Deep and surface learning of elementary calculus concepts in a blended learning environment

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Abstract: Poor results in mathematics at South African higher education institutions have been the centre of academic debate. Mathematics forms the core for engineering and science studies at institutions of higher learning. However, analysis of student's performance in mathematics tasks reveals that mastery of skills is not acquired. This paper focuses on students understanding of elementary calculus in a blended learning course at a University of Technology (UOT). Conventional lectures were integrated with the computer laboratory teaching environment to promote interactive and discovery learning. Projects were designed to support the development of calculus frames in conjunction with a theoretical framework that was used in analyzing students understanding of integral calculus concepts. The students in the blended learning mathematics course (experimental group) was also compared to students that were traditionally lectured (control group). Both groups were assessed by the modified Orton's battery of tests on integral calculus. The experimental group exhibited deep learning of concepts, while the control group possessed more surface structures.

Keywords: Blended Learning, Calculus, Deep and Surface Structures

I INTRODUCTION

Calculus is fundamental to further study of mathematics at a University of Technology. However, students that study engineering at the UOT, enter with low symbols in mathematics. They possess many misconceptions and have poor pre-knowledge frames in basic mathematics and calculus. Naidoo [1] deduced that, first year mathematics students that are taught traditionally, study by rules. Lecturers tend to teach mechanistically and do standard type problems and solutions. Tall [2] states that students develop coping strategies, like computational and manipulative skills, when faced with conceptual difficulties.

Research in teaching and learning using the computer laboratory method gave a measure of success, Naidoo [3], especially in graph construction and numerical solutions. Although students were performing better, they still made errors that include, amongst others inability to conclude that sequences converge; problems with rate of change of a curve; students lacked the ability to interpret symbols.

A mathematics research group had been established at the Durban University of Technology for over a decade. The aim of the local calculus reform research group was to research alternate ways of teaching elementary calculus. Students had access to mathematics laboratory sessions where project work in a computer-learning environment was encouraged. The learning environment was used by the students to investigate and explore concepts in calculus under the guidance of lecturers. This environment would help students to build their mental models to connect with aspects they meet during traditional lessons. These attempts hoped to develop interest in mathematics study and improve throughput rate within the University.

Findings of studies performed from another calculus reform group by Silverberg [4], showed a measure of success. His analysis of grades of traditional and reform cohorts produced the most compelling results. Significant improvements in results were noted between cohorts in contrast to the reform group that performed better after some time.

The need for alternate methods of instruction to enhance teaching and learning of calculus is essential. A Blended Learning (traditional and computer laboratory teaching) mathematics course was developed and implemented in an attempt to improve student's understanding of elementary calculus.

II BLENDED LEARNING

Researchers define Blended Learning in higher education as follows:

Blended learning is learning that is facilitated by the effective combination of different modes of delivery, models of teaching and styles of learning, and founded on transparent communication amongst all parties involved with a course.

According to Singh [5], blended learning mixes various event-based activities, including face-to-face classrooms, live e-learning, and self paced Web Based Learning (WBL). Blended Learning (BL) often is a mix of traditional instructor-led training, synchronous online conferencing or training, asynchronous self-paced study.
He propagates this type of learning since learning styles of each learner tend to be different, and hence, "a single mode of instructional delivery may not provide sufficient choices, engagement, social contact, relevance, and context needed to facilitate successful learning and performance".

Blended learning is viewed as midway along a continuum that at one extreme has conventional face-to-face instruction, and on the other end totally WBL. It is self paced, collaborative or an inquiry-based study. Blended learning should not be an "add on" to instruction, but as an integrated component of the course [6]. The current trend of research is to explore environments with a better balance between two extremes.

By using blended learning, we expect to enable students to easily move between

- listening to a lecture;
- engaging in class discussion;
- working collaboratively;
- using available software to investigate concepts or solve problems
- accessing their archived work

In this environment, linguistic, cultural, social and economic groups can interact within a group and among each other. The WBL was blended into the traditional lectures using the Wrap-Around Model of Mason [7].
The Blended Learning engineering mathematics one course was designed using 'Rule of Three' guiding principles which consist of graphical, numerical and analytical methods that are used to teach calculus concepts. The aim is to produce a course where the three points of view are balanced, and where students see each major idea from several angles which is necessary for an engineer [8].

III DEEP AND SURFACE STRUCTURES

Deep and surface level procedures of learning has been identified in many studies. Matz [9] states that surface level procedures are ordinary rules of algebra while deep learning serves the purpose of creating and modifying superficial-level rules or changing the control structure.

The "deep" approach requires higher order thinking skills that includes analysis and synthesis. "Deep" learners incorporate new ideas that they learn with existing knowledge and personal experience. Deep learning is encouraged by extending individual study time and time given for projects.

The blended mathematics course is guided by Campbell [10] who outlines the following methods to promote deep learning:

- encourage faculty/student interaction
- encourage student/student interaction
- use active and interactive teaching methods
- make links with existing student knowledge
- discussing/teaching learning skills explicitly
- link topics to student's lives and career aspirations
- encourage collaborative projects

Blended Learning discourages surface learning which focuses on comprehension and reproduction of knowledge (rote learning) as follows:

- excessive amount of material – Blended Learning releases the bare minimum until after interaction with students whose pre-conceptual frames determines the amount of learning material exposed to the student
- high lecture contact hours
- lack of opportunity to pursue subject in depth – Blended mathematics gives the opportunity to students to investigate the concepts by using the quiz or project or enrichment materials

Ramsden [11] summarized the deep and surface approach to learning as follows:

Deep	Surface
Focus is on "what is signified"	Focus is on the "signs"
Relates previous knowledge to new knowledge	Focus is on unrelated parts of the task
Relates knowledge from different courses	Information for assessment is simply memorized
Relates theoretical ideas to everyday experience	Facts and concepts are associated unreflectively
Relates and distinguishes evidence and argument	Principles are not distinguished from examples
Organizes and structures content to coherent whole	Task is treated as an external imposition
Emphasis is internal, from within the student	Emphasis is external, from demands of assessment

The point is how to get students to use the deep approaches rather than the surface approaches [12]. What students do when learning and why they do it is described as a 'congruent motive-strategy package' [13]. Case & Gunstone [14], describe metacognitive development as the move to greater knowledge, awareness and control of one's own learning.

Flavell [15], describes metacognition as 'one's knowledge concerning one's own cognitive processes and products or anything related to them'. It also includes 'the active monitoring and consequent regulation and orchestration' of information processing activities. The computer is a means of getting students to use deep approaches in their search for solutions. In the case of calculus solutions it also provides a visual aid to enhance comprehension. The importance of group work and problem-solving as a means of fostering the deep approach to learning should be noted. These are similar to the "active learning", "cooperative learning" and "problem-based instruction".

IV METHODOLOGY AND QUALITATIVE ANALYSIS

The qualitative analysis considered various sections in elementary calculus: sequences, limits and infinity, symbolism, area and integration to classify errors made by students. Students had to perform the project tasks that contributed to their part assessment for the mathematics course. Further investigations was performed to find out the strategy used by students with respect to the use of deep and surface structures to relate to the tasks presented to them.

Task A, required students to retrieve the pre-knowledge frames: area of trapezium = ½ (a + b) h, area of rectangle = l x b and area of triangle = $^1/_2$(b x h). Then using the appropriate values in the respective formulae they should find that area = ½ (a + b) h = [l x b + $^1/_2$(b x h)]. They should predict that this method gives the area under the line.

In Task B, the Zoom graph toolbar in Mathematica allows students to experiment with concepts by magnifying graphs, changing variables, etc. From this exercise they will see that as the number of rectangles under the graph is increased, the more accurate the area under the graph - the error is reduced.

Task C, tested the students understanding of area. Even though the result of finding the definite integral $area = \int_a^b f(x)dx$ is 0, there is clearly a region enclosed by the curve and the x axis. Learners had to reason that part of the curve lies below the x axis and if the Riemann integral is used the total area will not be equivalent to zero. Therefore, if area is considered positive both under and above the graph then the definite integral and Riemann sum should be equivalent to each other.

We further used a modified Orton's test [16] to elicit responses from a control (traditional learning group) and the experimental group. The tasks were modified to include heights of rectangles, area under graph, summation of area of rectangles and limit of sequence equals area under graph. This was related to 10 descriptive items and included in the project tasks. The control and experimental group were interviewed on the tasks output. Table 1 & 2 categorizes the learning into deep, surface and intermediate structures from the data.

Table 1: Cognitive Level: Experimental Group

Items	Deep	intermediate	Surface
1	82%	6%	12%
2	76%	12%	12%
3	46%	42%	12%
4	48%	45%	7%
5	82%	12%	9%
6	58%	3%	39%
7	39%	10%	51%
8	46%	13%	41%
9	31%	13%	56%
10	33%	33%	33%

Table 2: Cognitive level: Control Group

Items	Deep	intermediate	Surface
1	79%	8%	13%
2	69%	13%	18%
3	13%	41%	46%
4	44%	32%	24%
5	25%	12%	63%
6	28%	2%	70%
7	15 %	11%	74%
8	36%	6%	58%
9	27%	9%	64%
10	8%	53%	39%

V CONCLUSION

The results and analyses presented provide evidence that students with a lower level of cognitive maturity tend to engage in surface learning of calculus concepts. The frequency of errors made by the students indicates that their pre-knowledge frames were not well developed. With regards to elementary integral calculus, the poor understanding of pre-calculus concepts, contribute to a host of difficulties in the mind of the learner. Some of these difficulties were observed during the application of the modified battery of tests of Orton . Many factors need to be considered when referring to students understanding of integration.

The first factor relates to weak pre-knowledge frames. Students' had a poor mental image of area, summation and the limit concept. They were unable use these concepts and decipher the link between the Riemann Sums and the indefinite integral. Their problems were compounded when dealing with complex situation in integration. The analyses for deep, intermediate and surface structures show a clear distinction between the learning strategies employed by each group. It clearly showed that a sub-frame that was poorly developed in one task, reflected poorly again in a related task. This gives an indication that concepts in elementary calculus are difficult to grasp. Despite generally performing better than the control group, the experimental group still made a significant amount of errors [17]. This shows that the software by itself is not sufficient to address the pre-knowledge deficiencies that were prevalent. Also, many students were not technology inclined and struggled to adapt.

The second factor deals with reliance on algorithmic means to solve problems. This was evident in the "area" questions. Students develop coping strategies as described by Smith and Moore [18] to overcome their difficulty. In the learning of elementary calculus it is essential that a mechanistic application of a set of rules is not sufficient, rather the synthesis of the appropriate mental frames is needed to represent concepts and the procedures necessary to seek solutions. It is important that concepts be seen from several points of view. They must relate to the student's 'own environment' and 'world view'. The student in turn must build a web of connections to tackle real world problems. The analysis for deep, intermediate and surface structures using the Orton instrument showed that both groups struggled to connect meaningfully with applications. They had a very superficial understanding of the concepts in elementary integral calculus.

The third factor deals with errors made by students. A classification of the errors revealed that there were more structural and executive errors as compared to arbitrary errors. The experimental group made fewer errors in both categories as compared with the findings of Naidoo. Structural errors found in the study may be due to student's rote and mechanistic learning styles adopted in elementary integral calculus – lack of understanding of concepts since pre-knowledge frames were not developed. Students in the blended learning course made fewer structural errors than the control group. The experimental group scores indicate a deep cognitive understanding of integral calculus compared to the control group. This is so, since the Blended Learning mathematics course was designed to allow students the resources and flexibility to cater for their pre-knowledge frame deficiencies. This allows for reflection to understand and analyse a concept/problem in different ways.

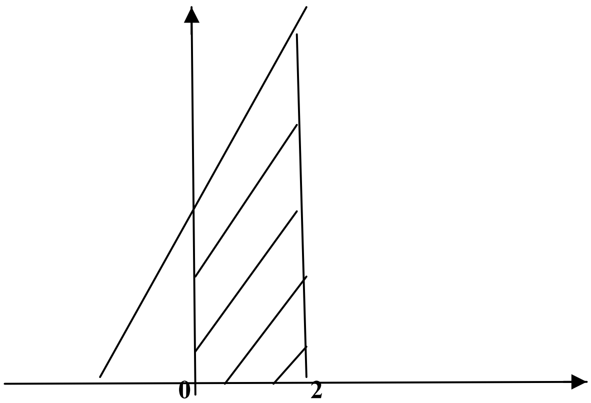
The fourth factor deals with symbolism in elementary calculus. Students lacked ability to interpret symbols and focused on superficial aspects of symbols thereby ignoring the meanings behind the symbols. The analysis for deep, intermediate and surface structures using the Orton instrument showed that both groups struggled to connect meaningfully with symbols. They had a very superficial understanding of the symbols in elementary differential calculus. This was consistent in the project task done by the experimental group as well.

It is clear from the tabular representation of the overall scores, for the experimental group and the control group, that the experimental group had a slight advantage of more developed frames in each of the tasks presented to them. However statistical tests suggests that there is a significant difference between the experimental and control groups. We believe that by modifying the *Blended Mathematics Course* greater improvements in learning of calculus concepts may be achieved.

APPENDIX I: PROJECT TASKS

Discuss the following problems by using the chat-room tool on the course. When solving, show all techniques, numerical tables, graphs and explain your answers fully.

TASK A

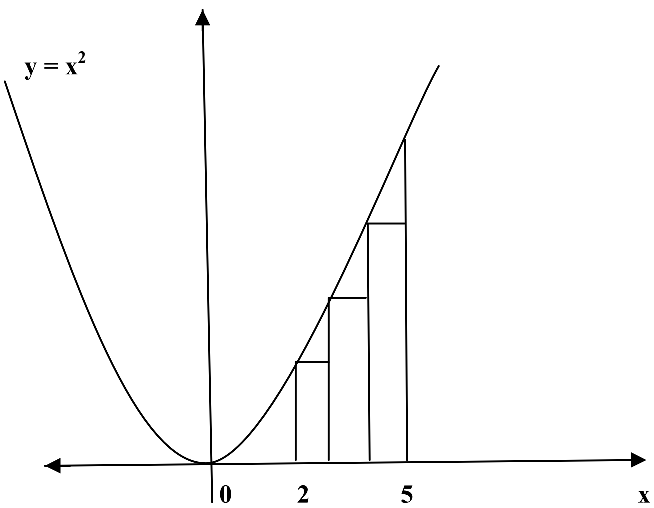


Find the area under $y = 6x + 3$ between $x = 0$ and $x = 2$ using the formulae for area of trapezium.

Can you verify this answer by using the formulae for area of a rectangle and area of a triangle?

Is it possible to find the area under a curve using this method, explain

TASK B



Using the Mathematica Graph toolbar (appendix 3) and the method of sums of rectangles under the curve $y = x^2$ above x axis, between $x = 2$ and $x = 5$, calculate the area if

i) there is one rectangle

ii) there are three rectangles

iii) there are six rectangles

iv) Explain what happens as the number of rectangles

under the curve is increased. Give a possible reason.

TASK C $_{\mathbf{x}}$

For the graph of $y = x^3$, find the area enclosed by the graph and the x axis between $x = -2$ and $x = 2$

i) using the definite integral

ii) using Riemann Sums

Is there any discrepancies? If so, explain.

APPENDIX II: EXEMPLARS FROM PROJECT WORK

TASK A

Area of trapezium = ½ (a + b)h = ½(3 + 15)2 = 18
Area under graph = area of rectangle + area of triangle
= l x b + $^1/_2$(b x h)
= (3 x 2) + $^1/_2$ (2 x 15)
= 6 + 15 = 21
…. but I think I made an error since Area of trapezium should equal area under graph

[Intermediate Structure - student is able to see the relationship between the area under the graph and the area of trapezium however made an executive error of substituting the incorrect height for the triangle, thereby resulting in the incorrect answer].

TASK B

i) Area of one rectangle under curve = l x h = 3 x $(2)^2 = 12$

ii) Area of 3 rectangles under curve = ∑ (l x h) = 1 x $((2)^2 + (3)^2 + (4)^2) = 29$

iii) Area of three rectangle under curve

$= \sum (l \times h) = {}^1/_2 \times ((2)^2) + (2{,}5)^2 + (3)^2$

$+ (3{,}5)^2 + (4)^2 + (4{,}5)^2) = 34$

iv) Using the Zoom Graph toolbar, we see that as the number of rectangles are increased, the area becomes closer and closer to the actual area under the graph which is given by

$$area = \int_2^5 x^2\,dx = x^3/3]_2^5 = (125/3) - (8/3) = 39$$

[Deep Structure – student exhibits all the appropriate pre-knowledge and lecture frames]

TASK C

i) $$area = \int_{-2}^{2} x^3\,dx = x^4/4]_{-2}^{2} = (16/4) - (16/4) = 0$$

ii) First, partition the intervals [-2,0] and [0,2] into n subintervals, each of length

$$\Delta x = \frac{b-a}{n} = \frac{2-0}{n} = \frac{2}{n}$$

Let's work with interval [0,2]. The implication for interval [-2,0] are the same. Because f is increasing in the interval [0,2], the minimum value on each subinterval occurs at the left endpoint, and the maximum value occurs at the right endpoint.

Using the left endpoint, $m_i = \frac{2(i-1)}{n}$, the lower sum is

$$s(n) = \sum_{i=1}^{n} f(m_i)(x_i - x_{i-1})$$
$$= \sum_{i=1}^{n} f[\frac{2(i-1)}{n}](\frac{2}{n})$$
$$= \sum_{i=1}^{n} [\frac{2(i-1)}{n}]^3 (\frac{2}{n})$$
$$= \sum_{i=1}^{n} [\frac{16}{n^4}](i^3 - 3i^2 + 3i - 1)$$
$$= \frac{16}{n^4}\sum_{i=1}^{n}(i^3 - 3i^2 + 3i - 1)$$

$$= \frac{16}{n^4}(\frac{n^2(n+1)^2}{4} - 3\frac{n(n+1)(2n+1)}{6} + 3\frac{n(n+1)}{2} + n)$$

$$= \frac{16}{n^4}(\frac{n^4 + 4n^3 + 7n^2 + 8n}{4})$$
$$= 4 - \frac{16}{n} + \frac{28}{n^2} + \frac{32}{n^3}$$

Using the right endpoints, $M_i = \frac{2i}{n}$, the upper sum is

$$S(n) = \sum_{i=1}^{n} f(M_i)(x_i - x_{i-1})$$
$$= \sum_{i=1}^{n} f(\frac{2i}{n})(\frac{2}{n})$$

$$= \sum_{i=1}^{n} (\frac{2i}{n})^3 (\frac{2}{n})$$
$$= \sum_{i=1}^{n} (\frac{16}{n^4})i^3$$
$$= (\frac{16}{n^4})\frac{n^2(n+1)^2}{4}$$
$$= 4 + \frac{8}{n} + \frac{4}{n^2}$$

The example illustrates that for any value of n, the lower sum is less than (or equal to) the upper sum.

$$s(n)=4-\frac{16}{n}+\frac{28}{n^2}+\frac{32}{n^3}\prec 4+\frac{8}{n}+\frac{4}{n^2}=S(n)$$

Also, the difference between these two sums lessens as n increases, so if we take the limit as $n\to\infty$, both the lower sum and upper sum approach 4

$$\lim_{n\to\infty} s(n)=\lim_{n\to\infty}\left(4-\frac{16}{n}+\frac{28}{n^2}+\frac{32}{n^3}\right)=4$$

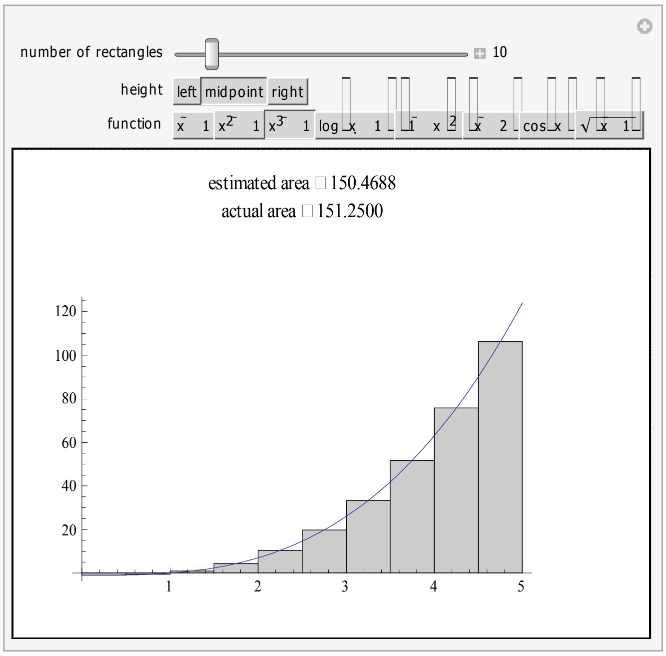
$$\lim_{n\to\infty} S(n)=\lim_{n\to\infty}\left(4+\frac{8}{n}+\frac{4}{n^2}\right)=4$$

So the area for the region [-2, 2] which are intervals [-2,0] and [0,2] is 4 + 4 = 8

iii) The definite integral and the Riemann Sum should have the same answer, but it is not the case since we are determining areas below and above the graph.

[Arbitrary Error/Deep structure – student was able to calculate the areas using the Riemann Sum and definite integral but did not consider that because areas below the axis are taken as negative, so the integral from -2 to 2 gives the area difference and not the sum].

APPENDIX III: MATHEMATICA DEMOSTRATION



The area under a curve can be approximated by a Riemann sum. The definite integral is the limit of that area as the width of the largest rectangle tends to zero. Observe that as the number of rectangles is increased, the estimated area approaches the actual area.

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