Design and Simulation Verification of Self-tuning Smith Predictors

V. Bobál, P. Chalupa, P. Dostál, and M. Kubalčík

Abstract**—**This paper deals with a design of algorithms for self-tuning digital control of processes with time-delay. The algorithms are based on the some modifications of the Smith Predictor (SP). One modification of the SP based on the digital PID controller was applied and it was compared with two new designed modifications based on polynomial approach (pole assignment and minimization of the quadratic criterion). The program system MATLAB/SIMULINK was used for simulation verification of these algorithms. Some of designed algorithms are suitable for implementation in real time conditions.

Keywords**—**Digital control, Polynomial approaches, Self-tuning control, Simulation of control loops, Smith predictor, Time-delay

I. Introduction

TIME- delay appear in many processes in industry and other fields, including economical and biological systems [1]. They are caused by some of the following phenomena:

- the time needed to transport mass, energy or information,
- the accumulation of time lags in a great numbers of low order systems connected in series,
- the required processing time for sensors, such as analyzers; controllers that need some time to implement a complicated control algorithms or process.

Consider a continuous time dynamical linear SISO (single input $u(t)$ – single output $y(t)$) system with time-delay T_d. The transfer function of a pure transportation lag is $e^{-T_d s}$, where s is complex variable. Overall transfer function with time-delay is in the form

$$G_d(s) = G(s)e^{-T_d s} \qquad (1)$$

where $G(s)$ is the transfer function without time-delay.

Manuscript received March 19, 2011. The authors wish to thank to the Ministry of Education of the Czech Republic (MSM7088352101) for financial support. This article was created with support of Operational Programme Research and Development for Innovations co-funded by the European Regional Development Fund (ERDF), national budget of Czech Republic within the framework of the Centre of Polymer Systems project (reg. number: CZ.1.05/2.1.00/03.0111) and grant 1M0567.

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Processes with significant time-delay are difficult to control using standard feedback controllers mainly because of the following [1]:

- the effect of the disturbances is not felt until a considerable time has elapsed;
- the effect of the control action requires some time to elapse;
- the control action that is applied based on the actual error tries to correct a situation that originated some time before.

The problem of controlling time-delay processes can be solved by some control methods using

- PID controllers;
- time-delay compensators;
- model predictive control techniques.

It is clear that many processes in industry are controlled by the PID controllers. When the process contains a time-delay, the tuning of the PID controller is difficult. The most popular tuning rules for processes with small time-delay were proposed by Ziegler and Nichols [2]. Several methods for new tuning rules were proposed for stable and unstable processes with time-delay. An algebraic approach for time-varying systems with time-delay is presented in [3, 4]. A presentation and review of some these methods are introduced in [5].

When a high performance of the control process is desired or the relative time-delay is very large, a predictive control strategy must be used [5]. The predictive control strategy includes a model of the process in the structure of the controller. The first time-delay compensation algorithm was proposed by Smith [6] in 1957. This control algorithm known as the Smith predictor (SP) contained a dynamic model of the time-delay process and it can be considered as the first model predictive algorithm.

Historically first modifications of time-delay algorithms were proposed for continuous-time (analogue) controllers. On the score of implementation problems, only the discrete versions are used in practice in this time.

The majority of processes met in the industrial practice have stochastic characteristics and eventually they embody nonlinear behaviour. Traditional controllers with fixed parameters are often unsuitable for such processes because their parameters change. One possible alternative for improving the quality of control for such processes is the use of adaptive control systems. Different approaches were proposed and utilized. One successful approach is represented by self-tuning controller (STC). The main idea of an STC is based on the combination of a recursive identification procedure and a selected controller synthesis. Some STC modifications of the digital Smith predictors are designed and verified by simulation in this paper.

The paper is organized in the following way. The general problem of a control of the time-delay systems is described in Section 1. The principle of the continuous-time Smith Predictor is introduced in Section 2 and digital version in Section 3. Three modifications of digital controllers that are used for self-tuning versions SPs are proposed in Section 4. Section 5 contains brief description of the recursive identification procedure. Simulation configuration is presented in Section 6. Results of simulation experiments are summed in Section 7.

II. Principle of Smith Predictor

The principle of the SP is shown in Fig. 1. It can be divided into two parts – the primary $G_c(s)$ controller and predictor part. This algorithm was primarily designed for continuous time PID controller. The predictor is composed of a model of the process without time delay and $G_m(s)$ (so called as the fast model) and a model of the time delay $e^{-T_d s}$. Then the complete process model is

$$G_p(s) = G_m(s)e^{-T_d s} \tag{2}$$

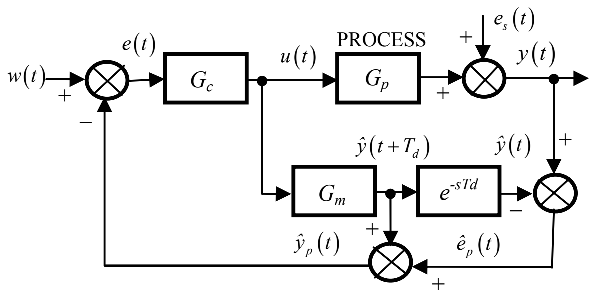

Fig. 1 Block diagram of an analogue Smith Predictor

The fast model $G_m(s)$ is used to compute an open-loop prediction. The difference between the output of the process $y(t)$ and the model including time delay $\hat{y}(t)$ is the predicted error $\hat{e}_p(t)$ as shown is in Fig. 2, where $u(t)$, $w(t)$, $e(t)$ and $n(t)$ are the control signal, reference signal, the error and the noise. If there are no modeling errors or disturbances, the error between the current process output and the model output will be null and the predictor output signal $\hat{y}_p(t)$ will be the time-delay-free output of the process. Under these conditions, the controller $G_c(s)$ can be tuned, at least in the nominal case, if the process had no time delay.

The Smith Predictor structure for the nominal case (without modelling errors) has three fundamental properties: time-delay compensation, prediction and dynamic compensation.

III. Digital Smith Predictor

Although time-delay compensators appeared in the mid 1950s, their implementation with analogue technique was very difficult and these were not used in industry. Since 1980s digital time-delay compensators can be implemented. In spite of the fact that all these algorithms are implemented on digital platforms, most works analyze only the continuous case. The digital time-delay compensators are presented e.g. in [7] – [9].

The discrete versions of the SP and its modifications are suitable for time-delay compensation in industrial practice. Most of authors designed the digital SP using discrete PID controllers with fixed parameters. However, the SP is more sensitive to process parameter variations and therefore requires an auto-tuning or adaptive approach in many practical applications [10], [11]. In [12], the structure of the discrete disturbance observer time-delay compensator is analyzed.

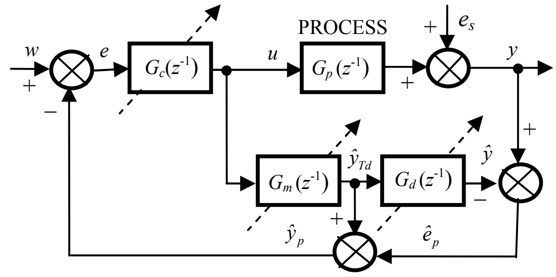

Fig. 2 Block diagram of a digital Smith Predictor with tuning

A. Structure of Digital SP

The digital SP (see [10] – [14]) is shown in Fig. 2. The function of the digital version is similar to the classical analogue version. The block $G_m(z^{-1})$ represents process dynamics without the time-delay and is used to compute an open-loop prediction. The difference between the output of the process y and the model including time- delay $\hat{y}$ is the predicted error $\hat{e}_p$ as shown is in Fig. 2, where u, w, e and e_s are the control signal, the reference signal, the error and the noise. If there are no modelling errors or disturbances, the error between the current process output and the model output will be null and the predictor output signal $\hat{y}_p$ will be the time-delay-free output of the process. Under these conditions, the controller $G_c(s)$ can be tuned, at least in the nominal case, as if the process had no time-delay. The primary (main) controller $G_c(z^{-1})$ can be designed by the different approaches (for example digital PID control or methods based on algebraic approach). The outward .feedback-loop through the block $G_d(z^{-1})$ in Fig. 1 is used to compensate for load disturbances and modelling errors. The dash arrows indicate the self-tuned parts of the Smith Predictor.

Most industrial processes can be approximated by a reduced order model with some pure time-delay. Consider the following second order linear model with a time-delay

$$G\left(z^{-1}\right)=\frac{B\left(z^{-1}\right)}{A\left(z^{-1}\right)}z^{-d}=\frac{b_1z^{-1}+b_2z^{-2}}{1+a_1z^{-1}+a_2z^{-2}}z^{-d} \tag{3}$$

for demonstration of some approaches to the design of the adaptive Smith Predictor. The term z^{-d} represents the pure discrete time-delay. The time-delay is equal to dT_0 where T_0 is the sampling period.

If the time-delay is not an exact multiple of the sampling period T_0, then dT_0 represents the largest integer multiple of the sampling period with remaining fractional deal absorbed into $B\left(z^{-1}\right)$ using the modified z-transformation [15].

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B. *Identification of Systems with Time-delay*

In this paper, the time-delay is assumed approximately known or possible to be obtained separately from an off-line identification using the least squares method (LSM) [16, 17]

$$\hat{\boldsymbol{\Theta}}=\left(\boldsymbol{F}^T\boldsymbol{F}\right)^{-1}\boldsymbol{F}^T\boldsymbol{y} \tag{4}$$

where the matrix $\boldsymbol{F}$ has dimension (N-n-d, $2n$), the vector $\boldsymbol{y}$ (N-n-d) and the vector of parameter model estimates $\hat{\boldsymbol{\Theta}}$ ($2n$). N is the number of samples of measured input and output data, n is the model order.

$$\boldsymbol{F}=\begin{bmatrix} -y(n+d) & -y(n+d-1) & \cdots & -y(d+1) & u(n) & u(n-1) & \cdots & u(1) \\ -y(n+d+1) & -y(n+d) & \cdots & -y(d+2) & u(n+1) & u(n) & \cdots & u(2) \\ \vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \cdots & \vdots \\ -y(N-1) & -y(N-2) & \cdots & -y(N-n) & u(N-d-1) & u(N-d-2) & \cdots & u(N-d-n) \end{bmatrix} \tag{5}$$

Equation (4) serves for a one-off calculation of the vector of parameter estimates $\hat{\boldsymbol{\Theta}}$ using N samples of measured data. The individual vectors and matrix in equation (4) have the form

$$\boldsymbol{y}^T=\left[y(n+d+1)\quad y(n+d+2)\quad\cdots\quad y(N)\right] \tag{6}$$

$$\hat{\boldsymbol{\Theta}}^T=\left[\hat{a}_1\quad \hat{a}_2\quad\cdots\quad \hat{a}_n\quad \hat{b}_1\quad \hat{b}_2\quad\cdots\quad \hat{b}_n\right] \tag{7}$$

Consider that model (3) is the deterministic part of the stochastic process described by the ARX (regression) model

$$y(k)=\frac{B\left(z^{-1}\right)z^{-d}}{A\left(z^{-1}\right)}u(k)+\frac{1}{A\left(z^{-1}\right)}e_s(k) \tag{8}$$

where $e_s(k)$ is the random nonmeasurable component

The ARX model (8) can be expressed as a stochastic difference equation

$$\begin{aligned} y(k)=&-a_1y(k-1)-a_2y(k-2)+\\ &+b_1y(k-1-d)+b_2y(k-2-d)+e_s(k) \end{aligned} \tag{9}$$

The vector of parameter model estimates is computed by solving equation (4)

$$\hat{\boldsymbol{\Theta}}^T(k)=\left[\hat{a}_1\quad \hat{a}_2\quad \hat{b}_1\quad \hat{b}_2\right] \tag{10}$$

and is used for computation of the prediction output

$$\begin{aligned} \hat{y}(k)=&-\hat{a}_1y(k-1)-\hat{a}_2y(k-2)+\\ &\hat{b}_1u(k-1-d)+\hat{b}_2u(k-2-d) \end{aligned} \tag{11}$$

The quality of ARX model can be judged by the prediction error, i.e. the deviation

$$\hat{e}(k)=y(k)-\hat{y}(k) \tag{12}$$

The prediction error plays a key role in identification of regressions model parameters derived from measured data. It is important for selecting the structure (order) of the regression model and a suitable sampling period. The quality of the model is also judged by the purpose for which it is used. In this case the prediction error was used for suitable choice of the time-delay dT_0.

IV. Algorithms of Digital Smith Predictors

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A. *Digital PID Smith Predictor (PIDSP)*

Hang *et. al.*[13, 14] used to design of the main controller $G_c\left(z^{-1}\right)$ the Dahlin PID algorithm [18]. This algorithm is based on the desired close-loop transfer function in the form

$$G_e\left(z^{-1}\right)=\frac{1-e^{-\alpha}}{1-z^{-1}} \tag{13}$$

where $\alpha={T_0}/{T_m}$ and T_m is desired time constant of the first order closed-loop response. It is not practical to set T_m to be small since it will demand a large control signal $u(k)$ which may easily exceed the saturation limit of the actuator. Then the individual parts of the controller are described by the transfer functions

$$G_c\left(z^{-1}\right)=\frac{\left(1-e^{-\alpha}\right)}{\left(1-z^{-1}\right)}\frac{\hat{A}\left(z^{-1}\right)}{\hat{B}(1)}\,; \quad G_m\left(z^{-1}\right)=\frac{z^{-1}\hat{B}(1)}{\hat{A}\left(z^{-1}\right)}$$

$$G_d\left(z^{-1}\right)=\frac{z^{-d}\hat{B}\left(z^{-1}\right)}{z^{-1}\hat{B}(1)} \tag{14}$$

where $B(1)=\hat{B}\left(z^{-1}\right)\Big|_{z=1}=\hat{b}_1+\hat{b}_2$.

Since $G_m\left(z^{-1}\right)$ is the second order transfer function, the main controller $G_c\left(z^{-1}\right)$ becomes a digital PID controller having the following form:

$$G_c\left(z^{-1}\right)=\frac{U(z)}{E(z)}=\frac{q_0+q_1z^{-1}+q_2z^{-2}}{1-z^{-1}} \tag{15}$$

where $q_0=\gamma, q_1=\hat{a}_1\gamma, q_2=\hat{a}_2\gamma$ using by the substitution $\gamma=\left(1-e^{-\alpha}\right)/\hat{B}(1)$. The PID controller output is given by

$$u(k)=q_0e(k)+q_1e(k-1)+q_2e(k-2)+u(k-1) \tag{16}$$

B. Digital Pole Assignment Smith Predictor (PASP)

Another two controllers applied in this paper were designed using a polynomial approach. Polynomial control theory is based on the apparatus and methods of a linear algebra (see e.g. [19] – [21]). The polynomials are the basic tool for a description of the transfer functions. They are expressed as the finite sequence of figures – the coefficients of a polynomial. Thus, the signals are expressed as infinite sequence of figures. The controller synthesis consists in the solving of linear polynomial (Diophantine) equations [22].

The design of the controller algorithm is based on the general block scheme of a closed-loop with two degrees of freedom (2DOF) according to Fig. 3.

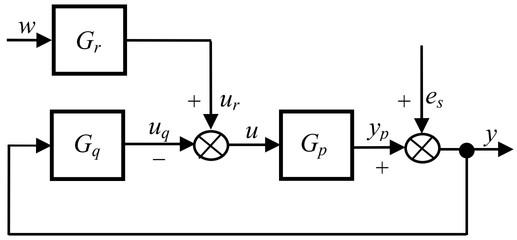

Fig. 3 Block diagram of a closed loop 2DOF control system

The controlled process is given by the transfer function in the form

$$G_p(z^{-1})=\frac{Y_p(z)}{U(z)}=\frac{B(z^{-1})}{A(z^{-1})} \tag{17}$$

where A and B are the second order polynomials. The controller contains the feedback part G_q and the feedforward part G_r. Then the digital controllers can be expressed in the form of a discrete transfer functions

$$G_r\left(z^{-1}\right)=\frac{R\left(z^{-1}\right)}{P\left(z^{-1}\right)}=\frac{r_0}{1+p_1z^{-1}} \tag{18}$$

$$G_q\left(z^{-1}\right)=\frac{Q\left(z^{-1}\right)}{P\left(z^{-1}\right)}=\frac{q_0+q_1z^{-1}+q_2z^{-2}}{\left(1+p_1z^{-1}\right)\left(1-z^{-1}\right)} \tag{19}$$

According to the scheme presented in Fig. 2 (for $e_s=0$),

$$Y(z^{-1})=\frac{B(z^{-1})R(z^{-1})}{A(z^{-1})P(z^{-1})+B(z^{-1})Q(z^{-1})}W(z^{-1}) \tag{21}$$

where

$$A(z^{-1})P(z^{-1})+B(z^{-1})Q(z^{-1})=D(z^{-1}) \tag{22}$$

is the characteristic polynomial.

The procedure leading to determination of polynomials Q, R and P in (21) and (23) can be briefly described as follows (see [23]). A feedback part of the controller is given by a solution of the polynomial Diophantine equation (2). An asymptotic tracking is provided by a feedforward part of the controller given by a solution of the polynomial Diophantine equation

$$S(z^{-1})D_w\left(z^{-1}\right)+B(z^{-1})R(z^{-1})=D(z^{-1}) \tag{23}$$

For a step-changing reference signal value $D_w\left(z^{-1}\right)=1-z^{-1}$ holds and S is an auxiliary polynomial which does not enter into controller design.

A feedback controller to control a second-order system without time-delay will be derived from Equation (22). A system of linear equations can be obtained using the uncertain coefficients method

$$\begin{bmatrix} \hat{b}_1 & 0 & 0 & 1 \\ \hat{b}_2 & \hat{b}_1 & 0 & \hat{a}_1-1 \\ 0 & \hat{b}_2 & \hat{b}_1 & \hat{a}_2-\hat{a}_1 \\ 0 & 0 & b_2 & -\hat{a}_2 \end{bmatrix}\begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ p_1 \end{bmatrix}=\begin{bmatrix} d_1+1-\hat{a}_1 \\ d_2+\hat{a}_1-\hat{a}_2 \\ d_3+\hat{a}_2 \\ d_4 \end{bmatrix} \tag{24}$$

where the characteristic polynomial is chosen as

$$D\left(z^{-1}\right)=1+d_1z^{-1}+d_2z^{-2}+d_3z^{-3}+d_4z^{-4} \tag{25}$$

For a step-changing reference signal value it is possible to solve Equation (23) by substituting $z=1$

$$R = r_0 = \frac{D(1)}{B(1)} = \frac{1 + d_1 + d_2 + d_3 + d_4}{b_1 + b_2} \tag{26}$$

The 2DOF controller output is given by

$$\begin{aligned} u(k) = r_0 w(k) - q_0 y(k) - q_1 y(k-1) - q_2 y(k-2) + \\ + (1 + p_1) u(k-1) + p_1 u(k-2) \end{aligned} \tag{27}$$

C. Digital Linear Quadratic Smith Predictor

The linear quadratic control methods try to minimize the quadratic criterion by penalization the controller output

$$J = \sum_{k=0}^{\infty} \left\{ \left[w(k) - y(k) \right]^2 + \lambda \left[u(k) \right]^2 \right\} \tag{28}$$

where λ is the so-called penalization constant, which gives the rate of the controller output on the value of the criterion (where the constant at the first element of the criterion is considered equal to one). The standard procedure of the minimization of the criterion (28) is based on the state description of the system and leads to the solution of the Riccati equation. In this paper, criterion minimization will be realized through the spectral factorization for an input-output description of the system. Spectral factorization of polynomials of the first and second order degree can be computed simply; the procedure for higher degrees must be performed iteratively.

For the coefficients of the second order characteristic polynomial $D\left(z^{-1}\right) = 1 + d_1 z^{-1} + d_2 z^{-2}$ of the closed loop were derived the expressions [20]

$$d_1 = \frac{m_1}{\delta + m_2}; \quad d_2 = \frac{m_2}{\delta} \tag{29}$$

The parameters m_1, m_2 and δ are computed as follows:

$$\begin{aligned} &\delta = \frac{\gamma + \sqrt{\lambda^2 - 4m_2^2}}{2}; \quad \gamma = \frac{m_0}{2} - m_2 + \sqrt{\left(\frac{m_0}{2} + m_2\right)^2 - m_1^2} \\ &m_0 = \lambda\,(1 + a_1^2 + a_2^2) + b_1^2 + b_2^2; \quad m_1 = \lambda(a_1 + a_1 a_2) + b_1 b_2 \\ &m_2 = \lambda\, a_2 \end{aligned} \tag{30}$$

The LQ controller of 2DOF structure has the same form as controller (27), only $d_3 = d_4 = 0$ in (24) - (26).

V. RECURSIVE IDENTIFFICATION PROCEDURE

The regression (ARX) model of the following form

$$y(k) = \boldsymbol{\Theta}^T(k) \boldsymbol{\Phi}(k) + e_s(k) \tag{31}$$

is used in the identification part of the designed controller algorithms, where

$$\boldsymbol{\Theta}^T(k) = \begin{bmatrix} a_1 & a_2 & b_1 & b_2 \end{bmatrix} \tag{32}$$

is the vector of model parameters and

$$\boldsymbol{\Phi}^T(k-1) = \begin{bmatrix} -y(k-1) & -y(k-2) & u(k-d-1) & u(k-d-2) \end{bmatrix} \tag{33}$$

is the regression vector. The non-measurable random component $e_s(k)$ is assumed to have zero mean value $E[e_s(k)] = 0$ and constant covariance (dispersion) $R = E[e_s^2(k)]$.

All digital self-tuning SP controllers use the algorithm of identification based on the Recursive Least Squares Method (RLSM) extended to include the technique of directional (adaptive) forgetting. Numerical stability is improved by means of the LD decomposition [23], [24]. This method is based on the idea of changing the influence of input-output data pairs to the current estimates. The weights are assigned according to amount of information carried by the data.

VI. SIMULATION VERIFICATION OF SELF-TUNING (ST) DIGITAL SP CONTROLLER ALGORITHMS

Simulation is a useful tool for the synthesis of control systems, allowing one not only to create mathematical models of a process but also to design virtual controllers in a computer. The mathematical models provided are sufficiently close to a real object that simulation can be used to verify the dynamic characteristics of control loops when the structure or parameters of the controller change. The models of the processes may also be excited by various random noise generators which can simulate the stochastic characteristics of the processes noise signals with similar properties to disturbance signals measured in the machinery. The simulation results are valuable for an implementation of a chosen controller (control algorithm) under laboratory and industrial conditions. It must be borne in mind, however, that the practical application of a controller verified by simulation can not be taken as a routine event. Obviously simulation and laboratory conditions can be quite different from those in real plants, and therefore we must verify its practicability with regard to the process dynamics and the required standard of control quality (for example maximum sufferable overshoot, accuracy, settling time, etc.).

The above mentioned SP controllers are not suitable for the control of unstable processes. Therefore, three types of processes were chosen for simulation verification of digital self-tuning SP controller algorithms.

Consider the following continuous-time transfer functions:

1) Stable non-oscillatory $G_1(s) = \dfrac{2}{(s+1)(4s+1)} e^{-4s}$

2) Stable oscillatory $G_2(s) = \dfrac{2}{4s^2 + 2s + 1} e^{-4s}$

3) With non-minimum phase $G_3(s) = \dfrac{-5s+1}{(s+1)(4s+1)} e^{-4s}$

Let us now discretize them a sampling period $T_0 = 2\,\mathrm{s}$. The discrete forms of these transfer functions are (see Equation (2))

$$G_1(z^{-1}) = \frac{0.4728z^{-1} + 0.2076z^{-2}}{1 - 0.7419z^{-1} + 0.0821z^{-2}} z^{-2}$$

$$G_2(z^{-1}) = \frac{0.6806z^{-1} + 0.4834z^{-2}}{1 - 0.7859z^{-1} + 0.3679z^{-2}} z^{-2}$$

$$G_3(z^{-1}) = \frac{-0.5489z^{-1} + 0.8897z^{-2}}{1 - 0.7419z^{-1} + 0.0821z^{-2}} z^{-2}$$

A simulation verification of proposed design was performed in MATLAB/SIMULINK environment. A typical control scheme used is depicted in Fig. 4.

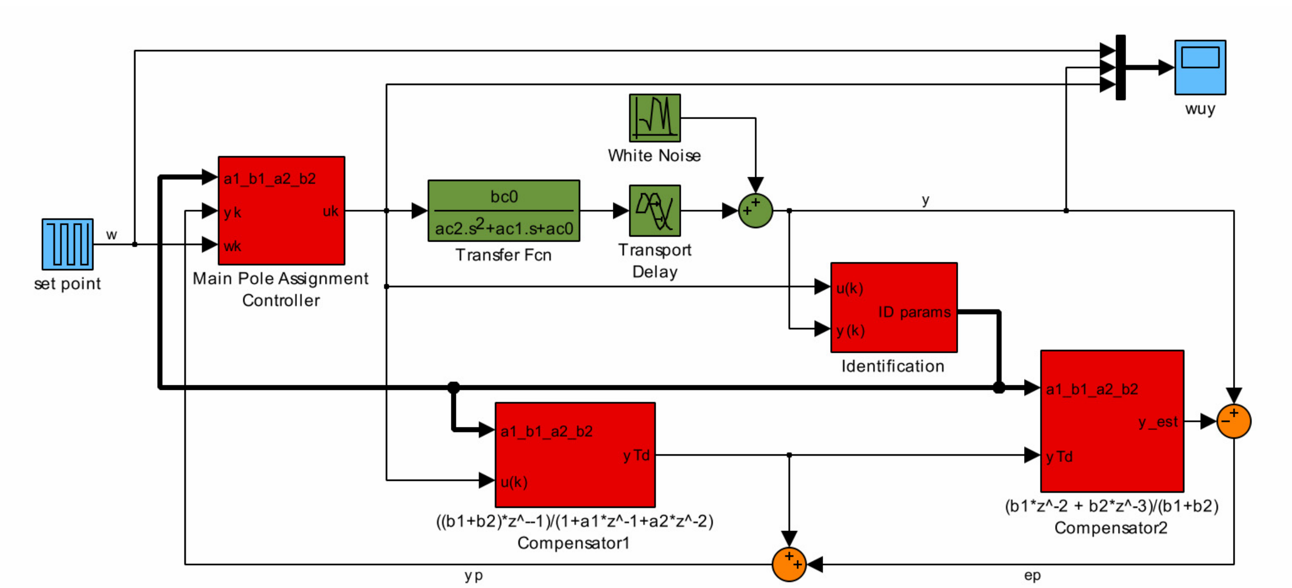

Fig. 4 Simulink control scheme

This scheme is used for systems with time-delay of two sample steps. Individual blocks of the Simulink scheme correspond to blocks of the general control scheme presented in Fig. 2. The green blocks represent the controlled system. Constants bc0, ac2, ac1, and ac0 are parameters of continuous-time system. Blocks Compensator 1 and Compensator 2 are parts of the Smith Predictor and they correspond to $G_m(z^{-1})$ and $G_d(z^{-1})$ blocks of Fig. 2 respectively. The control algorithm is encapsulated in Main Pole Assignment Controller which corresponds to $G_c(z^{-1})$ Fig. 2 block. The Identification block performs the on-line identification of controlled system and outputs the estimates of 2nd order ARX model (a1, b1, a2, b2) parameter.

The internal structure of the Main Pole Assignment Controller block is shown in Fig. 5.

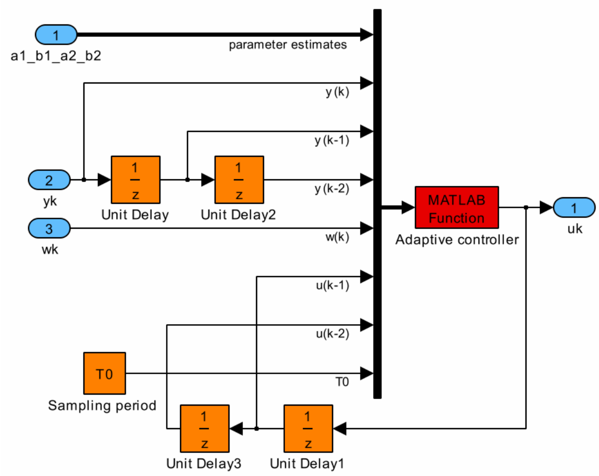

Fig. 5 Internal structure of the controller

Block MATLAB Function is the heart of the controller. The inputs to this function are current ARX estimates, current and previous values of process without time-delay, reference signal as well as previous control values and sample time. The MATLAB Fcn is a standard m-function which carries out desired control algorithm as described in Section 4.

The on-line identification part of the scheme, which is represented by block Identification block in Fig. 4, uses several parameters that are entered via standard SIMULINK dialog. This dialog is presented in Fig. 6.

The most important parameters form the point of view of the problem this papers is coping with are sample time, initial parameters estimations and dead time. The dead time is not entered in time units but in sample times. The other parameters affect the method used to compute ARX model and their detailed description can be found in [20] and [21].

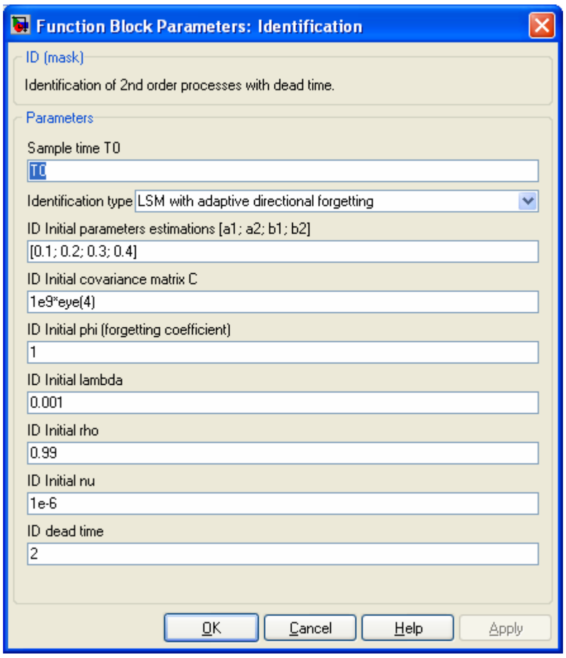

Fig. 6. Dialog for setting identification parameters

VII. Simulation Results

The configuration for simulation verification of the designed algorithms was chosen as follows:

- All three control loops were verified in the non-adaptive versions without a random noise.
- A suitable time constant T_m was chosen for the control using the PIDSP controller and the pole assignment of the closed-loop was calculated. These poles were used for the design of the PASP controller.
- A suitable penalization constant λ has been chosen for the control using the LQSP controller.
- All three control loops were verified in the adaptive versions with a random noise. Firstly, without a priori information (the initial values of the model parameter estimates were chosen randomly). Secondly, using a priori information (the initial estimates were chosen on the basis of the previous experiments).
- The outputs of the process models were influenced by White Noise Generator with mean value $E = 0$ and covariance $R = 10^{-4}$.

A. Simulation Verification of ST Digital PIDSP

Figs. 7 - 10 illustrate the simulation control performance using PIDSP controller (15), (16).

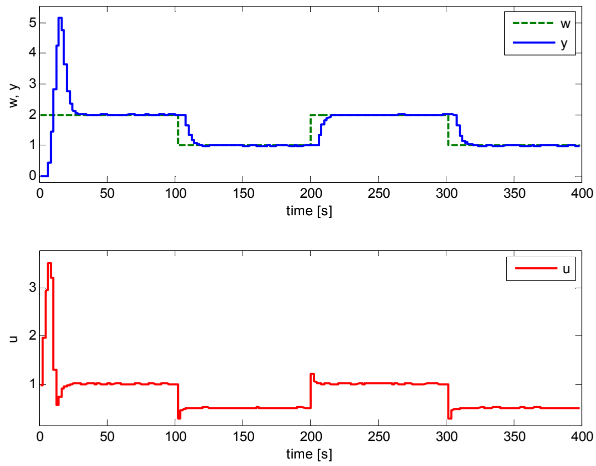

Fig. 7 Control of the model $G_1\left(z^{-1}\right)$, controller PIDSP (without a priori information)

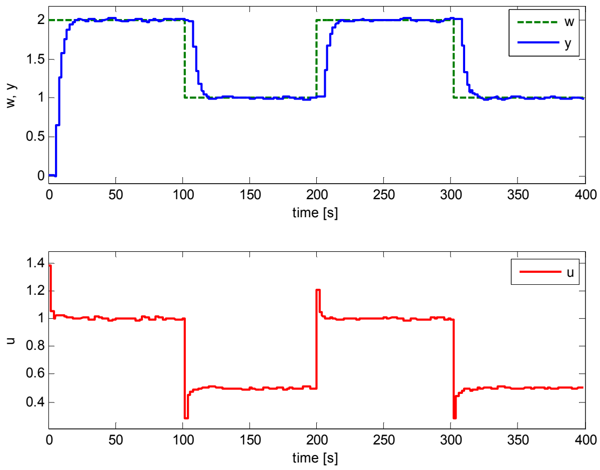

Fig. 8 Control of the model $G_1\left(z^{-1}\right)$, controller PIDSP (with a priori information)

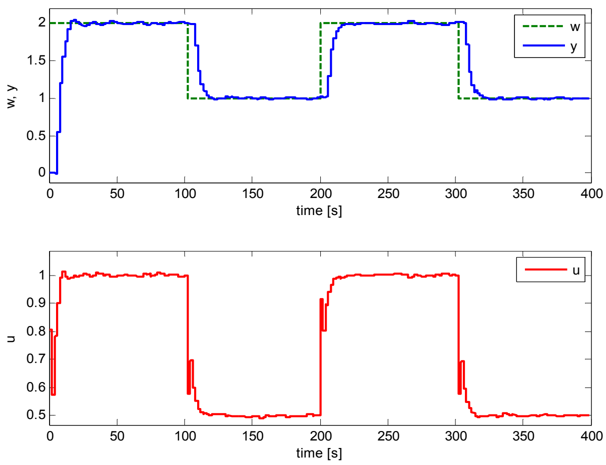

Fig. 9 Control of the model $G_2\left(z^{-1}\right)$, controller PIDSP (with a priori information)

It is obvious from Figs. 7 and 8 (the control of the stable model $G_1\left(z^{-1}\right)$) that the control process is dependent on knowledge of a priori information. The process output y has a large overshoot, when the initial model parameter estimates are chosen randomly. Using a priori information (the initial estimates were chosen based on the previous experiments) leads to very good control quality (without overshoot of y and with short settling time).

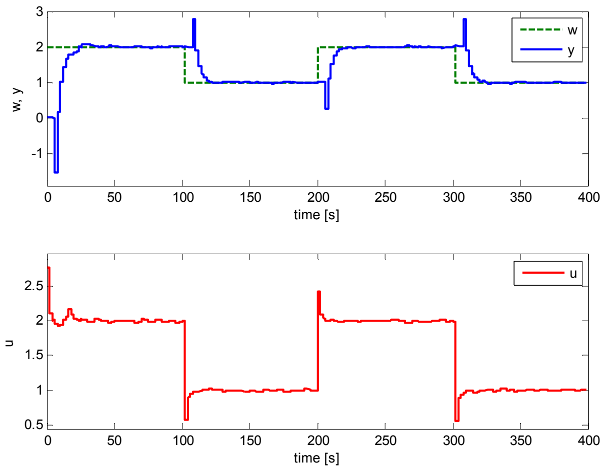

Fig. 10 Control of the model $G_3\left(z^{-1}\right)$, controller PIDSP (with a priori information)

Simulation results for the models $G_2\left(z^{-1}\right)$ (the stable oscillatory model) and $G_3\left(z^{-1}\right)$ (the non-minimum phase model) are shown in Figs. 9 and 10. The control quality (with a priori information) is very good.

B. Simulation Verification of ST Digital PASP

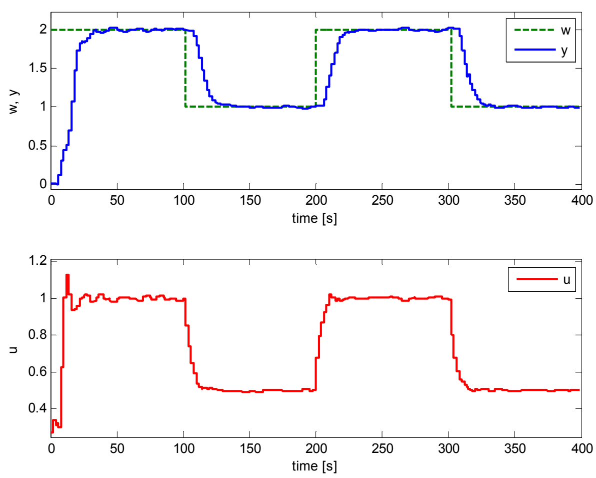

Fig. 11 Control of model $G_1\left(z^{-1}\right)$, controller PASP (without a priori information)

Figs. 11 - 13 illustrate the simulation control performance using PASP controller (24), (26) and (27). From Figs. 11 and 12 (the control of the stable model $G_1\left(z^{-1}\right)$), it is obvious that the control process is not dependent on knowledge of a priori information (the control courses in both cases are practically identical). In the case of choosing of the suitable closed-loop poles, the self-tuning PASP controller is more robust than the self-tuning PIDSP controller.

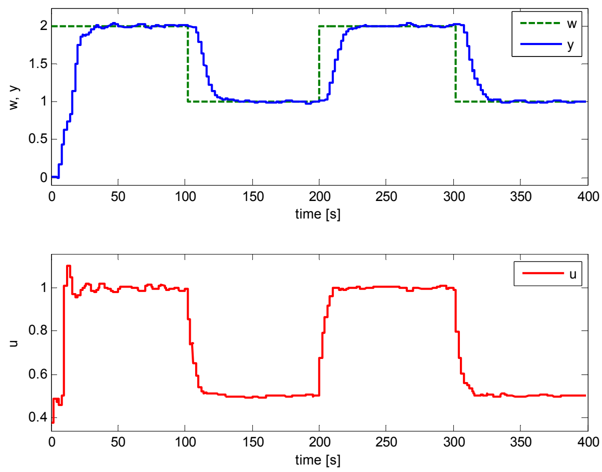

Fig. 12 Control of the model $G_1\left(z^{-1}\right)$, controller PASP (with a priori information)

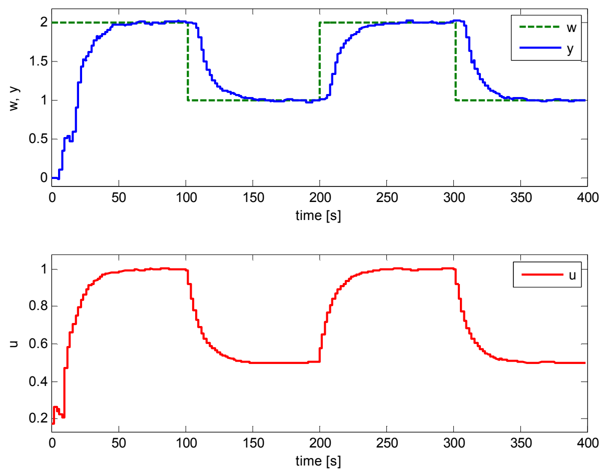

Fig. 13 Control of the model $G_2\left(z^{-1}\right)$, controller PASP (with a priori information)

Fig. 13 illustrates the simulation control performance of the stable oscillatory model $G_2\left(z^{-1}\right)$. The control process is relatively slow without overshot of y and u (it is the cautious adaptive controller).

Fig. 14 illustrates the simulation control performance of the non-minimum phase model $G_3\left(z^{-1}\right)$. The control process is good after initial part.

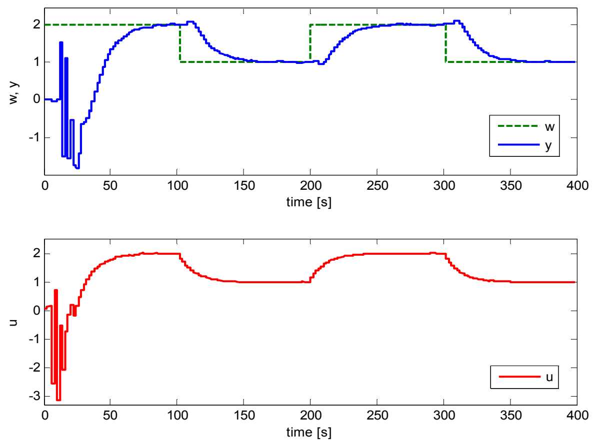

Fig. 14 Simulation results: control of the model $G_3\left(z^{-1}\right)$, controller PASP (with a priori information)

C. Simulation Verification of ST Digital LQSP

Figs. 15 - 17 illustrate the simulation control performance using LQSP controller (16), (29), (30). It is obvious from Figs. 14 and 15 that control process of the model $G_1\left(z^{-1}\right)$ is similar as that one using the controller PIDSP. The model $G_2\left(z^{-1}\right)$ is controlled with large overshoot of y in the initial part (see Fig. 17). The control of the non-minimum phase $G_3\left(z^{-1}\right)$ model was unstable.

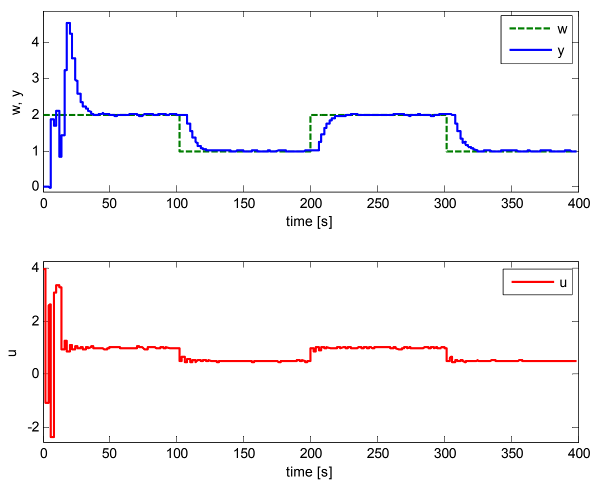

Fig. 15 Control of the model $G_1\left(z^{-1}\right)$, controller LQSP (without a priori information)

The relative low-quality control using the controller LQSP could be caused by choosing a second degree characteristic polynomial $D(z^{-1})$ (optimal polynomial is a fourth degree with four poles). The procedure for higher degrees than two must be performed iteratively.

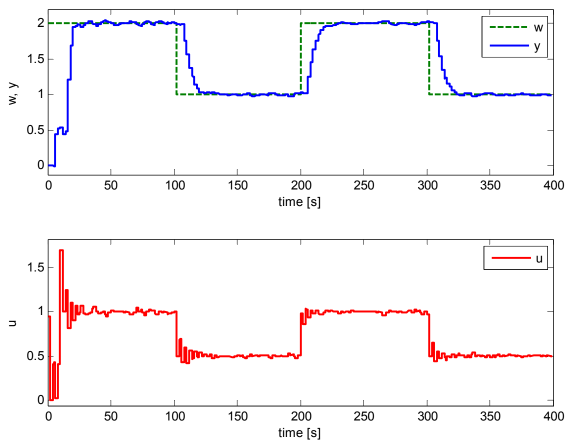

Fig. 16 Control of the model $G_1\left(z^{-1}\right)$, controller LQSP (with a priori information)

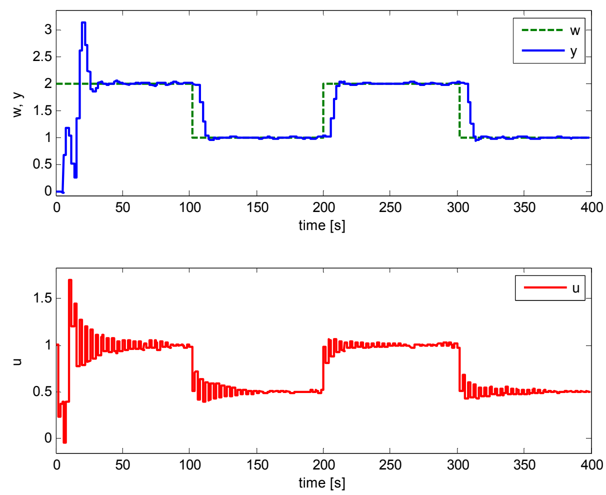

Fig. 17 Control of the model $G_2\left(z^{-1}\right)$, controller LQSP (with a priori information)

VIII. Conclusion

Adaptive Smith predictor algorithms for control of processes with time-delay based on polynomial design (pole assignment and linear quadratic control) were proposed. The polynomial controllers were derived purposely by analytical way (without utilization of numerical methods) to obtain algorithms with easy implementability in industrial practice. Both pole assignment and linear quadratic control algorithms were compared by simulation with adaptive digital Smith PID predictor. Three models of control processes were used for simulation verification (the stable non-oscillatory, the stable oscillatory and the non-minimum phase). Results of simulation verification demonstrated advantages and disadvantages of individual approaches for control of above mentioned processes with time-delay. The designed adaptive SP algorithms will be verified in real time conditions for a control of the laboratory heat exchanger [25].

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