that is $(n-p)R^2 \sim \chi^2_p$. The null hypothesis is rejected if the statistic $(n-p)R^2$ exceeds the Chi-square value at the selected value of α.

This paper considers multiple linear regression with two independent variables and autocorrelated errors.

$$Y_t = \beta_0 + \beta_1 X_{1t} + \beta_2 X_{2t} + \mu_t \qquad (5)$$

The length of the lag residual cannot be specified a proiri. Alciaturi et al. [1] proposed the use of the autocorrelation function with lag 1 residual in univariate and multivariate for model selection. Following their idea, we set the error term to follow the first-order autoregressive AR(1) scheme such that

$$u_t = \rho u_{t-1} + \varepsilon_t, \quad -1 < \rho < 1 \qquad (6)$$

The auxiliary regression to be examined is therefore translate to

$$\hat{u} = \hat{\alpha}_0 + \hat{\alpha}_1 X_{1t} + \hat{\alpha}_2 X_{2t} + \hat{\rho}\hat{u}_{t-1} + \varepsilon_t \qquad (7)$$

III. Numerical Examples

In this section we consider three examples to assess the performance of our newly proposed method.

A. *Churchill Downs Racetrack in Louiville Data*

The first example is the Chruchill Downs Racetrack in Louiville data taken from Shiffler and Adams [23]. It monitors the attendance and handle (amount of money wagered) during racing season. This data contains 17 observations which show that the average daily handle in millions of dollars (*Y*) is positively related to year of racing season (X_1) and average daily attendance in thousands of people (X_2). The data is shown in Table 1. This data is known to have autocorrelation problem. The bold values in parenthesis are the high leverage points.

Table 1: Original and Modified Indexes of Churchill Downs Racetrack in Louiville Data

Index	Y	X_1	X_2
1	0.734	1	9.623
2	0.719	2	8.740
3	0.736	3	8.358
4	0.833	4	9.050
5	0.728	5	8.067
6	0.850	6	9.001
7	0.850	7	8.887
8	0.982	8	9.387
9	1.107	9	9.975
10	1.080	10	9.644
11	1.095	11	9.532
12	1.060	12	9.425 **[12.20]**
13	0.861	13 **{25}**	7.105
14	0.806	14 **(26)**	6.909 **(12.50)**
15	1.077	15	8.950
16	1.139	16	9.810
17	1.240	17	10.385

Source: Louisville *Courier-Journal*, June 30, 1987.

Note: { } = high leverage point in X_1
[ ] = high leverage point in X_2
( ) = high leverage point in X_1 and X_2

Fig. 1 shows the index plot of residuals for the original data based on OLS estimation. The figure reveals that there is a cyclical pattern among the residuals suggesting autocorrelation problem in the residuals.

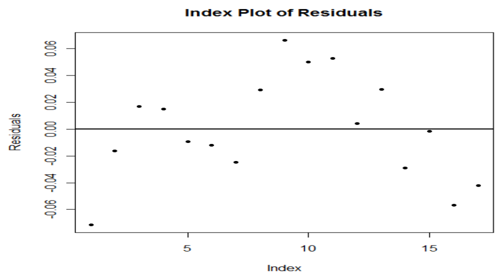

Figure 1: Index plot of residuals for Churchill Downs Racetrack in Louiville data

To see the effect of leverage points on the two tests, we purposely contaminate the data. Three types of contaminated data sets were considered in this study. The first type of the contaminated data is the data with one high leverage point in X_1. We randomly replace an observation in X_1 with a high leverage point. The second type of contaminated data is the data with one high leverage point in X_2. We arbitrarily replace an observation in X_2 with a high leverage point. The third type of contaminated data is the data with a high leverage point in X_1 and X_2 directions. For this case, we deliberately replace a good paired observation in X_1 and X_2 directions with a high leverage point. There are many classifications of high leverage points. In this study, we consider high leverage points as the values beyond the 3-deviation scopes from its mean. The bold values in parenthesis in Table 1 are the high leverage points.

The performances of BG and MBG tests are evaluated based on the *p*-values and the results are presented in Table 2.

Table 2: Autocorrelation Diagnostics for Average Daily Attendance Data

Test	BG (*p*-value)	MBG (*p*-value)
No High Leverage Point	1.242e-02	1.244e-02
One High Leverage Point in X_1	1.764e-01	1.782e-03
One High Leverage Point in X_2	3.383e-01	4.979e-02
One High Leverage Point in X_1 and X_2	5.875e-02	4.783e-02

We observe from this table that at $\alpha = 0.05$, the classical BG test is able to detect the autocorrelation if there is no high leverage point in the data. However, it fails to detect the autocorrelation problem when high leverage points occur in the data set. We now look at the results of the MBG test on the original and modified data. Unlike the BG test, the MBG test successfully detects the autocorrelation in the presence of high leverage point and consistently gives the lower significant *p*-value.

B. Boat Production Data

Our next example is the boat production data given by Newbold, Carlson and Thorne [19]. This data contains 24 observations and it shows the relationship between the number of boats produced data each year (*Y*), number of production stations (X_1) and number of workers used each year (X_2). Similar to Churchill data, we randomly replace a good observation with a high leverage point into the data set in order to get a modified data in X_1, X_2 and both X_1 and X_2 directions. The original and contaminated data are presented in Table 3.

Table 3: Original and Modified Indexes of Boat Production Data

Index	Y	X_1	X_2
1	40	1.0	2.0
2	45	1.2	2.1
3	52	1.2	2.7
4	57	1.1	3.0
5	65	2.0	3.1
6	75	3.0	3.6
7	86	4.0	4.0
8	95	4.5	6.0
9	100	4.5	7.1
10	130	4.5	8.5
11	161	4.1	8.9
12	215	6.0	10.0
13	260	8.1	13.9 **[34.5]**
14	265	7.9	16.1
15	275	11.0	14.0
16	282	12.0	14.0
17	300	13.2	15.6
18	340	14.0	17.0
19	370	14.8	18.0
20	405	15.0	20.9
21	430	16.0	21.0
22	440	16.0 **(26)**	21.4 **(33.5)**
23	460	17.0	21.8
24	472	17.0 **{37}**	22.1

Note: { } = high leverage point in X_1
[ ] = high leverage point in X_2
( ) = high leverage point in X_1 and X_2

Fig. 2 shows the scatter plot of the current residuals (Res1) versus lagged residuals (Res(-1)) for the original data based on OLS estimation. The plot shows that only a few residuals are clustered in the second and fourth quadrants, suggesting a positive autocorrelation in the residuals.

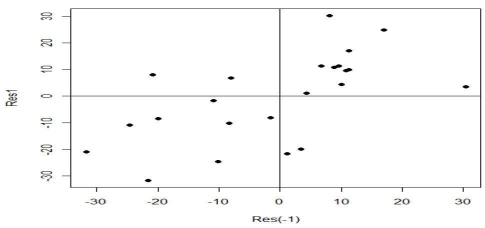

Figure 2: Current residuals (Res1) versus lagged residuals (Res(-1)) for boat production data

The results of the newly proposed MBG test and the classical BG test in detection of autocorrelation for the boat production are presented in Table 4. Similar results as for Churchill data were obtained. The results of the table 3 clearly show that the classical BG test can only correctly identify the autocorrelation problem when the data is free from contamination but it provides false indication in the presence of high leverage points in any respect. It is interesting to see that the MBG test still successfully detects the presence of autocorrelation problem with and without the presence of high leverage points in all respects.

Table 4: Autocorrelation Diagnostics for Boat Production Data

Test	BG (*p*-value)	MBG (*p*-value)
No High Leverage Point	3.252e-03	6.682e-04
One High Leverage Point in X_1	1.053e-01	2.644e-05
One High Leverage Point in X_2	5.399e-02	1.667e-02
One High Leverage Point in X_1 and X_2	4.664e-01	3.450e-02

C. *Apple Market Data*

The following example is apple market data. This data is provided by Kohler [13]. It contains 15 observations. From the OLS estimation, it is found that the quantity traded in millions of bushels (*Y*) has a reverse relationship with the disposable income in trillions of dollars (X_1), but it has a direct relationship with the rainfall in inches per year (X_2). The original and the contaminated data are presented in Table 5.

Table 5: Original and Modified Indexes of Apple Market Data

Index	Y	X_1	X_2
1	7.00	2.50	12.1
2	3.17	2.60	37.9
3	6.55	2.73	16.2
4	5.44	2.80 **{6.50}**	1 29.7
5	2.41	2.92	45.6
6	3.12	3.03	55.6
7	2.04	3.19	65.0
8	5.10	3.39	27.3
9	6.01	3.91 **(6.10)**	28.8 **(120)**
10	5.53	3.80	32.6
11	4.88	3.71	47.2
12	3.26	3.96	47.0
13	4.73	4.02	52.1 **[135]**
14	5.44	4.20	29.3
15	7.44	4.35	20.2

Note: { } = high leverage point in X_1
[ ] = high leverage point in X_2
( ) = high leverage point in X_1 and X_2

Fig. 3 shows the scatter plot of the current residuals (Res1) versus the lagged residuals (Res(-1)) for the original data. The residuals plot shows that there is no residual found in the third quadrant, indicating that there is a strong negative autocorrelation problem in the data set.

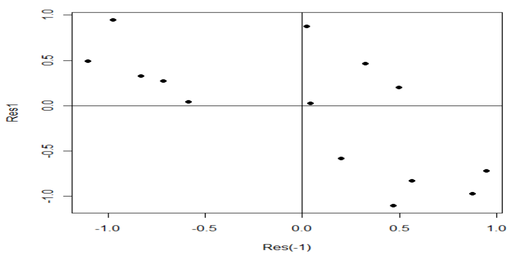

Figure 3: Current residuals versus lagged residuals for apple market data

The results of the newly proposed MBG test and the classical BG test in detection of autocorrelation for the apple market are shown in Table 6. The power of detection for BG and MBG tests are almost the same when there is no high leverage point in the data set. Again, the classical BG test can only identify the negative autocorrelation problem when the data is free from high leverage point but it gives a misleading indication in the presence of high leverage points. The MBG test is very reliable in identifying the negative autocorrelation problem in the occasions with and without the presence of high leverage points.

Table 6: Autocorrelation Diagnostics for Apple Market Data

Test	BG (*p*-value)	MBG (*p*-value)
No High Leverage Point	5.999e-03	5.534e-03
One High Leverage Point in X_1	3.260e-01	1.163e-02
One High Leverage Point in X_2	2.284e-01	1.933e-02
One High Leverage Point in X_1 and X_2	4.167e-01	1.851e-02

D. Quality Data

We have seen MBG test outperformed BG test in real time series data. As such, we would like to reaffirm the finding with a cross sectional data. The cross sectional data that we consider is quality data which is taken from McClave and James [14]. It shows the relationship between quality of finished product (*Y*), temperature in Fahrenheit (^0F) (X_1) and pressure in psi (X_2). Again, we randomly replace a good observation with a high leverage point into the data set in order to get a modified data in X_1, X_2 and both X_1 and X_2 directions. The original and modified data are exhibited in Table 7.

Table 7 : Original and Contaminated Indexes of Quality Data

Index	Y	X_1	X_2
1	50.8	80	50
2	50.7	80 **(140)**	50 **(100)**
3	49.4	80	50
4	93.7	80	55
5	90.9	80	55
6	90.9	80	55
7	74.5	80	60
8	73.0	80	60
9	71.2	80	60
10	63.4	90	50
11	61.6	90	50
12	63.4	90	50
13	93.8	90	55
14	92.1	90	55
15	97.4	90	55
16	70.9	90	60
17	68.8	90	60
18	71.3	90	60
19	46.6	100	50
20	49.1	100	50
21	46.4	100	50
22	69.8	100	55
23	72.5	100	55 **[135]**
24	73.2	100	55
25	38.7	100	60
26	42.5	100**{160}**	60
27	41.4	100	60

Note: { } = high leverage point in X_1
[ ] = high leverage point in X_2
( ) = high leverage point in X_1 and X_2

Fig. 4 shows the index plot of residuals for the original data based on OLS estimation. The residuals are forming some clusters among themselves and they are not randomly distributed. This indicates that there is an autocorrelation problem in the residuals.

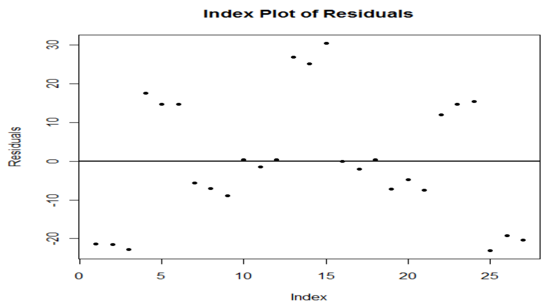

Figure 4: Index plot of residuals for quality data

Both tests were then applied to the data and the results are presented in Table 8. Similar results are obtained as in the previous examples. The BG test once again failed to detect the autocorrelation problem when there is high leverage point in the data set. However, the MBG maintain its autocorrelation detection power with the presence of high leverage points. The MBG test is also reliable in detecting the autocorrelation problem in cross sectional data.

Table 8: Autocorrelation Diagnostics for Quality Data

Test	BG (*p*-value)	MBG (*p*-value)
No High Leverage Point	3.351e-03	2.643e-04
One High Leverage Point in X_1	7.488e-02	2.924e-03
One High Leverage Point in X_2	5.737e-02	2.097e-03
One High Leverage Point in X_1 and X_2	5.585e-02	9.772e-04

IV. SIMULATION STUDY

We have seen the performance of the MBG test in real data. Now we want to reiterate the results by checking a Monte Carlo simulation experiment. In this study, we consider six different sample sizes n = 20, 40, 60, 80, 100 and 200, represent small (20 and 40) , medium (60 and 80) and large sample size (100 and 200) respectively. We consider the two possibilities of the directions for β_1 and β_2, either both parameters towards same direction or different directions.

A. Both Positive Directions for β_1 and β_2

For each sample size n = 20, 40, 60, 80, 100 and 200, n “good” data are generated according to the following relation:

$$Y = 1 + 2X_1 + 3X_2 + u \quad (6)$$

where all the values of X_1 and X_2 are generated from Uniform Distribution, U(0,10). The error terms u_t are generated by the first-order autoregressive scheme as follow:

$$u_t = 0.9u_{t-1} + \varepsilon_t \quad (7)$$

with an initial value of u_1 equals to 8. The White noise, ε_t is generated from Normal Distribution with mean 0 and standard deviation 0.1. This autoregressive scheme is repeated for every 10 observations to ensure the existence of autocorrelation problem.

We would like to compare the performance of BG and MBG tests with 5% and 10% high leverage point in X_1, X_2 and both X_1 and X_2. For each sample size, we generate high leverage points by deleting randomly the ‘good’ observations and replacing it with ‘bad’ data points. The BG test high leverage points in X_1, X_2 and both X_1 and X_2 are represented by a uniform distributed variate x_i from Uniform Distribution U(15,20). The significance level is set to 0.05 and in each simulation run, there were 10,000 simulations.

The ‘p-values’ of both BG and MBG tests are presented in Table 9. Again, the BG test performs miserably in the simulation. It can only detect autocorrelation problem in the normal data. However, when there are some high leverage points in the data, the BG test fails to diagnose the autocorrelation problem. On the other hand, the MBG test did a credible job. The results of Table 9 show that the MBG test has smaller and significant p-values for all contamination scenarios. Moreover, we can see that the detection power of the MBG test is higher with the increased in sample sizes.

Table 9: Simulation Results of Both Positive Parameters with Autocorrelation Problem

		p-value						
		No HLP	**5% of HLP**			**10% of HLP**		
			X_1	X_2	X_1 and X_2	X_1	X_2	X_1 and X_2
n = 20	**BG**	1.501e-02	4.063e-01	4.323e-01	4.239e-01	4.150e-01	4.150e-01	4.187e-01
	MBG	4.836e-03	4.195e-02	4.518e-02	4.864e-02	3.989e-02	3.788e-02	3.788e-02
n = 40	**BG**	1.803e-03	4.504e-01	4.700e-01	4.602e-01	4.571e-01	4.567e-01	4.616e-01
	MBG	3.437e-05	9.977e-04	1.054e-03	1.007e-03	1.310e-03	1.178e-03	1.089e-03
n = 60	**BG**	2.230e-04	4.572e-01	4.799e-01	4.714e-01	4.565e-01	4.700e-01	4.751e-01
	MBG	1.514e-07	2.753e-05	3.307e-05	3.146e-05	3.446e-05	3.499e-05	3.335e-05
n = 80	**BG**	2.779e-05	4.521e-01	4.845e-01	4.824e-01	4.563e-01	4.757e-01	4.737e-01
	MBG	1.462e-09	1.072e-06	1.259e-06	1.267e-06	3.342e-06	1.287e-06	1.558e-06
n = 100	**BG**	3.220e-06	4.513e-01	4.880e-01	4.812e-01	4.592e-01	4.768e-01	4.861e-01
	MBG	1.424e-11	4.271e-08	5.177e-08	5.498e-08	3.151e-07	6.141e-08	5.926e-08
n=200	**BG**	1.242e-10	4.148e-01	4.769e-01	4.840e-01	4.402e-01	4.806e-01	4.870e-01
	MBG	2.200e-16	5.996e-15	8.500e-15	1.037e-14	9.842e-15	1.164e-14	1.249e-14

B. One Positive and One Negative Direction for β_1 and β_2

Similar to both positive directions for β_1 and β_2, we generate X_1 and X_2 from Uniform Distribution, U(0,10) for samples sizes size *n* = 20, 40, 60, 80, 100 and 200. The *n* 'good" data are now simulated according to the following relation:

$$Y = 1 - 2X_1 + 5X_2 + u \quad (8)$$

The error terms u_t is again generated following the first-order autoregressive scheme stated in (7).

Once again, we compare the performance of BG and MBG tests with 5% and 10% high leverage point in X_1, X_2 and both X_1 and X_2 by repeating the same procedures of contamination for both positive parameters β_1 and β_2 stated above.

Table 10 exhibits the '*p*-values' of the BG and MBG tests. The classical BG tests only perform well when there is no high leverage point. This test fails to detect autocorrelation when there is high leverage point in any respect of data for all three sample sizes being studied. Similar to the previous results, the MBG test performs consistently throughout the whole simulation process. This test is reliable when data are contaminated with high leverage points. We can see from Table 10 that the autocorrelation detection power for MBG tests increases with the increased in sample sizes. Therefore, we can conclude that the MBG test performs more outstanding as compared to classical BG where both parameters β_1 and β_2 are in different directions.

V. CONCLUSION

In this paper, we show that the widely used BG test fails to identify autocorrelation problem when high leverage points are present in the data. To remedy this problem, we propose a robust modified Breusch-Godfrey test. The results of the numerical examples and simulations study show that the performance of the BG test are fairly closed to the MBG test when outliers are not present in the data. However, the BG test performs poorly in the presence of outliers. On the other hand, the MBG test is high leverage resistant. Hence, we recommend using the MBG test instead of using the classical Breusch-Godfrey test which is not reliable test in the presence of high leverage point.

Table 10: Simulation Results of One Positive and One Negative Parameters with Autocorrelation Problem

		p-value						
		No HLP	**5% of HLP**			**10% of HLP**		
			X_1	X_2	X_1 and X_2	X_1	X_2	X_1 and X_2
n = 20	**BG**	1.498e-02	4.196e-01	4.391e-01	3.855e-01	4.075e-01	4.148e-01	4.276e-01
	MBG	4.854e-03	3.402e-02	4.748e-02	4.418e-02	3.527e-02	3.749e-02	4.931e-02
n = 40	**BG**	1.903e-03	4.505e-01	4.675e-01	4.629e-01	4.466e-01	4.546e-01	4.713e-01
	MBG	3.332e-05	8.463e-04	1.070e-03	1.545e-03	1.078e-03	1.102e-03	2.817e-03
n = 60	**BG**	2.221e-04	4.582e-01	4.797e-01	4.862e-01	4.518e-01	4.740e-01	4.822e-01
	MBG	1.610e-07	2.513e-05	3.258e-05	6.220e-05	3.440e-05	3.338e-05	2.756e-04
n = 80	**BG**	2.961e-05	4.552e-01	4.865e-01	4.914e-01	4.548e-01	4.778e-01	4.880e-01
	MBG	1.466e-09	9.376e-07	1.314e-06	4.160e-06	2.182e-06	1.451e-06	2.243e-05
n = 100	**BG**	3.160e-06	4.526e-01	4.986e-01	4.925e-01	4.579e-01	4.757e-01	4.835e-01
	MBG	1.409e-11	3.954e-08	5.698e-08	2.081e-07	4.814e-08	5.816e-08	3.058e-06
n=200	**BG**	1.237e-10	4.119e-01	4.894e-01	4.831e-01	4.397e-01	4.806e-01	4.839e-01
	MBG	2.200e-16	5.191e-15	1.013e-14	1.149e-14	1.015e-14	1.164e-14	1.239e-14

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